A Toy Example for EM Algorithm $x \in \{0,1\}$ $P_{x}(0) \triangleq 0$, $P_{x}(1) \triangleq 0$, y ∈ {0.13 Prix (ili) = 0.19 i∈ {0.13 1. Single observation | y=1 x hidden aval: find 0 = (00, 0, 0010, 0011, 0110, 011) s.t. Py (y; 0) is maximized Solution: EM algo. Initialization: $\theta_0 = \theta_1^{(0)} = \frac{1}{2}$ $\theta_0|_{2} = 0.9 = \theta_1|_{1}$ $\theta_1|_{2} = 0.1 = \theta_0|_{1}$ These numbers are just mode up for the try example, can be compthing. $E-step: q_{(1)}^{(1)}(\cdot|\mathbf{p}) = P_{x|Y}(x|y)$ $P_{x(0)} = P_{x(0)}(0) = \frac{P_{x,y(0,1)}}{P_{y(0)}} = \frac{P_{x(0)} P_{y(10)}}{P_{x(0)} P_{y(10)} + P_{x(1)} P_{y(1)}}$ using $\theta = 0.1 \times \frac{1}{2}$ $0.1 \times \frac{1}{2}$ $0.1 \times \frac{1}{2} + 0.1 \times \frac{1}{2}$ similarly q(1) = 0.9 $M-step: \theta''= argmax E_{q'', 1} [log P_{x,y}(x, y; 0)]$ $(*) = \sum_{x=0}^{l} q^{(i)}(x) \log P_{x,y}(x, 1; \theta)$ = 9("(0) [log 80 + log 0,10] + 9("(1) [log 0, + log 0,1,] (**) want to maximize (**) subject to: Oot O1 = 1. Oo, O, O10, O11, & [0,1]. $\Rightarrow G_{0}^{(1)} = 0.1 \quad O_{0}^{(1)} = 0.9 \quad O_{0}^{(1)} = 1 \quad O_{0}^{(1)} = 0$

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use Lagrangian unitipliers. $O_{011} = 0$ $O_{111} = 1$

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2. Multiple (IID) Observations [y"=1, y"=0]
      \log P_{y}(y'', y''); \theta) = \log \sum_{x''} \sum_{x''} P_{x,y}(x'', x'', y''), y''; \theta)
          > Eq(x:)x:/y:/y:/y", [ log Px, T (x", x", y", y", y"); 0,7
E-step q(x_{\bullet}^{(i)}, x_{\bullet}^{(i)}, y^{(i)}) = P(x^{(i)}, x^{(i)} | y^{(i)}, y^{(i)})
                                                                due to
indepense both

(x'', y'')

2(x'', y'')
               = P(x" | y").P(x(2) | y(2))
      \Rightarrow 9,(0,0|1,0) = 910+10 0.1 × 0.9 = 0.09
                                                                  Computation of
           q(0, \phi(1), 0) = 0.1 \times 0.1 = 0.01
                                                                P(X (1) | Y (1)):
            9(1,0|1,0) = 0.9 \times 09 = 0.81
                                                                  See 1°
             9(1, 1| 1, 0) = 0.9 \times 0.1 = 0.09
   M-step: Equ., 1 you, you, I log Px, (x", x", y", y"); 0)
           = \sum_{x''=0}^{n} \sum_{x'''=0}^{n} q'''(x''', x''') \cdot \log P_{x,y}(x'', x''', 1, 0), 0)
           = 0.09 ( log 00+ log 00 + log 0010 + log 0010)
                                                                         0,1
             +0.01 ( lug 00 + log 0, + lug 0, 10 + lug 0011)
              +0.81 (log 0, + log 00 + log 0,1, + log 0012)
                                                                         1,0
              +0.09 ( log 0, + log 0p + log 0,1, + log 001,)
                                                                        ι, ι
          = light + light, + 0.9 light 0010 + 0.1 light 0.10 + 0.1 light 0011 + 0.9 light 11
          Subject to Oot 0, =1 Oolo + O110=1 Ool, + O11, =1
    Use Lagrangian multiplier \Rightarrow 00^{(1)} = \frac{1}{2} = 0^{(1)}
             O_{0|0} = O_{0|0} = O_{0|1} = O_{0|1} = O_{0|1} = O_{0|1}
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