CHAPTER 3

Vectors

Vectors

- 방향과 크기를 갖는 물리량
- Examples: 변위, 속도, 가속도

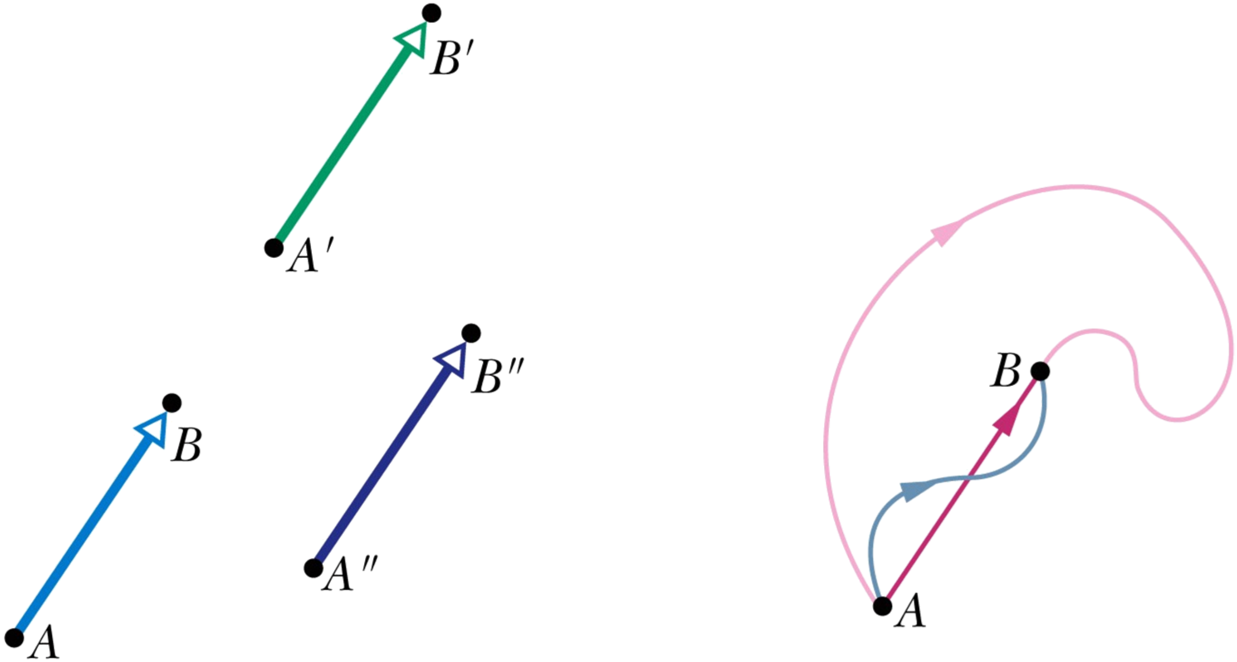
Scalars

- 크기만을 갖는 물리량
- Examples: 온도, 시간, 속력…

벡터의 표기는 굵은 알파벳 (**a**, **A**) 또는 화살표($\vec{a}$, $\vec{A}$) 사용

