Certificate in Quantitative Finance
Final Project Brief

January 2016 Cohort

Please note that Workshops will be held to review the project topics.

This document outlines each available topic together with submission requirements for the project report. By-step instructions offer a structure not a limit to what you can implement. You are welcome to submit the project earlier to gain extra time for the Final Exam preparation.

Exclusively for the use by current CQF delegates. No distribution.

Submission date is **Monday, 25 July 2016**

Submissions must match the Brief. There is no extension to the Final Project.

Projects without declaration or working code are incomplete and will be returned.

All projects are checked for originality. We reserve an option of *a viva voce.*

1 Instructions

Assessment for Module Six is carried out by the means of a programming project. It is designed to give an opportunity for further study of numerical methods required to implement and validate a quantitative model. Print out and focus on the section that covers your chosen topic.

Every project submitted has to be in accordance with the Brief of the cohort it is submitted for. To complete the project, you must implement **one** of the listed topics **plus** CVA component.

1. **Pricing Basket Credit Default Swap**
2. **Robust Portfolio Allocation with Advances**
3. **Time Series Analysis and Backtesting**
4. **LIBOR and OIS Rates - Volatility Modelling**

All. The mandatory CVA component does not serve as a second topic but balances exposure to the quant issues (interest rates, discounting) which would not be in focus otherwise.
CVA Calculation for an Interest Rate Swap

The Project Workshops, Brief, and Q&A are your primary guidance for the project. Please make sure you have (re-)watched relevant workshops and topical lectures with concentration. The Electives support the final project, explain concepts and some techniques. They give ideas on what to implement/write in Analysis and Discussion.

	Trading (Algo)	Comp. Methods	Regulation & Risk	Volatility Modelling	Advanced Portfolio	CP Risk
Credit		XX	X			XXX
Portfolio	XX				XXX	
Time Series	XX		X		XX	
Rates/LMM		XX		XX		

Table 1: Relating Topic Choice to CQF Electives on scale from X to XXX.

1.1 Submission Requirements

- Submit working code together with a well-written report and originality declaration.
- The report must be soft-bound in thesis style. Normal length is 25-40 pages, ex. code.
- The software must be provided on a flash drive/DVD in a plastic pocket securely attached to the inside back cover.
- E-mailed submissions not accepted. One copy must be posted on submission date at the latest to: Fitch Learning, 17th Floor, 25 Canada Square, London E14 5LQ, UK.

1.2 Project Code

- Traditionally, project code is written in Python, C++ or C# with .NET Framework, Java or VBA. Each of the production tools involves the specialised quant libraries. If you are using the coding language not common or functional please consult with a CQF Tutor.

- The aim of the project is to enable you to code numerical methods and develop model prototypes in a production environment. Minimise the use of Excel spreadsheet functions.

- Excel spreadsheets-only or-scripted solutions are below the expected standard for completion of the project. 'Scripted solution' means the ready functionality from toolboxes and libraries is called, but the amount of own functions and numerical methods re-coded is minimal or non-existent.

- Use of Mathematica/Matlab/R is encouraged, particularly where data presentation, time series or analytical mathematics is involved. CQF-supplied spreadsheets can be used as a starting point and to validate results but coding of numerical techniques is expected.

- To answer the question, "What should I code?" Delegates are expected to re-code numerical methods that are central to the model and exercise judgement in identifying them. Balanced use of libraries is allowed at the delegate's own discretion and subject to a description of limitations for ready functions/borrowed code (in the report).

- It is up to delegates to develop their own test cases, sensibility checks and validation. It is normal to observe irregularities when the model is implemented on real life data. If in doubt, reflect on the issue in the project report.

- The code must be thoroughly tested and well-documented: each function must be described, and comments must be used. Provide instructions on how to run the code.

1.3 Project Report

The main purpose of the report is to facilitate access to numerical methods' implementation (the code) and pricing results.

