1. NUMBER SYSTEM

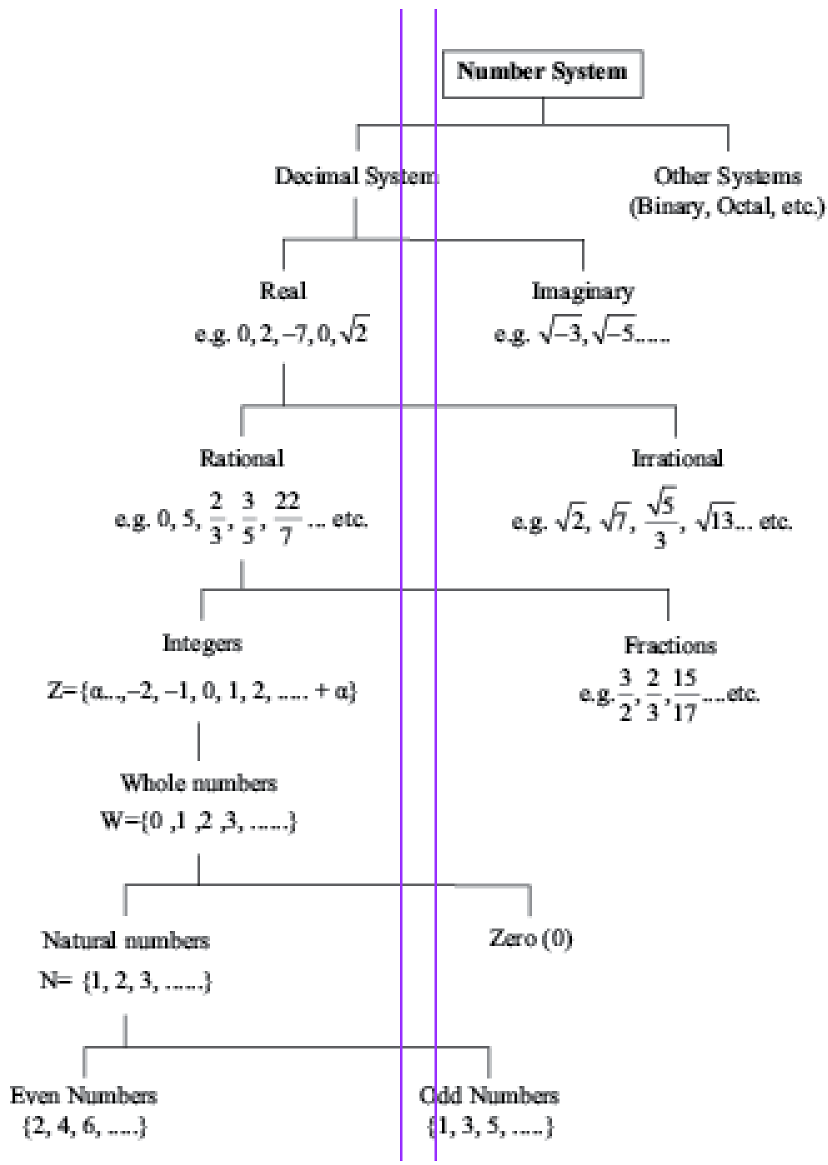
Introduction

The topic Number System deals with different operations on numbers like divisibility, remainders, finding the unit digit, HCF and LCM of different sets of numbers, Factors and multiples of given numbers etc., number in other base system like Octal, Hexadecimal etc. One can't escape from this topic as it form the foundation of the quantitative section.

Relevance to the Topic

Number System is a dominant topic in the Quantitative Aptitude section of the CAT exam. It usually accounts for about one fourth of the section. The sub topics from which questions are more frequently asked in the CAT exam are Prime Numbers, Remainders, Divisibility rules, HCF and LCM, Logarithm. To do well in this topic one should be well versed with the basic concepts as many questions in the past exam are from the basic concepts only. One should also focus on the different types of questions from each sub topic to do well in the exam. Also one should always try to solve the question in a alternate ways apart from the standard approaches.

The number Tree



Real Numbers: The numbers that can represent physical quantities in a complete manner are known as Real Numbers. All real numbers can be measured and can be represented on a number line. They are of two types:

Rational Numbers: The number that can be represented in the form $\frac{p}{q}$ of where p and q are integers and q is not equal to zero are called Rational Numbers.

Example: $\frac{2}{3}, \frac{5}{7}, \frac{3}{11}$ etc. They can be decimal numbers, whole numbers, integers, fractions, etc.

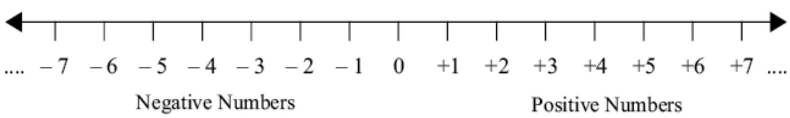
i.e. All terminating and recurring decimals are Rational numbers or they can be converted in the form of $\frac{p}{q}$.

Irrational Numbers: The number that cannot be represented in the form of $\frac{p}{q}$ where p and q are integers and q is not equal to zero are known as Irrational Number.

Example: $\sqrt{3}, \sqrt{11}, \sqrt{13}$ and π (pie) = 3.1416

The rational numbers can be classified into Integers and Fractions.

Integers: The set of numbers on the number line, with the natural numbers, zero and the negative numbers are knows as Integers, I = $\{\ldots. -3, -2, -1, 0, 1, 2, 3 \ldots.\}$



■ **Fractions:** A fraction denotes part of parts of an integer.

Example: $\frac{1}{4}$ represents $\frac{1}{4}$th part of the whole.

The type of fractions are:

Proper Fraction: The fractions where the numerator is less than the denominator value is known as a Proper Fraction. The value of a proper fraction is less than 1.

Example: , $\frac{1}{4}, \frac{3}{5}$ etc.

Improper Fractions: The fraction where the numerator is greater than or equal to the denominator value is known as an Improper Fractions. The value of an improper is always greater than or equal to 1.

Example: $\frac{8}{3}, \frac{6}{5}$ etc.

Mixed Fractions: A fraction which consists of an integer and a fractional part is known as a Mixed Fraction.

Example: $3\frac{2}{3}, 7\frac{1}{2}$

A mixed fraction can be written as a sum of integer part and the fraction part.

Example: $7\frac{2}{3} = 7 + \frac{2}{3} = \frac{23}{3}$

Also, $7\frac{2}{3} = \frac{3 \times 7 + 2}{3} = \frac{23}{3}$

The integers are classified into whole numbers and negative numbers.

Whole Numbers: The set of all positive numbers and '0' are known as Whole Numbers.

Example: 0, 1, 2, 3, 4, …..

Natural Numbers: The counting numbers 1, 2, 3, 4, 5.... are known as Natural Numbers.

Example: 1, 2, 3, 4, 5 ….. ∞

The natural numbers along with zero make the set of the whole numbers.

Even Numbers: The numbers which are divisible by 2 are known as Even Numbers.

Example: 2, 4, 6, 8, 10 etc.

Even number can be expressed in the form '$2n$' where n is an positive integer greater than 0.

Odd Numbers: The numbers which are not divisible by 2 are called Odd Numbers.

Example: 1, 3, 5, 7, 9 etc.

Odd numbers can be expressed in the form $(2n + 1)$ where n is an integer greater than equal to 0.

Composite Numbers: The numbers which have more than 2 factors are known as Composite Number.

Example: 4, 6, 8, 12, 36, etc.

Prime Numbers: The numbers are having only 2 factors *i.e.*, unity and itself is known as Prime Number.

Example: 2, 3, 5, 7, 11, 13 etc.

Some properties of Prime Numbers

- 2 is the only even prime number.
- 1 is neither prime nor composite.
- All prime numbers greater than 3 can be expressed in the form of $6n - 1$ or $6n + 1$, but all numbers of this form are not necessarily prime numbers.
- This is a necessary condition for a number to be prime but is not a sufficient condition.
- There are 15 prime numbers from 1 to 50 and 25 prime numbers from 1 to 100.
- Prime Numbers between 1 to 100 are:
  2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79,83, 89, 97.

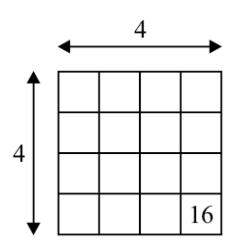
How to check whether a Number is Prime or not ?

I. Take the square root of the number.

II. Round of the square root to the immediately lower integer. Call the number z. For example, if you have to check for 141, its square root will be 11.
Hence, the value of z, in the case will be 11.

III. Check for divisibility of the number N by all prime numbers below z. If there is no prime number below the value of z which divides N then the number N will be prime.
So, we check for prime numbers less than 11 *i.e.* 2, 3, 5 & 7. Now 141 is divisible by 3, so it is not a Prime Number.

Perfect Square: The square of a whole number is called a perfect square. For example 16 is a perfect square because $4^2 = 16$



Some important points about perfect square

- Perfect Squares are always positive.
- Squares of even numbers are always even numbers and square of odd numbers are always odd.

- Squares of even numbers are always divisible by 4 whereas the square of odd numbers(other than 1) always
- Leaves a remainder of 1 when divided by 4.
- The number of zeros at the end of a perfect square is always even. In other words, a number ending in an odd number of zeros is never a perfect square.
- A number having 2, 3, 7 or 8 at unit's place is never a perfect square. In other words, Perfect Square numbers can never ends in 2, 3, 7 or 8.

What is a Recurring Decimal?

A decimal in which a digit or set of digits repeats continuously is known as a Recurring decimal. Recurring decimals are written in a shortened form by marking dots on the first and the last digit of the part which is repeated. The recurring nature of a decimal can also be represented by placing a bar over the set of digit(s) that recur.

For example, $\frac{1}{3} = 0.3333... = 0.\dot{3}$ or $0.\bar{3}$

and $\frac{1}{7} = 0.142857142857... = 0.\dot{1}4285\dot{7} = 0\ 0.\overline{142857}$

Recurring Decimals are of two types as pure recurring and mixed recurring.