변위벡터

- 길이: 변위의 크기
- 방향: 변위의 방향

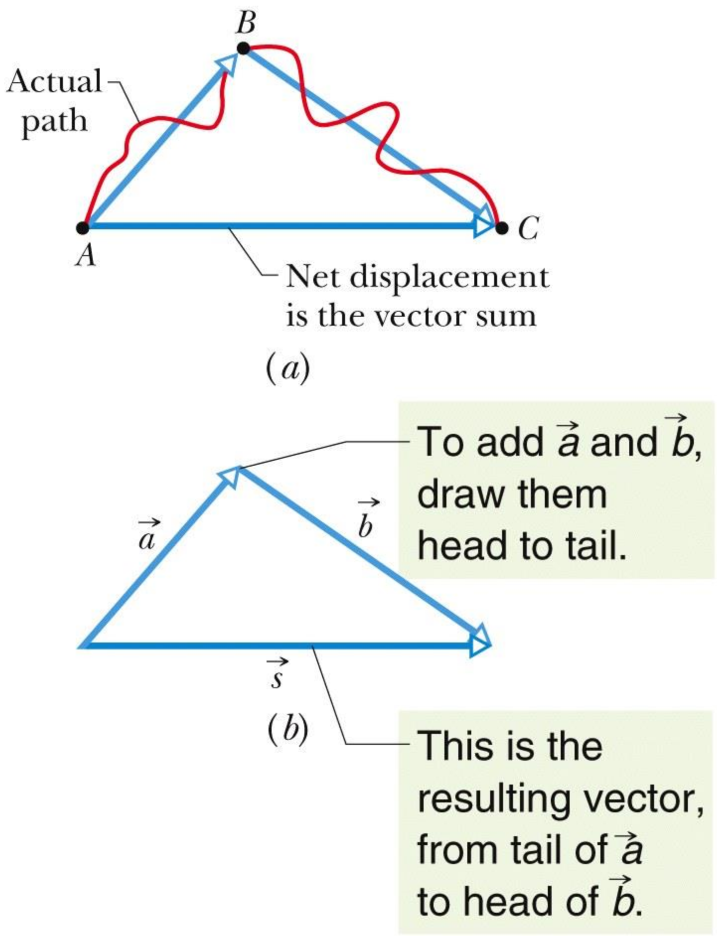


(a) (b)

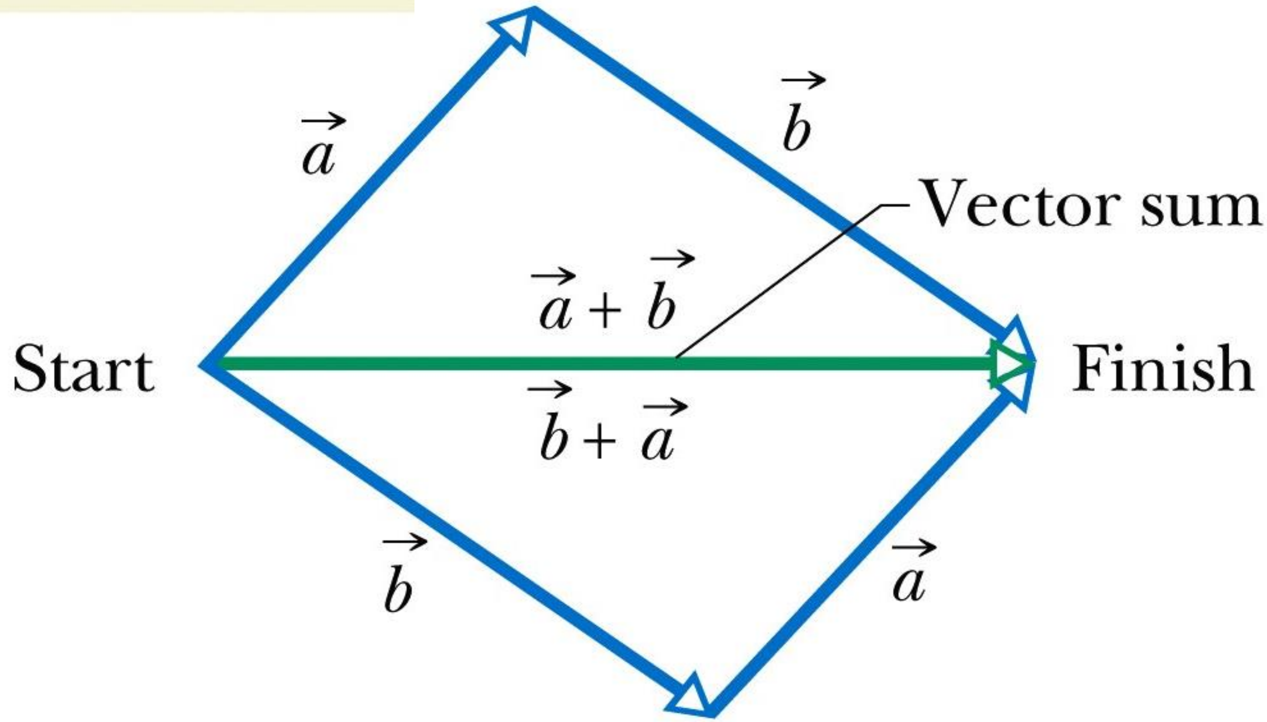
세 벡터는 모두 같은 변위를 표시함

변위벡터는 실제 경로와는 무관함.