- The report must contain a sufficient description of the mathematical model, numerical methods and their properties. In-depth study is welcome but report must be relevant.

- Identify numerical methods recorded and include their code/algorithms in an appendix.

- Please give due attention and space for presentation and discussion of your pricing results. Present explicit sensitivity and/or risk analysis. Use charts, test cases and comparison to research results where available.

- Mathematical sections of the report can be prepared using LaTeX or Equation Editor (Word). For Mathematica and Python notebooks, make sure they are presentable.

CVA Calculation for an Interest Rate Swap

Summary

To recognise the importance of credit and counterparty risk adjustments to the derivatives business we introduce this mandatory component which must be implemented with **each topic**.

Calculate the credit valuation adjustment (taken by Counterparty A) to the price of an interest rate swap using the credit spreads for Counterparty B. Plot MtM values (a good plot will show results from many simulations) and produce smoothed Expected Exposure profile using the mean of exposure distribution – distribution of Forward LIBORs at each time T_{i+1}.[1] Once that is done, produce Potential Future Exposure with the simulated L_{6M} taken from 97.5th percentile.

Provide a brief discussion of your observations, e.g., exposure over time, location of maximum exposure, impact of very small or negative rates. The advanced sensitivity analysis will illustrate the concept of the wrong-way risk.

Inputs and By-Step Instructions

The inputs for this CVA calculation are Forward LIBORs, discounting factors and default probabilities. You can bootstrap or make reasonable assumptions to supplement the data (e.g., flat credit spreads to 5Y tenor, approximate discounting factors).

- Probability of default is bootstrapped from credit spreads for a reference name (any reasonable set values) in $6M$ increment. Linear interpolation over spreads and use of ready PD bootstrapping spreadsheet are acceptable, $RR = 40\%$. CVA LGD - own choice.
- Assume the swap is written on 6M LIBOR L_{6M} expiring in 5Y.

  To simulate the future values of L_{6M} at times $T_1, T_2, T_3, \ldots$ take either HJM MC spreadsheet or ready implementation of a calibrated model for $r(t)$, such as Hull & White.

  At each time T_{i+1} there is a distribution of Forward LIBORs but we only require its mean (an average) to calculate the exposure. Notional $N = 1$ and payment frequency $\tau = 0.5$.
- Define MtM position as Floating Leg – Fixed Leg $= (L_{6M} - K)$ appropriately discounted. For simplicity, it is best to choose fixed leg (rate) K such that the exposure is positive.
- The maths of obtaining Forward LIBOR from a simulated forward curve is illustrated in the Yield Curve spreadsheet. Discounting factors to be taken from the OIS spot curve.

[1] Market practice is to use the mean (median) the positive side of distribution only (positive exposure).

Interest Rate Volatility and Derivatives

Summary

This advanced topic is for delegates <u>*with experience*</u> *in interest rates who would like a challenge. The topic does require access to caplet and/or swaption data, an alternative will be to make up the data by simulating caplet prices from a calibrated HJM Model and converting them to implied volatility terms using the Black formula.*

Discretised LMM SDE requires bootstrapped 3M caplet volatility data on input, to which volatility functions are fitted.

HJM model is a start to understand Forward Rates, including LIBOR as a simple annualised rate. HJM calibration requires fitting volatility from the historic data of interest rate changes, while the LMM is calibrated to the market data of cap prices (implied volatility). Both models rest on the no-arbitrage argument: the drift cannot be random and depends on volatility of forward rates.

Once the SDE is calibrated, simulations can be done for the purposes of pricing and risk evaluation. The outcomes are **a.** sensitivity analysis of caplet pricing *wrt* the market risks (ie, bumping the curve) and **b.** discussion of interest rate swaps and pricing of vanilla swaptions.

Above is classic LMM implementation. The market-driven alternative, formalised as Multi-curve LMM, starts with modelling the joint evolution of Forward OIS rates (available from BOE) and related LIBOR-OIS spreads for a given tenor. The twist of implementation is that the OIS rates and basis spreads as viewed factor and no market data of *volatilities* for OIS rates or bases is needed. Modelled explicitly, Forward LIBOROIS bases are guaranteed to be positive.

