CDS
JP Morgan formulation

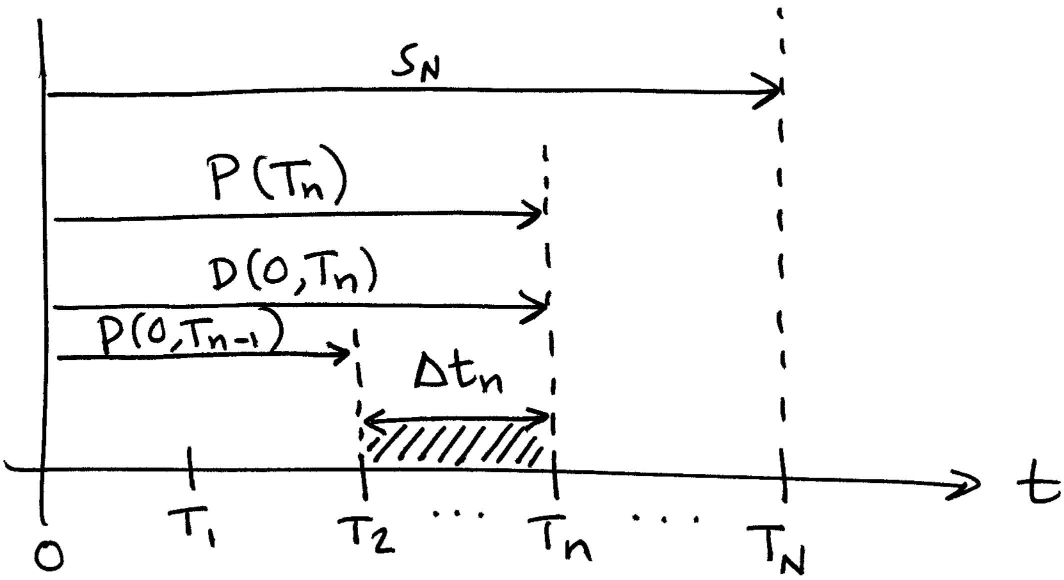
PREMIUM LEG

$$PL_N = S_N \sum_{n=1}^{N} D(0,T_n)\, P(T_n)\, \Delta t_n$$

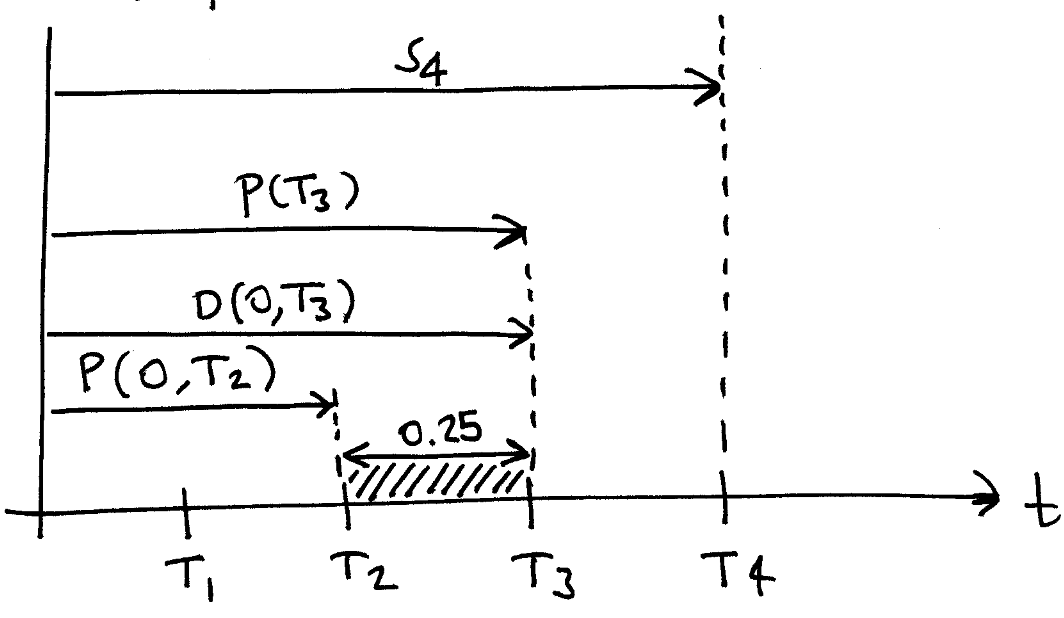
DEFAULT LEG

$$DL_N = (1-R) \sum_{n=1}^{N} D(0,T_n) \Big(P(T_{n-1}) - P(T_n) \Big)$$

based on time-grid:



for a quarterly CDS with maturity 1 year:
$\rightarrow N = 4$



CDS Bootstrapping

We assume that we have a vector of CDS market spreads for increasing maturities $[S_1, S_2, \dots, S_N]$. We now determine their associated survival probabilities $[P(T_1), P(T_2), \dots, P(T_N)]$.

N = 1

$$PL_N = S_N \sum_{n=1}^{N} \left(D(0,T_n) P(T_n) \Delta t_n \right)$$

$$PL_1 = S_1 \left(D(0,T_1) P(T_1) \Delta t_1 \right)$$

$$DL_N = (1-R) \sum_{n=1}^{N} \left(D(0,T_n) \left(P(T_{n-1}) - P(T_n) \right) \right)$$

$$DL_1 = (1-R)\, D(0,T_1) \left(P(T_0) - P(T_1) \right)$$

$$PL_1 = DL_1$$

$$S_1 D(0,T_1) P(T_1) \Delta t_1 = \underbrace{(1-R)}_{L} D(0,T_1) \left[P(T_0) - P(T_1) \right]$$

$$S_1 D(0,T_1) P(T_1) \Delta t_1 = L\, D(0,T_1) P(T_0) - L\, D(0,T_1) P(T_1)$$

