

$$x_i = \begin{bmatrix} x_{(i,1)} & x_{(i,2)} & \cdots & x_{(i,d)} \end{bmatrix}$$

$$Cov(Y(x_1), Y(x_2)) = K^{00}(x_1, x_2)$$

$$= \sigma^2 \prod_{k=1}^d g(x_{(1,k)}, x_{(2,k)}; l_k)$$

$$\frac{\partial}{\partial x_{(1,i)}} Cov(Y(x_1), Y(x_2)) = K^{i0}(x_1, x_2)$$

$$= \sigma^2 \frac{\partial}{\partial x_{(1,i)}} \prod_{k=1}^d g(x_{(1,k)}, x_{(2,k)}; l_k)$$

$$= \sigma^2 \left[\prod_{\substack{k=1 \\ k \neq i}}^d g(x_{(1,k)}, x_{(2,k)}; l_k) \right] \left[\frac{\partial}{\partial x_{(1,i)}} g(x_{(1,i)}, x_{(2,i)}; l_i) \right]$$

$$\frac{\partial}{\partial x_{(2,i)}} Cov(Y(x_1), Y(x_2)) = K^{0i}(x_1, x_2)$$

$$= -K^{i0}(x_1, x_2)$$

$$\frac{\partial^2}{\partial x_{(2,j)} \partial x_{(1,i)}} Cov(Y(x_1), Y(x_2)) = K^{ij}(x_1, x_2)$$

$$= \begin{cases} \sigma^2 \left[\prod_{\substack{k=1 \\ k \neq i,j}}^d g(x_k, x'_k; l_k) \right] \left[\frac{\partial^2}{\partial x_{(2,i)} \partial x_{(1,i)}} g(x_{(1,i)}, x_{(2,i)}; l_i) \right], i = j \\ \sigma^2 \left[\prod_{\substack{k=1 \\ k \neq i,j}}^d g(x_k, x'_k; l_k) \right] \left[\frac{\partial}{\partial x_{(2,j)}} g(x_{(1,j)}, x_{(2,j)}; l_j) \right] \left[\frac{\partial}{\partial x_{(1,i)}} g(x_{(1,i)}, x_{(2,i)}; l_i) \right], i \neq j \end{cases}$$