$$\left[y^*, y_1^\delta, y_2^\delta, y \middle| \sigma^2, l\right] \sim N(0, K)$$

$$K = \begin{bmatrix} K^{00}(x,x) & K^{00}(x^*,x)^T & K^{10}(x_1^\delta,x)^T & K^{10}(x_2^\delta,x)^T \\ K^{00}(x^*,x) & K^{00}(x^*,x^*) & K^{10}(x_1^\delta,x^*)^T & K^{10}(x_2^\delta,x^*)^T \\ K^{10}(x_1^\delta,x) & K^{10}(x_1^\delta,x^*) & K^{11}(x_1^\delta,x_1^\delta) & K^{11}(x_2^\delta,x_1^\delta)^T \\ K^{10}(x_2^\delta,x) & K^{10}(x_2^\delta,x^*) & K^{11}(x_2^\delta,x_1^\delta) & K^{11}(x_2^\delta,x_2^\delta) \end{bmatrix}$$

$$K = \begin{bmatrix} K^{00}(x^*,x^*) & K^{10}(x_1^\delta,x^*)^T & K^{10}(x_2^\delta,x^*)^T & K^{00}(x,x^*)^T \\ K^{10}(x_1^\delta,x^*) & K^{11}(x_1^\delta,x_1^\delta) & K^{11}(x_2^\delta,x_1^\delta)^T & K^{01}(x,x_1^\delta)^T \\ K^{10}(x_2^\delta,x^*) & K^{11}(x_2^\delta,x_1^\delta) & K^{11}(x_2^\delta,x_2^\delta) & K^{01}(x,x_2^\delta)^T \\ K^{00}(x,x^*) & K^{01}(x,x_1^\delta) & K^{01}(x,x_2^\delta) & K^{00}(x,x) \end{bmatrix}$$

$$\left[l, \sigma^2, y^*, y_1^\delta, y_2^\delta \middle| y\right] \propto \left[y, y^*, y_1^\delta, y_2^\delta \middle| \sigma^2, l\right]\left[\sigma^2, l\right]$$

$$\propto \left[y^*, y_1^\delta, y_2^\delta \middle| y, \sigma^2, l\right]\left[\left[y \middle| \sigma^2, l\right]\right]\left[\sigma^2, l\right]$$

$$\frac{\partial}{\partial x_k} y\left(x^\delta\right)$$

$$K^{00}(x,x') = Cov\left(y(x), y(x')\right) = \sigma^2 \prod_{k=1}^{d} g\left(x_k, x'_k; l_k\right)$$

$$K^{10}(x,x') = \frac{\partial}{\partial x_i} Cov\left(y(x), y(x')\right) = \sigma^2 \prod_{k=1}^{d} g\left(x_k, x'_k; l_k\right)$$

$$K^{11}(x,x') = \frac{\partial^2}{\partial x'_j \partial x_i} Cov\left(y(x), y(x')\right) = \sigma^2 \prod_{k=1}^{d} g\left(x_k, x'_k; l_k\right)$$

$$g(x,x') = \left(1 + \theta|x-x'| + \frac{1}{3}\theta^2|x-x'|^2\right)\exp\left\{-\theta|x-x'|\right\}$$

$$\frac{\partial}{\partial x} g(x,x') = -\frac{1}{3}\theta^2(x-x')\left[1+\theta|x-x'|\right]\exp\left\{-\theta|x-x'|\right\}$$

$$\frac{\partial^2}{\partial x' \partial x} g(x,x') = \frac{1}{3}\theta^2\left[1+\theta|x-x'| - \theta^2(x-x')^2\right]\exp\left\{-\theta|x-x'|\right\}$$

$$K^{10}(x,x') = K^{01}(x',x)^T$$

$$\left[y^*, y_1^\delta, y_2^\delta \middle| y, \sigma^2, l\right] = N(m, S)$$

$$K = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}$$

$$\Sigma_{11} = \begin{bmatrix} K^{00}(x^*,x^*) & K^{10}(x_1^\delta,x^*)^T & K^{10}(x_2^\delta,x^*)^T \\ K^{10}(x_1^\delta,x^*) & K^{11}(x_1^\delta,x_1^\delta) & K^{11}(x_2^\delta,x_1^\delta)^T \\ K^{10}(x_2^\delta,x^*) & K^{11}(x_2^\delta,x_1^\delta) & K^{11}(x_2^\delta,x_2^\delta) \end{bmatrix}$$

$$\Sigma_{12} = \Sigma_{21}^T = \begin{bmatrix} K^{00}(x,x^*)^T \\ K^{01}(x,x_1^\delta)^T \\ K^{01}(x,x_2^\delta)^T \end{bmatrix} = \begin{bmatrix} K^{00}(x^*,x) \\ K^{10}(x_1^\delta,x) \\ K^{10}(x_2^\delta,x) \end{bmatrix}$$

$$\Sigma_{22} = K^{00}(x,x)$$

$$m = 0 + \Sigma_{12}\Sigma_{22}^{-1}(y-0) = \begin{bmatrix} K^{00}(x^*,x) \\ K^{10}(x_1^\delta,x) \\ K^{10}(x_2^\delta,x) \end{bmatrix} K^{00}(x,x)^{-1} y$$

$$S = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$$

$$y(x) = 11x_1^{10} + 9x_1^8 + 7x_1^6 + 10x_2^9 + 8x_2^7$$

$$\frac{\partial}{\partial x_1} y(x) = 110x_1^9 + 72x_1^7 + 42x_1^5$$

$$\frac{\partial}{\partial x_2} y(x) = 90x_2^8 + 56x_2^6$$