$$
\begin{aligned}
\mathcal{L} \;=\;& \sum_a \sum_{b\in sink(a)} \sum_k \mathbb{E}_q \Bigg[z_{a\to b,k} z_{a\leftarrow b,k} log\, \beta_{k,0} + (1 - z_{a\to b,k} z_{a\leftarrow b,k}) log\, \epsilon \Bigg] \\
+& \sum_a \sum_{b \notin sink(a)} \sum_k \mathbb{E}_q \Bigg[z_{a\to b,k} z_{a\leftarrow b,k} log\, \beta_{k,1} + (1 - z_{a\to b,k} z_{a\leftarrow b,k}) log\, (1-\epsilon) \Bigg] \\
+& \sum_a \sum_{b\in sink(a)} \sum_k \mathbb{E}_q \Bigg[z_{a\to b,k} log\, \theta_{a,k} + z_{a\leftarrow b,k} log\, \theta_{b,k} \Bigg] \\
+& \sum_a \sum_{b\notin sink(a)} \sum_k \mathbb{E}_q \Bigg[z_{a\to b,k} log\, \theta_{a,k} + z_{a\leftarrow b,k} log\, \theta_{b,k} \Bigg] \\
-& \sum_a \sum_{b\in sink(a)} \sum_k \mathbb{E}_q \Bigg[z_{a\to b,k}\, log\, \phi_{a\to b,k} + z_{a\leftarrow b,k}\, log\, \phi_{a\leftarrow b,k} \Bigg] \\
-& \sum_a \sum_{b \notin sink(a)} \sum_k \mathbb{E}_q \Bigg[z_{a\to b,k} log\, \phi_{a\to b,k} + z_{a\leftarrow b,k} log\, \phi_{a\leftarrow b,k} \Bigg] \\
+& \sum_a log\, \Gamma(\sum_k \alpha_k) - \sum_a \sum_k log\, \Gamma(\alpha_k) + \sum_a \sum_k (\alpha_k - 1) \mathbb{E}_q \Bigg[log\, \theta_{a,k} \Bigg] \\
-& \sum_a log\, \Gamma(\sum_k \gamma_{a,k}) + \sum_a \sum_k log\, \Gamma(\gamma_{a,k}) - \sum_a \sum_k (\gamma_{a,k} - 1) \mathbb{E}_q \Bigg[log\, \theta_{a,k} \Bigg] \\
+& \sum_k log\, \Gamma(\eta_0 + \eta_1) - \sum_k log\, \Gamma(\eta_0) - \sum_k log\, \Gamma(\eta_1) + \sum_k (\eta_0 - 1) \mathbb{E}_q \Bigg[log\, \beta_{k,0} \Bigg] \\
+& \sum_k (\eta_1 - 1) \mathbb{E}_q \Bigg[log\, \beta_{k,1} \Bigg] \\
-& \sum_k log\, \Gamma(\tau_{k,0} + \tau_{k,1}) + \sum_k log\, \Gamma(\tau_{k,0}) + \sum_k log\, \Gamma(\tau_{k,1}) - \sum_k (\tau_{k,0} - 1) \mathbb{E}_q \Bigg[log\, \beta_{k,0} \Bigg] \\
-& \sum_k (\tau_{k,1} - 1) \mathbb{E}_q \Bigg[log\, \beta_{k,1} \Bigg] \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L} &= \sum_a \sum_{b \in sink(a)} \sum_k \Bigg(\phi_{a\rightarrow b,k}\phi_{a\leftarrow b,k}\Big(\Psi(\tau_{k0}) - \Psi(\tau_{k0}+\tau_{k1})\Big) + (1-\phi_{a\rightarrow b,k}\phi_{a\leftarrow b,k}) log\,\epsilon \Bigg) \\
&+ \sum_a \sum_{b \notin sink(a)} \sum_k \Bigg(\phi_{a\rightarrow b,k}\phi_{a\leftarrow b,k}\Big(\Psi(\tau_{k1}) - \Psi(\tau_{k0}+\tau_{k1})\Big) + (1-\phi_{a\rightarrow b,k}\phi_{a\leftarrow b,k}) log\,(1-\epsilon) \Bigg) \\
&+ \sum_a \sum_{b \in sink(a)} \sum_k \Bigg(\phi_{a\rightarrow b,k}\Big(\Psi(\gamma_{a,k}) - \Psi(\sum_k \gamma_{a,})\Big) + \phi_{a\leftarrow b,k}\Big(\Psi(\gamma_{b,k}) - \Psi(\sum_k \gamma_{b,})\Big) \Bigg) \\
&+ \sum_a \sum_{b \notin sink(a)} \sum_k \Bigg(\phi_{a\rightarrow b,k}\Big(\Psi(\gamma_{a,k}) - \Psi(\sum_k \gamma_{a,})\Big) + \phi_{a\leftarrow b,k}\Big(\Psi(\gamma_{b,k}) - \Psi(\sum_k \gamma_{b,})\Big) \Bigg) \\
&- \sum_a \sum_{b \in sink(a)} \sum_k \Bigg(\phi_{a\rightarrow b,k}\, log\, \phi_{a\rightarrow b,k} + \phi_{a\leftarrow b,k}\, log\, \phi_{a\leftarrow b,k} \Bigg) \\
&- \sum_a \sum_{b \notin sink(a)} \sum_k \Bigg(\phi_{a\rightarrow b,k}\, log\, \phi_{a\rightarrow b,k} + \phi_{a\leftarrow b,k}\, log\, \phi_{a\leftarrow b,k} \Bigg) \\
&+ \sum_a log\,\Gamma(\sum_k \alpha_k) - \sum_a \sum_k log\,\Gamma(\alpha_k) + \sum_a \sum_k (\alpha_k - 1)\Big(\Psi(\gamma_{a,k}) - \Psi(\sum_k \gamma_{a,})\Big) \\
&- \sum_a log\,\Gamma(\sum_k \gamma_{a,k}) + \sum_a \sum_k log\,\Gamma(\gamma_{a,k}) - \sum_a \sum_k (\gamma_{a,k} - 1)\Big(\Psi(\gamma_{a,k}) - \Psi(\sum_k \gamma_{a,})\Big) \\
&+ \sum_k log\,\Gamma(\eta_0+\eta_1) - \sum_k log\,\Gamma(\eta_0) - \sum_k log\,\Gamma(\eta_1) + \sum_k (\eta_0 - 1)\Big(\Psi(\tau_{k0}) - \Psi(\tau_{k0}+\tau_{k1})\Big) \\
&+ \sum_k (\eta_1 - 1)\Big(\Psi(\tau_{k1}) - \Psi(\tau_{k0}+\tau_{k1})\Big) \\
&- \sum_k log\,\Gamma(\tau_{k,0}+\tau_{k,1}) + \sum_k log\,\Gamma(\tau_{k,0}) + \sum_k log\,\Gamma(\tau_{k,1}) - \sum_k (\tau_{k,0} - 1)\Big(\Psi(\tau_{k0}) - \Psi(\tau_{k0}+\tau_{k1})\Big) \\
&- \sum_k (\tau_{k,1} - 1)\Big(\Psi(\tau_{k1}) - \Psi(\tau_{k0}+\tau_{k1})\Big) \qquad (2)
\end{aligned}
$$