$$\vec{s} = \vec{a} + \vec{b}$$

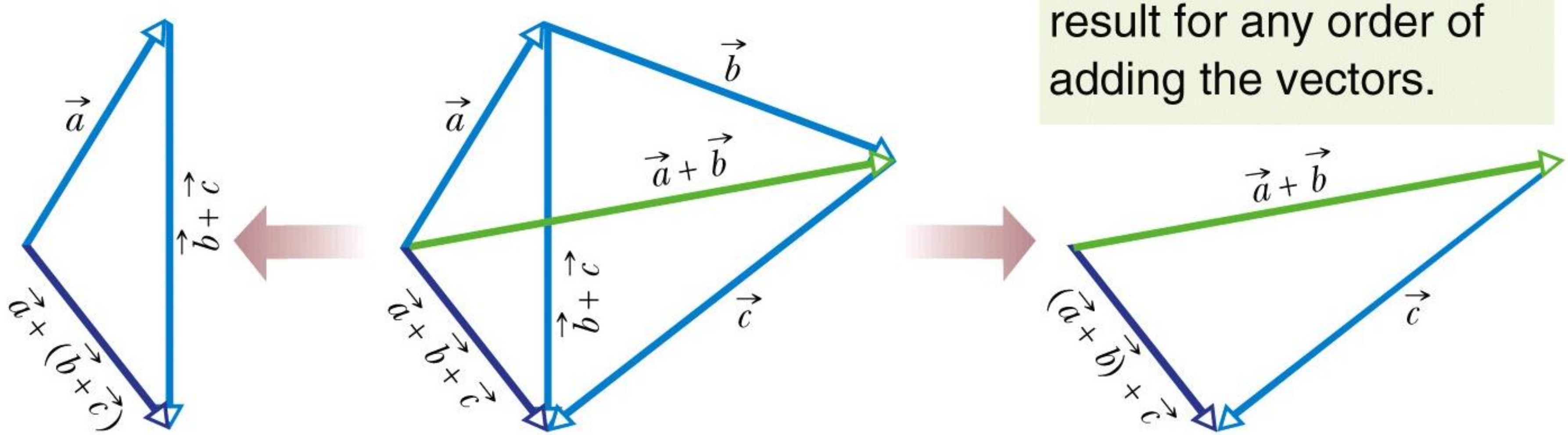


$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

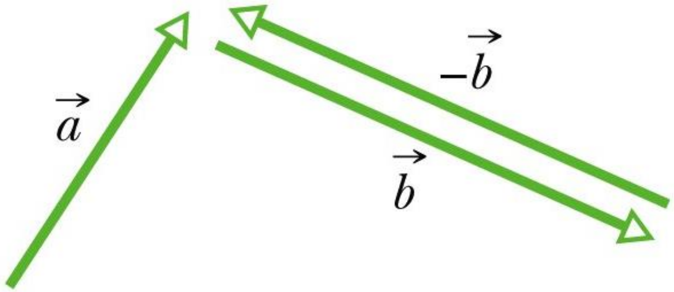


$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$$

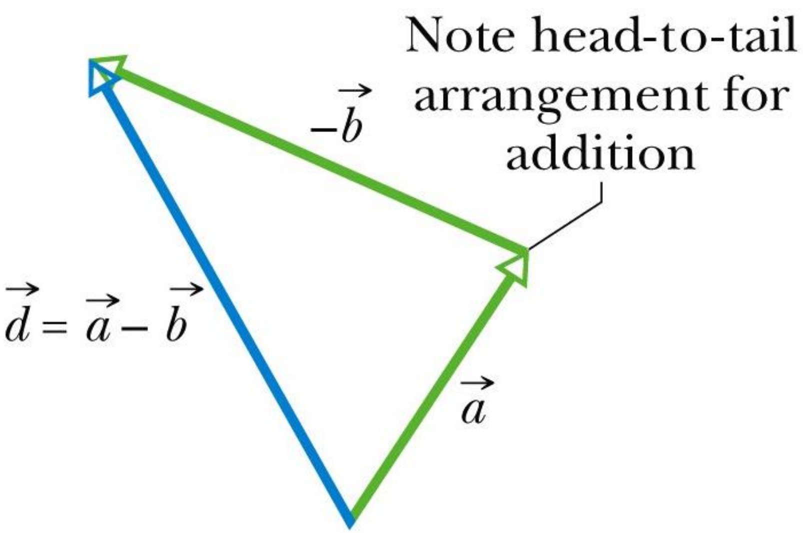
You get the same vector result for any order of adding the vectors.