$$S_1 D(0,T_1) P(T_1) \Delta t_1 + L D(0,T_1) P(T_1) = L D(0,T_1) P(T_0)$$

$$P(T_1)\left[S_1 D(0,T_1) \Delta t_1 + L D(0,T_1)\right] = L D(0,T_1) P(T_0)$$

$$P(T_1) \cancel{D(0,T_1)} \left[S_1 \Delta t_1 + L\right] = L \cancel{D(0,T_1)} P(T_0)$$

with $P(T_0) = 1$

$$P(T_1) = \frac{L}{S_1 \Delta t_1 + L}$$

$\underline{\underline{N = 2}}$

$$PL_N = S_N \sum_{n=1}^{N} \left(D(0,T_n) P(T_n) \Delta t_n \right)$$

$$PL_2 = S_2 \left[D(0,T_1) P(T_1) \Delta t_1 + D(0,T_2) P(T_2) \Delta t_2 \right]$$

$$DL_N = (1-R) \sum_{n=1}^{N} D(0,T_n) \left(P(T_{n-1}) - P(T_n) \right)$$

$$DL_2 = (1-R) \left[D(0,T_1) \left(P(T_0) - P(T_1) \right) + D(0,T_2) \left(P(T_1) - P(T_2) \right) \right]$$

$$PL_2 = DL_2$$

$$S_2\left[D(0,T_1)P(T_1)\Delta t_1 + D(0,T_2)P(T_2)\Delta t_2\right] = \underbrace{(1-R)}_{L}\left[D(0,T_1)\left(\overset{1}{\cancel{P(T_0)}} - P(T_1)\right) + D(0,T_2)\left(P(T_1)-P(T_2)\right)\right]$$

$$S_2 D(0,T_1)P(T_1)\Delta t_1 + S_2 D(0,T_2)P(T_2)\Delta t_2 = L\,D(0,T_1)\left(1-P(T_1)\right) + L\,D(0,T_2)\times\left(P(T_1)-P(T_2)\right)$$