We can translate these two lines:

$$
\begin{aligned}
&\sum_a \sum_{b \in sink(a)} \sum_k \Bigg(\phi_{a\rightarrow b,k}\Big(\Psi(\gamma_{a,k}) - \Psi(\sum_k \gamma_{a,})\Big) + \phi_{a\leftarrow b,k}\Big(\Psi(\gamma_{b,k}) - \Psi(\sum_k \gamma_{b,})\Big) \Bigg) \\
+ &\sum_a \sum_{b \notin sink(a)} \sum_k \Bigg(\phi_{a\rightarrow b,k}\Big(\Psi(\gamma_{a,k}) - \Psi(\sum_k \gamma_{a,})\Big) + \phi_{a\leftarrow b,k}\Big(\Psi(\gamma_{b,k}) - \Psi(\sum_k \gamma_{b,})\Big) \Bigg)
\end{aligned}
$$

to

$$\sum_{a} \sum_{\substack{b \in sink(a) \\ b \neq a}} \sum_{k} \Bigg(\phi_{a \to b,k} \Big(\Psi(\gamma_{a,k}) - \Psi(\sum_{k} \gamma_{a,}) \Bigg) + \sum_{a} \sum_{\substack{b \in sink(a) \\ b \neq a}} \sum_{k} \Bigg(\phi_{a \leftarrow b,k} \Big(\Psi(\gamma_{a,k}) - \Psi(\sum_{k}$$

$$+ \sum_{a} \sum_{\substack{b \notin sink(a) \\ b \neq a}} \sum_{k} \Bigg(\phi_{a \to b,k} \Big(\Psi(\gamma_{a,k}) - \Psi(\sum_{k} \gamma_{a,}) \Big) \Bigg) + \sum_{a} \sum_{\substack{b \notin sink(a) \\ b \neq a}} \sum_{k} \Bigg(\phi_{a \leftarrow b,k} \Big(\Psi(\gamma_{a,k}) - \Psi($$

so $\mathcal{L}_{\gamma_{a,k}}$:

$$\begin{aligned}
\mathcal{L}_{\gamma_{a,k}} &= \Big(\Psi(\gamma_{a,k}) - \Psi(\sum_{k} \gamma_{a,}) \Big) \Bigg(\sum_{b \neq a} \phi_{a \to b,k} + \sum_{b \neq a} \phi_{a \leftarrow b} + \alpha_k - \gamma_{a,k} \Bigg) \\
&- \; log\, \Gamma(\sum_{k} \gamma_{a,k}) + log\, \Gamma(\gamma_{a,k})
\end{aligned}$$

Then $\dfrac{\partial \mathcal{L}_{\gamma_{a,k}}}{\partial \gamma_{a,k}} = 0$ yields:

$$\gamma_{a,k} = \alpha_k + \sum_{b \neq a} \phi_{a \to b,k} + \sum_{b \neq a} \phi_{a \leftarrow b}$$