A decimal is called pure recurring if all the digits after the decimal are recurring (or repeating) like $0.\bar{3}$.

A decimal is Mixed Recurring if some of the digits after the decimal point are not recurring like $0.1\bar{6}$ (in this case only the digit 6 is recurring and 1 is not recurring) as $= 0.16666 \ldots$

As we have already discussed that every recurring decimal is a rational number as they can be represented in the $\frac{p}{q}$ form.

Before discussing the general rule of converting recurring decimals into fractions $\frac{p}{q}$ form, we will consider few examples so that we can understand the rule easily.

What is the $\frac{p}{q}$ form of 0.11111…. (or $0.\bar{1}$)

Let $x = 0.11111 \ldots$ (or $0.\bar{1}$) …. (1)

Multiply equation (1) by 10

$\Rightarrow 10x = 1.111 \ldots$ (or $1.\bar{1}$) …. (2)

Subtracting (1) from (2)

$9x = 1$

So, $x = \frac{1}{9}$

If $x = 0.313131 \ldots$ (or $0.\overline{31}$) … (1)

Multiply equation (1) by 100

$\Rightarrow 100x = 31.3131\ldots$ (or $31.\overline{31}$) … (2)

Subtracting (1) from (2)

$99x = 31$

So, $x = \frac{31}{99}$

In the first example we have multiplied the equation by 10 and in the second example by 100, as we need one more equation in which the recurring part is same, which is eliminated after the subtraction.

Let us take one Mixed Recurring decimal

Let $x = 0.2\bar{3}$

$\Rightarrow 10x = 2.\bar{3}$ … (1)

Now the right hand side (RHS) has become a pure recurring, so we can apply the same method that we have applied in pure recurring decimals.

Multiply equation (1) by 10

$100x = 23.\bar{3}$ … (2)

As equation (1) and (2) have the same recurring part, so

Subtracting (1) from (2)

$90x = 23 - 2$

$\Rightarrow x = \frac{21}{90}$ or $\frac{7}{30}$

Now we can write down the rule for converting a pure recurring decimal into the $\frac{p}{q}$ form as follows:

The $\frac{p}{q}$ form of a pure recurring decimal

$$= \frac{\text{The recurring part written once}}{\text{As many 9's as the number of digits in the recurring part}}$$

Thus, $0.\overline{31}$ can be straightaway written as $\frac{31}{99}$ & $0.\overline{217}$ can be written as $\frac{217}{999}$

Now Let's see the rule for converting a mixed recurring decimal into the $\frac{p}{q}$ form.

The $\frac{p}{q}$ form of a mixed recurring decimal

$$= \frac{\begin{array}{c}\text{(The non - recurring and the recurring part}\\ \text{written once} - \text{(the non - recurring part)}\end{array}}{\begin{array}{c}\text{As many 9's as the number of digits in recurring part followed}\\ \text{by as many 0's as the number of digits in the non - recurring part}\end{array}}$$

Thus, $0.6\overline{13}$ can be written as $\frac{613 - 6}{990} = \frac{607}{990}$

or $0.91\bar{3} = \frac{913 - 91}{900} = \frac{822}{900} = \frac{411}{450}$

Even/Odd Arithematic

$$\begin{cases} \text{Even} + \text{Even} = \text{Even} \\ \text{Even} - \text{Even} = \text{Even} \\ \text{Even} \times \text{Even} = \text{Even} \\ \text{Even} \div \text{Even} = \text{Even or Odd} \end{cases}$$

$$\begin{cases} \text{Odd} + \text{Odd} = \text{Even} \\ \text{Odd} - \text{Odd} = \text{Even} \\ \text{Odd} \times \text{Odd} = \text{Odd} \\ \text{Odd} \div \text{Odd} = \text{Odd} \end{cases}$$

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$$\begin{cases}\text{Odd} + \text{Even} = \text{Odd}\\ \text{Odd} - \text{Even} = \text{Odd}\\ \text{Odd} \times \text{Even} = \text{Even}\\ \text{Odd} \div \text{Even} = \text{(never divisible)}\end{cases}$$

$$\begin{cases}(\text{Even})^{\text{even / odd}} = \text{Even}\\ (\text{Odd})^{\text{odd / even}} = \text{Odd}\end{cases}$$

Example 1: Which of the following statement is correct?

a. Sum of two irrational numbers is always an irrational number.

b. Sum of a rational and an irrational number is always an irrational number.

c. Sum of two rational numbers is always an integer.

d. Square of an irrational number is always a rational number.

Solution:

a. Sum of two irrational numbers can be rational or irrational. For example, the sum of $7 + \sqrt{3}$ and $5 - \sqrt{3}$ is 12 (rational number) and the sum of $3 + \sqrt{5}$ and $12 + 2\sqrt{5}$ is $15 + 3\sqrt{5}$ (irrational number).

b. It is always true that the sum of a rational and an irrational number is always an irrational number because the irrational part cannot be cancelled by addition. For example, the sum of 5 and $3 + \sqrt{5}$ is $8 + \sqrt{5}$ (irrational)

c. The sum of two rational numbers may or may not be an integer. For example, the sum of 3 and $\frac{2}{3}$ is $\frac{11}{3}$ (fraction) and the sum of $\frac{2}{7}$ and $\frac{5}{7}$ is 1 (integer).

d. The square of an irrational number can be a rational or an irrational number. For example, the square of $\sqrt{3}$ is 3 (rational) and the square of $1 + \sqrt{3}$ is $4 + 2\sqrt{3}$ (irrational)

So, the answer is (b).

Example 2: Which one of the following numbers is a prime number?

a. 161 b. 221 c. 373 d. 437

Solution: (c)

In order to find whether a given number is prime or not, we need to check whether it is divisible by any of the prime number less than the square root of the given number or not.

Approximate square root of 161 would be 12.68. So, we will check for the prime numbers which are less than 12. The numbers are 2, 3, 5, 7, 8, 11.

Since 161 is divisible by 7. It is not a prime. Similarly for 221, the prime numbers less than 14 are considered which are 2, 3, 5, 7, 11 and 13.

Since 221 is divisible by 13. It is not a prime number. For 373 also prime numbers less than 19 are checked. The number 373 is not divisible by any prime number.

For, 437, prime numbers less than 20 are checked.

Since 437 is divisible by 19. So, it is also not a prime number.

\ 373 is a prime number.

Example 3: P_1, P_2, P_3, P_4 and P_5 all are prime numbers $P_1 \times P_2 \times P_3 \times P_4 \times P_5 = 2310$

Also, $P_1 < P_2 < P_3 < P_4 < P_5$. Then what is the value of P_1?

a. 2 b. 5

c. 7 d. Cannot be determined

Solution: (a)

The product of five prime numbers is given as even. So one of the number has to be even (2 only). Now P_1 has to be 2 as P_1 is the smallest of all the given prime numbers.

Example 4: If a, $a + 2$ and $a + 4$ are prime numbers, then the number of possible values for a is:

a. One b. Two c. Three d. More than three

[CAT 2003]

Solution: (a)

The only possible value of a is 3, such that a, $a + 2$ and $a + 4$ are prime numbers.

The prime number are 3, 5 and 7.

The number of possible solution for 'a' is one.

Example 5: Which of the following alternatives is incorrect if a is an even number and b is an odd number?

a. $(a + a^a)(b + b^b)$ is even

b. $(a + b) + (a^b + b^a) + (a^b + a^a)$ is odd

c. $a + b^a$ is odd

d. $(a + b) + (a^b + b) + a^b + b^a$ is odd

Solution:(b)

a. $a + a^a$ = even + even = even

$b + b^b$ = odd + odd = even. Now, even × even = even.

b. $(a + b) + (a^b + b^a) + (a^b + a^a)$ = odd + (even + odd) + (even + even) = odd + odd + even = even.

c. $a + b^a$ = even + odd = odd.

d. $(a + b) + (a^b + b) + a^b + b^a$ = odd + (even + odd) + even + odd

= odd + odd + even + odd = even + odd = odd.

Therefore, alternative (b) is incorrect.

Example 6: If $x + y = z$; x, y and z all are prime numbers and $y < x$. What is the value of y?

a. 1 b. 2 c. 3 d. None of these

Solution: (b)

$x + y = z$

Here z is given as a prime number. The value of z has to be odd as if it is even, then it has to be 2 and 2 is the smallest prime number and it cannot be written as the sum of two other prime numbers.

Now, as z is odd; so one of x and y should be even and the other has to be odd. As $y < x$, so y has to be 2, because 2 is also the smallest prime number.

Example 7: Let p and q be positive integers such that p is prime and q is composite. Then,

a. $q - p$ cannot be an even integer

b. pq cannot be an even integer

c. $\frac{(p+q)}{p}$ cannot be an even integer

d. None of these statements is true

[CAT 2003]

Solution: (d)

The best way to solve these questions is by taking values of the variables.

Option (a) : let $q = 4$; $p = 2$

$\therefore$ $(q - p)$ is an even integer. Hence, ruled out.

Option (b) : $q = 4$; $p = 2$

$\therefore$ pq is an even integer. Hence, ruled out.

Option (c): $\frac{(p+q)}{p}$. Let p = 2 ; q = 6

$\therefore$ $\frac{(2+6)}{2} = 4$ which is even. Hence, ruled out.

Hence, option (d) is correct answer.

