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Invited Paper

Learning Models: An Assessment of Progress, Challenges, and New Developments

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Learning models extend the traditional discrete choice framework by postulating that consumers have incomplete information about product attributes and that they learn about these attributes over time. In this survey we describe the literature on learning models that has developed over the past 20 years, using the model of Erdem and Keane as a unifying framework [Erdem T, Keane M (1996) Decision-making under uncertainty: Capturing dynamic brand choice processes in turbulent consumer goods markets. *Marketing Sci.* 15(1):1–20]. We describe how subsequent work has extended their modeling framework and applied learning models to a wide range of different products and markets. We argue that learning models have contributed greatly to our understanding of consumer behavior—in particular, in enhancing our understanding of brand loyalty and long-run advertising effects. We also discuss the limitations of existing learning models and potential extensions. One key challenge is to disentangle learning as a source of dynamics from other key mechanisms that may generate choice dynamics (inventories, habit persistence, etc.). Another is to enhance identification of learning models by collecting and using direct measures of signals, perceptions, and expectations.

Key words: learning models; choice modeling; dynamic programming; structural models; brand equity
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1. Introduction

In the field of discrete choice, the most widely used models are the multinomial logit and probit.[1] Of course, there has been substantial effort over the past 20 years to generalize these workhorse models to allow richer structures of consumer taste heterogeneity, serial correlation in preferences, dynamics, endogenous regressors, etc. However, with few exceptions, work within the traditional random utility framework maintains the strong assumption that *consumers know the attributes of their choice options perfectly*.

Learning models extend the traditional discrete choice framework by postulating that consumers may have incomplete information about product attributes. Thus, they make choices based on *perceived* attributes. Over time, consumers receive information signals that enable them to learn more about products. It is this inherent temporal aspect of learning models that distinguishes them from static choice under uncertainty models.

Within this general framework, different types of learning models can be distinguished along *four* key dimensions. One is whether consumers behave in a forward-looking manner. If attributes are uncertain and consumers are myopic, they choose the alternative with the highest expected current utility. But forward-looking consumers may (i) make trial purchases to enhance their information sets or (ii) actively search for information about products using a variety of other sources of information.

A second key distinction is whether utility is linear in attributes or whether consumers exhibit risk

[1] Until recently, the logit was much more popular than probit, largely because of its computational advantages. But advances in simulation methods, such as the GHK algorithm and Gibbs sampling (see Geweke and Keane 2001, McCulloch et al. 2000), have greatly increased the popularity of probit, particularly among Bayesians.

aversion. In the linear case, forward-looking consumers are willing to pay a premium for *un*familiar products, as they receive not only the expected utility of consumption but also the value of the information acquired by trial.[2] But with risk aversion, consumers are willing to pay a premium for a more familiar product. This can generate brand equity for well-known brands.

A third distinction involves sources of information. In the simplest learning models, trial is the only information source. In more sophisticated models, consumers can learn from a range of sources, such as advertising, word of mouth, price signals, salespeople, product ratings, social networks, and newspapers. A consumer must decide how much to use each available source. In particular, consumers may engage in *passive search*, using only information sources that arrive exogenously, or *active search*, exerting effort to gather information. They may also do both.

A fourth distinction is how consumers learn. They may be Bayesians, or they may update perceptions in some other way. For instance, consumers may over- or underweight new information relative to an optimal Bayesian rule, or they may forget information that was received too far in the past.

Learning models were first applied to marketing problems in pioneering work by Roberts and Urban (1988) and Eckstein et al. (1988).[3] Because of technical limitations of the time (both in computer speed and estimation algorithms), however, their models had to be quite simple. Roberts and Urban (1988) study how Bayesian consumers learn about a new product from word-of-mouth signals. Consumers in their model are risk averse but myopic, so there is no active search. In contrast, in Eckstein et al. (1988), consumers are forward-looking, and trial purchase is the (only) source of information. Yet utility is linear, so their model exhibits the "value of information" phenomenon but not the "brand equity" phenomenon created by risk aversion. For Roberts and Urban (1988), the converse is true. The strong simplifying assumptions of these early models, plus the difficulty of estimating even simple learning models 20 year ago, most likely explains why no further learning models appeared in the literature for several years after 1988.

The paper by Erdem and Keane (1996) represents a significant methodological advance because it greatly expanded the class of learning models that are feasible to estimate.[4] Their approach could handle forward-looking consumers, risk aversion, multiple information sources, and both active and passive search in one model. Thus, their model exhibits *both* the value of information and brand loyalty phenomena. In their empirical application, consumers were uncertain about the quality of brands and learned through use experience (active learning) *and* exogenously arriving advertising signals (passive learning).

Erdem and Keane (1996) assume that consumers processed quality signals as Bayesians. This assumption imposes a very special *structure* on how past choice history affects current choice probabilities.[5] A striking result in their paper is that this structurally motivated functional form for choice probabilities actually fits the data better than commonly used reduced-form specifications, such as Guadagni and Little's (1983) exponential weighted average of past purchases specification (the so-called loyalty variable).[6] Erdem and Keane (1996) also find strong evidence that advertising has important long-run effects on demand (via the total stock of advertising) but that short-run effects of recent advertising are negligible.

The Erdem and Keane (1996) paper is influential because (1) it provides a practical method for estimating complex learning models; (2) it shows that, far from imposing a "straitjacket" on the data, the Bayesian learning structure led to insights about the functional form for state dependence that improved model fit; (3) it generates interesting results about long- versus short-run effects of advertising; and (4) it gives an economic rationale for the brand loyalty observed in scanner panel data.[7] These results generated new interest in structural learning models (and

[2] Thus, learning models play havoc with traditional welfare analysis, as consumer surplus is no longer the area under the demand curve. Parameters of the demand curve are no longer structural parameters of preferences but depend on the information set (e.g., they can be shifted by advertising). See Erdem et al. (2008) for a discussion.

[3] Pioneering papers that first applied learning models in labor economics were Jovanovic (1979) and Miller (1984). Both were concerned with workers and firms learning about the quality of job matches.

[4] Their computational approach involves (1) using the method of Keane and Wolpin (1994) to obtain a fast but accurate approximate solution to the dynamic optimization problem of forward-looking agents and (2) using (then) recently developed simulation estimation methods (see, e.g., Keane 1994) to approximate the likelihood function.

[5] Specifically, only the total number of use experiences should matter, not their timing. The same is true for ad exposures. This structure changes with forgetting, an issue we will discuss in §3.1.1.

[6] In most applications, imposing structure involves sacrificing fit to some extent (i.e., not surprisingly, structural models usually fit worse than flexible reduced-form or statistical/descriptive models). The payoff of imposing the structure is (1) greater interpretability of parameter estimates and (2) the ability to do policy experiments. Erdem and Keane (1996) is a rare instance where a structural model fit better than popular competing reduced-form models.

[7] A key insight of Erdem and Keane (1996) is that uncertainty about quality combined with risk aversion could lead to brand-loyal behavior (i.e., persistence in brand choice over time). Loyalty emerges when consumers stick with familiar products (whose attributes are precisely known) to avoid risk. Given equal prices,

dynamic structural models more generally) within the fields of marketing and economics.

Nevertheless, there was a time lag of roughly five years from Erdem and Keane (1996) to the publication of many additional papers on learning models. But, starting in the early 2000s, there has been an explosion of new work in marketing and economics applying learning models to brand choice and many other problems. Other interesting applications include (i) demand for new products, (ii) choice of TV shows and movies, (iii) prescription drugs, (iv) durable goods, (v) insurance products, (vi) choice of tariffs (i.e., price/usage plans), (vii) fishing locations, (viii) career options, (ix) service quality, (x) child-care options, and (xi) medical procedures. Some of these applications are based rather closely on the Erdem and Keane framework with forward-looking Bayesian consumers, whereas other papers depart from or extend that framework in important ways (often along one of the four key dimensions noted above).

The outline of the survey is as follows: In §2 we describe the learning model of Erdem and Keane (1996) in detail. We treat their model as a unifying framework to discuss the rest of the literature. In general, later developments can be viewed as extending the Erdem–Keane model along certain dimensions (while typically restricting it on others to make those extensions feasible) or applying it in different contexts. Section 3 reviews the subsequent literature on learning models. It is divided in subsections that cover (i) more sophisticated learning models with myopic consumers, (ii) more sophisticated learning models with forward-looking consumers, (iii) models for product-level/market share data, and (iv) new or novel applications of learning models. Section 4 describes what we consider the key challenges for future research. In §5 we summarize and conclude.

2. The General Structure of the Erdem and Keane (1996) Model

As noted in §1, the papers by Roberts and Urban (1988) and Eckstein et al. (1988) are the first applications of learning models to marketing problems. The model of Erdem and Keane (1996; henceforth referred to as EK) nests those models in a more general framework. In this section we describe the EK model in some detail. Readers interested in additional information about the earlier models can refer to a detailed description in Online Appendix A (available as supplemental material at http://dx.doi.org/10.1287/mksc.2013.0805).

2.1. A Simple Dynamic Learning Model with Gains from Trial Information

The key feature of learning models is that consumers do not know the attributes of brands with certainty. Although this may be true of many attributes, most papers, including Erdem and Keane (1996), have focused on learning about brand quality. In the EK model, consumers receive signals about quality through both use experience and ad signals. But before receiving any information, consumers have a normal prior on brand quality:

$$Q_j \sim \mathrm{N}(Q_{j1}, \sigma_{j1}^2), \quad j = 1, \ldots, J. \tag{1}$$

This says that, before receiving any information, consumers perceive that the true quality of brand j, denoted as Q_j, is distributed normally with mean Q_{j1} and variance σ_{j1}^2. So in the first period, the consumer's information set is $I_1 = \{Q_{j1}, \sigma_{j1}^2 \; \forall j\}$. The values of Q_{j1} and σ_{j1}^2 may be influenced by many factors, such as reputation of the manufacturer, prelaunch advertising, etc.

Use experience does not fully reveal quality because of *inherent product variability*. This has multiple interpretations. First, the quality of different units of a product may vary. Second, a consumer's experience of a product may vary across use occasions. For instance, a cleaning product may be effective at removing a certain type of stain on most occasions but be ineffective on other occasions. Alternatively, there may be inherent randomness in psychophysical perception, e.g., the same cereal tastes better on some days than others.

Given inherent product variability, there is a distinction between experienced quality for brand j on purchase occasion t, which we denote as Q_{jt}^E, and true quality Q_j. Let us assume the experienced quality delivered by use experience is a noisy signal of true quality, as in

$$Q_{jt}^E = Q_j + \varepsilon_{jt}, \quad \text{where } \varepsilon_{jt} \sim \mathrm{N}(0, \sigma_\varepsilon^2) \quad \text{for } t = 1, \ldots, T. \tag{2}$$

Here, σ_ε^2 is the variance of inherent product variability, which we often refer to as experience variability. Of course, experience signals are specific to consumer i. But here and in later equations, we will suppress the i subscript whenever possible to save on notation.

Note that we have conjugate priors and signals, as both the prior on quality in (1) and the noise

a familiar brand may be chosen over a less familiar brand even if it has lower expected quality, provided consumers are sufficiently risk averse. In this framework, "loyalty" is the price premium that consumers are willing to pay for greater familiarity (lower risk). Keller (2002) refers to the general framework laid out in Erdem and Keane (and elucidated further in Erdem and Swait 1998) as the principle *economic* model of brand equity. There are also a number of psychology-based models (see Keller 2002 for an overview).

in the quality signals in (2) are assumed to be normal. This structure gives simple formulas for updating perceptions as new information arrives, as we will see below. This is precisely why we assume priors and signals are normal. Few other reasonable distributions would give simple expressions. Also, because signals are typically unobserved by the researcher, it is not clear that more flexible distributions would be identified from choice data alone.

Thus, the posterior for perceived quality, given a single use experience signal (received after the first purchase of brand j), is given by the simple Bayesian updating formulas:

$$Q_{j2} = \frac{\sigma_{j1}^2}{\sigma_{j1}^2 + \sigma_\varepsilon^2} Q_{j1}^E + \frac{\sigma_\varepsilon^2}{\sigma_{j1}^2 + \sigma_\varepsilon^2} Q_{j1}, \tag{3}$$

$$\sigma_{j2}^2 = \frac{1}{1/\sigma_{j1}^2 + 1/\sigma_\varepsilon^2}. \tag{4}$$

Equation (3) describes how a consumer's prior on quality of brand j is updated as a result of the experience signal Q_{j1}^E. The extent of updating is greater the more accurate the signal (i.e., the smaller the σ_ε^2). Equation (4) describes how a consumer's uncertainty declines when he or she receives the signal. The quantity σ_{j2}^2 is often referred to as the *perception error variance*.

Equations (3) and (4) generalize to multiple periods. Let $N_j(t)$ denote the total number of use experience signals received before the purchase at time t. Then we have

$$Q_{jt} = \frac{\sigma_{j1}^2}{N_j(t)\cdot\sigma_{j1}^2 + \sigma_\varepsilon^2} \sum_{s=1}^{t-1} Q_{js}^E d_{js} + \frac{\sigma_\varepsilon^2}{N_j(t)\cdot\sigma_{j1}^2 + \sigma_\varepsilon^2} Q_{j1} \quad \text{for } t = 2, \ldots, T, \tag{5}$$

$$\sigma_{jt}^2 = \frac{1}{1/\sigma_{j1}^2 + N_j(t)\cdot 1/\sigma_\varepsilon^2} \quad \text{for } t = 2, \ldots, T, \tag{6}$$

where d_{jt} is an indicator for whether brand j is bought or consumed at time t.