Data Sources

- Caplet or swaption data is usually maintained by trading desks and interdealer brokers. Data for certain markets is available from Thomson Reuters and Bloomberg. The simple input, a strip of market cap volatility prices (1M, 2M, 3M, 6M, 9M, 12M, 2Y, 3Y, etc.) can be taken from a textbook/research paper.

- LMM can also be calibrated to swaption volatility data (Option Maturity 1Y, 2Y, 3Y etc. vs. Swap Length 1Y, 2Y, 3Y, etc. that begins after maturity) – all at-the-money swaption volatilities (strike is equal to the rate on Forward Swap). That calibration is achieved via optimisation and called the Rebonato Method.

- For each tenor, you will need a discount factor and should make a decision on whether to use dual-curve discounting where DF is coming from the OIS curve. You will also need Forward LIBOR rates given deterministically by the forward curve *today.*

- Below are the links for Pound Sterling Bank Liability Curve (BLC) from LIBOR-linked instruments, OIS spot rates, Euro area curve (Government Liability) and FRB H.15 release

(Treasuries and Interest Rate Swaps, each instrument gives its own spot curve):

http://www.bankofengland.co.uk/statistics/Pages/yieldcurve/archive.aspx

http://www.bankofengland.co.uk/statistics/Documents/yieldcurve/ukois09_mdaily.xls

http://www.ecb.europa.eu/stats/money/yc/html/index.en.html

http://www.federalreserve.gov/releases/h15/data.htm

Step-by-Step Instructions

Part I: Data

1. You will need market price data – cap price and discount factor (two columns). Caps are quoted in terms of implied volatility $\sigma^{cap}(t, T)$ with $T = 1Y, 2Y, 3Y, \ldots$.
   (a) For pre-simulated caplet data (ie, from the HJM model) the Black formula is conventional means of converting the caplet's cashflow to the implied volatility number.
2. The second set of data to which model fitting can be done is swaptions, for which the deliverable asset is an interest rate swap.

Part II: Calibration (Volatility Stripping)

3. Strip 3M caplet volatilities $\sigma^{cap}(T_i, T_{i+3M})$ or market traded caps traded in one-year increment using simplifying assumptions, e.g., flat volatility and pre-interpolated volatilities $\sigma^{cap}(t, T + 3M)$ using definition of a cap:

   $$\text{cap}(t, 1Y) = \text{caplet}(t, T_{3M}, T_{6M}) + \text{caplet}(t, T_{6M}, T_{9M}) + \text{caplet}(t, T_{9M}, T_{12M}).$$

   To use Black formula you will need from *today's* curve, forward LIBOR f_i and strike K
   (a) The first step is to determine a strike for each caplet as a forward swap rate $S(t; T_{i-3M}, T_i)$.
   (b) The second step is to strip caplet volatilities.
4. Alternatively, volatilities can be calibrated from vanilla swaptions (European options on forward-starting swaps) where Rebonato method makes the Black formula suitable.
5. Fitting the *abcd* instantaneous volatility function is defined for each tenor as $\sigma_i(t)$. Coefficients *abcd* are estimated by optimisation that can be joint *wrt* caplet implied volatilities σ_i^{cap} and swaption implied volatilities υ_i. Goal is to minimise the squared differences between two implied volatilities (for the same tenor enumerated i).
   (a) Use a parametric function for correlation structure ρ_{ij}.

Part II: Pricing and Sensitivity Analysis

Pricing of swaptions has already been done in the process of calibration (stripping) of caplet volatilities because forward swap rates $S(t; T_{i-3M}, T_i)$ were calculated. Pricing of path-dependent options, such as Bermudans that give exercise flexibility on some or all payment dates $T_i, T_{i+1}, ..., T_m$, would require the modified Least Squares Monte-Carlo simulation. Make sure that calibrated LMM model returns cap prices similar to the input data. Sensitivity Analysis means modifying the input forward curve (today) and/or discounting factors to evaluate the impact on derivatives pricing.