Some formulas of Counting Numbers

- Sum of first n natural nos. = $\frac{n(n+1)}{2}$

  $(1 + 2 + 3 + \ldots\ldots\ldots n)$

- Sum of squares of first n natural nos. = $\frac{n(n+1)(2n+1)}{6}$

  $(1^2 + 2^2 + 3^2 + \ldots\ldots\ldots n^2)$

- Sum of cubes of first n natural nos. = $\left[\frac{n(n+1)}{2}\right]^2$

  $(1^3 + 2^3 + 3^3 + \ldots\ldots\ldots n^3)$

- Sum of first n odd nos. = n^2
- Sum of first n even nos. = $n(n + 1)$

Example 8: What is the sum of the squares of first 20 natural numbers?

a. 8610 b. 2870 c. 4620 d. 1540

Solution:

$1^2 + 2^2 + 3^2 + \ldots\ldots. 20^2$

As $1^2 + 2^2 + 3^2 + \ldots\ldots.$ n2 = $\frac{n(n+1)(2n+1)}{6}$

Here, $n = 20$

So, Sum = $\frac{20 \times 21 \times 41}{6} = 2870$

Example 9: What is the value of $1^2 + 3^2 + 5^2 + 7^2 + \ldots\ldots\ldots\ldots\ 21^2$

a. 3311 b. 1540 c. 2870 d. 1771

Solution:

$1^2 + 3^2 + 5^2 + 7^2 + \ldots\ldots. 21^2$

$= (1^2 + 2^2 + 3^2 + 4^2 + \ldots 20^2 + 21^2) - (2^2 + 4^2 + 6^2 + \ldots. 20^2)$

$= (1^2 + 2^2 + 3^2 + 4^2 + \ldots. 20^2 + 21^2) - [(1 \times 2)^2 + (2 \times 2)^2 + (2 \times 3)^2 .. (2 \times 10)^2]$

$= (1^2 + 2^2 + 3^2 + 4^2 + \ldots\ldots 20^2 + 21^2) - 2^2 (1^2 + 2^2 + 3^2 + \ldots. 10^2)$

$= \frac{21 \times 22 \times 43}{6} - \frac{4 \times 10 \times 11 \times 21}{6}$

$= \frac{21 \times 22}{6}(43 - 20) = \frac{21 \times 22 \times 23}{6} = 1771$

Surds and Indices

Indices: By the term 'a^m' we, mean $a \times a$ .... m times *i.e.* the product of m factors each equal to 'a' is represented by a^m. a is called the **base** and m is called the **power**.

$\sqrt[n]{a}$ or $a^{\frac{1}{n}}$ denotes the n^{th} root of a.

Fundamental Laws of Indices:

- $a^1 = a$
- $a^0 = 1$ (where $a \neq 0$)
- a– n = $\frac{1}{a^n}$
- a1/n = $\sqrt[n]{a}$
- $a^m \times a^n = a^{m+n}$

- $a^m \times a^n \times a^p \times \ldots = a^{m+n+p+\ldots}$
- $\dfrac{a^m}{a^n}$ = am – n
- $(a^m)^n = a^{mn}$
- ap/q $= \sqrt[q]{a^p}$, where ‘a’, ‘b’ and ‘p’ are real numbers and q ≠ 0
- $(a \times b)^m = a^m \times b^m$
- $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$
- ●If $a^m = a^n$ and $a \neq -1, 0, 1$; then $m = n$.
- ● $\left(a^b\right)^c 1\ a^{b^c}$

Surds: Any root of a rational number, which cannot be exactly calculated is called a Surd (an irrational number)

Example: $\sqrt{2}, \sqrt[3]{4}, 2 + \sqrt{2}$

For a surd $\sqrt[n]{a}$, a is called the radicand and n is called the order of the surd. √ is the radical sign.

Important points about Surds:

For $\sqrt[n]{a}$ to be a surd:

I. The radicand a should be a positive rational number.

II. n should be a natural number.

III. $\sqrt[n]{a}$ is an irrational number.

Laws of Surds:

- $\left(\sqrt[n]{a}\right)^n = \left(a^n\right)^{1/n} = a$
- $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b} = (ab)^{1/n}$
- $\left(\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}\right) = \left(\dfrac{a}{b}\right)^{1/n}$
- $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}} = \left[(a)^{1/n}\right]^{1/n}$
- $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m} = \left(a^n\right)^{1/n}$

Example 67: If a, b are the positive integers ($b > 1$) such that $a^b = 121$, then value of $(a-1)^{b+1}$ is:

a. 12321 b. 1 c. 1000 d. 11

Solution: (c)

\ $a^b = 121$ and $b > 1$

\ $(11)^2 = 121$

Þ $a = 11$ and $b = 2$

Hence $(a-1)^{b+1} = (10)^3 = 1000$

Example 62: The value of $2^{2^m} = 16^{2^{3m}}$, then m is equal to

a. – 1 b. 0 c. 1 d. None of these

Solution: (a)

$2^{2^m} = 16^{2^{3m}}$

$\Rightarrow 2^{2^m} = \left(2^4\right)^{2^{3m}}$

$\Rightarrow 2^{2^m} = 2^{4.2^{3m}} = 2^{2^{3m+2}}$

$\Rightarrow 2^{2^m} = 2^{2^{3m+2}}$ [Since base in both sides is equal]

$\Rightarrow 2^m = 2^{3m+2}$

$\Rightarrow m = 3m + 2$

$\Rightarrow 2m = -2$

$\Rightarrow m = -1$

Alternate method:

Put values and check.

Example 59: If $x^{x^{\frac{2}{3}}} = \left(x^{\frac{2}{3}}\right)^x$, then the value of ‘x’ is:

a. $\frac{2}{3}$ b. $\frac{9}{4}$ c. $\frac{16}{25}$ d. $\frac{8}{27}$

Solution: (b)

$x^{x^{\frac{2}{3}}} = \left(x^{\frac{2}{3}}\right)^x$

$\Rightarrow x^{x^{\frac{2}{3}}} = x^{\left(\frac{2}{3}\right)x}$

$\Rightarrow x^{\frac{2}{3}} = \frac{2}{3}x$

$\Rightarrow x^{\frac{1}{2}} = \frac{3}{2} \Rightarrow \mathbf{x}\frac{9}{4}$

Example 63: If , $\dfrac{9^q \times 3^2 \times \left(3^{\frac{-q}{2}}\right)^{-2} - (27)^q}{3^{3p} \times 2^3} = \dfrac{1}{27}$ then the value of (p – q) is:

a. – 1 b. 1 c. 2 d. – 2

Solution: (a)

$$\frac{9^q \times 3^2 \times \left(3^{\frac{-q}{2}}\right)^{-2} - (27)^q}{3^{3p} \times 2^3} = \frac{1}{27}$$

$$\Rightarrow \frac{3^{2q} \times 3^2 \times 3^q - 3^{3q}}{3^{3p} \times 2^3} = \frac{1}{3^3}$$

$$\Rightarrow \frac{3^{2q+2+q} - 3^{3q}}{8 \times 3^{3p}} = \frac{1}{3^3}$$

$$\Rightarrow \frac{3^{3q+2} - 3^{3q}}{3^{3p} \times 8} = 3^{-3}$$

$$\Rightarrow \frac{3^{3q}(3^2-1)}{3^{3p}\times 8} = 3^{-3}$$

$\Rightarrow 3^{3q} - 3^{3p} = 3^{-3}$

$\Rightarrow 3q - 3p = -3$

$\Rightarrow p - q = 1$

Example 60: If $x^p = y^q = z^r$ and $y^2 = zx$, then the value of $\frac{1}{p} + \frac{1}{r}$ is:

a. $\frac{q}{2}$ b. $\frac{9}{4}$ c. $\frac{2}{q}$ d. $2p$

Solution: (c)

If $x^p = y^q = z^r$ and $y^2 = zx$

Let $x^p = y^q = z^r = k$

$$\Rightarrow x = k^{\frac{1}{p}},\ y = k^{\frac{1}{q}},\ z = k^{\frac{1}{r}}$$

Now, $\because$ $y^2 = zx$

$$\therefore \left(k^{\frac{1}{q}}\right)^2 = \left(k^{\frac{1}{r}}\right).\left(k^{\frac{1}{p}}\right)$$

$$\Rightarrow k^{\frac{2}{q}} = k^{\frac{1}{r}+\frac{1}{p}}$$

$$\Rightarrow \frac{1}{p} + \frac{1}{r} = \frac{2}{q}$$

Example 61: The value of

$$\frac{1}{1+x^{q-p}+x^{r-p}} + \frac{1}{1+x^{r-q}+x^{p-q}} + \frac{1}{1+x^{p-r}+x^{q-r}}$$

a. 0 b. x^{pqr} c. 1 d. $x^{(p+q+r)}$

Solution: (c)

$$= \frac{1}{1+x^{q-p}+x^{r-p}} + \frac{1}{1+x^{r-q}+x^{p-q}} + \frac{1}{1+x^{p-r}+x^{q-r}}$$

$$= \frac{1}{1+\frac{x^q}{x^p}+\frac{x^r}{x^p}} + \frac{1}{1+\frac{x^r}{x^q}+\frac{x^p}{x^q}} + \frac{1}{1+\frac{x^p}{x^r}+\frac{x^q}{x^r}}$$

$$= \frac{x^p}{x^p+x^q+x^r} + \frac{x^q}{x^q+x^r+x^p} + \frac{x^r}{x^r+x^p+x^q}$$

$$= \frac{x^p+x^q+x^r}{x^p+x^q+x^r} = 1$$

Example 64: Which one is smaller out of $\sqrt[3]{2}$ and $\sqrt[4]{3}$?

a. $\sqrt[4]{3}$ b. $\sqrt[3]{2}$

c. Both are equal d. Cannot be determined

Solution: (b)

Here we have, $(2)^{1/3}$ and $(3)^{1/4}$

(Taking the LCM of 3 and 4)

LCM of 3 and 4 = 12

So, we will make both the surds of order 12.

$\sqrt[3]{2} = (2)^{\frac{1}{3}} = 2^{\frac{4}{12}} = (16)^{\frac{1}{12}}$

and $\sqrt[4]{3} = (3)^{\frac{1}{4}} = 3^{\frac{3}{12}} = (27)^{\frac{1}{12}}$

Hence $\sqrt[3]{2} < \sqrt[4]{3}$

Example 65: Which of the following is largest?