In (5), the perceived quality of brand j at time t, Q_{jt}, is a weighted average of the prior and all quality signals received up until the beginning of time t, $\sum_{s=1}^{t-1} Q_{js}^E d_{js}$. It is crucial to note that this is a random variable across consumers, as some will, by chance, receive better-quality signals than others. Thus, the learning model endogenously generates heterogeneity across consumers in perceived quality of products (even starting from identical priors). This aspect of the model is appealing. It seems unlikely that people are born with brand preferences (as standard models of heterogeneity implicitly assume), but rather they arrive at their views through heterogeneous experience.

Of course, as Equation (6) indicates, the variance of perceived quality around true quality declines as more signals are received and in the limit perceived quality converges to true quality. Still, heterogeneity in perceptions will persist over time, for several reasons: (i) both brands and consumers are finitely lived, (ii) there is a flow of new brands and consumers entering a market, and (iii) as people gather more information, the value of trial diminishes, and the incentive to learn about unfamiliar products decreases. Intuitively, once a consumer is familiar with a substantial subset of brands, there is rarely much marginal benefit to learning about all the rest.

In general, learning models must be solved by dynamic programming (DP) because today's purchase affects tomorrow's information set, which affects future utility. The key idea of DP is that, at each time t, the value of choosing option j consists of an immediate payoff *plus* the expected present value of the future payoff stream that arises from period $t+1$ onward. This forward-looking term is *conditional* on the option j chosen at time t, as the choice at j alters the consumer's information set, which in turn affects the choices that he or she makes in the future.

In our notation, the information set is I_t, the value of choosing alternative j at time t is $V(j, t \mid I_t)$, the current payoff is a context-specific utility function, and the expected present value of future payoffs (conditional on I_t and j), or "future component," is $\mathrm{EV}(I_{t+1} \mid I_t, j)$.

If choices convey not just utility but also information, it may not be optimal to choose the brand with the highest *perceived* quality in the *current* period. To see this, consider the special case where the choice is between an old familiar brand (whose attributes are known with certainty) and a new brand. Denote these by $j = o, n$ (for old and new). The information set is $I_t = \{Q_{nt}, \sigma_{nt}^2\}$, where we suppress the values for the old brand, which are Q_o and $\sigma_{ot}^2 = 0$. Prices are given by P_{jt} for $j = o, n$. Then values of choosing each brand in the current period are

$$V(n, t \mid I_t) = \mathrm{E}[U(Q_{nt}^E, P_{nt}) \mid I_t] + \beta\, \mathrm{EV}(I_{t+1} \mid I_t, n), \quad \text{where } I_{t+1} = \{Q_{n,t+1}, \sigma_{n,t+1}^2\}; \tag{7}$$

$$V(o, t \mid I_t) = \mathrm{E}[U(Q_{ot}^E, P_{ot})] + \beta\, \mathrm{EV}(I_{t+1} \mid I_t, o), \quad \text{where } I_{t+1} = I_t. \tag{8}$$

At time t, a consumer chooses the brand with the highest V, which is not necessarily the brand with the highest expected utility. It is important to understand why. Purchase of the old familiar brand gives expected utility $\mathrm{E}[U(Q_{ot}^E, P_{ot}) \mid I_t]$. This is increasing in true quality Q_o, which is known. It is decreasing in experience variability σ_ε^2, assuming consumers are risk averse. On the other hand, purchase of the new brand delivers expected utility

$E[U(Q^E_{nt}, P_{nt}) \mid I_t]$, which is increasing in Q_{nt} and decreasing in *both* experience variability and the perception error variance σ^2_{nt}. Purchase of the new brand also increases next period's expected value function. That is, $EV(I_{t+1} \mid I_t, n) > EV(I_{t+1} \mid I_t, o) = EV(I_t \mid I_t, o)$ because I_{t+1} contains better information. As a result, it may be optimal to try the new brand even if $E[U(Q^E_{nt}, P_{nt}) \mid I_t] < E[U(Q^E_{ot}, P_{ot})]$.

To gain further insight, consider the special case where, as in Eckstein et al. (1988), utility is linear in experienced quality Q^E_{jt}, thus abstracting from risk aversion, and where utility is also linear in price. In that case, (7) and (8) simplify to

$$V(n, t \mid I_t) = Q_{nt} - P_{nt} + e_{nt} + \beta\, EV(I_{t+1} \mid I_t, n), \quad \text{where } I_{t+1} = \{Q_{n,t+1}, \sigma^2_{n,t+1}\}; \tag{9}$$

$$V(o, t \mid I_t) = Q_o - P_{ot} + e_{ot} + \beta\, EV(I_{t+1} \mid I_t, o), \quad \text{where } I_{t+1} = I_t. \tag{10}$$

Here, the e_{jt} for $j = o, n$ are stochastic terms in the utility function that represent purely idiosyncratic tastes for the two brands. These play the same role as the brand-specific stochastic terms in traditional discrete choice models such as logit and probit.[8]

A consumer will now choose the new brand over the familiar brand if the value function in Equation (9) exceeds that in (10). This means that $V^*_{nt} > 0$, where

$$V^*_{nt} \equiv V(n, t \mid I_t) - V(o, t \mid I_t) = (Q_{nt} - P_{nt}) - (Q_o - P_{ot}) + (e_{nt} - e_{ot}) + G_t, \tag{11}$$

$$G_t \equiv \beta\big[EV(I_{t+1} \mid I_t, n) - EV(I_{t+1} \mid I_t, o)\big]. \tag{12}$$

We will refer to G_t as the *gain from trial*. It is the increase in expected present value of utility from $t+1$ until the terminal period T that arises because the consumer obtains information by trying the new brand at time t.

Intuitively, the gain from trial comes from two sources. Most obvious is that the consumer may learn that the new brand is better than the old brand. A more subtle source is that evidence may indicate that the new brand is inferior to the familiar brand. There is, nevertheless, a large enough price differential such that the consumer will choose the new brand over the familiar brand if the new brand is cheaper by at least that amount. More precise information about the quality of the new brand enables the consumer to set this reservation price differential more accurately.

Here, we give a sketch of a proof that $EV(I_{t+1} \mid I_t, n) > EV(I_{t+1} \mid I_t, o)$ and hence that G_t is positive. This is a general result of information economics, but it is easiest to show in the linear case. It is also easiest to consider a finite-horizon problem with terminal period T. As there is no future, the consumer at time T simply chooses the brand with highest expected utility. Thus, the expected utility a consumer with incomplete information (i.e., $Q_{nT} \neq Q_n$) receives at T is simply

$$EV_T\big(I_T = \{Q_{nT}, \sigma^2_{nT}\}\big) = \max\big\{Q_{nT} - P_{nT} + e_{nT},\, Q_o - P_{oT} + e_{oT}\big\}.$$

On the other hand, a consumer with complete information would receive (with certainty) the utility

$$V_T\big(I_T = \{Q_n, 0\}\big) = \max\big\{Q_n - P_{nT} + e_{nT},\, Q_o - P_{oT} + e_{oT}\big\}.$$

This depends on true quality Q_n, not on perceived quality Q_{nT}. Thus, a consumer with incomplete information is, in effect, making decisions at T using the "wrong" decision rule, so in general, he or she will make suboptimal decisions. More formally stated, letting a^* be a noisy measure of a, we have $E\{V \mid V = \max(a, b)\} > E\{V \mid V = a \text{ if } a^* = \max(a^*, b),\ V = b \text{ otherwise}\}$. This is the key intuition for why information is valuable. A complete proof involves two more steps. First, one needs to show that as Q_{nT} becomes more accurate, the consumer's decisions become closer to optimal, so that $EV_T(I_T = \{Q_{nT}, \sigma^2_{nT}\})$ is decreasing in the perception error variance σ^2_{nT}. Second, by backwards induction, it can be shown that this is true back to any period t.

Although $G_t > 0$ (i.e., more information is better), it is notable that G_t is smaller if (i) the consumer has more information (σ^2_{nt} smaller) or (ii) use experience signals are less accurate (σ^2_ε larger). Both lower the value of trial. Note also that (11) can be rewritten as "choose brand n if"

$$(Q_{nt} - P_{nt}) + G_t + e_{nt} > (Q_o - P_{ot}) + e_{ot}. \tag{13}$$

This shows that the trial value G_t *augments* the perceived value of the new brand $(Q_{nt} - P_{nt})$. Thus, ceteris paribus, the new brand can command a price premium over the old brand because it delivers valuable trial information. So the model with linear utility (i.e., no risk aversion) generates a value of information effect that is *opposite* to the conventional brand

[8] Without these terms, choice would be deterministic (conditional on Q_{nt}, Q_o, P_{nt}, and P_{ot}). But in contrast to standard discrete choice models, it is not strictly necessary to introduce the $\{e_{jt}\}$ terms to generate choice probabilities. This is because perceived quality Q_{nt} is random from the perspective of the econometrician. We feel it is advisable to include the $\{e_{jt}\}$ terms regardless because, in their absence, choice becomes deterministic conditional on price as experience with the new brand grows. Yet both introspection and simple data analysis suggest that consumers may switch brands for purely idiosyncratic reasons even under full information.

loyalty phenomenon. (In the Online Appendix A, we give some details on estimation of this model.)

2.2. Introducing Risk Aversion and Exogenous Signals

Next, we introduce two key features of the Erdem and Keane (1996) model that generalize the simple setting described above. First, we introduce exogenous signals of quality (e.g., advertising) as an additional source of information besides use experience. Second, we consider utility functions that exhibit risk aversion with respect to variation in brand attributes (focusing again on quality). We should note that both of these features were already present in Roberts and Urban (1988) but in a static choice context.

There are numerous ways one can obtain information about a brand other than trial purchase.[9] Examples include advertising, word of mouth, magazine articles, and dealer visits. For simplicity, we will refer to these as *exogenous* signals, as we may think of them as arriving randomly from the outside environment. (Of course, a consumer may also actively seek out such signals, an extension we discuss below.) For frequently purchased goods, the most important source of information is probably advertising, which is the source that Erdem and Keane (1996) consider.

Let A_{jt} denote an exogenous signal (advertising, word of mouth, etc.) that a consumer receives about brand j at time t (before the time t purchase decision). We further assume that

$$A_{jt} = Q_j + \varepsilon^A_{jt}, \quad \text{where } \varepsilon^A_{jt} \sim \mathrm{N}(0, \sigma^2_A) \quad \text{for } t = 2, \ldots, T. \tag{14}$$

This says the signals A_{jt} provide unbiased but noisy information about brand quality, where the noise has variance σ^2_A. The noise is assumed to be normal to maintain conjugacy with the prior in (1).

It is important to compare (14) with (2). The noise in trial experience is from inherent product variability, which is largely a feature of the product itself. The noise in a signal such as advertising or word of mouth is, in contrast, largely a function of the medium. Presumably, some media convey information more accurately than others, and no medium is as accurate as direct use experience. We also stress that the noise in (14) differs fundamentally from that in (2), as inherent product variability affects a consumer's *experienced* utility from consuming the product, whereas exogenous quality signals do not. Nevertheless, both types of signals enter the consumer's learning process in the same way. Given *only* the exogenous signal A_{jt}, we can rewrite (5) and (6) as

$$Q_{jt} = \frac{\sigma^2_{j1}}{N^A_j(t)\cdot\sigma^2_{j1} + \sigma^2_A}\sum_{s=2}^{t} A_{js}d^A_{js} + \frac{\sigma^2_A}{N^A_j(t)\cdot\sigma^2_{j1} + \sigma^2_A} Q_{j1} \quad \text{for } t = 2, \ldots, T, \tag{15}$$

$$\sigma^2_{jt} = \frac{1}{(1/\sigma^2_{j1}) + N^A_j(t)\cdot(1/\sigma^2_A)} \quad \text{for } t = 2, \ldots, T, \tag{16}$$

where d^A_{jt} is an indicator for whether a signal for brand j is received at time t, and $N^A_j(t)$ is the total number of ad signals received for brand j up through and including time t.

It is simple to extend the Bayesian updating rules in (5) and (6) and (15) and (16) to allow for two types of signals—i.e., both use experience and exogenous signals. Our timing convention is that time t ad signals are received *before* the time t purchase decision, whereas time t experience signals are received afterwards. For example, perceived quality before the purchase decision at $t = 2$ is $Q_{j2} = \sigma^2_{j2}[(1/\sigma^2_\varepsilon)Q^E_{j1} + (1/\sigma^2_A)A_{j2} + (1/\sigma^2_{j1})Q_{j1}]$. Generalizing to multiple periods, we have

$$Q_{jt} = \frac{1/\sigma^2_\varepsilon}{S_j(t)}\sum_{s=1}^{t-1} Q^E_{js}d_{js} + \frac{1/\sigma^2_A}{S_j(t)}\sum_{s=2}^{t} A_{js}d^A_{js} + \frac{1/\sigma^2_{j1}}{S_j(t)} Q_{j1} \quad \text{for } t = 2, \ldots, T, \tag{17}$$

$$\sigma^2_{jt} = \frac{1}{1/\sigma^2_{j1} + N_j(t)\cdot 1/\sigma^2_\varepsilon + N^A_j(t)\cdot 1/\sigma^2_A} \quad \text{for } t = 2, \ldots, T, \tag{18}$$

where $S_j(t) \equiv 1/\sigma^2_{jt}$. Note that the recency of signals does not matter: in (17) and (18), only the total stock of signals determines a consumer's state. Furthermore, receiving N signals with variance σ^2 affects the perception variance in the same way as receiving one signal with variance σ^2/N.

