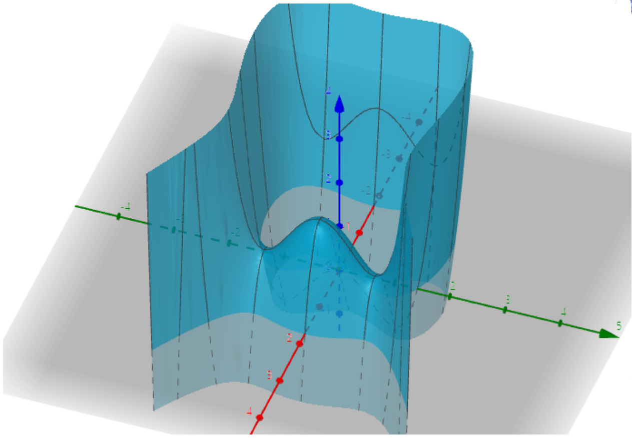
풀3.

#1 $z=f(x,y)=3x-x^3-2y^2+y^4$



$f_x=3-3x^2 \longrightarrow x^2=1 \quad \underline{x=\pm 1}$

$f_y=-4y+4y^3 \longrightarrow y^3-y=0$

$y(y^2-1)=0 \quad \underline{y=\pm 1,\ 0}$

$D=f_{xx}f_{yy}-(f_{xy})^2$ 이므로.

따라서

$f_{xx}=-6x$

$f_{yy}=-4+12y^2$

$f_{xy}=0$

$(1,0) \Rightarrow D>0 \quad f_{xx}<0 \longrightarrow$ local max

$(-1,0) \Rightarrow \underline{D<0}$ 안장점

$(1,1) \Rightarrow \underline{D<0}$ 〃

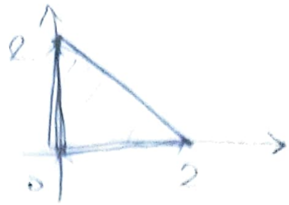
$(-1,1) \Rightarrow D>0 \quad f_{xx}>0 \rightarrow$ local min

$(1,-1) \Rightarrow \underline{D<0}$ 안장점.

$(-1,-1) \Rightarrow D>0 \quad f_{xx}>0 \longrightarrow$ local min.

#2

$z = f(x,y) = (x-1)^2 + y^2 - 1$



$f_x = 2x-2$

$f_y = 2y$

$\nabla f = \lambda \nabla g$

① $(x+y-2=0) = g$ → $g_x = 1$ $f_x = 2x-2$

$g_y = 1$ $f_y = 2y$

$y = \frac{1}{2}, x = \frac{3}{2}$

② $y=0$ → $g_x = 0$ $f_x = 2x-2$

$g_y = 1$ $f_y = 2y$

→ $\lambda = 0$ $(1 = 2y - 1)$ → $0 = -1$ → 불능 (X)

→ $x=1$ $\therefore (1,0)$

③ $x=0$ → $g_x = 1$ $f_x = 2x-2$

$g_y = 0$ $f_y = 2y$

→ $\lambda = -\frac{1}{2}$

→ $\lambda = 0$ (불능) (X)

→ $y=0$ $(0,0)$

$\therefore (0,0) = 0$

$(1,0) = -1$

$\left(\frac{3}{2}, \frac{1}{2}\right) = \frac{1}{4} + \frac{1}{4} - 1 = -\frac{1}{2}$

max = 0

min = −1

#4

$x^2 - z^2 = 1 \quad = f$

$(0, 0, 0)$

$(x)^2 + (y)^2 + (z)^2 = d = g$

$f_x = 2x \qquad g_x = 2x$

$f_y = 0 \qquad g_y = 2y$

$f_z = -2z \qquad g_z = 2z$

$\to f = \lambda \cdot g$

$f_y = \lambda \cdot g_y \to 0 = \lambda \cdot 2y$

① $\lambda = 0 \to$ $x^2 = 1 + z^2$, $\underline{1 + z^2 + y^2 = d}$ $\qquad 2z^2 + y^2 \geq 0$ (최소 : 0) $\rightsquigarrow \underline{d = 1}$ //

② $y = 0 \to$ $x^2 - z^2 = 1$, $x^2 + z^2 = d$. $\leftarrow x^2 = 1 + z^2$

$2z^2 + 1 = d$

최소여야하므로 $z^2 = 0$ 이어야 함

$\underline{d = 1}$ //

$f_z = \lambda \cdot g_z \to -2z = \lambda \cdot z$

① $\lambda = -1$, $x = 0$, $y = 0 \Rightarrow \underline{z^2 = -1}$ (X)
불능

② $z = 0$ $\to$ $\underline{x^2 = 1}$, $\underline{y = 0}$ $\qquad \underline{1 = d}$ //

$(1, 0, 0) \quad (-1, 0, 0)$

$\therefore$ $d = 1$ 최소이다 //

#4

$x^2 - z^2 = 1 \quad = f$

$(0,0,0)$

$(x)^2 + (y)^2 + (z)^2 = d(\cancel{x,z}) = g$

$f_x = 2x$

$f_y = 0$

$f_z = -2z$

$g_x = 2x$

$g_y = 2y$

$g_z = 2z$

$\rightarrow f = \lambda \cdot g$

$f_y = \lambda \cdot g_y \rightarrow 0 = \lambda \cdot 2y$

① $\lambda = 0 \rightarrow$ $x^2 = 1 + z^2$, $1 + 2z^2 + y^2 = d$

$2z^2 + y^2 = 0$ (최소 : 0) $\rightsquigarrow d = 1$

② $y = 0 \rightarrow$ $x^2 - z^2 = 1$, $x^2 + z^2 = d$, $x^2 = 1 + z^2$

$2z^2 + 1 = d$

d가 최소여야하므로 $z^2 = 0$ 이어야함.

$d = 1$

$f_z = \lambda \cdot g_z \rightarrow -2z = \lambda \cdot z$

① $\lambda = -1$, $x = 0$, $y = 0 \rightarrow -z^2 = 1$ 불능 (X)

② $z = 0$ $\rightarrow$ $x^2 = 1$, $y = 0$ $\quad 1 = d$

$(1,0,0)$ $(-1,0,0)$

$\therefore d = 1$ 최소이다