$$\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b})$$

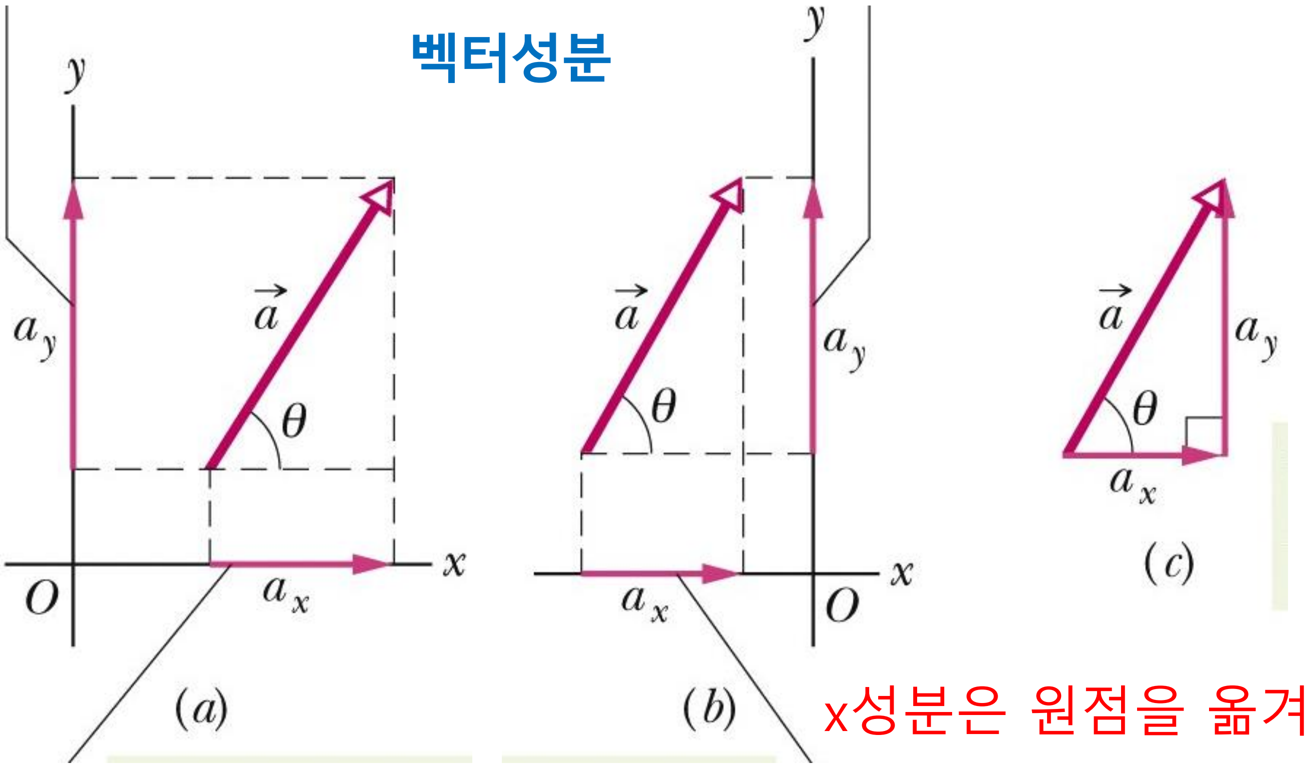


(a)



(b)