$\sqrt{3} - \sqrt{2}, \sqrt{4} - \sqrt{3}, \sqrt{5} - \sqrt{4}, \sqrt{2} - 1$

Solution:

$= \sqrt{3} - \sqrt{2} = \dfrac{\sqrt{3} - \sqrt{2}}{1} \times \dfrac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}}$

$= \dfrac{3 - 2}{\sqrt{3} + \sqrt{2}} = \dfrac{1}{\sqrt{3} + \sqrt{2}}$

Similarly, $\sqrt{4} - \sqrt{3} = \dfrac{1}{\sqrt{4} + \sqrt{3}}, \sqrt{5} - \sqrt{4} = \dfrac{1}{\sqrt{5} + \sqrt{4}}$

and $\sqrt{2} - 1 = \dfrac{1}{\sqrt{2} - 1}$

As we know, if the numerator is same then the fraction whose denominator is larger the fraction will be lower. Hence the correct order of descending is

$\left(\sqrt{2} - 1\right) > \left(\sqrt{3} - \sqrt{2}\right) > \left(\sqrt{4} - \sqrt{3}\right) > \left(\sqrt{5} - \sqrt{4}\right)$

Alternate Method:

The difference between square root values of two consecutive numbers always decreases.

So, $\sqrt{2} - 1$ will be the largest.

Hence, $\sqrt{2} - 1$ is the largest.

Logarithms

The logarithm of any number to a given base is the index or the power to which the base must be raised in order to be equal to the given number,

Thus,

$$\text{if } a^x = N, \text{ then } x = \log_a N$$

Some Important Rules in Logarithms

- $\log_a (mn) = \log_a m + \log_a n$
- $\log_a (m/n) = \log_a m - \log_a n$
- $\log_a b = \dfrac{1}{\log_b a}$
- $\log_a m^p = p \times \log_a m$
- $\log_a m = \dfrac{\log_b m}{\log_b a}$ [Inclusion of a new base]
- $\log_{a^q} m^p = \dfrac{p}{q} \log_a m$
- $a^{\log_a N} = N$

- $\log_a a = 1$ (logarithm of any number to the same base is 1)
- $\log_a 1 = 0$ (log of 1 to any base is 0)
- For a given an equation $\log_a M = \log_b N$,

I. If $M = N$, then a will be equal to b; if $M \neq 1$ and $N \neq 1$.

II. If $a = b$, then M will be equal to N; if $a \neq 0$ and $b \neq 0$.

Example : What is the value of log4 $\sqrt{128}$?

a. $\frac{3}{2}$ b. $\frac{7}{2}$ c. $\frac{7}{4}$ d. $\frac{7}{8}$

Solution: (c)

$\log_4 \sqrt{128} = \sqrt{2^7} \log 4 = \log_{2^2} 2^{7/2}$

$= \frac{7/2}{2} \log_2^2 = \frac{7}{4}$

Example : If $\log_3 (x - 2) = 4$, then what is the value of x?

a. 79 b. 81 c. 83 d. 85

Solution: (c)

$\log_3 (x - 2) = 4$

so, $x - 2 = 3^4$

$x = 81 + 2 = 83$

Example : Find the number of digits in 2^{64} if $\log_2 = 0.30103$

a. 19 b. 20 c. 21 d. 22

Solution: (b)

The characteristic of a number is one less than the number of digits in number

$\log 2^{64} = 64 \times \log 2 = 64 \times 0.30103$

$= 19.26592$ (characteristics)

So, the number of digits in 2^{64} is (19 + 1) *i.e.*, 20.

No. of digits = Characteristic + 1

Divisibility Rules

By 1 : Any number is divisible by 1.

By 2 : A number is divisible 2 when the digit in the units place is 0 or an even number. (*i.e.* 2, 4, 6 and 8)
e.g. 10, 938, 438, etc. are divisible by 2.

By 3 : A number is divisible by 3 when the sum of the digits of the given number is divisible by 3.
e.g. the number 316521.
Sum of the digits = 3 + 1 + 6 + 5 + 2 + 1 = 18
18 is divisible by 3.
$\therefore$ The number 316521 is divisible by 3.

By 4 : A number is divisible by 4 when the last two digits of the number are zeros or are divisible by 4.
e.g. 100, 120, 1100, 246896, etc. are divisible by 4.

By 5 : A number is divisible by 5 when its units digit is 0 or 5.
e.g. 50, 235, 6855, etc. are divisible by 5.

By 6 : A number is divisible by 6 if it is divisible by both 2 and 3,

e.g. 864, last digit is 4,
$\therefore$ 864 is divisible by 2.
Sum of digits = (8 + 6 + 4) = 18 is divisible by 3.
$\therefore$ 864 is divisible by 3.
Hence, 864 is divisible by 6.

By 8 : A number is divisible by 8 if the last three digits of ($8 = 2^3$) the number are zeros or are divisible by 8.
e.g. 1000, 12000, 135192, etc. are divisible by 8.

By 9 : A number is divisible by 9, if the sum of its digits is divisible by 9.
e.g. 207, 8649, 91827.

By 10 : A number is divisible by 10, when its unit digit is zero.
e.g. 110, 2100, 8270, etc. are divisible by 10.

By 11 : A number is divisible by 11 if the difference between the sum of the digits in the odd places and in the even places of the number is either zero or a multiple of 11.
e.g.
$1001 \rightarrow (1 + 0) - (0 + 1) = 0$
$24211 \rightarrow (1 + 2 + 2) - (1 + 4) = 0$
$60907 \rightarrow (7 + 9 + 6) - (0 + 0) = 22$

By 12 : A number is divisible 12 if it is divisible by both 3 and by 4,
e.g.
5496: last two digits are 96
$\therefore$ 5496 is divisible by 4.
Sum of digits = (5 + 4 + 9 + 6) = 24 is divisible by 3.
$\therefore$ 5496 is divisible by 3. Hence, 5496 is divisible by 12.

Divisibility by a higher composite number

- Express the composite number as a product of minimum number of factors that are co-prime.
- Check the divisibility rules for each factor.
- If the rules hold true for each of the factors the number is also divisible by the original number.

To check for divisibility by
Example 1: $12 = 3 \times 4$
Example 2: $72 = 8 \times 9$

Important points of Divisibility

- $x^n + y^n$ is always divisible by $x + y$ when n is odd.
- $x^n - y^n$ is always divisible by $(x - y)$ if n is odd.
- $x^n - y^n$ is always divisible by $(x + y)$ and $(x - y)$ if 'n' is even.
- When any number with an even number of digits is added to its reverse, the sum is always divisible by 11.

e.g. 1234 + 4321 = 5555 which is divisible by 11.

- When any number with an odd number of digits is Subtracted from its reverse, the absolute difference is always divisible by 11.

e.g. 68951 – 15986 = 52965 which is divisible by 11.

- If a digit repeats itself three times successively, the resultant number is exactly divisible by 3 and 37.

e.g. 555 is divisible by 3 and 37.

Example 5: What least number must be subtracted from 1000 to get a number which is exactly divisible by 17?

a. 16 b. 16 c. 14 d. 3

Solution: (c)

On dividing 1000 by 17, we get 14 as remainder.

$\therefore$ Required number to be subtracted is 14.

Example 6: Find the number which is nearest to 2115 and exactly divisible by 21?

a. 2100 b. 2121 c. 2142 d. None of these

Solution: (b)

On dividing 2115 by 21, we get 15 as remainder.

So, if we increase 2115 by 6(21 – 15) it will become divisible by 21.

So, Required number = 2115 + 6 = 2121

Example 7: What is the maximum and minimum value of $X + Y$ if $46X3Y4$ is divisible by 9?

a. 8, 0 b. 1, 10 c. 3, 5 d. None of these

Solution: (b)

For the number $46X\,3Y\,4$ to be divisible by 9, where X and Y are single positive digits, sum of the digits $(4 + 6 + X + 3 + Y + 4)$ has to be divisible by 9.

$\therefore$ $4 + 6 + X + 3 + Y + 4 = 9\,K$ Þ $17 + X + Y = 9\,K$

Substituting $K = 1$

$X + Y = -8$ ….. not possible. \ $K = 2$,

$10 + X + Y = 17$ Þ $X + Y = 1$ (minimum value)

If $K = 3$, $17 + X + Y = 27$ Þ $X + Y = 10$ (maximum value)

Example 8: Which of the following numbers is exactly divisible by 99?

a. 111111 b. 110011 c. 100011 d. None of these

Solution: (d)

A number is divisible by 99 if it is divisible by both 9 and 11.

None of the first three options satisfy the divisibility rules of both 9 and 11.

Example 9: How many even integers n, where $100 \le n \le 200$, are divisible neither by 7 nor by 9?

a. 40 b. 37 c. 39 d. 38

[CAT 2003]

Solution: (c)

There are 51 even integers between 100 & 200. [100, 102, 104, 106, ….. 200]

There are 14 numbers between 100 & 200 that are divisible by 7.

[105, 112, 119, 126, 133, 140, 147, 154, ….196]

There are 7 even numbers among the above numbers. [112, 126, 140, ….196]

There 6 even numbers between 100 & 200 that are divisible by 9. [108, 126, ….198]

The even number divisible by both 7 & 9 is 126

Numbers which are divisible neither by 7 nor 9 = 51 – 7 – 6 + 1 = 39.

Example 10: The number of positive integers n in the range $12 \le n \le 40$ such that the product $(n-1)(n-2)\ldots..3.2.1$ is not divisible by n is:

a. 5 b. 7 c. 13 d. 14

[CAT 2003]

Solution: (b)

$$(n-1)(n-2)\ldots..3.2.1 = \frac{n(n-1)(n-2).......3.2.1}{n.n} = \frac{n!}{n^2}$$

$n!$ is not divisible by n^2 when n is a prime number. [13, 17, 19, 23, 29, 31, 37]

Therefore there are 7 values of n so as to satisfy the given condition.