Another question is to explore how the LIBOR Market Model has to be modified with the introduction of OIS discounting. The change of numeraire to the discounting factor given by the OIS curve leads to an adjustment to the drift term of the LMM SDE.

Fair Spread for Credit Portfolio

Summary

Price a fair spread, for a Basket CDS with at least 5 reference names, as an expectation over the joint distribution of default times. The distribution is unknown analytically and so the copula approach is applied. Sampling from a copula algorithm provides simulated co-dependent uniform variables, which are converted to default times using a term structure of hazard rates (separately for each name). Copula is calibrated by estimating the appropriate default correlation, preferably from the historical CDS data but other sources acceptable. Initial results are histograms (uniformity checks) and scatter plots (co-dependence checks). Substantial result is sensitivity analysis that involves repricing.

A successful project will implement sampling from both, Gaussian and t copulae, pricing all k-th to default instruments (1st to 5th) as well as elaborate sensitivity analysis. Key numerical aspect is the use of low discrepancy sequences (e.g., Halton, Sobol).

Data Requirements

Two **separate** datasets required, together with matching discounting curve data for each.

1. **A snapshot of credit curves** (on a particular day). Each debt issuer has a USD/EUR CDS curve – from which a term structure of hazard rates is bootstrapped and utilised to obtain exact default times, $u_i \rightarrow \tau_i$. Values can be visually stripped from financial media with reasonable assumptions, for instance, if you have 5Y CDS single value assume monotonically increasing values fro $1Y, 2Y, \ldots, 4Y$ for a concave down curve (positive slope).

2. **Historical credit spreads data** usually taken at the most liquid tenor $5Y$ for each of five reference names. The five columns of that sample data give the calculation input for 5×5 default correlation matrix. <u>Default correlation can be obtained from different sources</u>, in particular correlating equity returns is useful in the absence of historical credit spreads.

   Corporate credit spreads can be obtained from Bloomberg or Reuters terminals (via your firm or a colleague) – they are unlikely to be in open access. For sovereign credit spreads, time series of ready bootstrapped PD_{5Y} can be downloaded from DB Research: click on Excel icon to download file, *rename extension to .xls*, and check rows for missing values.
   `http://www.dbresearch.com/servlet/reweb2.ReWEB?rwnode=DBR_INTERNET_EN-PROD\$EM\&rwobj=CDS.calias\&rwsite=DBR_INTERNET_EN-PROD`

   Even if sovereign CDS_{5Y} and PD_{5Y} series are available with daily frequency, co-movement in daily changes might reflect market noise more than correlation of default events that are relatively rare to observe. Weekly changes give more appropriate input for estimation of such fundamental quantity as default correlation however that would entail using 2-3 years of historical data given that we need at least 100 data points to estimate correlation.

Step-by-Step Instructions

1. For each reference name, bootstrap implied default probabilities from quoted CDS and convert them to a term structure of hazard rates, $\tau \sim Exp(\hat{\lambda}_{1Y}, \ldots, \hat{\lambda}_{5Y})$.

2. Estimate default correlation matrix (different for Gaussian and t) and d.f. parameter (ie, calibrate copulæ). You will need to implement pricing by Gaussian and t copulæseparately.

3. Using sampling form copula algorithm, repeat the following routine (simulation):

   (a) Generate a vector of correlated uniform random variable.

   (b) For each reference name, use its term structure of hazard rates to calculate exact time of default (or use semi-annual accrual).

   (c) Calculate the discounted values of premium and default legs for every instrument from *1st* to *5th*-to-default. Conduct MC separately or use one big simulated dataset.

4. Average premium and default legs across simulations separately. Calculate the fair spread.

Model Validation

- The fair spread for k*th*-to-default Basket CDS should be less than k-1 to default. Why?