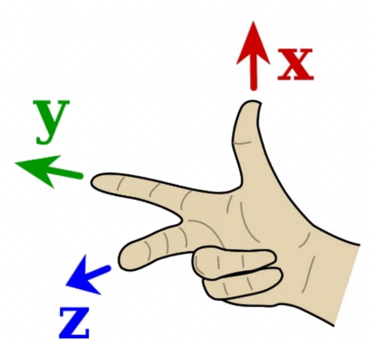
벡터성분

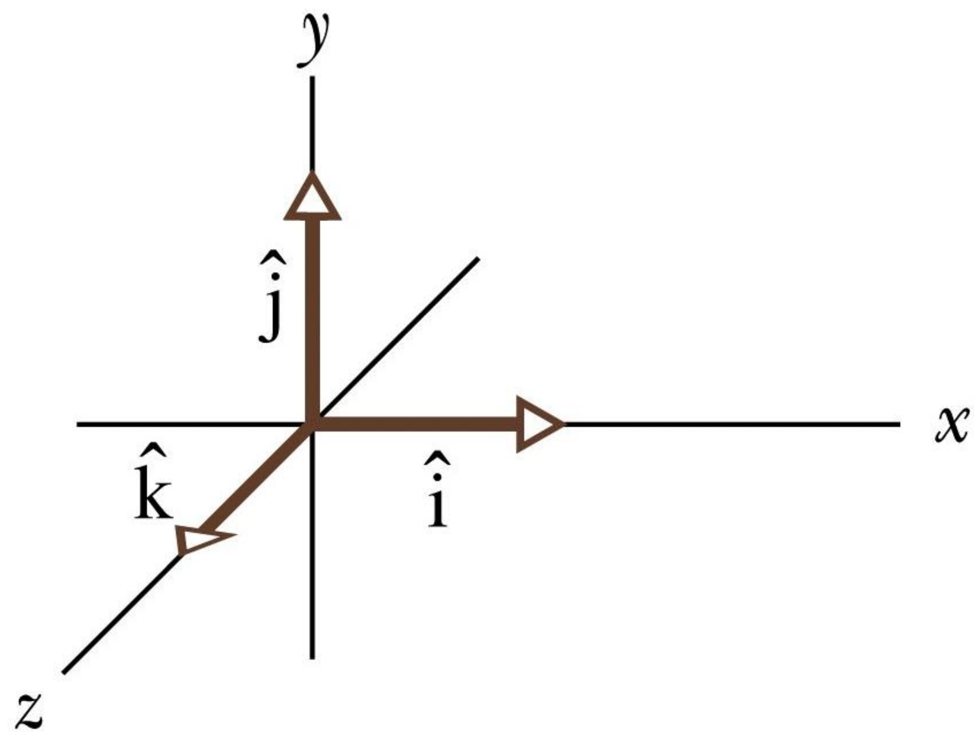


x성분은 원점을 옮겨도 변하지 않음.

$$\vec{a} = \vec{a}_x + \vec{a}_y,$$
$$a_x = a\cos\theta,\ a_y = a\sin\theta$$
$$a = \sqrt{{a_x}^2 + {a_y}^2},\ \tan\theta = \frac{a_y}{a_x},\ \theta = \tan^{-1}\frac{a_y}{a_x} = \arctan\frac{a_y}{a_x}$$

- 단위길이
- 오른손법칙





$$\vec{a} = \vec{a}_x + \vec{a}_y + \vec{a}_z = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$$

If $\vec{r} = \vec{a} + \vec{b}$, then

$$r_x = a_x + b_x$$
$$r_y = a_y + b_y$$
$$r_z = a_z + b_z$$

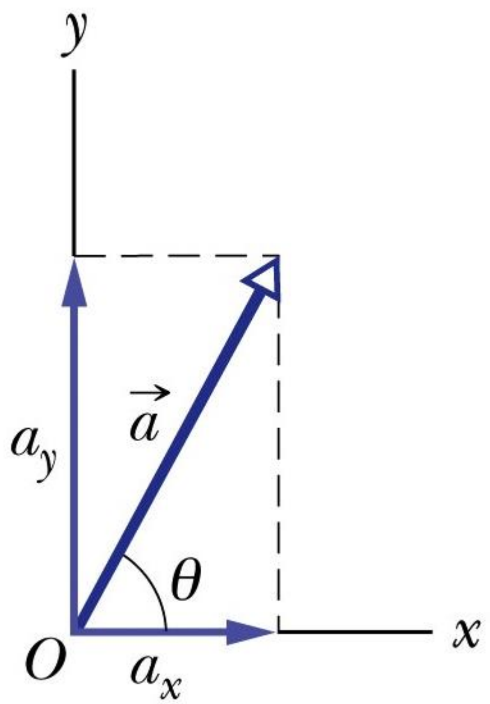
두 벡터의 성분이 같으면 두 벡터는 같다.

벡터 빼기: $\vec{d} = \vec{a} - \vec{b}$,

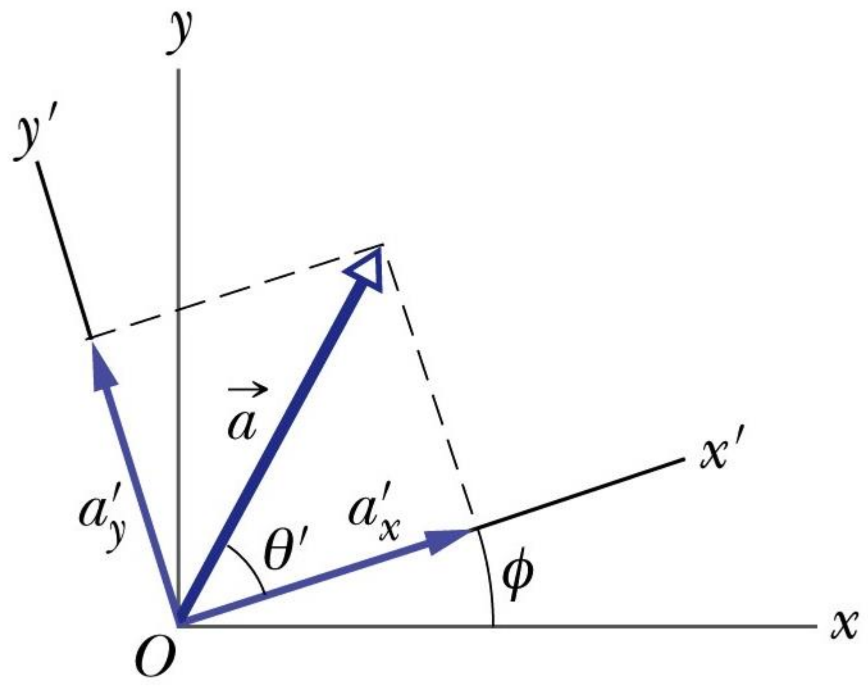
$$d_x = a_x - b_x, \quad d_y = a_y - b_y, \quad \text{and} \quad d_z = a_z - b_z$$

$$\vec{d} = d_x\hat{\mathrm{i}} + d_y\hat{\mathrm{j}} + d_z\hat{\mathrm{k}}$$

물리법칙은 좌표계와 무관하게 성립한다.