As we will see, these properties are important for simplifying the solution to consumers' dynamic optimization problem. This is because the consumer's level of uncertainty, as captured by the $\{\sigma^2_{jt}\}^J_{j=1}$, depends only on the number of signals received, not the order or timing with which they were received. One could imagine scenarios where more recent signals are more salient, or, conversely, where first impressions are most important. These are important potential extensions of the model, but they would make computation *much* more difficult.

To progress further and develop a model that can be taken to the data, one must specify a particular functional form for the utility function. Of course, many functions are possible. Erdem and Keane (1996) assume a utility function of the form:

$$U(Q^E_{jt}, P_{jt}) = w_Q Q^E_{jt} - w_Q r (Q^E_{jt})^2 + w_P(X - P_{jt}) + e_{jt}. \tag{19}$$

[9] Indeed, when considering a durable good as opposed to a frequently purchased good, trial purchase is no longer even a relevant consideration (assuming that one cannot return a purchase to get a refund).

Here, utility is quadratic in the experienced quality of brand j at time t and linear in consumption of the composite outside good $C_t = X - P_{jt}$, where X is income. The parameter w_Q is the weight on quality, r is the risk coefficient, w_P is the marginal utility of the outside good, and e_{jt} is an idiosyncratic brand and time-specific error term.[10] Note that, as choices only depend on utility differences, and as income is the same regardless of which brand is chosen, income drops out of the model. So we can think of w_p as the price coefficient.

Given (19), combined with (2) and (18), expected utility is given by

$$\mathrm{E}[U(Q_{jt}^E, P_{jt}) \mid I_t] = w_Q Q_{jt} - w_Q r Q_{jt}^2 - w_Q r(\sigma_{jt}^2 + \sigma_\varepsilon^2) - w_P P_{jt} + e_{jt}, \quad j = 1, \ldots, J. \tag{20}$$

Also, as the EK model was meant to be applied to weekly data, and as consumers may not buy every week, a utility of the no-purchase option must also be specified. Erdem and Keane (1996) write this as $\mathrm{E}[U_0 \mid I_t] = \Phi_0 + \Phi_1 t + e_{0t}$. The time trend captures in a simple way the possibility of changing value of substitutes for the category in question.

We have now specified the complete EK model, and we are in a position to formally state the consumer's problem. Consumers are assumed to be forward-looking, making choices to maximize value functions of the form:

$$V(j, t \mid I_t) = \mathrm{E}[U(Q_{jt}^E, P_{jt}) \mid I_t] + \beta\, \mathrm{EV}(I_{t+1} \mid I_t, j) \quad \text{for } j = 0, \ldots, J, \tag{21}$$

where the consumer's information set is given by

$$I_{t+1} = \{Q_{1,\,t+1}, \ldots, Q_{J,\,t+1}, \sigma_{1,\,t+1}^2, \ldots, \sigma_{J,\,t+1}^2\}. \tag{22}$$

A key point is that a consumer's information about a brand may be updated between t and $t+1$ for one of two reasons: (i) the consumer buys the brand, or (ii) the consumer receives an exogenous signal about the brand. Henceforth, we simply refer to these as *ad signals*. In forming $\mathrm{EV}(I_{t+1} \mid I_t, j)$, we allow for both sources of information. We describe the process in detail in the next section.

With the introduction of risk aversion, the EK model can capture *both* gains from trial and brand loyalty phenomena. As we discussed earlier, the $\mathrm{EV}(I_{t+1} \mid I_t, j)$ terms in a dynamic learning model capture the gain from trial information. These are greater for less familiar brands, where the gain from trial is greater. At the same time, the risk terms σ_{jt}^2 are also greater for such brands. These two forces work against each other; which dominates determines whether a consumer is more or less likely to try a new unfamiliar brand over a familiar brand. In categories where risk aversion dominates, we would expect to see a high degree of brand loyalty (i.e., persistence in choice behavior). In categories where the gains from trial dominate, we would expect to see a high degree of brand switching (as a result of experimentation).[11]

2.3. Solving the Dynamic Optimization Problem

Here, we show how to solve a consumer's DP problem. Solving the DP problem is computationally difficult for two reasons: (i) the expected value functions in (21) are high-dimensional integrals, and (ii) these integrals must be evaluated at many state points.

The expected value functions in (21) have the form:

$$\mathrm{EV}(I_{t+1} | I_t, j) = E_t \max\{V(0, t+1 | I_{t+1}), \ldots, V(J, t+1 | I_{t+1})\}. \tag{23}$$

That is, the consumer at time t knows that, at time $t+1$, he or she will choose from among the J options the one with the highest value function. The consumer can form the expected maximum over these value functions because his or her information set and decision at time t (i.e., the (I_t, j)) generate a distribution of I_{t+1} in the manner described earlier.

However, it is not immediately obvious how (23) helps us to solve the consumers' optimization problem. The Vs on the right-hand side of (23) themselves contain expected value functions dated at $t+2$—that is, functions of the form $\mathrm{EV}(I_{t+2} \mid I_{t+1}, j)$. So it seems we have only pushed the problem one period ahead. One key insight for solving a dynamic programming problem is the assumption that there exists a terminal period T beyond which a consumer does not plan. At T, the consumer will simply choose the option with highest expected utility. Thus, we have that

$$V(j, T | I_T) = \mathrm{E}[U(Q_{jT}^E, P_{jT}) | I_T] \quad \text{for } j = 0, \ldots, J; \tag{24}$$

$$\mathrm{EV}(I_T | I_{T-1}, j) = \mathrm{E}_{T-1} \max\{\mathrm{E}[U_0 | I_T], \mathrm{E}[U(Q_{1T}^E, P_{1T}) | I_T], \ldots, \mathrm{E}[U(Q_{JT}^E, P_{JT}) | I_T]\}. \tag{25}$$

[10] A common assumption is that utility is linear in the consumption of the outside good, so there are no income effects when dealing with inexpensive items such as frequently purchased consumer goods. This also means that the marginal utility of consumption w_p is constant within the range of outside good consumption levels spanned by different brand choices. However, it is likely that the marginal utility of consumption w_p would be lower for households at higher wealth levels. And the assumption of no income effects would not be tenable for expensive durable goods.

[11] We discuss the identification of learning models in detail in §2.5. Here, we note that it is not possible to determine from raw data patterns the magnitudes of risk aversion versus gains from trial. Such an inference can only be made conditional on an assumed model structure.

The integral in (25) is feasible to evaluate. Suppose that, hypothetically, the I_T and P_T were known at $T-1$. Then, as we see from (20), the only unknowns appearing in (25) would be the logistic errors $\{e_{0T}, \ldots, e_{JT}\}$. In that case, (25) would have a simple closed form given by the well-known nested logit "inclusive value" formula (see Rust 1994). This illustrates the point that estimating a finite-horizon dynamic model is very much like estimating a nested logit model, if one thinks of moving down the nesting structure as a process that plays out over time.

Of course, evaluating (25) in the EK model is more difficult because the I_T and P_T are not known at $T-1$. Both experience signals and ad signals may arrive between $T-1$ and T, causing the consumer to update his or her information set. The expectation in (25) must be taken over the possible I_T that may arise as a result of these signals. Specifically, we must (i) update $\sigma^2_{j,T-1}$ to $\sigma^2_{j,T}$ using (18) to account for additional use experience, (ii) integrate over possible values of the use experience signal in (2) to take the expectation over possible realizations of Q_{jT}, (iii) integrate over possible ad exposures that may arrive between $T-1$ and T (i.e., over realizations of d^A_{jT} for $j=1,\ldots,J$) to account for ad-induced changes in the $\{\sigma^2_{jT}\}$, and (iv) integrate over possible values of the ad signals in (14), as these will lead to different values of the $\{Q_{jT}\}$.[12]

Clearly, the integrals in (25) are high dimensional, and simulation methods are needed. That is, we *integrate by simulation* over draws from the distributions of the signal processes. The computational burden increases if consumers learn about multiple brands and/or have more than one source of information. Memory is also an issue, as all the $\mathrm{EV}(I_T \mid I_{T-1}, j)$ must be saved.

Having calculated the values of $\mathrm{EV}(I_T \mid I_{T-1}, j)$ for every possible (I_{T-1}, j), and saving the results (a point we return to below), we can move back to time $T-1$, where (21) becomes

$$V(j, T-1 \mid I_{T-1}) = \mathrm{E}\big[U(Q^E_{jT-1}, P_{jT-1}) \mid I_{T-1}\big] + \beta\,\mathrm{EV}(I_T \mid I_{T-1}, j). \qquad (26)$$

Note that (26) is just like (24), except for the $\mathrm{EV}(I_T \mid I_{T-1}, j)$ terms that are appended. But we have already solved for these and saved them in memory, so they are just numbers. So we can now construct the $V(j, T-1 \mid I_{T-1})$. This enables us to proceed backwards and calculate the time $T-1$ version of (25), and we can thus obtain the $\mathrm{EV}(I_{T-1} \mid I_{T-2}, j)$. Then we can work back again and obtain the $V(j, T-2 \mid I_{T-2})$. This backwards induction process is repeated until we have solved the entire dynamic programming problem back to $t=1$. Detailed descriptions of this process, known as "backsolving," are contained in many sources (see, for instance, Keane et al. 2011).

In practice, T is generally chosen to be some time beyond the end of the sample period. This can be chosen far enough out so that results are not sensitive to the exact value of T.

Unfortunately, the above description is oversimplified because it assumes that it is feasible to calculate the value $\mathrm{EV}(I_T \mid I_{T-1}, j)$ *for every possible* (I_{T-1}, j). Note that the number of variables that characterize the state of an agent in (22) is $2\cdot J$. Solving a dynamic programming problem exactly requires that one solve the expected value function integrals at *every* point in the state space; this is clearly not feasible here because there are too many state variables. Of course, as the state variables in (22) are continuous, it would be impossible to solve for the expected value functions at every state point (because the number of points is infinite). A common approach is to discretize continuous state variables using a fairly fine grid. Suppose we use G grid points for each state variable.[13] As we have $2\cdot J$ state variables, this gives $G^{2\cdot J}$ grid points, which is impractically large even for modest G and J. This is known as the "curse of dimensionality." A number of ways to deal with this problem have been proposed.

To solve the optimization problem in their model (that is, to construct the $\mathrm{EV}(I_{t+1} \mid I_t, j)$ in (21)), Erdem and Keane (1996) use an approximate solution method developed in Keane and Wolpin (1994). The idea is to evaluate the expected value function integrals at a randomly selected subset of state points (where this set is relatively small compared with the size of the total state space).[14] The expected value functions are then constructed at other points via interpolation. For instance, one can run a regression of the value functions on the state variables (at the random subset of state points) and use the regression to predict the value functions at other points. We give more detail on how to apply this method to estimate learning models in Online Appendix B.

[12] It is also necessary to integrate over future price realizations for all brands. To make this integration as simple as possible, Erdem and Keane (1996) assume that the price of each brand is distributed normally around a brand-specific mean, with no serial correlation other than that induced by these mean differences.

[13] Note that range of the discretization needs to big enough to cover the true Q_j's (or $\sigma^2_{j,0}$'s), which are unknown to researchers a priori. In Online Appendix B, we outline a procedure to determine the bounds.

[14] In most applications, the expected value function integrals are simulated using Monte Carlo methods rather than evaluated numerically. This makes it practical to deal with the three aspects of integration described below in Equation (31): integration over content of use experience signals, integration over exposure to ads, and integration over the content of ads.

The Keane–Wolpin approximation method (Keane and Wolpin 1994), or variants on its basic idea, has become widely used in both economics and marketing in the past 15 years to solve many types of dynamic models. This has greatly increased the richness and complexity of the dynamic models that it is feasible to estimate. We will not give details of these computational methods here but refer the reader to surveys by Keane et al. (2011), Aguirregebirria and Mira (2010), Geweke and Keane (2001), and Rust (1994).

Finally, a common question is, how can we solve the DP problem when we do not know the true parameter values for the utility function or the stochastic processes that generate signals? The answer is that the DP problem must be solved at each trial parameter value considered during the search process for the maximum of the likelihood function. In other words, the DP solution is nested within the likelihood evaluation. We consider the construction of the likelihood function in the next section.