Remainders

Rule 1: Remainders are additive in nature. If x_1, x_2, x_3 …. are divided by '*d*' and leaves the remainders r_1, r_2, r_3…. *i.e.* $\frac{x_1}{d} \rightarrow r_1, \frac{x_2}{d} \rightarrow r_2, \frac{x_3}{d} \rightarrow r_3$ then x1 + x2 + x3 + ……. when divided by 'd' will give the remainder same as obtained when (r1 + r2 + r3 +….) is divided by '*d*'.

Example 1: Find the remainder when $10^1 + 10^2 + 10^3 + 10^4 + 10^5$ is divided by 6?

a. 4 b. 6 c. 8 d. 2

Solution: (d) $\frac{10 + 10^2 + 10^3 + 10^4 + 10^5}{6} = \frac{111110}{6}$

$= 2 \ \frac{10}{6} \rightarrow 4, \frac{10^2}{6} \rightarrow 4, \frac{10^3}{6} \rightarrow 4, \frac{10^4}{6} \rightarrow 4, \frac{10^5}{6} \rightarrow 4$

Now, if we divide $\frac{4 + 4 + 4 + 4 + 4}{6}$, the remainder is 2.

Rule 2: Remainders are multiplicative in nature. If a_1, a_2, a_3 …. are divide by '*d*' and leaves the remainder r_1, r_2, r_3…. *i.e.*, $\frac{a_1}{d} \rightarrow r_1, \frac{a_2}{d} \rightarrow r_2, \frac{a_3}{d} \rightarrow r_3$ then $a_1 \times a_2 \times a_3$… when divided by 'd' will give the remainder same as obtained when $r_1 \times r_2 \times r_3$…. is divided by 'd'.

Example 2: Find the remainder when 67 × 95 is divided by 8?

a. 1 b. 3 c. 5 d. 7

Solution:

$\frac{67 \times 95}{8} = \frac{6365}{8} = 5$

$\frac{67}{8} \rightarrow 3, \frac{95}{8} \rightarrow 7$

Now, if we divide $\frac{3 \times 7}{8} = \frac{21}{8}$, the remainder is 5.

Example 14: Find the remainder when 1701 × 1703 × 1705 × 1707 × 1709 is divided by 18?

a. 9 b. 12 c. 15 d. 6

Solution: (a)

Instead of multiplying and then dividing by 18, we can find the remainder of the multipliers.

Thus we find the remainder of the remainders.

Hence, the remainder of $\frac{1701 \times 1703 \times 1705 \times 1707 \times 1709}{18}$

$= \frac{9 \times 11 \times 13 \times 15 \times 17}{18}$

(9, 11, 13, 15, 17 are the remainder when the given numbers are divided by 18.)

$= \frac{99 \times 13 \times 15 \times 17}{18}$

$= \frac{9 \times 13 \times 255}{18} = \frac{117 \times 255}{18}$

$= \frac{9 \times 3}{18} = \frac{27}{18} = 9$

Concept of Negative Remainder

Let 19 is divided by 5 then the remainder is 4 or – 1.
Remainder is – 1 because 19 is 1 less than 20 which is completely divisible by 5.
If 231 is divided by 8 then the remainder is 7 or – 1 by the same reason.

Important Points of Remainder

- $\frac{(x+1)^n}{x}$ always gives remainder 1.
- $\frac{(x-1)^n}{x}$ always gives remainder 1 when 'n' is even and gives the remainder –1 or (x – 1) when '*n'* is odd, where '*x*' is any integer and '*n*' being the positive integer.

Example 13: Find the remainder when 3^{22} is divided by 5?

a. 2 b. 3 c. 4 d. 1

Solution: (c)

$$\frac{3^{22}}{5} = \frac{(3 \times 3) \times (3 \times 3) \times ....11 \text{ pair}}{5}$$

Remainder of $\frac{3 \times 3}{5}$ is – 1.

$\therefore$ Remainder of $\frac{(-1) \times (-1) \times .....11 \text{ pairs}}{5} = -1$

$\Rightarrow$ Remainder of 3^{22} divided by 5 is – 1 + 5 *i.e.* 5

Example 15: What is the remainder when 3^{1235} is divided by 80.

a. 25 b. 27 c. 29 d. 31

Solution: (b)

$$\frac{3^{1235}}{80} = \frac{3^{1232} \times 3^3}{80}$$

$$= \frac{\left(3^4\right)^{308} \times 3^3}{80}$$ [As 81 when divided by 80, the remainder is 1]

$$= \frac{(81)^{308} \times 3^3}{80}$$

$$= \frac{1 \times 27}{80} = \frac{27}{80}$$

Thus the remainder is 27.

Example 16: What is the remainder when $(3)^{67!}$ is divided by 80.

a. 0 b. 1 c. 2 d. Can't say

Solution: (b)

$\because$ $3^4 = 81$ so $\frac{3^{4n}}{80}$ gives remainder 1.

Thus $\frac{3^{67!}}{80}$ will also give the remainder 1.

Since $67! = 4n$ for a positive integer n. (Since $67!$ is always divisible by 4 for any positive integer n)

Example 17: Find the remainder of $\dfrac{888^{222} + 222^{888}}{5}$ is.

Solution:

$$\frac{888^{222} + 222^{888}}{5}$$

$$= \frac{888^{222}}{5} + \frac{222^{888}}{5}$$

$$= \frac{(3^4)^{55} \times 3^2}{5} + \frac{(2^4)^{222}}{5}$$

$$= \frac{1 \times 9}{5} + \frac{1}{5}$$

$$= \frac{4}{5} + \frac{1}{5} = \frac{4+1}{5} \Rightarrow \frac{5}{5}$$

Thus the remainder is zero.

Alternate Method:

[To check the divisibility by 5 just see the sum of the unit digits which is 10 (= 4 + 6)

$\because$ $8^{222} \rightarrow 4$ (unit digit)

and $2^{222} \rightarrow 6$ (unit digit)

Hence it is divisible. So there is no remainder.

Unit's Digit Cyclicity

When a number N is raised to any integral power 'm', the digit in the units place of the resulting value can be determined without actually evaluating the power.

Each number when raised to powers will give values in which the digits of the unit's place follow a cyclic pattern.

Cyclicity of 2

$$\left.\begin{aligned} 2^1 &= 2 \\ 2^2 &= 4 \\ 2^3 &= 8 \\ 2^4 &= 16 \end{aligned}\right\} \text{After every 4 steps unit's digit repeat}$$

$$\left.\begin{aligned} 2^5 &= 32 \\ 2^6 &= 64 \\ 2^7 &= 128 \\ 2^8 &= 256 \end{aligned}\right\} \text{After every 4 steps unit's digit repeat}$$

$$\left.\begin{aligned} 2^9 &= 512 \\ 2^{10} &= 1024 \\ 2^{11} &= 2048 \\ 2^{12} &= 4096 \end{aligned}\right\} \text{After every 4 steps unit's digit repeat}$$

The cyclicity of 2 is 4 as each unit digit repeats itself in a cycle of 4.

General Formula	**Unit Digit**

$2^{4x + 1}$	2
$2^{4x + 2}$	4
$2^{4x + 3}$	8
$2^{4x + 4}$	6

Similarly, we can find cyclicity and the cyclic pattern of other digits as well.

Digit	**Cyclicity**	**Cyclic Pattern**
0	1	0
1	1	1
2	4	2, 4, 8, 6
3	4	3, 9, 7, 1
4	2	4, 6
5	1	5
6	1	6
7	4	7, 9, 3, 1
8	4	8, 4, 2, 6
9	2	9, 1

Example 18: Find the unit digit for 317^{983}.

a. 7 b. 9 c. 3 d. 1

Solution: (c)

$N = 317$

Unit Digit of $N = 7$

$983 = 4n + 3$

$\Rightarrow$ $(7)^{983} = (7)^{4n}\,(7)^3$ will give the unit digit.

$(7)^{4n}$ completes the cycle of 4 and $(7)^3$ is the third step of the next cycle.

$(7)^3 = 343$

$\therefore$ Unit digit is 3.

Example 19: The rightmost non-zero digit of the number 230^{2720} is:

a. 1 b. 3 c. 7 d. 9

Solution: (a)

$230^{2720} = 23^{2720} \times 10^{2720}$

Last digit before 0 is the last digit of 23^{2720} *i.e.* 1.

Example 20: Find the unit digit of $(138)^{23}$?

a. 8 b. 2 c. 4 d. 6

Solution: (b)

Last digit of 8^{23} = last digit of 8^3 as $8^{23} = 8^{4K + 3}$

Last digit of $(138)^{23} = 2$

Example 21: What will be the unit digit of $21^n - 1$?

a. 1 b. 9 c. 0 d. 2

Solution: (c)

Since $21^n - 1$ will be exactly divisible by (21 – 1) or 20. Hence, its last digit will be zero.

Example 20: Find the unit digit of product of all prime numbers.

a. 0 b. 2 c. 5 d. 7

Solution: (a)

Unit digit of $2 \times 3 \times 5 \times 7 \times 9 \ldots.. = 0$

2×5 present in the product will give 10.

Factors and Multiples

When two or more integer are multiplied together then the resultant value is called their product:

For example:

$3 \times 5 = 15$, $2 \times 3 \times 5 = 30$, $2 \times 5 \times 11 = 110$

Here, 15, 30 and 110 are called the products.

But we see that $15 = 5 \times 3$, it means 5 and 3 are the factors of 15.

Now $30 = 2 \times 3 \times 5$

where 2, 3, 5 are known as factors of 30.

Again, 15 is called as the multiple of 3 or multiple of 5.

Similarly, 30 is called as the multiple of 2 or 3 or 5 or 6 or 10 or 15.

Thus, a number which divides a given number exactly is called factor (or divisor) of that given number and the given number

Some important points of Factors and Multiples

- 1 is a factor of every number.
- Every number is a factor of itself.
- Every number, except 1 has atleast two factors viz., 1 and itself.
- Every factor of a number is less than or equal to that number.
- Every multiple of a number is greater than or equal to itself.
- Every number has infinite number of its multiples.
- Every number is a multiple of itself.

