Certificate in Quantitative Finance
Final Project Brief

January 2016 Cohort

Please note that Workshops will be held to review the project topics.

This document outlines each available topic together with submission requirements for the project report. By-step instructions offer a structure not a limit to what you can implement. You are welcome to submit the project earlier to gain extra time for the Final Exam preparation.

Exclusively for the use by current CQF delegates. <u>No distribution.</u>

Submission date is <u>**Monday, 25 July 2016**</u>

Submissions must match the Brief. There is no extension to the Final Project.

Projects without declaration or working code are incomplete and will be returned.

All projects are checked for originality. We reserve an option of *a viva voce.*

1 Instructions

Assessment for Module Six is carried out by the means of a programming project. It is designed to give an opportunity for further study of numerical methods required to implement and validate a quantitative model. Print out and focus on the section that covers your chosen topic.

Every project submitted has to be in accordance with the Brief of the cohort it is submitted for. To complete the project, you must implement **one** of the listed topics **plus** CVA component.

1. **Pricing Basket Credit Default Swap**
2. **Robust Portfolio Allocation with Advances**
3. **Time Series Analysis and Backtesting**
4. **LIBOR and OIS Rates - Volatility Modelling**

All. The mandatory CVA component does not serve as a second topic but balances exposure to the quant issues (interest rates, discounting) which would not be in focus otherwise.
CVA Calculation for an Interest Rate Swap

The Project Workshops, Brief, and Q&A are your primary guidance for the project. Please make sure you have (re-)watched relevant workshops and topical lectures with concentration. The Electives support the final project, explain concepts and some techniques. They give ideas on what to implement/write in Analysis and Discussion.

	Trading (Algo)	Comp. Methods	Regulation & Risk	Volatility Modelling	Advanced Portfolio	CP Risk
Credit		XX	X			XXX
Portfolio	XX				XXX	
Time Series	XX		X		XX	
Rates/LMM		XX		XX		

Table 1: Relating Topic Choice to CQF Electives on scale from X to XXX.

1.1 Submission Requirements

- Submit working code together with a well-written report and originality declaration.
- The report must be soft-bound in thesis style. Normal length is 25-40 pages, ex. code.
- The software must be provided on a flash drive/DVD in a plastic pocket securely attached to the inside back cover.
- E-mailed submissions not accepted. One copy must be posted on submission date at the latest to: Fitch Learning, 17th Floor, 25 Canada Square, London E14 5LQ, UK.

1.2 Project Code

- Traditionally, project code is written in Python, C++ or C# with .NET Framework, Java or VBA. Each of the production tools involves the specialised quant libraries. If you are using the coding language not common or functional please consult with a CQF Tutor.

- The aim of the project is to enable you to code numerical methods and develop model prototypes in a production environment. Minimise the use of Excel spreadsheet functions.

- Excel spreadsheets-only or-scripted solutions are below the expected standard for completion of the project. 'Scripted solution' means the ready functionality from toolboxes and libraries is called, but the amount of own functions and numerical methods re-coded is minimal or non-existent.

- Use of Mathematica/Matlab/R is encouraged, particularly where data presentation, time series or analytical mathematics is involved. CQF-supplied spreadsheets can be used as a starting point and to validate results but coding of numerical techniques is expected.

- To answer the question, "What should I code?" Delegates are expected to re-code numerical methods that are central to the model and exercise judgement in identifying them. Balanced use of libraries is allowed <u>at the delegate's own discretion</u> and subject to a description of limitations for ready functions/borrowed code (in the report).

- It is up to delegates to develop their own test cases, sensibility checks and validation. It is normal to observe irregularities when the model is implemented on real life data. If in doubt, <u>reflect on the issue in the project report</u>.

- The code must be thoroughly tested and well-documented: each function must be described, and comments must be used. Provide instructions on how to run the code.

1.3 Project Report

The main purpose of the report is to facilitate access to numerical methods' implementation (the code) and pricing results.

- The report must contain a sufficient description of the mathematical model, numerical methods and their properties. In-depth study is welcome but report must be relevant.

- Identify numerical methods recorded and include their code/algorithms in an appendix.

- Please give due attention and space for presentation and discussion of your pricing results. Present explicit sensitivity and/or risk analysis. Use charts, test cases and comparison to research results where available.

- Mathematical sections of the report can be prepared using LaTeX or Equation Editor (Word). For Mathematica and Python notebooks, make sure they are presentable.