(a)



(b)

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{a_x'^2 + a_y'^2} \qquad \theta = \theta' + \phi$$

벡터에 스칼라 곱하기

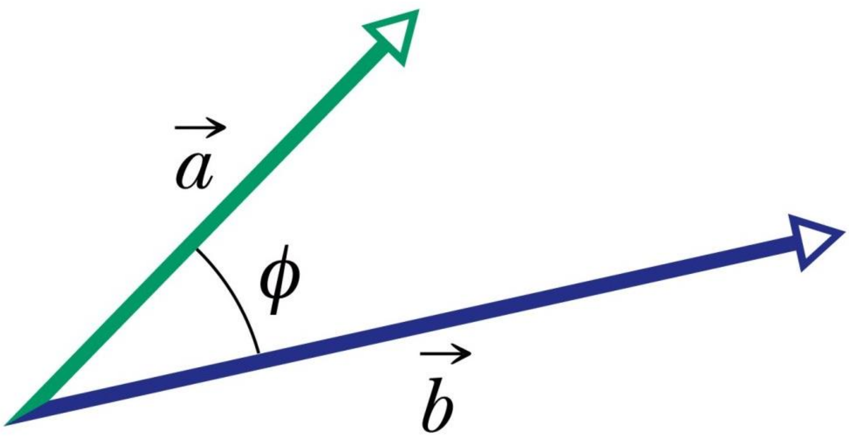
$$\vec{a} \times s = s\vec{a}$$

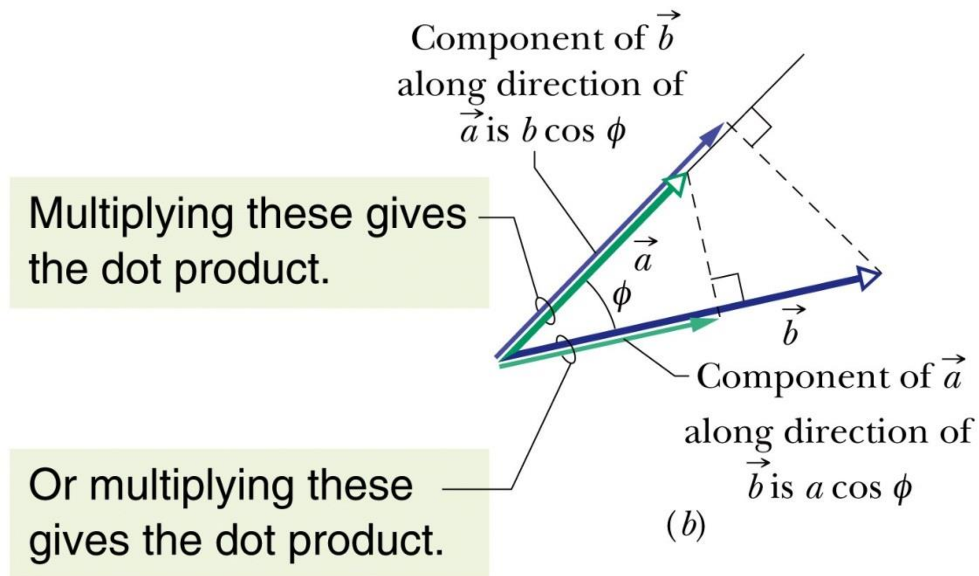
크기: $\vec{a}$의 크기와 s의 절대값의 곱

방향: $\vec{a}$의 방향 (s가 음수면 반대 방향)