Number of factors of a given number

Prime numbers have exactly two factors but a composite number can have any number of factors. it is possible to find the number of factors of any composite number without listing all the factors. This process of finding the factors is known as factorization.

For example, lets' take 24 for instance, it can be expressed in the price factorized form as $2^3 \times 3^1$.

Here the power of 2 can take four values as (0, 1, 2, 3) and the power of 3 can take two values as (0, 1).

So, total number of combination of we take the two as combination in $4 \times 2 = 8$

As each combination of the powers of 2 and 3 gives us a distinctly different factor, so there are 8 distinct factors of the number 24.

General Method:

Let there be a composite number N and its prime factors be $a, b, c, d, \ldots.$ etc. and $p, q, r, s, \ldots.$ etc. be the indices (or powers) of the $a, b, c, d, \ldots$ etc respectively *i.e.*, if N can be expressed as

$N = a^p \times b^q \times c^r \times d^s \ldots.$

Then the number of total divisors or factors of N (including 1 and N) is

$(p + 1) \times (q + 1) \times (r + 1) \times (s + 1)\ldots.$

For example, $1200 = 2^4 \times 3^1 \times 5^2$

Total number of factors = (4 + 1)(1 + 1)(2 + 1)

= 5 × 2 × 3 = 30

(Please note that the figure arrived at by using the above formula includes 1 and the number N as they are also the factors of N. So, if you want to find the factors of 1200 excluding 1 and the number 1200 itself, then the answer will be 30 – 2 = 28 factors.)

Number of Odd and Even Factors

Let the number $N = 2^p \times b^q \times c^r$ ………. where b and c are prime numbers.

Total number of even factors = $p(q + 1)(r + 1)$

Total number of odd factors = Total number of factors – Total number of even factors.

Sum of factors of a given number

Let N be the composite number and a, b, c, d… be its prime factors and p, q, r, s be the indices (or powers) of a, b, c, d *i.e.*, if N can be expressed as

$N = a^p \times b^q \times c^r \times d^s$ ….

then the sum of all the divisors (or factors) of N

$$= \frac{(a^{p+1}-1)(b^{q+1}-1)(c^{r+1}-1)(d^{s+1}-1)}{(a-1)(b-1)(c-1)(d-1)}$$

Product of factors of a given number

The product of factors of composite number $N = N^{n/2}$, where n is the total number of factors of N.

Example 24: Find the total number of factors of 240:

a. 24 b. 20 c. 30 d. None of these

Solution: (a)

240 = 2 × 2 × 2 × 2 × 3 × 5

or $240 = 2^4 \times 3^1 \times 5^1$

Total number of factors of 240 is (4 + 1)(1 + 1)(1 + 1) = 20.

Example 25: Find the total number of divisors of 7500 except 1 and itself.

a. 32 b. 30 c. 28 d. 26

Solution: (c)

$7500 = 2 \times 2 \times 3 \times 5 \times 5 \times 5 \times 5 = 2^2 \times 3^1 \times 5^4$

$\therefore$ Total number of divisors of 7500 is

(2 + 1)(1 + 1)(4 + 1) = 30

But we have to exclude 1 and 7500

So, there are only 30 – 2 = 28 factors of 7500 except 1 and 7500. So (c) is the correct option.

Example 26: Find the sum of factors of 360.

a. 2340 b. 1170 c. $(360)^{15}$ d. None of these

Solution: (b)

$360 = 2^3 \times 3^2 \times 5$

$$\therefore \text{ Sum of factors of 360} = \frac{(2^{3+1}-1)(3^{2+1}-1)(5^{1+1}-1)}{(2-1)(3-1)(5-1)}$$

$$= \frac{15 \times 26 \times 24}{1 \times 2 \times 4} = 1170$$

Example 27: Product of divisors of 7056 is:

a. $(84)^{48}$ b. $(84)^{44}$ c. $(84)^{45}$ d. None of these

Solution: (c)

$\because$ $7056 = 2^4 \times 3^2 \times 7^2$

$\therefore$ Number of factors/divisors of 7056 = (4 + 1)(2 + 1)(2 + 1)

= 45

$\therefore$ Product of factors = $(7056)^{45/2} = (84)^{45}$

Highest Common Factor (H.C.F.)

Consider two natural numbers n_1, and n_2.

If the number n_1 and n_2 are exactly divisible by the same number x, then x is a common divisor of n_1 and n_2.

The highest of all the common divisor of n_1 and n_2 is called as the Greatest Common Divisor (GCD) or the Highest Common Factor (H.C.F). This is denoted as GCD (n_1, n_2).

Basically there are two methods of finding the HCF.

(i) Factor Method

(ii) Division Method

(i) **Factor Method:** In this method first we break (or resolve) the numbers into prime factors then take the product of all the common factors. This obtained resultant product is known as the HCF of the given numbers.

Example 28: Find the HCF of 250, 5400 and 8100.

a. 81 b. 90 c. 100 d. 50

Solution: (d)

250 = 2 × 5 × 5 × 5

5400 = 2 × 2 × 2 × 3 × 3 × 3 × 5 × 5

8100 = 2 × 2 × 3 × 3 × 3 × 3 × 5 × 5

So, the product of common factors = 2 × 5 × 5 = 50

Hence, the HCF of 250, 5400 and 8100 is 50.

(ii) **HCF by Division Method:** Consider two numbers for which HCF is to be found, we continue this process as we divide the third lowest number by the last divisor obtained in the above process.

Example 29: Find the HCF of 210, 495 and 980.

a. 2 b. 10 c. 12 d. 5

Solution:

First we consider the two smallest number *i.e.*, 210 and 495.

$$
\begin{array}{l}
210)\overline{495}(2 \\
\quad \underline{420} \\
\quad 75)\overline{210}(2 \\
\qquad \underline{150} \\
\qquad 60)\overline{75}(1 \\
\qquad\quad \underline{60} \\
\qquad\quad 15)\overline{60}(4 \\
\qquad\qquad \underline{60} \\
\qquad\qquad \times
\end{array}
\qquad\qquad
\begin{array}{l}
15)\overline{980}(65 \\
\quad \underline{975} \\
\quad 5)\overline{15}(3 \\
\qquad \underline{15} \\
\qquad \times
\end{array}
$$

Hence the HCF is 5.

HCF with Remainders

Case 1: Find the greatest possible number with which when we divide 37 and 58, it leaves the respective remainder of 2 and 3 respectively.

Solution:

Since when we divided 37 and 58 by the same number then we get remainders 2 and 3 respectively.

So, (37 – 2) and (58 – 3) must be divisible by the required number and will leave the remainders zero.

It means 35 and 55 both are divisible by the number. So, the HCF of 35 and 55 is 5.

Hence the greatest possible number is 5.

Case 2: Find the largest possible number with which when 60 and 98 are divided it leaves the remainder 3 in each case.

Solution:

Since 60 and 98 both leave the remainders 3 when divided by such a number.

Thus 57(= 63 – 3) and 95(= 98 – 3) will be divisible by the same number without leaving any remainder. So, the HCF of 57 and 95 is 19. Hence 19 is the highest possible number.

Case 3: Find the greatest number which is such that when 76, 151 and 226 are divided by it, the remainders are all alike. Also find the common remainder.

Solution:

Let k be the remainder, then the number $(76 - k)$, $(151 - k)$ and $(226 - k)$ are exactly divisible by the required number. Now, we know that if two numbers are divisible by a certain number, then their difference is also divisible by the number. Hence, the numbers $(151 - k) - (76 - k)$, $(226 - k) - (151 - k)$ and $(226 - k) - (76 - k)$ or 75, 75 and 150 are divisible by the required number. *i.e*.

Therefore, the required number = HCF common factor of 75, 75 and 150 = 75

And the remainder will be found after dividing 76 by 75, as 1.

Least Common Multiple (L.C.M.)

Let n_1 and n_2 be two natural numbers distinct from each other. The smallest natural number n that is exactly divisible by n_1 and n_2 is called Least Common Multiple (LCM) of n_1 and n_2 and is designated as LCM (n_1, n_2).

Basically, there are two methods to find the LCM.

(i) Factor Method

(ii) Division Method

(i) **Factor Method:** Resolve the given numbers into their prime factors, then take the product of all the prime factors of the first number with those prime factors of second number which are not common to the prime factors of the first number.

Now this resultant product can be multiplied with those prime factors of the third number which are not common to the factors of the previous product and this process can be continued for further numbers if any.

Example 30: Find the LCM of 48, 72 and 140.

a. 5040 b. 10080 c. 80640 d. 2520

Solution: (a)

We can write the given numbers as:

$48 = 2 \times 2 \times 2 \times 2 \times 3 \quad = 2 \times 2 \times 2 \times 3 \times 2$

$72 = 2 \times 2 \times 2 \times 3 \times 3 \quad = 2 \times 2 \times 2 \times 3 \times 3$

$140 = 2 \times 2 \times 5 \times 7 \quad = 2 \times 2 \times 5 \times 7$

So, the LCM of 48, 72 and 140 is:

$= 2 \times 2 \times 2 \times 3 \times 2 \times 3 \times 5 \times 7 = 5040$

(ii) **Division Method:** First of all write down all the given numbers in a line separating by the comma (,) then divide these numbers by the least common prime factors say 2, 3, 5, 7, 11... to the given numbers, then write the quotients just below the actual numbers separating by comma (,). If any number is not divisible by such a prime factor then write this number as it is just below itself, then continue this process of division by considering higher prime factors, if the division is complete by lower prime factor, till the quotient in the last line is 1. Then take the numbers (or quotient) in different lines (or steps). This product will be the LCM of the given numbers.