- Project Report on this topic should have a section on **Risk and Sensitivity Analysis** of the fair spread *w.r.t.*

  1. default correlation among reference names: either stress-test by constant high/low correlation or $\pm$ percentage change in correlation from the actual estimated levels.
  2. credit quality of each individual name (change in credit spread, credit delta) as well as recovery rate.

  Make sure you discuss and compare sensitivities for all five instruments.

- Ensure that you explain historical sampling of default correlation matrix and copula fit (ie, uniformity of pseudo-samples) – that is, Correlations Experiment and Distribution Fitting Experiment (will be described at the Module Five Workshop). Use histograms.

  Convergence and robustness of spread pricing depend on correct copula fitting.

Introduction to Copula

MathWorks have tutorials on copula fitting and applications – I can recommend one from their older CD (dated 2005). Please search for *Market Risk using GARCH, EVT and Copula.*

If access to historical credit spreads poses a problem remember, default correlation matrix can be estimated from historic equity returns or debt yields.

Portfolio Construction

Summary

Construct a portfolio of diverse assets using sufficient historic time series and estimate allocations for multiple levels of risk aversion together with the key risk measures (VaR, Expected Shortfall) and satisfaction indices (Sharpe Ratio). Utilise the key point of the Black-Litterman model, which is an introduction of analysts' recommendations into portfolio construction. You will have to identify sensible recommendations and formalise them as BL model views.

The effective advanced implementation will (a) improve on the sample covariance matrix estimator and (b) carefully diagnose and treat the optimisation problem, choosing from mean-variance, mean-TE, or mean-ES kinds of optimisation.[2] It is necessary to study the robustness of allocations (ie, what makes allocations to change) under different assumptions and constraints *ceteris paribus*. The study needs to be detailed and well-illustrated.

A successful project will have matrix form calculations and other numerical techniques coded (rather than spreadsheet calculations), robust optimisation results for different levels of risk aversion, and extensive graphic presentation. Compare your allocations to market benchmark weights and explore diversification. <u>A naive mean-variance optimisation on sample mean and covariance is of little value.</u>

Data Requirements (Portfolio Choice)

The objective is to come up with either **a.** a multi-asset diversified portfolio that can be seen as a global tactical allocation or **b.** specialised portfolio that focuses on an industry, emerging market(s), credit, etc. The first kind of portfolio can have under 8-10 diverse assets and sub-assets, while the specialised portfolio usually has more. If you follow portfolio choice from a reference please provide the source.

- Main asset classes are equity, fixed income, credit and volatility. Commodities, real estate and other assets are optional. ETFs can be used to represent the asset classes for which the data is hard to obtain (e.g., bonds, futures).
- Replication of broad equity indices is convenient.[3] However, the multi-asset approach comes from the need to account for most factors that drive performance – an equity index might have hundreds of names, for which performance is driven by only a few factors.
- You can consider investment into a factor(s) directly. The factor is a time series of returns of a long-short portfolio (eg, value factor, momentum factor, cointegrated combination).

[2]TE stands for Tracking Error and ES stands for Expected Shortfall mathematically known as CVaR.

[3]Because index weights can be converted into equilibrium weights. Without a ready market index/benchmark, the market cap approach can be used to obtain equilibrium weights.

The minimum historical time period is 2-3 years (for daily returns) though you might use the shorter periods of 1-3 months for variance estimation; that would require robust estimation and exclusion of stressed periods. Portfolios that are tradeable strategies themselves might require the higher frequency data (1-10min). A starting source for historical daily close prices of US equities and ETFs is Yahoo!Finance. Mean-variance optimisation is specified for *excess* simple returns (not compound). The technical challenges, such as the risk-free rate changing over time can be dealt using simplified assumptions.

Step-by-Step Instructions

Part I: The Black-Litterman Model

1. Construct the prior (reference distribution): equilibrium returns can come from a benchmark index, while covariance is estimated from historical data.