2.4. Evaluating the Likelihood Function

In this section we discuss how to form the likelihood function for the EK learning model. Let $\theta = \{w_Q, w_P, r, \{Q_{j1}, \sigma_{j1}^2\}, \sigma_\varepsilon^2, \sigma_A^2\}$ denote the entire vector of model parameters. Combining Equations (20) and (21), we have the choice-specific value functions:

$$V(j,t|I_t) = w_Q Q_{jt} - w_Q r Q_{jt}^2 - w_Q r(\sigma_{jt}^2 + \sigma_\varepsilon^2) - w_P P_{jt} + e_{jt} + \beta \mathrm{EV}(I_{t+1}|I_t, j) \quad j = 1, \ldots, J, \tag{27a}$$

$$V(0,t|I_t) \equiv \Phi_0 + \Phi_1 t + e_{0t} + \beta \mathrm{EV}(I_{t+1}|I_t, 0). \tag{27b}$$

Erdem and Keane (1996) assume that the idiosyncratic brand- and time-specific error terms e_{jt} in (27) are independent and identically distributed extreme value. In this case, the choice probabilities have a simple multinomial logit form:

$$P(j \mid I_t) = \frac{\exp(V_j(\theta))}{\exp(V_0(\theta)) + \sum_{k=1}^{J} \exp(V_k(\theta))}, \tag{28}$$

where

$$V_j(\theta) \equiv w_Q Q_{jt} - w_Q r Q_{jt}^2 - w_Q r(\sigma_{jt}^2 + \sigma_\varepsilon^2) - w_P P_{jt} + \beta \mathrm{EV}(I_{t+1} \mid I_t, j) \quad j = 1, \ldots, J, \tag{29a}$$

$$V_0(\theta) \equiv \Phi_0 + \Phi_1 t + \beta \mathrm{EV}(I_{t+1} \mid I_t, 0). \tag{29b}$$

Equations (28) and (29) illustrate the point, stressed by Keane et al. (2011), that choice probabilities in dynamic discrete choice models look exactly like those in static discrete choice models (multinomial logit in the present case), *except* that the $V_j(\theta)$ terms in the dynamic model include the extra $\mathrm{EV}(I_{t+1} \mid I_t, j)$ terms. However, once one has solved the DP problem, these extra terms are merely numbers that one can look up in a table that is saved in computer memory (i.e., a table that lists expected value functions at every point in the state space). Alternatively, if one has used an interpolating method rather than saving every value, the appropriate $\mathrm{EV}(I_{t+1} \mid I_t, j)$ may be constructed as needed using the interpolating function. Erdem and Keane (1996) use the latter procedure because the number of possible states in their model is so large.

To proceed in constructing the likelihood, we need some definitions. Let $j(t)$ denote the choice actually made at time t (we continue to suppress the i subscripts to conserve on notation). Let $D_{t-1} \equiv \{j(1), \ldots, j(t-1)\}$ denote the history of purchases made before time t. Similarly, let $D_t^A \equiv \{d_2^A, \ldots, d_t^A\}$ denote the history of ads received up through time t, where $d_t^A \equiv \{d_{jt}^A\}_{j=1}^J$. Let $A_t \equiv \{A_{2t} d_{2t}^A, \ldots, A_{Jt} d_{Jt}^A\}$ denote the content of ads seen at time t. Also, let $Q_{t-1}^E \equiv \{Q_{j(s),s}^E\}_{s=1}^{t-1}$ and $\check{A}_t \equiv \{A_s\}_{s=2}^t$ denote the actual *content* of all experience and ad signals received up through t. (Recall that D_{t-1} and D_t^A are observed by the econometrician, whereas Q_{t-1}^E and $\check{A}_t$ are not.)

Finally, we define $P(j(t) \mid I_1, D_{t-1}, D_t^A, Q_{t-1}^E, \check{A}_t)$ as the probability of a person's choice at time t given his or her initial state (I_1), the history of use experience before time t, and advertising exposures up through time t, as well as the content of those signals. It is worth emphasizing the timing convention that the ads at time t are observed before the time t choice is made.

Unfortunately, we cannot observe the actual *content* of ad and experience signals. Thus, we must *integrate* over that content to obtain unconditional probabilities $P(j(t) \mid I_1, D_{t-1}, D_t^A)$. Thus, the probability of a choice history for an individual takes the following form:

$$\begin{aligned} P(\{j(t)\}_{t=1}^T) &= \prod_{t=1}^T P(j(t) \mid I_t) = \prod_{t=1}^T P(j(t) \mid I_1, D_{t-1}, D_t^A) \\ &= \int_{\{Q_{j(t-1),t-1}^E, \check{A}_t\}_{t=1}^T} \prod_{t=1}^T P\big(j(t) \mid I_1, D_{t-1}, D_t^A, Q_{t-1}^E, \check{A}_t\big) \cdot \\ &\quad d\mu\{Q_{j(t-1),t-1}^E, \check{A}_t\}_{t=1}^T. \end{aligned} \tag{30}$$

In (30) we integrate over all experience and advertising signals that the consumer may have received from $t = 1, \ldots, T$. That is, we integrate over the distribution of $\{Q_{j(t-1),t-1}^E, \check{A}_t\}_{t=1}^T$. Clearly, the required order of integration is substantial.[15]

To deal with this problem, Erdem and Keane (1996) use simulated maximum likelihood (see, e.g.,

[15] It is worth emphasizing that this high-order integration problem arises even in a static learning model (i.e., with myopic agents) as long as the contents of signals are not observed.

Keane 1993, 1994). Specifically, draw D sets of signals

$$S^d = \{Q^d_{j(t-1),\,t-1}, \breve{A}^d_t\}_{t=1}^T \quad \text{for } d = 1, \ldots, D,$$

using the distributions defined in (2) and (14). Then form the simulated probability:

$$P(\{j(t)\}_{t=1}^T) = \frac{1}{D}\sum_{d=1}^{D}\prod_{t=1}^{T} P\big(j(t) \mid I_1, D_{t-1}, D_t^A, S^d\big). \quad (31)$$

Finally, sum the logs of these probabilities across individuals $i = 1, \ldots, N$.

A key complication is that consumer purchase histories and ad exposures are not usually observed before the start of the sample period. This creates an "initial conditions problem." A consumer who likes a particular brand will have bought it often *before* the sample period starts. Thus, brand preference is correlated with the information set at $t = 1$. The usual consequence is to exaggerate the impact of lagged purchases on current choices. An exact solution to the initial conditions problem requires integrating over all possible initial conditions when forming the likelihood, but in most cases, this is not computationally feasible. Thus, a number of approximate (ad hoc) approaches have been proposed. For example, Erdem and Keane (1996) have three years of scanner data available, but they use the first two years to estimate the initial conditions for each consumer at the start of the third year and then use only the third year in estimation.

2.5. Identification

The discussion of "identification" can be confusing, as the word has multiple meanings. It can mean showing that the parameters of a model are identified *given the assumed model structure*. This may involve formal proof as well as intuitive discussion of what data patterns drive the estimates. We discuss identification in this "narrow" sense in §2.5.1.

Identification can also mean analysis of which assumptions are *necessary* to estimate a model, or just convenient.[16] For example, can assumptions such as Bayesian updating or normal signals be relaxed? Even more generally, can one distinguish the learning model from other plausible models that also generate state dependence? How can we tell if consumers are forward-looking? We discuss identification in this "broad" sense in §2.5.2.

Finally, some parameters may be formally identified but difficult to pinpoint in finite samples. We examine this issue in §2.6, when we discuss the estimates of the EK model.

2.5.1. Identification of Learning Model Parameters (Given the Model Structure). Some key points about identification become apparent from examining (27)–(29). First, suppose that consumers have complete information about all brands. Then we have $\mathrm{EV}(I_{t+1} \mid I_t, j) = \mathrm{EV}(I_t) = k$ for $j = 1, \ldots, J$, where k is a constant. That is, there is no updating of information sets based on choice, and so the $\beta\,\mathrm{EV}(I_{t+1} \mid I_t, j)$ terms drop out of the model (just like any term that is constant across choices in a discrete choice model). What remains is a static model where

$$\begin{aligned} V\big(j, t \mid I_t = \{Q_1, \ldots, Q_J, 0, \ldots, 0\}\big) \\ \equiv w_Q Q_j - w_Q r Q_j^2 - w_Q r \sigma_\varepsilon^2 - w_P P_{jt} + e_{jt}. \quad (32)\end{aligned}$$

Here, we have set $\sigma_{jt}^2 = 0$ and $Q_{jt} = Q_j$ because there is no uncertainty about quality.

Obviously, we cannot identify β, σ_A^2, or the priors $\{Q_{j1}, \sigma_{j1}^2\}$ as they drop out of the model. We also cannot identify σ_ε^2, as it is constant across alternatives $j = 1, \ldots, J$.[17] Careful inspection of (32) reveals that r is not identified either, as it cannot be disentangled from the scaling of Q_j. (If Q_j had a known scale, this would not be a problem.) Thus, r, β, σ_A^2, σ_ε^2, and the priors $\{Q_{j1}, \sigma_{j1}^2\}$ only affect choice probabilities through the $\mathrm{EV}(I_{t+1} \mid I_t, j)$ terms.

So in an environment of complete information, all that can be identified are the price coefficient w_p, the products $w_Q Q_j$, and the terms Φ_0 and Φ_1 in the value of the no-purchase option.[18] Furthermore, as only utility differences matter for choice, we need a normalization to establish a reference alternative. Erdem and Keane (1996) set $Q_j = 1$ for one brand, so the quality of all other brands is measured relative to brand j.[19]

[16] This is known as "nonparametric" identification analysis. Unfortunately, this literature has been misinterpreted by many researchers as suggesting that it may be possible to obtain "model-free evidence" about behavior. In fact, the approach of the nonparametric identification literature is to make a priori assumptions about certain parts of a model and then show that some other part (e.g., the functional form of utility or an error distribution) is identified without further assumptions. Thus, what is nonparametrically identified is just a part of the model, not all of it. For instance, Matzkin (2007) says the "ideal" of nonparametric estimation is to *start* with a structural model and then impose only restrictions implied by theory (e.g., continuity, monotonicity, homogeneity, equilibrium conditions). One then uses the data to identify functional forms and distributions that are not pinned down by theory. A related point is that observing data patterns that seem consistent or inconsistent with a model can make that model seem more or less plausible, given our priors. Yet they can never provide nonparametric evidence that a model is correct. In §4.5 we give two examples to illustrate these points.

[17] Note that σ_ε^2 does not enter the value of the no-purchase option. However, any shift in σ_ε^2 can be undone by a shift in Φ_0, leaving utility differences unchanged.

[18] The scale normalization on utility is imposed by assuming that the scale parameter of the extreme value errors is 1.

[19] It is worth noting that an alternative normalization would be to set $Q_j = 0$ for one brand. However, this would not let one

Alternatively, one could fix w_Q. To summarize, by observing consumers with essentially complete information (i.e., those with a great deal of experience with all brands), we can identify w_Q and the $\{Q_j\}$, given normalization, as well as w_p, Φ_0, and Φ_1.

The identification of the parameters $\beta, \sigma_A^2, \sigma_\varepsilon^2$, and r, as well as the priors $\{Q_{j1}, \sigma_{j1}^2\}$, requires that incomplete information actually exists. In that case, variation in $\mathrm{EV}(I_{t+1} \mid I_t, j)$ and σ_{jt}^2 across consumers is generated by variation in the information sets I_{it}. Intuitively, the parameters $\beta, \sigma_A^2, \sigma_\varepsilon^2$, and r, as well as $\{Q_{j1}, \sigma_{j1}^2\}$, are identified by the extent to which, ceteris paribus, consumers with different information sets are observed to have different choice probabilities. For instance, by comparing (27a) and (32), we can clearly see that variation in σ_{jt}^2 across consumers, arising from variation in use experience and ad exposures, enables us to identify r. (This is because w_Q is already identified from consumers with complete information, as noted earlier.)

Similarly, variation of I_{it} within consumers over time is also relevant. The learning parameters $\sigma_A^2, \sigma_\varepsilon^2$, and $\{Q_{j1}, \sigma_{j1}^2\}$ determine how the arrival of ad and use experience signals change σ_{jt}^2 and $\mathrm{EV}(I_{t+1} \mid I_t, j)$. Thus, these parameters are pinned down by the extent to which the arrival of signals alters behavior over time. For instance, if behavior is greatly altered by the arrival of one use experience signal, it implies that σ_{j1}^2 is large and σ_ε^2 is small.

It is worth stressing that this argument for identification based on comparing behavior of consumers with different amounts of information applies in both dynamic and static models. Indeed, this is the source of identification of the learning-related parameters in the static Bayesian learning model of Roberts and Urban (1988). Also note that variation in ad exposures and in prices are plausibly exogenous sources of variation in the I_{it}.

Now we turn to dynamic considerations. Recall from §2.1 that consumers will only engage in strategic trial if $\beta > 0$. But in the typical scanner data set, we cannot observe whether a purchase is a "trial." Thus, aside from the functional forms of utility and the EV functions, the discount factor β is pinned down by variables that affect the $\mathrm{EV}(I_{t+1} \mid I_t, j)$ but do not affect current utility. In the EK model, there are two exogenous variables that play this role. These are the brand-specific price variances and advertising frequencies. There is no reason for these variables to affect behavior in a static model. In such a model one cares only about the current price and the current stock of information, not the likelihood of future deals or future information arrival.

2.5.2. Identification in the General or Nonparametric Sense. As Erdem and Keane (1996) discuss in some detail, the Bayesian learning model implies a particular form of state dependence and serial correlation in the errors. This can be seen from careful examination of Equations (17) and (18) and (27). A frequently asked question is how learning behavior can be distinguished from other forms of state or serial dependence.

In his fundamentally important paper on panel data, Chamberlain (1984) defines the relationship between two variables y_t and x_t as "static" conditional on a latent variable c if (i) y_t is independent of lagged x conditional on x_t and c, and (ii) x_t is *strictly* exogenous with respect to y conditional on c (i.e., y_t does not cause future x). As Chamberlain shows, this static condition is actually *stronger* than the condition in which there is no structural state dependence (see Heckman 1981), as the latter does not require strict exogeneity.[20]

Rather remarkably, Chamberlain (1984) also shows that in nonlinear models (such as discrete choice models), one can always find a distribution of the latent variable c such that the relationship between y_t and x_t is static. In simple terms, one can always find a sufficiently flexible/complex heterogeneity distribution such that state dependence is not needed to explain the data. The key implication is that one cannot construct a nonparametric test of whether state dependence exists.