CVA Calculation for an Interest Rate Swap

Summary

To recognise the importance of credit and counterparty risk adjustments to the derivatives business we introduce this mandatory component which must be implemented with **each topic**.

Calculate the credit valuation adjustment (taken by Counterparty A) to the price of an interest rate swap using the credit spreads for Counterparty B. Plot MtM values (a good plot will show results from many simulations) and produce smoothed Expected Exposure profile using the mean of exposure distribution – distribution of Forward LIBORs at each time T_{i+1}.[1] Once that is done, produce Potential Future Exposure with the simulated L_{6M} taken from 97.5th percentile.

Provide a brief discussion of your observations, e.g., exposure over time, location of maximum exposure, impact of very small or negative rates. The advanced sensitivity analysis will illustrate the concept of the wrong-way risk.

Inputs and By-Step Instructions

The inputs for this CVA calculation are Forward LIBORs, discounting factors and default probabilities. You can bootstrap or make reasonable assumptions to supplement the data (e.g., flat credit spreads to 5Y tenor, approximate discounting factors).

- Probability of default is bootstrapped from credit spreads for a reference name (any reasonable set values) in $6M$ increment. Linear interpolation over spreads and use of ready PD bootstrapping spreadsheet are acceptable, $RR = 40\%$. CVA LGD - own choice.
- Assume the swap is written on 6M LIBOR L_{6M} expiring in 5Y.

  To simulate the future values of L_{6M} at times $T_1, T_2, T_3, \ldots$ take either HJM MC spreadsheet or ready implementation of a calibrated model for $r(t)$, such as Hull & White.

  At each time T_{i+1} there is a distribution of Forward LIBORs but we only require its mean (an average) to calculate the exposure. Notional $N = 1$ and payment frequency $\tau = 0.5$.
- Define MtM position as Floating Leg − Fixed Leg $= (L_{6M} - K)$ appropriately discounted. For simplicity, it is best to choose fixed leg (rate) K such that the exposure is positive.
- The maths of obtaining Forward LIBOR from a simulated forward curve is illustrated in the Yield Curve spreadsheet. Discounting factors to be taken from the OIS spot curve.

[1] Market practice is to use the mean (median) the positive side of distribution only (positive exposure).

Time Series Analysis and Backtesting

Summary

The aim to this topic is an estimation and analysis of tradeable relationships between two or more financial time series. Identifying and backtesting a robust cointegrated relationship means exposing a factor that drives both (or many) asset prices. The factor is traded by entering the long-short position given by cointegrating weights.

Through implementation you will have a hands-on introduction to Vector Autoregression (for returns) and Error Correction (for prices) models, which are the main variations of the multivariate regression. The econometric applications of such a regression are **a.** forecast, **b.** impulse response analysis, and **c.** (Granger) causality analysis. However, none of these applications serves the needs of quantitative analytics/portfolio allocation/hedge fund investing and trading. Instead a range of techniques and considerations applied known as 'backtesting'.

A project that solely runs pre-programmed statistical tests on data is a preparation work, not the complete project. The project should have coding of necessary statistical tests from the first principles (explicit regression equations) by yourself. The least deliverables are **a.** implemented Engle-Granger procedure, **b.** statistical diagnosis and backtesting (split dataset in half or compute rolling estimates), and **c.** market factor backtesting via regressing returns from your strategy on market index returns or another factor. These are in addition to the underlying numerical methods on matrices and vector autoregression.[5]

Backtesting

The following notes offer choices to implement in aspects and questions of backtesting:

- All project designs (whether learning-level or in-depth) should include backtesting of a strategy. The strategy is realised by using cointegrating coefficients $\boldsymbol{\beta}_{Coint}$ as allocations $\boldsymbol{w}$. That creates a long-short portfolio that generates a mean-reverting spread (cointegrated residual).

- Does cumulative P&L behave as expected (for a cointegration trade)? Is P&L coming from a few or lot of trades/time period? What are the SR/Maximum Drawdown? Behaviour of risk measures (volatility/VaR)? Concentration in assets and attribution?

- Impact of transaction costs (plot an average P&L value vs. number of transactions).

- Optionally, introduce liquidity and algorithmic flow considerations (a model of order flow). How would you be entering and accumulating the position? What impact bid-ask spread and transaction costs will make?

[5] It is recommended that you use the environment with facilities for matrix and time series manipulation (R, Matlab) or code in Python/C++ with the use of quant libraries. The use of VBA will be cumbersome.

