

$$\underbrace{\underline{X}}_{n_i \times d} = \begin{bmatrix} x_1 \\ \vdots \\ x_{n_i} \end{bmatrix} = \begin{bmatrix} x_{(1,1)} & \dots & x_{(1,d)} \\ \vdots & \ddots & \vdots \\ x_{(n_i,1)} & \dots & x_{(n_i,d)} \end{bmatrix}$$

$$\underbrace{K^{d_1 d_2} \left(\underbrace{\widehat{\underline{X}}}_{n_1 \times d}, \underbrace{\widehat{\underline{X'}}}_{n_2 \times d} \right)}_{n_1 \times n_2} = K^{d_1 d_2} \left(\begin{bmatrix} x_1 \\ \vdots \\ x_{n_1} \end{bmatrix}, \begin{bmatrix} x_1' \\ \vdots \\ x_{n_2}' \end{bmatrix} \right) = \left[k^{d_1 d_2} (x_i, x_j') \right]_{ij}$$

$$K^{d_1 d_2} (X, X') = X^{d_1 d_1} (X', X)^T$$

$$[y^*, y_1^\delta, y_2^\delta, \dots, y_d^\delta, y | \sigma^2, l] \sim N(0, \Sigma)$$

$$\Sigma = \begin{bmatrix} K^{00}(X^*, X^*) & K^{01}(X^*, X_1^\delta) & K^{02}(X^*, X_2^\delta) & \dots & K^{0d}(X^*, X_d^\delta) & K^{00}(X^*, X) \\ K^{10}(X_1^\delta, X^*) & K^{11}(X_1^\delta, X_1^\delta) & K^{12}(X_1^\delta, X_2^\delta) & \dots & K^{10}(X_1^\delta, X_d^\delta) & K^{10}(X_1^\delta, X) \\ K^{20}(X_2^\delta, X^*) & K^{21}(X_2^\delta, X_1^\delta) & K^{22}(X_2^\delta, X_2^\delta) & \dots & K^{10}(X_2^\delta, X_d^\delta) & K^{20}(X_2^\delta, X) \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ K^{d0}(X_d^\delta, X^*) & K^{d1}(X_d^\delta, X_1^\delta) & K^{d2}(X_d^\delta, X_2^\delta) & \dots & K^{dd}(X_d^\delta, X_d^\delta) & K^{d0}(X_d^\delta, X) \\ K^{00}(X, X^*) & K^{01}(X, X_1^\delta) & K^{02}(X, X_2^\delta) & \dots & K^{0d}(X, X_d^\delta) & K^{00}(X, X) \end{bmatrix}$$