Obviously, if one cannot form a nonparametric test of whether state dependence exists, then it is true a fortiori that one cannot form a nonparametric test of whether any *particular* form of state dependence (such as learning) exists (i.e., if heterogeneity can account for general state dependence, it can also account for any particular form of state dependence). Nor can we form nonparametric tests to distinguish among competing forms of state dependence (e.g., learning versus inventories versus adjustment costs).

Chamberlain's (1984) result is an instance of the Cowles Foundation view that one cannot deduce interesting economic relationships from the data alone. One needs a priori identifying assumptions, regardless of what sort of idealized variation is

disentangle the $w_Q Q_j$ products. Thus, Erdem and Keane (1996) instead set $Q_j = 1$ for one brand. Also, with quadratic utility, it is desirable to constrain the largest Q_j to fall in the region of increasing utility. Erdem and Keane impose this constraint in estimation by updating the level of Q_j at each step (while keeping relative Q values fixed).

[20] Note that a dependence of y_t on a lagged x is the defining characteristic of structural state dependence. This is ruled out by condition (i), but condition (ii) is additional. In the learning model, the exogenous x variables correspond to advertising exposures and prices. These variables affect purchase decisions by shifting the I_{it} and budget constraints.

present in the data.[21] Thus, our interpretation of data will always be subjective, as it is contingent on our model. To be concrete, both the extent and the nature of any state dependence we find in discrete choice data will depend on the assumed functional forms for state dependence and heterogeneity (see, e.g., Keane 1997).

As described in §2.5.1, in the parameterized EK learning model, we identify parameters that describe dynamics from variation in choice behavior *across* consumers with different information sets (I_{it}) and *within* consumers as their information sets change over time. This variation in I_{it} arises from different histories of use experience, ad exposures, and prices. However, Chamberlain's (1984) results imply that differences in behavior stemming from differences in history (i.e., state dependence) cannot be distinguished nonparametrically from differences in behavior stemming from a completely general form of heterogeneity. Nor can learning behavior be distinguished nonparametrically from other mechanisms that may induce state dependence.

Thus, functional forms of both state dependence and heterogeneity must be constrained for the learning model to be identified. But this is true of *any* nonlinear dynamic model. For a structural econometrician, this is not a limitation—a model that specifies very general forms of state dependence and/or heterogeneity so as to obtain a good fit to the data is merely a statistical model with no structural/behavioral interpretation. Such a model cannot be used for policy experiments. Furthermore, Occam's razor suggests that we do not wish to work with such general models. What we seek are parsimonious models that fit well, give useful insights into the data, and can be used for policy experiments.

Recognizing the impossibility of completely nonparametric identification of learning effects, we can still give some contingent answers to the questions we asked at the start of §2.5. First, note that the Bayesian updating and normal signaling assumptions can be relaxed. We discuss some papers that do this in §3.1.1.

Second, in principle, one can distinguish learning from other plausible mechanisms that may generate state dependence (such as inventories or switching costs), but only if one is willing to specify parametric forms for the competing models. This is consistent with the Bayesian decision theory view that one "needs a model to beat a model." We return to this point in §4.

Third, the questions of whether we can identify the discount factor and whether we can test if consumers are forward-looking are obviously closely related. What is interesting, however, is that Ching et al. (2012a) show that, in the learning model, one can identify whether consumers are forward-looking using only (i) the laws of motion of the state variables and (ii) the form of current utility. Yet identification of the discount rate requires assumptions about the full structure (i.e., expectation formation) so that one can construct the expected value functions.

To see this, suppose we adopt the Geweke and Keane (2000) method to estimate dynamic models without the need to solve agents' DP problem and without imposing the full structure of the model. To implement their method, we take the value function in (21):

$$V(j,t\mid I_t) = \mathrm{E}[U(Q^E_{jt}, P_{jt})\mid I_t] + \beta\,\mathrm{EV}(I_{t+1}\mid I_t, j) \quad \text{for } j=0,\ldots,J, \tag{21$'$}$$

and replace it with the equation:

$$V(j,t\mid I_t) = \mathrm{E}[U(Q^E_{jt}, P_{jt})\mid I_t] + F[I_{t+1}(I_t, j)\mid \pi_t] \quad \text{for } j=0,\ldots,J. \tag{33}$$

Here, $F[I_{t+1}(I_t,j)\mid\pi_t] \approx \beta\,\mathrm{EV}(I_{t+1}\mid I_t, j)$ is a polynomial in the state variables that approximates the future component of the value function, and π_t is a vector of reduced-form parameters that characterize the future component. The idea of the Geweke–Keane method is to estimate the π_t jointly with the structural parameters that enter the current-period expected utility function.

Notice that, as F is just a flexible function of the state variables, all that is assumed is that consumers understand the laws of motion of the state variables (i.e., how $(I_{t+1}\mid I_t, j)$ is formed). They need not form expectations based on the true model. The approach is also agnostic about whether consumers use Bayesian updating or some other method. In general, identification of π_t requires exclusion restrictions such that some variable enters F but not U.[22]

[21] As Koopmans et al. (1950, pp. 63–64) state, "Suppose… [investigator] B is faced with the problem of identifying…the structural equations that alone reflect specified laws of economic behavior…. Statistical observation will in favorable circumstances permit him to estimate…the probability distribution of the variables. Under no circumstances whatever will passive statistical observation permit him to distinguish between different mathematically equivalent ways of writing down that distribution…. The only way in which he can hope to identify and measure individual structural equations…is with the help of a priori specifications of the form of each structural equation."

[22] Geweke and Keane (2000) point out that in the absence of exclusion restrictions, one must observe current payoffs (at least partially) to identify F. In labor economics, researchers argue that wages capture much of the current payoff (e.g., Houser 2003). Researchers can also control current payoffs in a lab experiment (e.g., Houser et al. 2004). Recently, Yao et al. (2012) propose another strategy to identify the discount factor. They argued that if a data

We see from (33) that, when the full structure is not imposed, one cost is that we lose identification of the discount factor. The β is subsumed as a scaling factor for the parameters π_t of the F function.[23] On the other hand, we can test whether $\pi_t = 0$, which is a test for forward-looking behavior. Although the test makes weak assumptions about F, it is *not* nonparametric, as a functional form must be chosen for the current payoff function. As Ching et al. (2012a) show, given the current payoff function, the π_t are identified in the learning model because different current choices lead to different values of next period's state variables.

2.6. Key Substantive Results of Erdem and Keane (1996)

Erdem and Keane (1996) estimate their model on Nielsen scanner panel data on liquid detergent from Sioux Falls, South Dakota. The sample period spanned 1986 to 1988, but only the last 51 weeks were used; at that time, telemeters were attached to panelists' TVs to measure household-specific ad exposures. The data include seven brands. A nice feature of the data is that three brands were introduced during the period, generating variability in consumers' familiarity with the brands. The estimation sample contained 167 households who met various criteria, such as having a working telemeter.

Some key issues that arose in estimation are worth discussing because they are common across many applications of dynamic learning models. First, Erdem and Keane (1996) had difficulty obtaining a precise estimate of the weekly discount factor, and so they pegged it at 0.995.[24] Identification of the discount factor is often a practical problem in dynamic models, even when it is formally identified. (We discuss this further in §4.3.) Second, the authors also found it difficult to pinpoint the prior mean of quality. Hence, they constrained it to equal the average true quality level across all brands. This implies peoples' priors are correct *on average*. They also constrained the prior uncertainty to be equal across brands, $\sigma_{j1} = \sigma_1$, as allowing it to differ did not significantly improve the fit.

Aside from the dynamic learning model, Erdem and Keane (1996) estimate two other models for comparison. These are a myopic learning model ($\beta = 0$) and a reduced-form model similar to that of Guadagni and Little (1983; henceforth referred to as GL). The latter is a multinomial logit with an exponentially smoothed weighted average of past purchases (the *loyalty* variable), a similar variable for ad exposures, a price coefficient, brand intercepts, and trends for values of no purchase and small brands.

It is striking that Erdem and Keane (1996) find that both structural learning models fit substantially better than the GL model.[25] This is surprising, as GL specifies flexible (albeit ad hoc) functional forms for effects of past use and ad exposures on current choice probabilities, whereas the Bayesian learning models impose a special structure. Specifically, as we saw in (17) and (18), only the *sum* of past experience or ad exposures matter in the Bayesian models, not the timing of signals.

Another striking result is that advertising is not significant in the GL model, implying that advertising has no effect on brand choice. In the EK model, there is no one coefficient to capture the effect of advertising. The parameter r is significant and positive, so consumers are risk averse with respect to quality, whereas the σ_1, σ_ε, and σ_A imply that (i) consumers have rather precise priors about new brands in the detergent category, and (ii) experience signals are much more accurate than ad signals. Yet the effect of advertising can only be assessed via simulations.

Erdem and Keane (1996) use their model to simulate an increase in ad frequency for Surf from 23% to 70%.[26] The simulation was also done for a hypothetical *new* brand with the characteristics of Surf. The results imply that an increase in advertising has little effect on market share for about four months, but the impact is substantial after about seven or

set consists of two regimes, a static environment and a dynamic environment, one can first estimate the parameters of the current payoff function using the static environment data and then hold them fixed when estimating the discount factor using the dynamic environment data. Their approach requires the assumption that the current payoff function remains unchanged across regimes.

[23] Several papers have explored using exclusion restrictions to estimate the discount factor. Chevalier and Goolsbee (2009) and, more recently, Ishihara and Ching (2012) use the resale value of a used good as an exclusion restriction in estimating dynamic demand models for new and used goods. In a dynamic store choice model, Ching et al. (2012b) use cumulative points earned via a reward program as an exclusion restriction. In a study of salesperson productivity, Chung et al. (2013) use cumulative sales as an exclusion restriction. The ideas presented in Ching et al. (2012b) and Chung et al. (2013) are similar: cumulative points (or sales) do not affect current payoffs until they reach certain cutoffs so that customers (or sales representatives) can receive a bonus. Fang and Wang (2010) show that even parameters of quasi-hyperbolic discounting can be identified if a dynamic model has exclusion restrictions and one has panel data with at least three periods.

[24] In trying to estimate the weekly discount factor, they obtained 1.001 with a standard error of 0.02. This standard error implies a large range of annual discount factors. It is also worth noting that Erdem and Keane (1996) set the terminal period for the DP problem at $T = 100$, which is 50 weeks past the end of the data set.

[25] The dynamic learning model had 16 parameters, whereas the other two models both had 15. Erdem and Keane (1996) obtain Bayesian information criterion values of 7,531, 7,384, and 7,378 for GL and the myopic and forward-looking learning models, respectively.

[26] "Ad frequency" is the weekly probability of a household seeing an ad for a brand. In the data this was 23% for Surf.

eight months. Thus, the model implies that advertising has little impact in the short run, but sustained advertising is important in the long run. As expected, the impact of advertising is much greater for a new brand (as there is more scope for learning).[27]

The advertising simulation results are not surprising in light of the parameter estimates. As consumers have rather precise priors about brands in the detergent category, and as ad signals are imprecise, it takes sustained advertising over a long period to move priors and/or reduce perceived risk of a brand to a significant degree.[28] A clear prediction is that the higher prior uncertainty is, and the more precise ad signals are, the larger the advertising effects will be, and the quicker they will become noticeable. Thus, an important agenda for the literature on learning models is to catalogue the magnitudes of prior uncertainty and signal variances across categories.

3. A Review of the Recent Literature on Learning Models

Here, we review developments in learning models subsequent to the foundational work discussed in §2. Almost all this work is post-2000, but it already forms a large literature. We divide the review into (i) more complex learning models with myopic agents, (ii) more complex learning models with forward-looking consumers, (iii) learning models for product-level/market share data, and (iv) new applications of learning models (beyond brand choice). We should note that our survey focuses on *empirical* structural learning models where agents are uncertain about product attributes. (There is a literature on dynamic games where agents learn how to play equilibrium strategies or how to coordinate in multiple equilibria settings including social learning environments. This literature is beyond the scope of our survey.)

3.1. Models with Myopic Agents

One stream of literature has focused on extending learning models by allowing for more complex learning mechanisms. To make such extensions feasible, it is often necessary to assume that consumers are myopic. We consider such models in the next two subsections that cover (i) models with more complex learning mechanisms and (ii) models with correlated learning.

3.1.1. More Complex Learning Mechanisms. Mehta et al. (2004) extend the Bayesian model to account for forgetting. Consumers imperfectly recall prior brand experiences, and the extent of forgetting increases with time. Therefore, a consumer's state depends on the timing of signals, not just the total number (as in Equation (6)). Thus, it is necessary to assume myopia to make modeling forgetting feasible.

Deighton (1984) propose that advertising has a transformative effect whereby it alters consumer assessment of the consumption experience. Mehta et al. (2008) include this effect in a learning model. They allow information signals from advertising to be biased, and this bias can change how consumers interpret their consumption experience. The identification of such a model is challenging. Mehta et al. (2008) can achieve identification because their data set includes consumers who rarely watch TV commercials. These consumers' choices allow one to identify true brand quality levels because their experience signals are not "contaminated" by the transformative effect of advertising. After controlling for the true mean brand qualities, the choices of the consumers who do watch TV commercials allow the authors to identify the bias of the advertising signals and the transformative effects of advertising.

