Certificate in Quantitative Finance

Model Implementation and Robust Estimation in Rates, Credit and Portfolio Construction

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This Version: February 16, 2016

Instructions

These comprehensive notes address the issues encountered in final project implementation. Coding and model validation—that is, checking that a model provides sensible numbers—is part of independent project work.

Please print selected sections/pages of this document for your reference when working on the CQF Final Project.

Monte Carlo Methodology (All Topics)

Improvement for Credit Spread Pricing

Please see the section on Averaging PL and DL. The number of simulations required for convergence (with naive Excel random numbers) can reach into $100,000$s, particularly for the higher kth-to-default. Check sensibility of your pricing result (is it as expected, is its variance reducing) even if it *appears* to converge after $10,000$s simulations.

The key reference for simulation design and sampling strategies, particularly when sampling from copula required, is a textbook on *Monte-Carlo Methods in Finance* by Peter Jaekel.

Improvement for HJM Simulations

With $k = 3\ldots5$ factors (volatility functions) used to simulate the curve, the dimensionality of Monte Carlo simulation is sufficiently low. Potentially, fractal patterns in random numbers across the depth of multiple dimensions can lead to pricing patterns that can be arbitraged.

There is no requirement to simulate the curve for up to 25 years into the 'future time' (with $\Delta t = 0.01$ that would be 2500 rows on the HJM Spreadsheet). You can start with a 5-year future period – that will give enough data for most caplet pricing illustrations.

However, for each simulated 'table' of forward rates one can observe that the same kind of simulated curve propagates into the future time. That raises an issue about importance of re-sampling when pricing by Monte Carlo.

[Insert 3D plots here]

Random Numbers Generation (for faster convergence)

There are following RN generation methods that you can adopt from ready implementations (eg, NAG Library) to give professional quality to the Monte Carlo within your project. Low discrepancy generators provide statistically dependent numbers with improved evenness in the multidimensional space.

1. Mersenne Twister (Pseudo RN)

2. Halton Numbers (low discrepancy, Quasi RN)

3. Sobol Numbers (low discrepancy, Quasi RN)

The convergence provided by low discrepancy RN generators is the order of $c(d)\dfrac{(\ln N)^d}{N}$ vs. $\dfrac{1}{\sqrt{N}}$ for quasi RN generators such as Excel's *RAND()* and Mersenne Twister. N refers to the number of simulations, d is number of dimensions (eg, $d = 3$ factors in HJM SDE and $d = 5$ reference names in Basket CDS), and $c(d)$ is some scaling function.

Example for HJM implementation: while Excel-generated random numbers might not give convergence even after $2,000$ simulations, Monte Carlo with proper low discrepancy RNs gives a good (low variance) estimate right after 200 simulations and satisfactory convergence after about 600 simulations. The second problem brought by this example is that **the use of Excel's pseudo random numbers over-estimated the price of a derivative.**

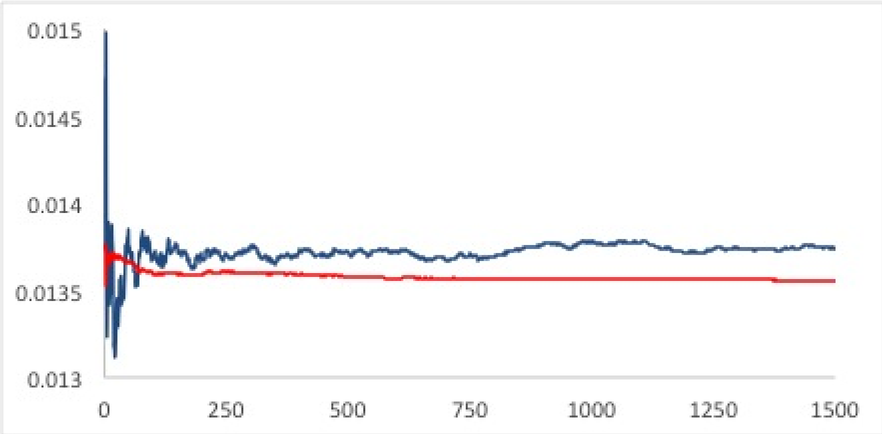

Figure 4: Dark blue line is a derivative price using *RAND()*, while red line result is for the same derivative obtained using low-discrepancy RNs.

Variance Reduction

Monte-Carlo simulation is evaluated by the speed (computational criterion) as well as variance of the estimator. That is, if you are estimating bond price or caplet price by Monte Carlo, you might would like to plot running standard deviation (after each 100 of simulations added).