$$S_2 D(0,T_1)P(T_1)\Delta t_1 + S_2 D(0,T_2)\underline{P(T_2)}\Delta t_2 = L\,D(0,T_1) - L\,D(0,T_1)P(T_1) + L\,D(0,T_2)P(T_1) - L\,D(0,T_2)\underline{P(T_2)}$$

$$S_2 D(0,T_2)P(T_2)\Delta t_2 + L\,D(0,T_2)P(T_2) = L\,D(0,T_1) - L\,D(0,T_1)P(T_1) + L\,D(0,T_2)P(T_1) - S_2 D(0,T_1)P(T_1)\Delta t_1$$

$$P(T_2)\left[S_2 D(0,T_2)\Delta t_2 + L\,D(0,T_2)\right] = \ldots$$

$$P(T_2)\left[D(0,T_2)\left(S_2\Delta t_2 + L\right)\right] = D(0,T_1)\left(L - L\,P(T_1) - S_2 P(T_1)\Delta t_1\right) + D(0,T_2)\,L\,P(T_1)$$

$$P(T_2)\left[D(0,T_2)\left(S_2\Delta t_2 + L\right)\right] = D(0,T_1)\left[L - P(T_1)\left(L + S_2\Delta t_1\right)\right] + D(0,T_2)\,L\,P(T_1)$$

$$P(T_2) = \frac{D(0,T_1)\left[L - P(T_1)\left(L + S_2\Delta t_1\right)\right]}{D(0,T_2)\left(S_2\Delta t_2 + L\right)} + \frac{\cancel{D(0,T_2)}\,L\,P(T_1)}{\cancel{D(0,T_2)}\left(S_2\Delta t_2 + L\right)}$$

$$P(T_2) = \frac{D(0,T_1)\left[L - P(T_1)\left(L + S_2\Delta t_1\right)\right]}{D(0,T_2)\left(L + S_2\Delta t_2\right)} + \frac{P(T_1)\,L}{L + S_2\Delta t_2}$$

__N = 3__

$$P(T_N) = \frac{\sum_{n=1}^{N-1} D(0,T_n)\left[L P(T_{n-1}) - (L+\Delta t_n S_N) P(T_n)\right]}{D(0,T_N)(L+\Delta t_n S_N)} + \frac{P(T_{N-1})\, L}{(L+\Delta t_N S_N)}$$

$$P(T_3) = \frac{\sum_{n=1}^{2} D(0,T_n)\left[L P(T_{n-1}) - (L+\Delta t_n S_3) P(T_n)\right]}{D(0,T_3)(L+\Delta t_n S_3)} + \frac{P(T_2)\, L}{(L+\Delta t_3 S_3)}$$

Note: The bootstrapping formulas above are implemented in the XLS file:
ImprovedBootstrappingExample.xls

PROBABILITY OF DEFAULT

Recovery Rate	50%

$$\underline{\underline{N=1}}$$

$$P(T_1) = \frac{L}{s_1 \Delta t_1 + L}$$

TIME (Years)	dt	MARKET SPREAD	DF	IMPLIED SURVIVAL PROB	first term	second term	third term	fourth term	sum	quotient	first term	last term
0				100.00%								
1	1	29.00	0.9803	99.42%								
2	1	39.00	0.9514	98.45%	- 0.0010				- 0.0010	0.4794	- 0.0020	0.9865
3	1	46.00	0.9159	97.26%	- 0.0017	0.0003			- 0.0013	0.4622	- 0.0029	0.9755
4	1	52.00	0.8756	95.88%	- 0.0022	- 0.0002	0.0008		- 0.0017	0.4424	- 0.0038	0.9626
5	1	57.00	0.8328	94.37%	- 0.0027	- 0.0007	0.0004	0.0013	- 0.0018	0.4211	- 0.0043	0.9480

$$\underline{\underline{N=5}}$$

$$P(T_5) = \frac{\sum_{n=1}^{4} \left(\blacksquare + \blacksquare + \blacksquare + \blacksquare \right)}{\blacksquare} + \frac{P(T_4)\, L}{L + \Delta t_5 S_5}$$