2. Define input views of both kinds, relative and absolute.

3. Estimate the posterior distribution of excess returns using the Black-Litterman formulae.

Part II: Robust Allocation

4. Choose at least one/two more optimisation designs in addition to variance minimisation. Experiment with sensible constraints in addition to views, for example, 'no short positions in bonds', 'no short positions at all'. Obtain allocations for three levels of risk aversion.

5. Prepare the sample covariance matrix and optionally, the matrix based on original correlations and GARCH-smoothed variances. Use both as input to optimisation and compare.[4]

6. Explore model behaviour (allocations resulting from the varied optimisation/covariance input), compare against the market benchmark weights, and check for common pitfalls such as 'corner solutions'. Formulate and answer questions such as: Are you over or underweight compared to a market? Can you think of the common themes and explanations? Are allocations artefacts of the model choice or make sense in terms of market factors?

A naive mean-variance optimisation on statistical mean and variance of returns is of little value without constraints, views and preferably robustness checks.

[4]Usually, GARCH is applied to forecast next day's volatility, which can be expressed in annualised terms. But, you are unlikely to rebalance a model and reallocate the portfolio every day, therefore, it makes sense to explore estimations over the longer horizons, e.g., estimate covariance using weekly/monthly returns.

Time Series Analysis and Backtesting

Summary

The aim to this topic is an estimation and analysis of tradeable relationships between two or more financial time series. Identifying and backtesting a robust cointegrated relationship means exposing a factor that drives both (or many) asset prices. The factor is traded by entering the long-short position given by cointegrating weights.

Through implementation you will have a hands-on introduction to Vector Autoregression (for returns) and Error Correction (for prices) models, which are the main variations of the multivariate regression. The econometric applications of such a regression are **a.** forecast, **b.** impulse response analysis, and **c.** (Granger) causality analysis. However, none of these applications serves the needs of quantitative analytics/portfolio allocation/hedge fund investing and trading. Instead a range of techniques and considerations applied known as 'backtesting'.

A project that solely runs pre-programmed statistical tests on data is a preparation work, not the complete project. The project should have coding of necessary statistical tests from the first principles (explicit regression equations) by yourself. The least deliverables are **a.** implemented Engle-Granger procedure, **b.** statistical diagnosis and backtesting (split dataset in half or compute rolling estimates), and **c.** market factor backtesting via regressing returns from your strategy on market index returns or another factor. These are in addition to the underlying numerical methods on matrices and vector autoregression.[5]

Backtesting

The following notes offer choices to implement in aspects and questions of backtesting:

- All project designs (whether learning-level or in-depth) should include backtesting of a strategy. The strategy is realised by using cointegrating coefficients $\boldsymbol{\beta}_{Coint}$ as allocations $\boldsymbol{w}$. That creates a long-short portfolio that generates a mean-reverting spread (cointegrated residual).
- Does cumulative P&L behave as expected (for a cointegration trade)? Is P&L coming from a few or lot of trades/time period? What are the SR/Maximum Drawdown? Behaviour of risk measures (volatility/VaR)? Concentration in assets and attribution?
- Impact of transaction costs (plot an average P&L value vs. number of transactions).
- Optionally, introduce liquidity and algorithmic flow considerations (a model of order flow). How would you be entering and accumulating the position? What impact bid-ask spread and transaction costs will make?

[5]It is recommended that you use the environment with facilities for matrix and time series manipulation (R, Matlab) or code in Python/C++ with the use of quant libraries. The use of VBA will be cumbersome.

Step-by-Steps Designs

Design 1: 'Learning' and Cointegration in Pairs

An understanding-level design can use the ready specification tests, but matrix form regression estimation must be re-coded. The project can rely on the Engle-Granger procedure for cointegration testing among pairs.

1. Implement *concise matrix form* estimation for multivariate regression and conduct model specification tests for: (a) identifying optimal lag p and (b) stability check.