Camacho et al. (2011) also model perception biases, but in a simpler framework. They argue that some types of experience may be more salient in certain contexts. For example, a physician may pay special attention to feedback from patients who have just switched treatment. The authors modify the standard Bayesian model by introducing a salience parameter to capture the extra weight physicians may attach to signals in that case. Using data on asthma drugs, they find evidence that feedback from switching patients receives 7–10 times more weight in physician learning than feedback from other patients.

Zhao et al. (2011) allow for consumer uncertainty about the precision of quality signals. In their model, consumers update their perception of this precision over time. In particular, consumers who receive a very negative experience signal may change their perception of signal variance. They estimate the model using scanner data that span the period of a product-harm crisis affecting Kraft Australia's peanut butter division in June 1996. Their model fits the data better than a standard learning model, which assumes consumers know the true signal variance.

3.1.2. Models of Correlated Learning. Another stream of literature models information spillover across brands, or correlated learning. By *correlated learning*, we mean learning about a brand in one

[27] We notice that Figure 1 in Erdem and Keane (1996) contains a typo. The scale on the y axis in Figure 1, which reports results for the new brand with the myopic model, is incorrectly labeled. It should be labeled in the same way as Figure 5. This does not affect any of the results discussed here.

[28] The simulation results also clarify why the GL model fails to find significant advertising effects. The *loyalty* variable tends to put more weight on recent advertising, and it discounts advertising from several months in the past. But the simulations show that advertising in the past few months does little to move market shares.

category by using the same brand in another category and/or learning about one attribute (e.g., drug potency) from another (e.g., side effects). This occurs if priors and/or signals are correlated across products or attributes.

Erdem (1998) considers a model where priors are correlated across "umbrella brands" (i.e., brands that operate in multiple categories). She finds evidence that consumers learn from experience across umbrella brands in the toothpaste and toothbrush categories. She shows that brand dilutions can occur if a brand in the parent category (toothbrush) is extended to a new product in a different category (toothpaste) and the new product is not well received. This framework has been extended to study decisions about fishing locations (Marcoul and Weninger 2008) and adoption of organic food products (Sridhar et al. 2012).[29]

Other papers have extended learning models to multiattribute settings where consumers use experience of one attribute to draw inferences about other attributes. Prescription drugs are a good example. Coscelli and Shum (2004) estimate a diffusion model for omeprazole, an anti-ulcer drug. The drug can treat (i) heartburn, (ii) hypersecretory conditions, and (iii) peptic ulcers, and it can provide (iv) maintenance therapy. In the model, physicians know how signals are correlated across the four conditions. In each patient–physician encounter, a physician only observes a signal of the condition being treated, but he or she uses it to update his or her multidimensional prior belief.

Chan et al. (2013) also apply a multiattribute learning model to the drug market. They assume experience signals are correlated on the two dimensions of side effects and effectiveness. They achieve identification by supplementing revealed preference data with data on self-reported reasons for switching: side effects or ineffectiveness. Interestingly, they find detailing visits are more effective in reducing uncertainty about effectiveness than side effects.

3.2. More Sophisticated Learning Models with Forward-Looking Consumers

Following Erdem and Keane (1996), several papers have made significant contributions in the area of learning models with forward-looking consumers. We discuss these in turn.

Ackerberg (2003) deviates from Erdem and Keane in several dimensions. Most notably, he models both informative and persuasive effects of advertising.[30] The persuasive effect is modeled as advertising intensity shifting consumer utility directly. The informational effect is modeled by allowing consumers to draw inferences about brand quality based on advertising intensity.[31] This is quite different from the information mechanism in Erdem and Keane (1996), where ad content provides noisy signals of quality. Other differences are as follows: (i) Ackerberg is primarily interested in learning about a new product, and his model allows for heterogeneity in consumers' match value with the new product; and (ii) he assumes it takes only one trial for consumers to learn the true match value. Estimating the model on scanner data for yogurt, Ackerberg finds a strong, positive informational effect of advertising. However, the persuasive effect is not significant.

The key innovation of Crawford and Shum (2005) is allowing for multiattribute learning. In an application to prescription drugs, they argue that panel data allow them to identify two effects: (i) symptomatic effects, which affect a patient's per-period utility via symptom relief; and (ii) curative effects, which alter the probability of recovery. They allowed physicians and patients to have uncertainty along both dimensions (although they abstract from correlated learning). They also endogenize the length of treatment by allowing patients to recover. Their estimates imply substantial patient heterogeneity in drug efficacy. They go on to study the welfare cost of uncertainty relative to the first-best environment with no uncertainty. Welfare questions cannot be addressed without a structural model. However, after estimating their model, Crawford and Shum can simulate the removal of uncertainty by setting the initial prior variance to zero and setting each consumer's prior match value to be the true match value. By conducting this experiment, they find that consumer learning allows consumers to dramatically reduce the costs of uncertainty.

[29] Hendricks and Sorensen (2009) use a similar idea of information spillover to explain the skewness of music CD sales. They find evidence that a successful new album release by an artist increases the likelihood that consumers purchase that same artist's older albums.

[30] The separate identification of informative and persuasive effects of advertising relies on this qualitative implication of learning models: as consumers gather more information over time, the marginal benefits of informative advertising must fall. Therefore, if advertising has any impact on brand choice in the long run, it is due to persuasive advertising. To our knowledge, Leffler (1981) is the first paper that proposes this identification strategy. He implements it in a reduced-form model using product-level sales data for new and old prescription drugs. Narayanan et al. (2005) make use of the same identification argument when estimating their structural model using product-level data. Recently, Ching and Ishihara (2012) propose a new identification strategy to attack this problem: they argue that informative advertising should affect all products that share the same features/ingredients equally, but persuasive advertising should be brand specific. Ching and Ishihara implement their identification strategy in a prescription drug market where some drugs have the same active ingredient but are marketed under different brand names.

[31] That is, ad frequency itself signals brand quality, as in the theoretical literature on "advertising as burning money" (which only high-quality brands can afford to do) (Kihlstrom and Riordan 1984).

Erdem et al. (2008) is the first paper to our knowledge to model the quality signaling role of price in the context of frequently purchased goods. They also allow both advertising frequency and advertising content to signal quality (combining the features of Ackerberg 2003 and Erdem and Keane 1996). And they allow use experience to signal quality such that consumers may engage in strategic sampling. Thus, this is the only paper that allows for these four key sources of information simultaneously. In the ketchup category, they find that use experience provides the most precise information, followed by price, then advertising. The direct information provided by ad signals is found to be more precise than the indirect information provided by ad frequency. The main finding of Erdem et al. (2008), obtained via simulation of their model, is that when price signals quality, frequent price promotions can erode brand equity in the long run. They note that there is a striking similarity between the effect of price cuts in their model and in an inventory model. In each case, frequent price cuts reduce consumers' willingness to pay for a product: in the signaling case, it is done by reducing perceived quality; in the inventory case, by making it optimal to wait for discounts. We return to this issue in §4.

Osborne (2011) is the first paper to allow for both learning and switching costs as sources of state dependence in a forward-looking learning model. This is important because learning is the only source of brand loyalty in Erdem and Keane (1996). So it is possible they only found learning to be important as a result of omitted switching costs. However, Osborne finds evidence that both learning and switching costs are present in the laundry detergent category. When learning is ignored, cross elasticities are underestimated by up to 45%.[32]

Erdem et al. (2005) represents a significant extension of previous learning models, as it is the first paper where consumers actively decide how much effort to devote to search before buying a durable. This contrasts with Roberts and Urban (1988), where word-of-mouth (WOM) signals are assumed to arrive exogenously, or Erdem and Keane (1996), where ad signals arrive exogenously. Another novel feature of the paper is that there are several information sources to choose from (WOM, advertisements, magazine articles, etc.). Also, in each period, consumers decide how many of these sources to use.[33] In their application, Erdem et al. (2005) consider technology adoption (Apple/Mac versus Windows) in personal computer markets where there is quality and price uncertainty. As in the brand choice problem, consumers are not perfectly informed about competing technologies. A special aspect of high-tech durables, however, is rapid technical progress. This causes the price of PCs to fall rapidly over time. Thus, there are two incentives to delay purchase: (i) to get a better price and (ii) to search for more information about the technologies. Delay, however, implies a forgone utility of consumption. Erdem et al. (2005) estimate their model on survey panel data collected from consumers who are in the market for a PC. Their results indicate that consumers defer purchases both to gather more information and to get a better price. But, perhaps surprisingly, simulations of their model imply that learning is the more important reason for purchase delay.

Another way for consumers to learn is by observing other consumers' choices instead of their opinions, i.e., observational learning (Banerjee 1992). To capture this idea, Zhang (2010) develops a new product adoption model with observational learning. In her model, consumers are forward-looking and heterogeneous. One interesting implication of her model is that observational learning would lead to slower product adoption compared with full informational sharing (i.e., all experience signals are common knowledge). She estimates her model using data from the U.S. kidney market and, using a counterfactual experiment, quantifies the extent of inefficiency caused by observational learning (compared with full informational sharing).[34]

Che et al. (2011) develop a forward-looking consumer brand choice model with spillover effects in learning and changing consumer needs over time. Theirs is the first paper to model correlated learning with forward-looking consumers. They estimate their model using scanner data for the disposable diapers category, where consumers have to go up a size periodically to accommodate growing babies. This leads to an increase in strategic trial around the time of needing to change sizes. Their results imply that consumer experience of a particular size of a brand provides a quality signal for other sizes, and consumer quality sensitivities are lower and price sensitivities higher for larger sizes than they are for smaller sizes.

Finally, Dickstein (2012) considers a model in which forward-looking physicians are uncertain

[32] Osborne (2011) allows for a continuous distribution of consumer types. Of course, it is impossible to solve the DP problem for each type (which is why the DP literature usually assumes discrete types). Thus, some approximation is necessary here. Osborne estimates his model by adapting the Markov chain Monte Carlo algorithm developed by Imai et al. (2009) and extended by Norets (2009) to accommodate serially correlated errors. It is worth noting that Narayanan and Manchanda (2009) also estimate a learning model with continuous distribution of consumer types. To estimate their model, however, they needed to assume that agents are myopic.

[33] Chintagunta et al. (2012) model how physicians learn about drugs using both patients' experiences and detailing.

[34] Cai et al. (2009) provide interesting evidence for observational learning by studying a natural experiment where customers of a restaurant are given a ranking of some popular dishes.

about patients' intrinsic preferences for *multiple* drug attributes.[35] This is the first model of forward-looking agents that allows for information spillover across *alternatives*. A physician uses patients' utility of consuming a drug at time t to update his or her belief about patients' preferences parameters. The Bayesian updating procedure for physicians is similar to Bayesian inference in a linear regression model. An interesting implication of this approach is that, after seeing a negative outcome of drug A, physicians may want to avoid other drugs that share some of the attributes of drug A.[36]

3.3. Modeling Consumer Learning Using Product-Level Data

The estimation technique developed by Berry et al. (1995) (henceforth referred to as BLP) led to a large body of demand analysis that applies static discrete choice models primarily to product-level or market share data.[37] In general, however, BLP cannot be used to estimate the demand systems generated by consumer learning models. Learning models are always dynamic in that current sales affect future demand regardless of whether consumers are forward-looking or myopic. Demand for *one* brand in a learning model depends on the whole distribution (across consumers) of perceived quality for *all* brands. We are skeptical about whether individual heterogeneity distributions can be *credibly* identified from aggregate (i.e., product-level) data. Furthermore, it is difficult to combine such a complex demand system with a supply side model.

If one wants to estimate a dynamic demand system with consumer learning and only market share data are available, clearly one has to abstract from the endogenous consumer heterogeneity generated by individual-level purchase histories (see §2.1, Equation (5)). To address this issue, Narayanan et al. (2005) and Ching (2000, 2010a) propose two related modifications of the EK framework. Narayanan et al. (2005) assume that every agent has an identical purchase history; i.e., for each brand, the quantity purchased is equally distributed across agents in each period. Ching (2000, 2010a) assume that consumers can learn from each other's experiences via social networks or information-gathering institutions (e.g., physician networks). As a result, consumers use the same set of experience signals to update their beliefs, and all consumers share a common belief at any point of time.[38] This assumption eliminates the distribution of consumers across different states as the state variable for firms. Both Narayanan et al. (2005) and Ching (2000, 2010a) parsimoniously capture consumer learning, so that, when combined with an oligopolistic supply side, the size of the state space is manageable. Both papers applied their frameworks to study the demand for prescription drugs.[39,40]

Because a demand system with consumer learning is always dynamic, we would expect firms to be forward-looking when choosing their marketing mix. In an initial attempt to address this issue, Ching (2010b) extends Ching (2010a) by combining a social learning demand model with a dynamic oligopolistic supply-side model. As far as we know, this is the first empirical paper to combine a dynamic demand system with forward-looking firms and to endogenize price. In the model, consumers *and* firms are uncertain about the quality of generic drugs, but they rely on the same information set to update their belief over time, and hence perceived quality and variance are common for consumers and firms. Equilibrium is Markov-perfect Nash, as in Maskin and Tirole (1988) and Ericson and Pakes (1995). The model is tailored to study the competition between brand-name and generic drugs. Ching (2010b) applies his model to the market for clonidine. Simulations of the model show that it can rationalize two important stylized facts: (i) that the diffusion of generic drugs is slow and (ii) that brand-name firms *slowly* raise their prices after generic entry.[41]

Extending Ching's (2010a) model, the model of Ching and Ishihara (2010; hereafter abbreviated to CI) seeks to explain an important stylized fact about the drug market: that the effectiveness of detailing

[35] Learning about preferences may appear to be different from learning about attributes. Yet the two are equivalent as long as one assumes that the utility function is linear in attributes and preference weights.