스칼라곱(점곱, dot product, 내적)

$$\vec{a} \cdot \vec{b} = ab\cos\phi$$

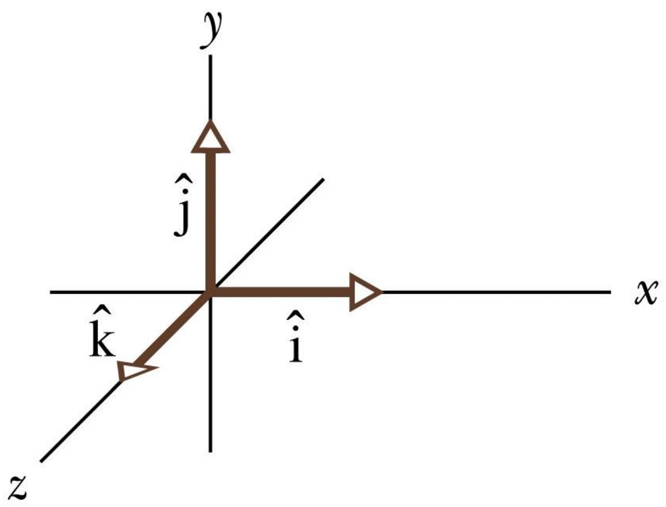




스칼라곱(점곱, dot product, 내적)

$$\hat{i}\cdot\hat{i}=\hat{j}\cdot\hat{j}=\hat{k}\cdot\hat{k}=1$$

$$\hat{i}\cdot\hat{j}=\hat{j}\cdot\hat{k}=\hat{k}\cdot\hat{i}=0$$



$$\vec{a}\cdot\vec{b}=(a_x\hat{i}+a_y\hat{j}+a_z\hat{k})\cdot(b_x\hat{i}+b_y\hat{j}+b_z\hat{k})$$

$$=a_x\hat{i}\cdot b_x\hat{i}+a_x\hat{i}\cdot b_y\hat{j}+a_x\hat{i}\cdot b_z\hat{k}+a_y\hat{j}\cdot b_x\hat{i}+a_y\hat{j}\cdot b_y\hat{j}+a_y\hat{j}\cdot b_z\hat{k}$$

$$+a_z\hat{k}\cdot b_x\hat{i}+a_z\hat{k}\cdot b_y\hat{j}+a_z\hat{k}\cdot b_z\hat{k}$$

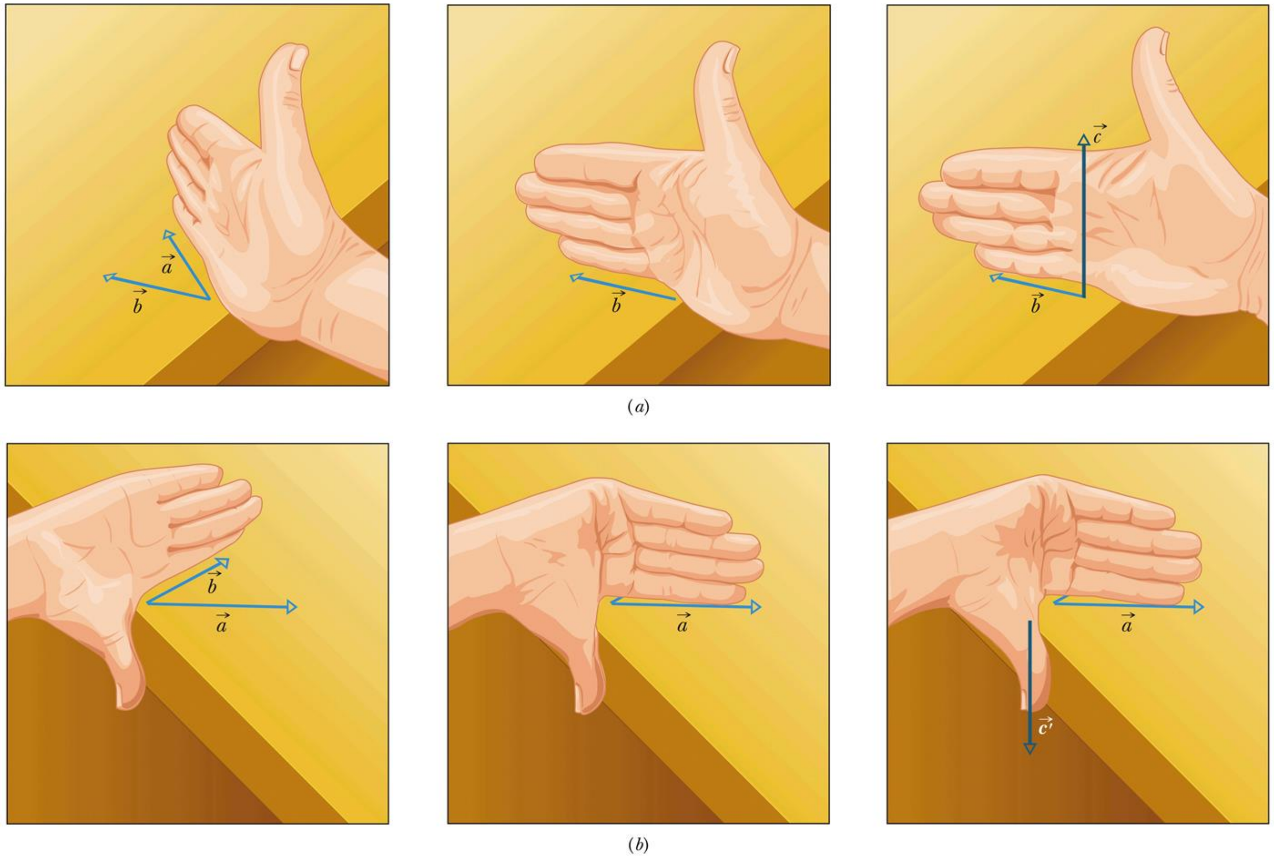
$$=a_xb_x\hat{i}\cdot\hat{i}+a_yb_y\hat{j}\cdot\hat{j}+a_zb_z\hat{k}\cdot\hat{k}=a_xb_x+a_yb_y+a_zb_z$$

