$$\left[y^*,y_1^\delta,y_2^\delta,y\middle|\sigma^2,l\right]\sim N(0,K)$$

$$K=\begin{bmatrix} K^{00}\left(x^*,x^*\right) & K^{10}\left(x_1^\delta,x^*\right)^T & K^{20}\left(x_2^\delta,x^*\right)^T & K^{00}\left(x,x^*\right)^T \\ K^{10}\left(x_1^\delta,x^*\right) & K^{11}\left(x_1^\delta,x_1^\delta\right) & K^{21}\left(x_2^\delta,x_1^\delta\right)^T & K^{01}\left(x,x_1^\delta\right)^T \\ K^{20}\left(x_2^\delta,x^*\right) & K^{21}\left(x_2^\delta,x_1^\delta\right) & K^{22}\left(x_2^\delta,x_2^\delta\right) & K^{02}\left(x,x_2^\delta\right)^T \\ K^{00}\left(x,x^*\right) & K^{01}\left(x,x_1^\delta\right) & K^{02}\left(x,x_2^\delta\right) & K^{00}\left(x,x\right) \end{bmatrix}$$

$$\in\left\{0,1,2,...,\dim(x)\right\}$$

$$K^{d_1d_2}\left(x_1,x_2\right)$$

$$\left[l,\sigma^2,y^*,y_1^\delta,y_2^\delta\middle|y\right]\propto\left[y,y^*,y_1^\delta,y_2^\delta\middle|\sigma^2,l\right]\left[\sigma^2,l\right]$$

$$\propto\left[y^*,y_1^\delta,y_2^\delta\middle|y,\sigma^2,l\right]\left[\left[y\middle|\sigma^2,l\right]\right]\left[\sigma^2,l\right]$$

$$\frac{\partial}{\partial x_k}y\left(x^\delta\right)$$

$$K^{00}\left(x,x'\right)=Cov\left(y(x),y(x')\right)=\sigma^2\prod_{k=1}^{d}g\left(x_k,x'_k;l_k\right)$$

$$K^{10}\left(x,x'\right)=\frac{\partial}{\partial x_i}Cov\left(y(x),y(x')\right)=\sigma^2\prod_{k=1}^{d}g\left(x_k,x'_k;l_k\right)$$

$$K^{11}\left(x,x'\right)=\frac{\partial^2}{\partial x'_j\,\partial x_i}Cov\left(y(x),y(x')\right)=\sigma^2\prod_{k=1}^{d}g\left(x_k,x'_k;l_k\right)$$

$$g\left(x,x'\right)=\left(1+\theta|x-x'|+\frac{1}{3}\theta^2|x-x'|^2\right)\exp\left\{-\theta|x-x'|\right\}$$

$$\frac{\partial}{\partial x}g\left(x,x'\right)=-\frac{1}{3}\theta^2\left(x-x'\right)\left[1+\theta|x-x'|\right]\exp\left\{-\theta|x-x'|\right\}$$

$$\frac{\partial^2}{\partial x'\partial x}g\left(x,x'\right)=\frac{1}{3}\theta^2\left[1+\theta|x-x'|-\theta^2\left(x-x'\right)^2\right]\exp\left\{-\theta|x-x'|\right\}$$

$$K^{10}\left(x,x'\right)=K^{01}\left(x',x\right)^T$$

$$\left[y^*,y_1^\delta,y_2^\delta\middle|y,\sigma^2,l\right]=N(m,S)$$

$$K=\begin{bmatrix}\Sigma_{11} & \Sigma_{12}\\ \Sigma_{21} & \Sigma_{22}\end{bmatrix}$$

$$\Sigma_{11}=\begin{bmatrix} K^{00}\left(x^*,x^*\right) & K^{10}\left(x_1^\delta,x^*\right)^T & K^{10}\left(x_2^\delta,x^*\right)^T \\ K^{10}\left(x_1^\delta,x^*\right) & K^{11}\left(x_1^\delta,x_1^\delta\right) & K^{11}\left(x_2^\delta,x_1^\delta\right)^T \\ K^{10}\left(x_2^\delta,x^*\right) & K^{11}\left(x_2^\delta,x_1^\delta\right) & K^{11}\left(x_2^\delta,x_2^\delta\right) \end{bmatrix}$$

$$\Sigma_{12}=\Sigma_{21}^T=\begin{bmatrix} K^{00}\left(x,x^*\right)^T \\ K^{01}\left(x,x_1^\delta\right)^T \\ K^{01}\left(x,x_2^\delta\right)^T \end{bmatrix}=\begin{bmatrix} K^{00}\left(x^*,x\right) \\ K^{10}\left(x_1^\delta,x\right) \\ K^{10}\left(x_2^\delta,x\right) \end{bmatrix}$$

$$\Sigma_{22}=K^{00}\left(x,x\right)$$

$$m=0+\Sigma_{12}\Sigma_{22}^{-1}\left(y-0\right)=\begin{bmatrix} K^{00}\left(x^*,x\right) \\ K^{10}\left(x_1^\delta,x\right) \\ K^{10}\left(x_2^\delta,x\right) \end{bmatrix}K^{00}\left(x,x\right)^{-1}y$$

$$S=\Sigma_{11}-\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$$

$$y\left(x\right)=11x_1^{10}+9x_1^8+7x_1^6+10x_2^9+8x_2^7$$

$$\frac{\partial}{\partial x_1}y\left(x\right)=110x_1^9+72x_1^7+42x_1^5$$

$$\frac{\partial}{\partial x_2}y\left(x\right)=90x_2^8+56x_2^6$$