2. Optionally, test forecasting capability of regression with IRF and Granger Causality.

3. Implement Engle-Granger procedure and use it to identify a cointegrated pair. Estimate relationships both ways to select the appropriate lead variable.

4. ADF test for unit root must be coded and used.

Design 2: 'In-depth' and Multivariate Cointegration with Trends

The advanced implementation will re-code the essential tests (selection of lag p and unit root) and offer the multivariate estimation of cointegration. Analysis of relationships suitable for trading has to extend to exploring the quality of mean-reversion in cointegrated residual.

1. Implement *concise matrix form* estimation for multivariate regression and conduct your own coded tests for: (a) identifying optimal lag p and (b) stability check.

2. Apply Maximum Likelihood Estimation (Johansen Procedure) for multivariate cointegration on asset price data (in levels, not returns).

3. This is subject to (a) prior testing for unit roots by ADF test and (b) specification of deterministic trends for cointegrating equation. Use simple trends and check for overfitting.

4. Present clear analysis of results for *Maximum Eigenvalue* and *Trace* statistical tests, both are based on Likelihood Ratio test principle.

Before own implementation, consider R/Matlab functionality for Johansen Procedure and especially the tests for the number of cointegrated relationship in a multivariate system.

Strategy Backtest (both designs)

5. Use cointegration analysis to identify candidates for the long-short portfolios as given by cointegrating weights.[6]

6. Backtest a statistical arbitrage trade (or several) by investigating a mean-version in cointegrated residual $\boldsymbol{e_t} = \boldsymbol{\beta'_{Coint} Y_t}$. Use the fitting to the OU process and give the speed of reversion, number of trades as well as other properties of P&L. Rely on backtesting considerations given above.

[6] Within an asset class, a robust cointegration relationship exposes *a factor*.

Resources

Reading List: CVA

- CQF Lecture(s) on *Credit Value Adjustment*. For an alternative implementation example, `http://www.cvacentral.com/books/credit-value-adjustment/spreadsheets/`

Reading List: Rates Volatility

- Review *Methods for Constructing a Yield Curve* by Pat Hagan and Graeme West – best to start with version published in WILMOTT (May-June 2008).
- *The LIBOR Market Model in Practice* specialised textbook by Gatarek, et al. (2006) gives technical detail on cailbration from caplets and swaptions (Chapters 7 and 9 respectively) that will be useful to those working with LIBOR derivatives. (Please email the tutor.)

Reading List: Credit Portfolio

- The starting source on *sampling from copula* algorithm is *Monte Carlo Methods in Finance* textbook by Peter Jaekel (2002). Please see Chapter 5, particularly pages 46-50.
- Most likely, you will need to re-visit *CDO Lecture* material, particularly about slides 48-52 that illustrate Elliptical copula densities and discuss factorisation.
- Rank correlation coefficients are introduced *Correlation Sensitivity Lecture* and Jaekel (2002) as above. Project Q&A document gives the clarified formulae and explanations.
- Bootstrapping of survival probabilities is covered in *Credit Default Swaps Lecture*.

Reading List: Portfolio Construction

- CQF Lecture on *Fundamentals of Optimization and Application to Portfolio Selection*
- *A Step-by-step Guide to The Black-Litterman Model* (Incorporating user-specified confidence levels). Thomas Idzorek, 2002
- *The Black-Litterman Approach: Original Model and Extensions* Attilio Meucci, 2010. `http://ssrn.com/abstract=1117574`

Reading List: Time Series

- CQF Lectures on *Cointegration and Statistical Arbitrage* and *Statistical Methods for PD.*
- *Explaining Cointegration Analysis: Parts I and II*, by David Hendry and Katarina Juselius, *Energy Journal* 2000 and 2001.
- *Learning and Trusting Cointegration in Statistical Arbitrage* by Richard Diamond, *WILMOTT Magazine*, November 2013.
- *User Guide for Johansens Method* by Kyungho Jang and Masao Ogaki, 2001.