벡터곱(가위곱, cross product, 외적)

$$\vec{c} = \vec{a} \times \vec{b}$$

$$c = ab \sin \phi$$

방향: 오른손 법칙



(a)

(b)

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벡터곱(가위곱, cross product, 외적)

$$\vec{c} = \vec{a} \times \vec{b}$$

$$c = ab \sin \phi$$

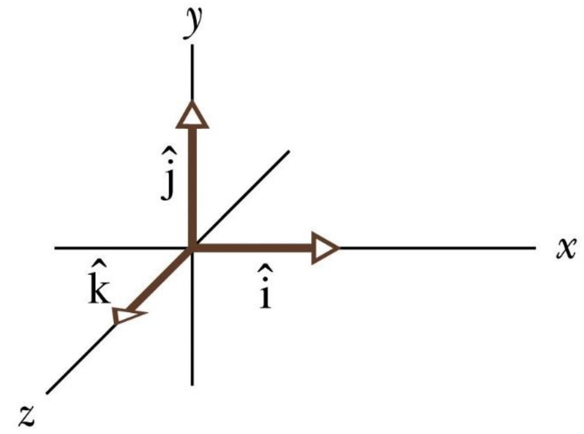
방향: 오른손 법칙

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

$$\hat{i} \times \hat{j} = \hat{k}, \ \hat{j} \times \hat{k} = \hat{i}, \ \hat{k} \times \hat{i} = \hat{j}$$

$$\hat{j} \times \hat{i} = -\hat{k}, \ \hat{k} \times \hat{j} = -\hat{i}, \ \hat{i} \times \hat{k} = -\hat{j}$$

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$



$$\vec{a}\times\vec{b} = (a_x\hat{i} + a_y\hat{j} + a_z\hat{k})\times(b_x\hat{i} + b_y\hat{j} + b_z\hat{k})$$

$$= a_x\hat{i}\times b_x\hat{i} + a_x\hat{i}\times b_y\hat{j} + a_x\hat{i}\times b_z\hat{k}$$

$$+a_y\hat{j}\times b_x\hat{i} + a_y\hat{j}\times b_y\hat{j} + a_y\hat{j}\times b_z\hat{k}$$

$$+a_z\hat{k}\times b_x\hat{i} + a_z\hat{k}\times b_y\hat{j} + a_z\hat{k}\times b_z\hat{k}$$

$$= (a_y b_z - a_z b_y)\hat{i} + (a_z b_x - a_x b_z)\hat{j} + (a_x b_y - a_y b_x)\hat{k}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

행렬 (matrix)

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & j \end{pmatrix} \qquad \begin{pmatrix} a & b & c & d \\ e & f & g & h \\ j & k & l & m \\ n & p & q & r \end{pmatrix}$$

행렬식 (determinant)

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & j \end{vmatrix} = aej + bfg + cdh - gec - hfa - jdb$$

$$\vec{a}\times\vec{b} = (a_x\hat{i} + a_y\hat{j} + a_z\hat{k})\times(b_x\hat{i} + b_y\hat{j} + b_z\hat{k})$$

$$= a_x\hat{i}\times b_x\hat{i} + a_x\hat{i}\times b_y\hat{j} + a_x\hat{i}\times b_z\hat{k}$$

$$+a_y\hat{j}\times b_x\hat{i} + a_y\hat{j}\times b_y\hat{j} + a_y\hat{j}\times b_z\hat{k}$$

$$+a_z\hat{k}\times b_x\hat{i} + a_z\hat{k}\times b_y\hat{j} + a_z\hat{k}\times b_z\hat{k}$$

$$= (a_yb_z - a_zb_y)\hat{i} + (a_zb_x - a_xb_z)\hat{j} + (a_xb_y - a_yb_x)\hat{k}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$