The most important trick to reduce variance is to identify which simulated inputs increase the standard deviation of the estimate. For example in Basket CDS projects very small u imply default times τ_i which are less than one quarter. Because each default time is a stand-alone output (inter-arrival times are conditionally independent) it possible to remove those early defaults without breaking the continuity of Monte-Carlo. On the other hand, you will not be able to do the same thing when simulating a log-normal asset price (i.e., removing price levels at will).

Between Excel RAND() and Quasi RN, the variance of the estimator can vary threefold!

The note is an excerpt from the Q&A document on *Model Implementation and Robust Estimation* by Dr Richard Diamond, CQF.

Financial Time Series Analysis

These notes give early discussion of causality analysis for Vector Autoregression and Equilibrium Correction. All the **new content**, including Backtesting, Cointegration Case in R (spot rates market data), as well as *Learning and Trusting Cointegration in Statistical Arbitrage* to be presented in the relevant lectures and the Project Workshop. The updated Vector Autoregression and Johansen MLE Procedure notes can be found in *TS Workings* document that comes with the Cointegration Lecture.

Granger Causality

In addition to a decomposition of feedback relationship between two variables, the test is useful to determine whether a variable can be treated as exogenous, and therefore, dropped from VAR(p) system. Here is the original paper by Granger (1969) – the proofs, illustrations and discussion cover the case of two variables. The useful feature of his proof is the decomposition of feedback relationship between two variables in arms (A to B and B to A).
`http://www.sonoma.edu/users/c/cuellar/econ411/Granger.pdf`

Algorithm

Actual implementation of Granger Causality in software packages (MATLAB open-source econometrics toolboxes) forms the full VAR(1) matrix, calculate the unrestricted regression, then drop a variable and calculate the restricted regression. For example, for one unrestricted regression for row 1 (Y1,t) there are four restricted regressions (removing Y1,t-1, Y2,t-1, Y3,t-1, Y4,t-1). Repeat for row 2 (Y2,t).

Y1,t-1 Y2,t-1 Y3,t-1 Y4, t-1
Y1,t
Y2,t
Y3,t
Y4,t

Conditional Statement

For a case of more than two variables, the system would require a modified proof and statement about joint impact and conditioning. But econometrics literature says that keeping other variables in restricted and unrestricted regression approximates the bi-variate test.

I suggest that when Granger causality was introduced, the understanding of statistical models (a regression) as conditional statements was yet in development. In my opinion, using additional variables increases the degree of conditionality and limits the test. In other words the effect of a variable is evaluated in presence of other variables this sounds as a positive quality but, a simple VAR system does not represent the dynamics between financial time series well. Therefore, increase in conditionality actually limits the model which could be useful in simpler formulation and times when there are clear trends in the market.

Examples

There is an example of the use of Granger causality by Economist at the link below. Controversies apart, the statistical testing and results presentation were carried out with care see Charts 3a and 3b. `http://www.economist.com/node/21554185`

Adding lags to Granger causality testing increases statistical significance of links already identified with VAR(1) but could also introduce weaker links that appear comparable to stronger links in their significance. If partial autocorrelations do not suggest an autoregressive dependence beyond lag 1 then there would be a little point testing further. Information Criteria tests for lag p usually indicate the same as autocorrelations.

High correlation across time series could also be a problem: Granger causality testing would show that two highly correlated variables X, Y 'cause a third variable without identifying whether X or Y has the stronger impact. Not only such construction can fall apart but if we consider residuals: removing X will not reduce residuals much (Y would still be a good predictor) and so we conclude that X does not matter; same goes for Y and we are in Catch 22.

Pairs Trading [Earlier Notes]

Instead of solely relying on confirming significance of equilibrium correction (the small term $(1 - alpha)e_{t-1}$, the trade design should explore the properties of mean-reversion in the spread by the fitting to the OU process. Beware of the regime changes in non-stationary series: a series can shift in response to shocks leading to the change in equilibrium level or dismantling of a cointegrating relationship altogether.

Spread trading applications of cointegration work well in the presence of the a common factor or term structure (i.e., inflation and rates).

1. Once a cointegrated relationship in the pair is identified, establish the level of equilibrium (intercept) and plot the mean-reverting spread.

2. It will be prudent to study the joint distribution of *the common stochastic process*. It should consist of *i.i.d.* random variables, however if they produce a skew it is a dangerous sign. If it does not follow the Normal distribution, other problems arise: high-peaked and thin-tailed distribution indicates difficulty to find entry point, whereas fat-tailed distribution produces more entry points but potentially low P&L.

3. For trade design, long waiting times mean cost of carry (financing of the leverage) and increase the risk of regime change in the co-integrating relationship. Estimate half-life time of a co-integrating relationship returning to its equilibrium and explore trading partial half life.