[36] To estimate his model, Dickstein (2012) uses the Gittins index approach (Gittins and Jones 1979). Yet, as in Eckstein et al. (1988), who also use that approach, he needed to assume consumers are risk neutral. It is worth noting that Ferreyra and Kosenok (2011) use a method similar to the Gittins index to estimate a simpler dynamic learning problem.

[37] Ackerberg et al. (2007) provide an excellent survey of this area. Note that product- or market-level data are more readily available than scanner data for many industries.

[38] An interesting paper that studies both across and within consumer learning was written by Chintagunta et al. (2009). They apply their model to doctors' prescribing decisions for Cox-2 inhibitors, a new class of pain killers, and find that both types of learning are important.

[39] Chen et al. (2013) and Moretti (2011) use a similar framework to study the impact of WOM on movie sales.

[40] The BLP estimation method can be applied to the demand model of Narayanan et al. (2005), but it cannot be applied to the model of Ching (2000, 2010a). Because of space constraints, we will not discuss the details here. Interested readers may refer to Ching (2010a) for a detailed discussion.

[41] Ching's model includes heterogeneity in consumer price sensitivity. As learning takes place, an increasing proportion of price-sensitive consumers switch to generics. Hence, the demand faced by the brand-name firms becomes more inelastic over time. This is why they gradually increase price.

changes when new information arrives (e.g., if a new clinical trial is positive, effectiveness of detailing increases). Standard Bayesian learning models such as EK cannot generate this pattern because they imply that the marginal impact of information signals falls over time (see Equations (17) and (18)). Thus, the CI model deviates from the EK framework by introducing three new features: (i) both consumers and firms are uncertain about the quality of the product, (ii) social learning takes place via an intermediary (opinion leader, consumer watch group, etc.) who updates the information set for each brand, and (iii) detailing is designed to build a stock of physicians who are familiar with the most recent information set of the promoted brand.[42] The model generates heterogeneity in information sets, as the fraction of physicians with the most up-to-date information about a brand is a function of its cumulative detailing. Using this framework, Ching and Ishihara (2010) are able to quantify how effectiveness of detailing changes when a new clinical trial outcome is released.

Lim and Ching (2012) extend the CI framework to a multidimensional learning model with correlated beliefs, and they apply it to study demand for the major class of anticholesterol drugs (i.e., statins). The CI model may be applicable to settings other than drugs where the interaction between news and informative advertising is of first-order importance. The model is parsimonious and could, in principle, be combined with an oligopolistic supply side, but this has not yet been done.

An interesting paper by Hitsch (2006) studies firm learning about the demand for a new product. He abstracts from consumer learning by using a reduced-form demand model. That is, unlike other papers we have discussed, consumers have no uncertainty about new products. Still, a firm needs to learn the true demand parameters. Hitsch considers a one-side learning equilibrium model and also abstracts from competition. These simplifications significantly reduce the computational burden of the estimation, and yet the model delivers important new insights to the product launch and exit problem. Finally, it is worth noting that, because of computational burden, no paper has yet estimated a model with both forward-looking consumers and firms.

3.4. Other Applications of Learning Models: Services, Insurance, Media, Tariffs, etc.

Learning models have been applied to many problems other than choice among different product brands. In particular, Bayesian learning models have been applied to choice among services, insurance plans, media, and tariffs, for example. Here, we discuss these types of applications.

Israel (2005) uses a learning model to study the customer–firm relationship in the auto insurance market. This environment is well suited for studying learning because opportunities to learn arrive exogenously when an accident happens. Presumably, when consumers file a claim, they learn something about the customer service of the insurance company. If a consumer leaves the company after filing a claim, it may indicate that they had a negative experience.

Chan and Hamilton (2006) is a surprising application of an Erdem and Keane-style learning model to study a clinical trial of HIV treatments. The natural experiment school of econometrics views clinical trials as the gold standard to which economics should aspire so as to avoid structural modeling (see Keane 2010a). Yet Chan and Hamilton show how a structural learning model helps to evaluate clinical trial outcomes by correcting for endogenous attrition. In their model, patients in a trial are uncertain about the effectiveness and side effects of treatment, but they learn from experience signals. In each period, they need to decide whether to quit based on costs (side effects) or to stay to reap the expected benefits of continuing the trial. The authors found that a structural interpretation of clinical trial results can be very different from the standard approach. For example, a low CD4 cell count implies a weaker immune system. A treatment that is less effective in increasing CD4 counts may still be preferable because it has fewer side effects. Fernandez (2013) extends Chan and Hamilton by allowing patients to be uncertain about whether they are assigned to a treatment or control group.

Tariff choice is another area where Bayesian learning models have been useful. It is widely believed that consumers are irrational when they choose between flat-rate and per-use plans. Several studies have found that many consumers could save by switching to a per-use option. Yet these papers tend to look at behavior over a short period. By using a longer period, Miravete (2003) finds strong evidence to contradict the irrational consumer view. Using data from the 1986 Kentucky tariff experiment, he provides evidence that consumers learn their actual usage rates over time and switch plans to minimize their monthly bills.

Narayanan et al. (2007) interpret the same data using a Bayesian learning model with myopic consumers. To explain why consumers make mistakes in choosing an initial plan, the authors assume that consumers are uncertain about their actual usage. The structural approach allows them to quantify changes in consumer welfare under different counterfactual experiments. Iyengar et al. (2007) develop

[42] This is in contrast to standard models where advertising is modeled as a noisy signal of true mean brand quality.

another myopic learning model that is closely related, but they also allow consumers to be uncertain about the quality of service.

Goettler and Clay (2011) use a Bayesian learning model to infer switching costs for tariff plans. They do not observe consumers switching plans in their data. Identification of switching costs is achieved by assuming that consumers are forward-looking, have rational expectations about their own match value, and make plan choice decisions every period after their initial enrollment. The implied cost to rationalize no switching is quite high ($208 per month).

Grubb and Osborne (2012) provide an alternative explanation for infrequent plan switching, i.e., that consumers do not consider plan choice every period (as consideration is costly and/or time consuming). They model the consideration decision using the "price consideration model" of Ching et al. (2009) and the switching decision as a Bayesian learning model. Their rich data set allows them to investigate prior mean bias, projection bias, and overconfidence.

Finally, learning models have also been extended to study the value of certification systems (Chernew et al. 2008, Xiao 2010), TV program choices (Anand and Shachar 2011), fertility decisions (Mira 2007), spousal interactions (Yang et al. 2010), bargaining problems (Watanabe 2009), a manager's job assignment problem (Pastorino 2012), human capital investment problems (Stange 2012, Stinebrickner and Stinebrickner 2012, Hoffman and Burks 2013), and voters' decision problems (Knight and Schiff 2010).

4. Limitations of the Existing Literature and Directions for Future Work

In this section we discuss limitations of existing learning models and directions for future research. In our view, the four main limitations are as follows: (1) It is difficult to identify complex models with rich specifications of consumer behavior. (2) It is difficult to disentangle different sources of dynamics. (3) There is no clear consensus on forward-looking versus myopic consumers. And (4) more work is needed on how to estimate equilibrium models with consumer learning.

4.1. Identification in Behaviorally Rich Specifications

In §2.5.1 we discussed the formal identification of learning models. This topic is also addressed in a number of other papers such as Erdem et al. (2008). By *formal identification*, we refer to a proof that a parameter is identified, given the structure of the model, as well as a discussion of any normalizations that are needed to achieve identification. However, an important point (see §2.6) is that it is common in complex models for a parameter to be formally identified, and yet (i) the intuition for what data patterns actually pin it down are not clear, and/or (ii) the likelihood is so close to flat in the parameter that it is not practical to estimate it in practice (what Keane 1992, p. 193, called "fragile identification"). These problems are not unique to dynamic learning models, but they deserve further attention in this context.

As noted in §2.5.1, in the EK model, the true qualities of brands, utility weights, and the price coefficient are identified from choices of households with sufficient experience of all brands so that their learning has effectively ceased; i.e., their choice behavior is stationary. In contrast, the learning parameters (risk aversion, prior uncertainty, signal variability) are pinned down by choice behavior of households with less experience (and how it differs from more experienced households). However, when one extends the basic setup of the EK learning model, it becomes more difficult to understand what data patterns help identify the structural parameters.

As noted in §3, there is a healthy trend toward specifying behaviorally richer, and hence more complex, learning models. Examples are models where consumers have multiple sources of information or learn about multiple objects or models that incorporate findings from psychology/behavioral economics. But this added complexity creates three problems: First, formal identification of complex models may be difficult. Second, even if formally identified, complex models may suffer from "fragile" identification in practice. Third, it may be difficult to understand what sources of variation in the data pin down certain parameters.

Of particular importance is that, given only revealed preference (RP) data on purchase decisions and signal exposures, it may be hard to identify models with complex learning mechanisms or to distinguish among alternative learning mechanisms (i.e., multiple mechanisms may fit the data almost equally well). A promising solution to this problem is to combine RP data with stated preference (SP) data that attempt to directly measure the learning process (also known as process data). For example, consider the paper by Erdem et al. (2005) on how consumers learn about computers. In addition to RP data, they also have data on how people rated each brand in each period leading up to purchase. They treat the SP data as providing noisy measures of consumer's perceptions. This enables them to identify variances of different information sources. Intuitively, if people's ratings tend to move a lot after seeing an information source (and their perceived uncertainty tends to fall a lot), it implies that the information source is perceived as accurate. Another paper that combines RP and SP data to aid in identification is Shin et al. (2012), which is an attempt to disentangle preference heterogeneity from learning.

An alternative approach is to combine choice data with direct measures of information signals. Roberts and Urban (1988) do this in their original paper. Ching and Ishihara (2010) and Lim and Ching (2012) use results of clinical trials to measure the content of signals received by physicians and incorporate this into their structural learning models. In a reduced-form study, Ching et al. (2011) use data on media coverage of prescription drugs and found evidence that when patients learn that anticholesterol drugs can reduce heart disease risk, they become more likely to adopt them. Kalra et al. (2011) attempt to pin down the content of information signals by examining news articles. In a study of doctor's prescribing decisions for Cox-2 inhibitors, Chintagunta et al. (2009) use patient diary data that record actual use experience.[43] In sum, there has been some work in this area, but there is obviously much room for further progress.[44]

4.2. Distinguishing Among Different Sources of Dynamics

Learning is one of many mechanisms that may cause structural state dependence. Other potential sources of state dependence include inertia, switching costs, habit persistence, and inventories. In this section we discuss attempts to distinguish among these sources of dynamics. We particularly emphasize the problem of distinguishing between learning and inventories, because most dynamic structural models in marketing have assumed that one of these mechanisms is the source of dynamics. Furthermore, and perhaps surprisingly, the behavioral patterns generated by learning can be quite similar to those generated by inventories. Thus it can be very difficult to identify which mechanism generates the state dependence we see in the data.

Learning and inventory models generate dynamics in very different ways. Learning models generate persistence in choices (brand loyalty) as risk aversion leads consumers to stay with "familiar" brands. This familiarity arises endogenously via information signals that cause consumers to gravitate toward particular brands early in the choice process. Inventory models, in contrast, do not generate persistence in brand choices. Rather, they must assume the existence of a priori consumer taste heterogeneity to generate loyalty. Obviously, a great appeal of learning models is that they provide a behavioral explanation for the emergence of brand loyalty.

However, once we introduce unobserved taste heterogeneity, the dynamics generated by learning and inventory models are rather hard to distinguish empirically. The similarity of the two models is discussed extensively by Erdem et al. (2008). They fit a learning model to essentially the same data used in the inventory model of Erdem et al. (2003). They find that both models fit the data almost equally well and make very similar predictions about choice dynamics. For instance, both models predict that, in response to a price cut, much of the increase in a brand's sales is due to purchase acceleration rather than brand switching.

The similarity of the two models is even greater if we allow for price as a signal of quality. Then, both models predict that frequent price promotion will reduce consumers' willingness to pay for a product; in the signaling case, it is done by reducing perceived quality, and in the inventory case, it is done by changing price expectations and making it optimal to wait for discounts.

An important avenue for future research is the determination of whether learning or inventory effects are of primary importance for explaining consumer choice behavior or, indeed, if both mechanisms are important. Unfortunately, however, computational limitations make it infeasible to estimate models with both learning and inventories. There are simply too many state variables—levels of perceived quality and uncertainty for all brands, inventory of all brands, current and lagged prices of all brands—to make a solution and estimation feasible. This makes it impossible to nest learning and inventory models and assess the quantitative importance of each mechanism. Advances in computation may remove this barrier in the future.

Meanwhile, some authors have proposed simpler approaches to test whether learning or inventories (or both) generate choice dynamics. Ching et al. (2012a) present a new quasi-structural approach that lets one estimate models with both learning and inventory effects while also testing for forward-looking behavior (strategic trial). The idea is to approximate the Emax functions in (23) using simple functions of state variables. The learning model generates a natural exclusion restriction: the Emax function associated with the choice of brand j contains the *updated* perception variance for brand j, whereas the current payoff and the Emax functions for all brands $k \neq j$ contain the *current* perception variance

[43] One limitation of their paper is that they treat the discrete signals from patients' diaries as a continuous variable. Chernew et al. (2008) show how to use data of this type by estimating a model with a discrete learning process.

