HJM_caplet_pricing_v2

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1 Caplet pricing with HJM model

This report summarises the results of Question 5 regarding the pricing of a **caplet option** written on 6M LIBOR starting 6 months from today using the formula:

$$DF_{OIS}(0,1) \times \max(L(0;,0.5,1) - K, 0) \times \tau \times N \tag{1}$$

with the following parameters:

- Strike (agreed rate) $K = 3.5\%$
- Notional $N = 100,000$
- Tenor $\tau = 0.5$
- Discount factor $DF_{OIS}(0,1) = 0.996$ taken from a curve built from traded OIS swaps

1.1 Monte Carlo

The was done using **Python 2.7** and the code has been attached separately (in the CQF portal exam submission documents).

For the Monte Carlo simulation the following parameters were used:

- Time step $dt = 0.01$
- Number of simulations $I = 6,000$
- The drift and volatilities of the forward rate were taken from the 'HJM Model - MC - Caplet v2.xlsm' spreadsheet
- The initial value for the forward rate $f(t = 0, \tau)$ was taken as the last row in 'HJM Model - PCA.xlsm'
- The random number generator used is python's numpy standard normal random number generator ('np.random.standard_normal') which draws samples from the standard normal distribution (mean=0, stdev=1)
- The antithetic variance reduction technique is used to increase the accuracy of the caplet price at minimal computational cost

To obtain an expectation of Libor rate in the future $L(t; T_i, T_{i+1})$, the forward rate is selected from the corresponding tenor column $\tau = T_{i+1} - T_i$ of the HJM output, for the correct simulated time t. We then convert to Libor rate using $L = m(e^{f/m} - 1)$ where m is the compounding frequency per year.

For the caplet option written on 6M Libor starting 6 months from today this means:

- $T_i = 0.5$ (6 months Libor)
- $T_{i+1} = 1.0$ (cashflow paid)
- Hence $\tau = T_{i+1} - T_i = 0.5$
- $m = 1/\tau = 1/0.5 = 2.0$

Hence, for each MC simulation we look for the forward rate $f(0; \tau = 0.5)$ and plug this into equation (1) above.

After 6,000 simulations, the caplet price with the above parameters converges to:

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Caplet price is:
Std. MC = 472.068922926 (0.144284561179)
Antithetic MC = 472.763692789 (0.175699863066)
Average = 472.416307857 (0.0199682961394)
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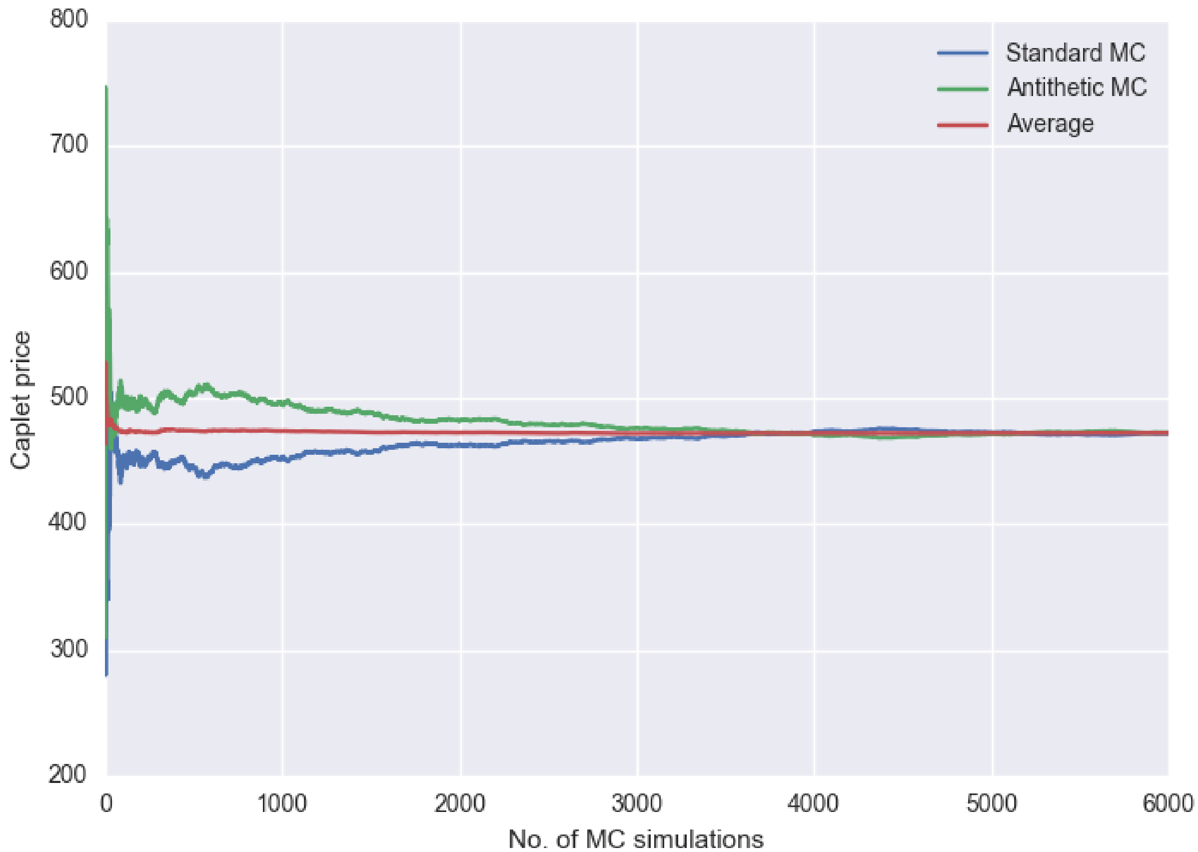
where in brackets we see the MC error from the number of simulations (std/sqrt(N))

1.1.1 Convergence diagram



The figure shows in blue the standard MC result and it's antithetic in green. They appear to be roughly the mirror image of each other about their average (the variance-corrected result). The caplet prices without the variance correction show robustness only after $\sim 3,500$ simulations, where they both appear converge to the average. The variance reduction therefore has a dramatic effect in reducing the simulation error even at early stages of the simulation. If this was the only source of error, one could get a very good estimate of the caplet price with just under ~ 500 simulations using the variance reduction technique, which would result in significantly faster computational times. However, MC integration has additional sources of error.

1.1.2 Sources of error

There are two main sources of error in the MC:

- Simulation error: this is because we are only simulating a finite number of realizations (possible paths). As discussed, this is calculated as $\sigma_N/\sqrt(N)$ where σ_N is the standard deviation of the sample, which is a good approximation as $N \to \infty$
- Time step error: an error is introduced by the step size dt by virtue of the discrete approximation to continuous events. Given the HJM model SDE to model the forward rate, this error is $\mathcal{O}(dt)$ (although this could perhaps be reduced by adding a second order term like in the Milstein scheme)
- Other sources of error come from for example using a PCA model to calculate the volatility and having used only the 3 largest eigenvalues