Example 31: Find the LCM of 420, 9009 and 6270.

a. 114110 b. 3423420 c. 1711710 d. 1141140

Solution: (b)

2	420, 9009, 6270
2	210, 9009, 3135
3	105, 9009, 3135
3	35, 3003, 1045
5	35, 1001, 1045
7	7, 1001, 209
11	1, 143, 19
13	1, 13, 19
19	1, 1, 19
	1, 1, 1

\ LCM of 420, 9009 and 6270 is

$= 2 \times 2 \times 3 \times 3 \times 5 \times 7 \times 11 \times 13 \times 19 = 3423420$

Example 32: Find the least possible number which can be divided by 32, 36 and 40.

a. 120 b. 144 c. 1440 d. 720

Solution: (c)

The number which is divisible by 32, 36 and 40 it must be the common multiple of all the given numbers. Since we need such a least number. So, we will have to find out the LCM of 32, 36 and 40.

The LCM of 32, 36 and 40 = 1440

Hence, the least possible number is 1440, which is divisible by all the given numbers.

LCM with Remainders

Case 1: When the remainders are same for all the divisors.

Example 33: What is the least number which when divided by 24, 32 and 42 and leaves remainder 5 in each case?

Solution:

As the positive remainder is same *i.e.* 5 for all the divisors. So, the required number will be 5 more than the LCM.

Since we know that the LCM of 24, 32 and 42 is divisible by the given numbers. So, the required number is

= (LCM of 24, 32, 42) + (5) = 672 + 5 = 677

Hence, the a least number is 677.

General term = $672\,K + 5$

Case 2: When the remainders are different for different divisors but the respective difference between the divisors and the remainders remains constant.

Example 34: What is the least possible number which when divided by 2, 3, 4, 5, 6 it leaves the remainder 1, 2, 3, 4, 5 respectively?

Solution:

As the negative remainder is same for all the divisors *i.e.* – 1. So, the required number will be 1 less than the LCM.

Since the difference is same as

$(2 – 1) = (3 – 2) = (4 – 3) = (5 – 4)(6 – 5) = 1$

Hence the required number = (LCM of 2, 3, 4, 5, 6) – 1

= 60 – 1 = 59

General term = 60 K + 59

Case 3: When neither the divisors are same nor the respective difference between divisors and the remainders remains constant.

Example 35: What is the least possible number which when divided by 13 leaves the remainder 3 and when it is divided by 5 it leaves the remainder 2?

Solution:

Let the required number be N then it can be expressed as follows

$N = 13k + 3$ ....(1)

and $N = 5l + 2$ ....(2)

Where k and l are the quotients belong to the set of integers.

Thus $5l + 2 = 13k + 3$

$\Rightarrow 5l = 13k + 1 = 10k + 3k + 1$

$\Rightarrow l = 2k + \frac{3k + 1}{5}$

Now we put the value of k such that numerator will be divisible by 5 or l must be integer. So, considering k = 1, 2, 3, ... we find that k = 3, l becomes 8. So the number $N = 5 \times 8 + 2 = 42$

Thus the least possible number = 42

To get the higher numbers which satisfy the given conditions in the above problem we just add the LCM of the given divisors (*i.e.*, 13 and 5) to the least possible number (*i.e.*, 42).

General Term = 65 K + 42

HCF and LCM of Decimals

First convert (of necessary) the same number of decimal places in each of the given numbers; then find their HCF/LCM as if they are integers and then mark off the result as many decimal places as there are in each of the given numbers.

e.g. HCF and LCM of 3.6, 2.16 and 6.0

Step 1: The given numbers are equivalent to 3.60, 2.16 and 6.00

Step 2: Now we will find the HCF and LCM of 360, 216 and 600
HCF of 360, 216 and 600 = 24 and
LCM of 360, 216 and 600 = 5400

Step 3: So, HCF = .24 and LCM = 54.00 *i.e.* 54

HCF and LCM of Fractions

First convert (of necessary) the fractions into their simplest form.

Now, HCF of fractions = $\frac{\text{HCF of Numerators}}{\text{LCM of Denominators}}$

and LCM of fractions = $\dfrac{\text{LCM of Numerators}}{\text{HCF of Denominators}}$

e.g. HCF and LCM of $\dfrac{4}{5}, \dfrac{5}{6}$ and $\dfrac{7}{15}$

HCF of given fractions = $\dfrac{\text{HCF of (4, 5 and 7)}}{\text{LCM of (5, 6 and 15)}} = \dfrac{1}{30}$

LCM of given fractions = $\dfrac{\text{LCM of (4, 5 and 7)}}{\text{HCF of (5, 6 and 15)}} = \dfrac{140}{1} = 140$

Important Rules

- HCF (n_1, n_2) × LCM (n_1, n_2) = $n_1 \times n_2$ (where n_1 and n_2 are positive numbers.)
- HCF should be a factor of LCM for any set of numbers.
- The HCF of a number of fractions is always a fraction
  (but this is not true with LCM)

Example 36: HCF of 2 numbers is 20 and their product is 800. Find the LCM of two numbers.

a. 160 b. 40 c. 20 d. 400

Solution: (b)

Product of 2 numbers = HCF × LCM

800 = 20 × LCM

$\Rightarrow$ LCM = 40

Example 37: HCF of 2 numbers is 12 and LCM is 360. How many such number of pairs exists.

a. 1 b. 2 c. 3 d. 4

Solution: (d)

HCF = 12

Thus, $N_1 = 12x$ and $N_2 = 12y$

Also, HCF × LCM = $N_1 \times N_2$

$12 \times 360 = 12x \times 12y$

$xy = 30$

No. of different co-prime pairs = 4(1 × 30, 2 × 15, 3 × 10, 5 × 6)

Þ Such number of pairs whose HCF is 12 and LCM is 360.

Example 38: HCF of 2 numbers is 20 and their product is 500. Find the LCM of two numbers.

a. 25 b. 50

c. 200 d. Data Inconsistent

Solution: (d)

Product of 2 numbers = HCF × LCM

500 = 20 × LCM

LCM = 25 which is wrong

This is because LCM is not a multiple of HCF.

$\Rightarrow$ No such pair of numbers exists.

Applications of HCF and LCM

Example 39: What is the greatest length which can be used to measure exactly the following lengths: 20 ft., 13 ft. 9 inches, 17 ft. 6 inches, 21 ft. 3 inches?

a. 1 ft. 3 inches b. 1 ft. 4 inches
c. 1 ft. d. 14 inches

Solution: (a)

We must express these lengths in the same denomination and find their greatest common divisor. Expressed in inches we have 240 inches, 165 inches, 210 inches and 255 inches. G.C.D of these values are 15 inches, or 1 ft. 3 inches.

Example 40: The sides of a hexagonal building are 216, 423, 1215, 1422, 2169 and 2223 meters. Find the greatest length of tape that would be able to exactly measure each of these sides without having to use fractions/parts of the tape?

a. 24 b. 18 c. 9 d. 15

Solution: (c)

In this question we are required to identify the HCF of the numbers 216, 423, 1215, 1422, 2169 and 2223 as the greatest length has to divide the given values.

In order to do that, we first find the smallest differece between any two of these numbers. It can be seen that the difference between 2223 – 2169 = 54. Thus, the required HCF would be a factor of the number 54.

The factors of 54 are 1, 2, 3, 6, 9, 18, 27 and 54

1×54

2×27

3×18

6×9

One of these 8 numbers one has to be the HCF of the 6 numbers. 54 cannot be the HCF because the numbers 423 and 2223 being odd numbers would not be divisible by any even number. Thus, we do not need to check any even numbers in the list.

27 does not divide 423 and hence cannot be the HCF. 18 can be skipped as it is even.

9 divides 216, 423, 1215, 1422 and 2169. Hence, it would become the HCF. (Note: we do not need to check 2223 once we know that 2169 is divisible by 9).

Example 41: A shop has 363, 429 & 693 chocolates respectively of 3 distinct varieties. It is desired to place these chocolates in straight rows of chocolates where each row has only one variety, so that the number of rows required is the minimum. What is the size of each row and how many rows would be required?

a. 33, 45 b. 45, 33 c. 11, 15 d. 15, 11

Solution: (a)

The size of each row would be the HCF of 363, 429 and 693. Difference between 363 and 429 = 66. Factors of 66 are 66, 33, 22, 11, 6, 3, 2, 1.

66 need not be checked as it is even and 363 is odd. 33 divides 363, hence would automatically divide 429 and also divides 693. Hence, 33 is the correct answer for the size of each row.

To find how many rows would be required, we need to follow the following process:

Minimum number of rows required =

$$\frac{363}{33} + \frac{429}{33} + \frac{693}{33} = 11 + 13 + 21 = 45 \text{ rows}$$

Example 42: The traffic lights at three different road-crossing, blinks after every 24 seconds, 27 seconds and 120 seconds respectively. If they all blink simultaneously at 10:54:00 hour, then at what time will they blink next simultaneously?

a. 10:57:00 b. 10:59:30
c. 11:00:00 d. 12:00:00

Solution: (c)

Interval for blink = LCM of (24, 72, 120) sec = 360 sec.
The lights will blink simultaneously after every 360 seconds, *i.e.*, 6 minutes 00 seconds.
Next simultaneous blink will take place at 11:00:00 hour.

Example 43: Find the least perfect square number which is exactly divisible by 4, 5, 6, 15 and 18.
a. 1024 b. 900 c. 1600 d. 225
Solution: (b)
First we need to find the LCM of 4, 5, 6, 15 and 18.
$4 = 2^2$
$5 = 5^0$
$6 = 2^0 \times 3$
$15 = 3 \times 5$
$18 = 2 \times 3^2$
$\therefore$ LCM $= 2^2 \times 3^2 \times 5 = 180$
$\therefore$ The number 180 is the least number that is divisible by all the individual numbers 4, 5, 6, 15 and 18.
Now,
$180 = 2^2 \times 3^2 \times 5$
$\therefore$ In order to get a perfect square that is a multiple of '180', we must include one power of 5.
So, required number $= 2^2 \times 3^2 \times 5^2 = 900$
$\therefore$ The least such perfect square number is 900.