[44] Hu et al. (2013) propose using eye-movement data collected in lab experiments as a proxy for preferences. By combining it with RP data, they are able to relax the Bayesian learning assumption and infer learning rules in a flexible way. And Ching et al. (2010) propose a new way to measure the extent to which consumers know the true quality of quasi-durable products by extending the price consideration model.

for brand j. Ching et al. (2012a) apply the method to the diaper category, which is ideal for studying learning because an exogenous event—in this case, the birth of a first child—triggers entry into the market. Their results suggest that learning and strategic trial are quite important, whereas inventories are a much less important source of dynamics for diapers.

Erdem et al. (2010) propose a simple test of the relative importance of learning versus inventories. They consider the learning mechanism where consumers use price as a signal of quality. They also exploit the fact that inventory models generate "reference" price effects (i.e., choices are based on the current price *relative* to the reference price of a brand). Their test relies on the interaction between a use experience term and the reference price (operationalized as an average of past prices). In a learning model, greater use experience should be associated with reduced use of price as a quality signal. Based on this test, they find evidence for both learning and inventory (i.e., reference price) effects for two frequently purchased goods (ketchup and diapers).

A simple alternative to nesting learning with other models of dynamics is to estimate a structural learning model and include a lagged choice variable in the payoff functions to capture any "leftover" state dependence in a reduced-form way. This model is identified, as lagged choice does not enter the EK learning model (only cumulative choices matter). However, it is difficult to interpret the lag coefficient. Osborne (2011) adopts this approach and calls the lag coefficient "switching costs." But there are many possible explanations, including inertia, inventories, habit persistence, and recency effects in learning. Suppose the standard Bayesian model is not literally true, and consumers put extra weight on recent signals. Then a lagged purchase variable may just absorb the misspecification of the learning process. In general, we are skeptical that including nonstructural elements in a learning model can be informative about the importance of learning versus other mechanisms that generate dynamics. We believe that the nesting of learning and other mechanisms and the incorporation of process data are needed to make progress.

4.3. Forward-Looking vs. Myopic Consumers

As we discussed in §2, the key distinction between forward-looking and myopic models is whether consumers engage in strategic trial. But the evidence on whether consumers are forward-looking is mixed. Indeed, in many applications, researchers have found it difficult to identify the discount factor because the likelihood is rather flat in this parameter. For instance, in the detergent category, Erdem and Keane (1996) find that increasing the discount factor from 0 (a myopic model) to 0.995 improved likelihood by only six points. That was significant, but if the likelihood is so flat in the discount factor, it may be hard to discern forward-looking behavior.[45]

Because forward-looking models may not provide substantial fit improvements (and because they are much harder to estimate), it is not surprising that many researchers have adopted myopic models, as we saw in §3.1. But before taking this path, it is important to emphasize that strategic trial is the distinguishing feature of forward-looking models. In a mature category, consumers may have nearly complete information, leaving little to gain from trial purchase. Then a forward-looking consumer will behave much like a myopic one. It is impossible to tell the two types apart, and the discount factor is not identified.

Given this observation, the small likelihood improvement that Erdem and Keane (1996) found may not be surprising; subjective prior uncertainty is fairly low for detergent, so perceived gains from trial are small. In contrast, in a market with significant uncertainty about product attributes, the rate of trial would be higher.[46] Forward-looking models may provide a superior fit in such markets.

Thus, we think it would be a mistake to infer from results on relatively mature categories that forward-looking models are unnecessary. This decision should be made on a case-by-case basis given the characteristics of the category under consideration. A key agenda item for the literature on learning models is to compare the degree of prior uncertainty across categories to determine when forward-looking behavior is most important.

Furthermore, we believe developing simple tests for forward-looking behavior (such as the test in the Ching et al. 2012a quasi-structural model) should be an important topic for future research. However, we also believe such tests must be derived explicitly from a theoretical model. There has been a recent trend whereby researchers seek to develop "model-free" tests for the assumptions of a structural model. We comment on this trend in §4.5.

4.4. Integration of Learning Models with Supply-Side Models

As we discussed in §3.3, there has been significant progress in developing dynamic demand systems with consumer learning that can be estimated using product-level data. Examples include Ching (2000, 2010a), Ching and Ishihara (2010), and Narayanan et al. (2005). Nevertheless, there is clearly a large discontinuity between their models and those that are

[45] Other examples include Chintagunta et al. (2012), Dickstein (2012), and Yang and Ching (2013).

[46] Note that the higher the discount factor, the higher the rate of strategic trial.

applied to individual-level data. All the models in §3.3 abstract from self-learning and, for reasons of tractability, assume the existence of an information aggregator.[47] In our opinion, self-learning is still an important source of information for frequently purchased goods despite the advance of social networks. Moreover, none of the models in §3.3 allows for forward-looking consumers. Therefore, we believe that developing a richer aggregate dynamic demand system with learning remains a challenging and important area for future research.

Most of the demand analyses that use product-level data are motivated by the ultimate goal of combining it with firms' problems to build an equilibrium model in order to study the long-term outcomes of certain policy changes (e.g., advertising regulations, anticompetitive pricing regulation, merger analysis). The key challenge is how to model the firms' problem when facing such a complex dynamic demand system. The demand system generated by the EK framework (or similar models) is so complex that it is very difficult to analyze even in a monopoly situation. It is unclear whether the fully rational approach to modeling firms' decisions is possible given computational costs of keeping track of such a complicated state space.

Thus, an important avenue for future research is the development of a dynamic demand system with learning that is not too costly for firms to use yet can capture the potential forward-looking and strategic trial behavior of consumers. Hendel and Nevo (2013) take this research direction, but in the context of storable, not experience, goods. Their demand model is motivated by the dynamic stockpiling models in Erdem et al. (2003) and Hendel and Nevo (2006) but is much simpler to estimate and tractable to combine with forward-looking firms. They show how their model can be used to study intertemporal price discrimination empirically.

4.5. Model-Free Evidence on the Validity of Structural Models?

Structural models in general, and learning models in particular, are often criticized on the grounds that they make a large number of assumptions (e.g., about how consumers learn and form expectations, the functional form of utility). Identification of these models relies on these functional form assumptions. Critics of structural models often argue that we should prefer simple methods and/or model-free evidence. The debate on this topic is extensive and beyond the scope of this survey. For further discussion, we refer the reader to articles such as Heckman (1997), Keane (2010a, b), and Rust (2010). These authors argue that drawing inferences from data always relies on some set of maintained assumptions. They also argue that simple reduced-form or statistical models typically rely on just as many assumptions as structural models, the main difference being that the simple models leave many assumptions implicit. Here, instead of repeating their general arguments, we illustrate our point by discussing two papers that use such simple approaches to test for learning behavior.[48]

Example 1 (Chintagunta et al. 2012). Chintagunta et al. (2012) present reduced-form evidence of forward-looking behavior by physicians. More specifically, when a new drug is introduced, they focus on the set of physicians who have not yet been exposed to detailing. They run a logit model to predict whether a physician will prescribe the new drug to a patient. The key point is they include *future detailing* as a regressor. Suppose there is some risk involved in experimenting with the drug now; future detailing is an opportunity to learn without risk. The author argue that if physicians are forward-looking, they will be more likely to respond to future detailing and less likely to prescribe the drug now. So a negative coefficient on future detailing suggests that physicians are forward-looking.

However, this model-free test implicitly assumes that there is no physician heterogeneity in receptivity to detailing. Yet it is plausible that some physicians are more skeptical about sales rep presentations, so they require more detailing to be convinced. This could cause sales reps to spend more time with less receptive physicians. Then, the coefficient on future detailing may be negative even if physicians are myopic. More generally, including a future variable in a regression is a Sims strict exogeneity test (Sims 1972). It may just be that a current prescription reduces future detailing. Thus, although the test result is interesting, it is difficult to interpret.

Example 2 (Dubé et al. 2010). We now turn to our second example. In an attempt to distinguish learning from other sources of state dependence such as switching costs, inertia, or habit persistence, Dubé et al. (2010) estimate the following simple discrete choice model:

$$V_{jt} = \alpha_j - w_P P_{jt} + \gamma_1 d_{j,\,t-1} + \gamma_2 d_{j,\,t-1} \cdot N_j(t) + \gamma_3 N_j(t) + e_{jt}. \tag{34}$$

Note that (34) contains lagged choice, cumulative use experience $N_j(t)$, and their interaction. So it could be

[47] This is a good illustration of how structural modeling often involves a trade-off between richness and tractability.

[48] For another example, see Ching (2013) for a critique on Moretti (2011).

viewed as a linear approximation of the more complex nonlinear form implied by the learning model (see Equations (5), (6), and (27)).

Now suppose the learning model is correct. Dubé et al. (2010) argue that, for experienced consumers who have complete information, lagged choice should not be a predictor of current choice.[49] When cumulative experience is large, the additional impact of more experience on the perceived variance of a brand is trivial (see Equation (6)).[50] More generally, the fact that use experience $N_j(t)$ reduces the effect of lagged purchase implies that the interaction coefficient γ_2 should be negative. However, using data on margarine and frozen orange juice, Dubé et al. find that more experience does not reduce the lagged choice effect ($\gamma_2 \approx 0$). They interpret this as evidence against consumer learning.

It is tempting to treat this as a model-free test because it does not impose the functional form assumptions required to estimate a fully specified learning model. Yet this interpretation is incorrect. First, the test fails to account for a key feature of the Bayesian learning model: when $N_j(t)$ is sufficiently large such that a consumer knows almost everything about brand j, any further increase in $N_j(t)$ has a negligible impact on utility. However, Equation (34) does not allow for this possibility, as $\partial V/\partial N = \gamma_3 + \gamma_2 d$, which is independent of $N_j(t)$. Second, when $N_j(t)$ is small, the impact of $d_{j,t-1}$ can be positive or negative depending on the realization of the experience signal relative to one's prior. Therefore, the sign of the interaction term is ambiguous. Combining these two points, it is possible that γ_2 may be close to zero, even if there is consumer learning in the data. So again, the test result is interesting, but it is difficult to interpret.

We believe that searching for data patterns that are potentially consistent or inconsistent with a structural model is a useful exercise. It can often provide valuable insights, and it can be a useful part of the process of building, validating, and improving structural models. However, we do not believe that simple models and/or model-free evidence can ever replace structural models or the key role of theory in empirical work more generally.

[49] Given controls for taste heterogeneity (α_j), the lagged choice variable $d_{j,t-1}$ can matter for several reasons, such as inertia, switching costs, habit persistence, inventories, or learning. Finding that lagged choice is significant for consumers with complete information may simply mean that sources of dynamics besides learning are also present.

[50] It is important to note that not all learning models imply that choice behavior becomes stationary given sufficient use experience. For instance, as discussed in §3.1.1, Mehta et al. (2004) extend the basic model to allow forgetting. It is also possible that product attributes change over time. Thus, it is conceptually straightforward to construct learning models where recent experience is more salient for a variety of reasons.

It is important to remember that truly model-free evidence cannot exist. The simple empirical work that promises to deliver such evidence always relies on some assumptions. These assumptions are often left implicit as a result of failure to present an explicit model. Often these implicit assumptions are (i) not obvious, (ii) hard to understand, and (iii) very strong. One of the main contributions of structural learning models to both marketing science and economics has been to generate increasing interest in the structural paradigm. We hope this will be a long-term trend regardless of future evaluations on the usefulness of the learning model per se.

5. Summary and Conclusion

In this survey we laid out the basic Bayesian learning model of brand choice, pioneered by Eckstein et al. (1988), Roberts and Urban (1988), and Erdem and Keane (1996). We described how subsequent work has extended the model in important ways. For instance, we now have models where consumers learn about multiple product attributes and/or use multiple information sources, and they can even learn from others via social networks. The model has also been applied to many interesting topics well beyond the case of brand choice, such as how consumers learn about different services, tariffs, forms of entertainment, medical treatments, and drugs.

We also identified some limitations of the existing literature. Clearly, an important avenue for future research is the development of richer models of learning behavior. For instance, it would be desirable to develop models that allow for consumer forgetting, changes in product attributes over time, a greater variety of information sources, and the like. Yet such extensions present computational problems and problems of identification. We suggest it would be desirable to augment RP data with direct measures of consumer perceptions and direct measures of signal content to help resolve these identification problems.

One clear limitation of the existing literature has been the difficulty of precisely estimating the discount factor in dynamic learning models. This makes it difficult to distinguish forward-looking and myopic behavior. We discussed the search for exclusion restrictions (i.e., variables that affect future, but not current, payoffs) to help resolve this issue.

Another key challenge for future research is to develop models that combine learning with other potentially important sources of dynamics, such as inventories or habit persistence. We noted that it has not been possible to build inventories into dynamic learning models because of computational limitations. However, this course of research is important because the dynamics generated by inventories can be quite similar to those generated by learning. Thus, it is important to try to distinguish between the two mechanisms. The identification of different sources of dynamics is also a challenge, and we again conclude that progress would be aided by the combination of RP and SP data.

Finally, we point out that integrating learning models of demand with supply-side models remains underexplored. This should be another important area for future research.

In summary, it is clear that learning models have contributed greatly to our understanding of consumer behavior over the past 20 years. Two of the best examples still come from the original Erdem and Keane (1996) paper: (1) when viewed through the lens of a simple Bayesian learning model, the data are consistent with strong long-run advertising effects; and (2) a Bayesian learning model can do an excellent job of capturing observed patterns of brand loyalty. Future work will reveal whether such key findings are robust to the extension of these models to include multiple sources of dynamics and behaviorally richer models of learning behavior.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/mksc.2013.0805.

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