Step-by-Steps Designs

Design 1: 'Learning' and Cointegration in Pairs

An understanding-level design can use the ready specification tests, but matrix form regression estimation must be re-coded. The project can rely on the Engle-Granger procedure for cointegration testing among pairs.

1. Implement *concise matrix form* estimation for multivariate regression and conduct model specification tests for: (a) identifying optimal lag p and (b) stability check.
2. Optionally, test forecasting capability of regression with IRF and Granger Causality.
3. Implement Engle-Granger procedure and use it to identify a cointegrated pair. Estimate relationships both ways to select the appropriate lead variable.
4. ADF test for unit root must be coded and used.

Design 2: 'In-depth' and Multivariate Cointegration with Trends

The advanced implementation will re-code the essential tests (selection of lag p and unit root) and offer the multivariate estimation of cointegration. Analysis of relationships suitable for trading has to extend to exploring the quality of mean-reversion in cointegrated residual.

1. Implement *concise matrix form* estimation for multivariate regression and conduct your own coded tests for: (a) identifying optimal lag p and (b) stability check.
2. Apply Maximum Likelihood Estimation (Johansen Procedure) for multivariate cointegration on asset price data (in levels, not returns).
3. This is subject to (a) prior testing for unit roots by ADF test and (b) specification of deterministic trends for cointegrating equation. Use simple trends and check for overfitting.
4. Present clear analysis of results for *Maximum Eigenvalue* and *Trace* statistical tests, both are based on Likelihood Ratio test principle.

Before own implementation, consider R/Matlab functionality for Johansen Procedure and especially the tests for the number of cointegrated relationship in a multivariate system.

Strategy Backtest (both designs)

5. Use cointegration analysis to identify candidates for the long-short portfolios as given by cointegrating weights.[6]
6. Backtest a statistical arbitrage trade (or several) by investigating a mean-version in cointegrated residual $\boldsymbol{e_t} = \boldsymbol{\beta'_{Coint} Y_t}$. Use the fitting to the OU process and give the speed of reversion, number of trades as well as other properties of P&L. Rely on backtesting considerations given above.

[6] Within an asset class, a robust cointegration relationship exposes *a factor*.

Resources

Reading List: CVA

- CQF Lecture(s) on *Credit Value Adjustment*. For an alternative implementation example, `http://www.cvacentral.com/books/credit-value-adjustment/spreadsheets/`

Reading List: Rates Volatility

- Review *Methods for Constructing a Yield Curve* by Pat Hagan and Graeme West – best to start with version published in WILMOTT (May-June 2008).
- *The LIBOR Market Model in Practice* specialised textbook by Gatarek, et al. (2006) gives technical detail on cailbration from caplets and swaptions (Chapters 7 and 9 respectively) that will be useful to those working with LIBOR derivatives. (Please email the tutor.)

Reading List: Credit Portfolio

- The starting source on *sampling from copula* algorithm is *Monte Carlo Methods in Finance* textbook by Peter Jaekel (2002). Please see Chapter 5, particularly pages 46-50.
- Most likely, you will need to re-visit *CDO Lecture* material, particularly about slides 48-52 that illustrate Elliptical copula densities and discuss factorisation.
- Rank correlation coefficients are introduced *Correlation Sensitivity Lecture* and Jaekel (2002) as above. Project Q&A document gives the clarified formulae and explanations.
- Bootstrapping of survival probabilities is covered in *Credit Default Swaps Lecture*.

Reading List: Portfolio Construction

- CQF Lecture on *Fundamentals of Optimization and Application to Portfolio Selection*
- *A Step-by-step Guide to The Black-Litterman Model* (Incorporating user-specified confidence levels). Thomas Idzorek, 2002
- *The Black-Litterman Approach: Original Model and Extensions* Attilio Meucci, 2010. `http://ssrn.com/abstract=1117574`

Reading List: Time Series

- CQF Lectures on *Cointegration and Statistical Arbitrage* and *Statistical Methods for PD*.
- *Explaining Cointegration Analysis: Parts I and II*, by David Hendry and Katarina Juselius, *Energy Journal* 2000 and 2001.
- *Learning and Trusting Cointegration in Statistical Arbitrage* by Richard Diamond, *WILMOTT Magazine*, November 2013.
- *User Guide for Johansens Method* by Kyungho Jang and Masao Ogaki, 2001.