Example 45: A gardner had a number of shrubs to plant in rows. At first he tried to plant 5 in each row, then 6, then 8 and then 12, but always has 1 shrub left over. On planting 13 shrubs per row, he had none left. What is the smallest number of shrubs that he could have had?
a. 481 b. 468 c. 469 d. 480
Solution: (a)
Let the number of shrubs be 'N'
According to the given problem, N leaves a remainder of 1 when divided by the numbers 5, 6, 8 or 12.
LCM of 5, 6, 8 and 12 is 120.
$\therefore$ $N = 120K + 1$ where $K \in N$
It is given that N = 13 K_1^1 $\therefore$ 120 K + 1 = 13 K_1^1
It is seen that the least value of K that satisfies the above condition is 4.

$$\therefore\ 120 \times 4 + 1 = 13\ K_1^1 \Rightarrow K_1^1 = \frac{481}{13} = 37$$

$\therefore$ The number $N = 120 \times 4 + 1 = 481$.

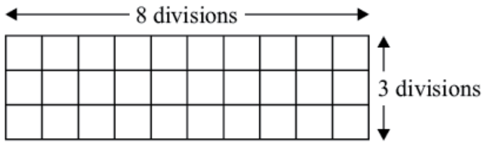
Example 46: A rectangular piece of cloth has dimension 16 m × 6 m. How many square pieces, all of the same size, can be cut from it such that no cloth is wasted and the side of each square is of the maximum possible length?
a. 8 b. 16 c. 24 d. 30
Solution: (c)
The side of the required square = HCF of the dimensions of the cloth.
$\therefore$ The side of the square = HCF of 16 and 6 *i.e.* 2
$\therefore$ The number of divisions along the length of the rectangle.

$$= \frac{\text{HCF of Numerators}}{\text{LCM of Denominators}} = 8$$

Similarly, the number of divisions along with width of the rectangle

$$= \frac{\text{breadth of rectangle}}{\text{side of square}} = \frac{6}{2} = 3$$

The above is illustrated by:



$\therefore$ The number of (2×2) squares $= 8 \times 3 = 24$.
This is also the minimum number of squares that can be cut from this piece of cloth without wasting any part.

Successive Division

In Successive division the quotient in a division is further used as a dividend for the next divisor and again the latest obtained quotient is used as a dividend for another divisor and so on.

Example 47: If 80 is successively divided by 6, 5 and 4, what will be the remainder in each case?

a. 4, 3, 2 b. 1, 2, 3 c. 2, 1, 2 d. 2, 3, 2

Solution: (d)

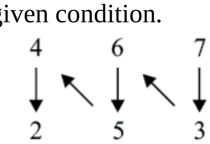
First step:	**Second step:**	**Third step:**
$6\overline{)80}(13$ 78 2	$5\overline{)13}(2$ 10 3	$4\overline{)2}(0$ 0 2
Quotient = 13 Remainder = 2	Quotient = 2 Remainder = 3	Quotient = 0 Remainder = 2

Example 48: A number, when successively divided by 4, 6 and 7, leaves remainders 2, 5 and 3. What will be the remainder if the original number is divided by 42?

a. 10 b. 11 c. 17 d. 13

Solution: (a)

We will first find the smallest number that satisfies the given condition.



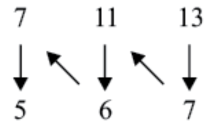
The number is $= 4(6(7a + 3) + 5) + 2$
$= 168a + 94 = 42(4a) + 84 + 10$
$= 42(4a + 2) + 10$
When divided by 42, it will leave a remainder of 10.

Example 49: How many numbers lie between 100 and 10000 which when successively divided by 7, 11 and 13 leaves the respective remainders of 5, 6 and 7?

a. 8 b. 10 c. 9 d. 11

Solution: (b)

The least possible number can be obtained as



$((7 \times 11) + 6)\ 7 + 5) = (77 + 6)\ 7 + 5)$
$= (83 \times 7 + 5) = (581 + 5) = 586$
The general form for the higher numbers is
$(7 \times 11 \times 13)\ m + 586 = (1001)\ m + 586$
So, the numbers can be obtained by considering $m = 0, 1, 2, 3, ...$ so the first number is 586 and the last number is 9595 which can be attained at $m = 9$. So there are total 10 such numbers lying between 100 and 10000.

Factorial

The factorial of a number is a number obtained by the multi- plication of all natural numbers starting from 1 till the number itself.
The factorial of a number 'n' is represented by n! or $\lfloor n$
Therefore, $n! = n \times (n-1) \times (n-2) \times ..... \times 3 \times 2 \times 1$
e.g. $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

Important Points

- $0! = 1$
- Factorial is defined only for whole numbers.
- $n! = n \times (n-1)!$

Example 52: The HCF and LCM of 13! and 31! are respectively:
a. 12! and 32! b. 13! and 31!
c. 26 and 403 d. Cannot be determined
Solution: (b)
Since 13! is contained in 31! so the LCM is 31! and 13! is common in 13! and 31!, so the HCF is 13!.

Number of Zeroes

We know that $10 = 5 \times 2$, $100 = 5^2 \times 2^2$, $1000 = 5^3 \times 2^3$ etc. So we can say that for '*n'* number of zeros at the end of the product we need exactly '*n'* combination of '5×2'.

Example 54: Find the number of zeros at the end of the product of the expression $10 \times 100 \times 1000 \times 10000 \times ....10000000000$.
a. 10 b. 100 c. 50 d. 55
Solution: (d)
$10 \times 100 \times 1000 \times 10000 \times ....\ 10000000000$
$= 10^1 \times 10^2 \times 10^3 \times\ .... \times 10^{10}$
$= 10^{(1 + 2 + 3 + ... + 10)} = 10^{55}$

Example 53: Find the number of zeros in the product of 20!.
a. 5 b. 3 c. 2 d. 4
Solution: (d)
$20! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times ..... \times 20$
It is obvious from the above expression that there are only two 4's and eighteen 2's.
Since the number of 5's are less (than the number of 2's) so the number of 5's will be effective to form the combination of '5×2'.
Thus there are only 4 zeros at the end of the product of 20!.

Example 55: Find the number of zeros at the end of the following expression $(5!)^{5!} + (10!)^{10!} + (50!)^{50!} + (100!)^{100!}$.

a. 165 b. 120 c. 125 d. None of these

Solution: (b)

The number of zeros at the end of $(5!)^{5!} = 120$

[$\because$ 5! = 120 and thus $(120)^{120}$ will give 120 zeros] and the number of zeros at the end of the $(10!)^{10!}$, $(50!)^{50!}$ and $(100!)^{100!}$ will be greater than 120.

Now since the number of zeros at the end of the whole expression will depend on the number which has least number of zeros at the end of the number among other given numbers.

So, the number of zeros at the end of the given expression is 120.

Divisibility of a factorial number by the largest power of any number

Suppose we have to find the highest power of k that can exactly divided $n!$, we divide n by k, n by k^2, n by k^3 .... and so on till we get $\left[\frac{n}{k^x}\right]$ equal to 1 (where, [P] means the greatest integer

less than or equal to P) and then add up as $\left[\frac{n}{k}\right] + \left[\frac{n}{k^2}\right] + \left[\frac{n}{k^3}\right] + \left[\frac{n}{k^4}\right] + ... + \left[\frac{n}{k^x}\right]$

.

Example 56: Find the largest power of 5 contained in 114!

a. 26 b. 25 c. 27 d. 28

Solution: (a)

$\left[\frac{114}{5}\right] + \left[\frac{114}{5^2}\right] = 22 + 4 = 26$

[We cannot do it further since 114 is not divisible by 5^3]

Hence, there are 26 times 5 involved as a factor in 114!

Example 57: Find the largest power of 3 that can exactly divide 333!

a. 111 b. 120 c. 165 d. 180

Solution: (c)

$$\begin{array}{c|ll} 3 & 333 & \\ \hline 3 & 111 & \rightarrow \\ \hline 3 & 37 & \rightarrow \\ \hline 3 & 12 & \rightarrow \\ \hline 3 & 4 & \rightarrow \\ \hline 3 & 1 & \rightarrow \\ \hline & 0 & \end{array} \left.\begin{array}{c} \\ \\ \\ \\ \\ \end{array}\right\} 165$$

Thus the highest power of 3 is 165 by which 333! can be divided.

Calendars

A ordinary year consists of 365 days (52 weeks and 1 odd day). An extra day is added once in every fourth year which was called the leap year, which has 366 days (52 weeks and 2 odd days). An year which is divisible by 4 is a leap year e.g. 1896, but if the year is a century year, then it should be divisible by 400, only then it would be a leap year. e.g. 1700 is not a leap year whereas 1600 is a leap year.

To find the day of any given date of the year, you need to understand the calender calculations :

I. First thing to remember, first January 1 AD was Monday therefore, we must count days from Sunday. This means the 0^{th} day was Sunday, so the 7^{th} day was Sunday again and so on and so forth.

II. The day gets repeated after every seventh day (concept of a week), if today is Monday, then 28^{th} day from now will also be Monday, then 28^{th} day from now will also be Monday as it a multiple of 7(28/7 = 4, so four weeks). Here the 30 day will be calculated by 30/7, which is 4 weeks and 2 days, these two day are called odd days. With starting day as Monday and two odd days, the day will be Wednesday; this point is the most critical in calenders. The other way to look at it is since the 28^{th} day is Monday, so the 30^{th} day will be Wednesday. But you have to understand and use the concept of odd days as the question may be about thousands of years.

III. In an normal year there are 365 days. So 52 weeks and 1 odd day, whereas in a leap year there are 366 days. So 52 weeks and 2 odd days.

IV. In 100 years there are 24 leap years and 76 normal years, so the number of odd days are 24(2) + 76(1) = 124, which is 17 weeks + 5 odd days, so 100 years have 5 odd days.

V. In 200 years the number of odd days is twice the number in 100 years which is 10, which is one week and 3 odd days, so 200 years have 3 odd days. In 300 years, the number of odd days is 15, which is two weeks and 1 odd day, so 300 years have one odd day.

VI. In 400 years, the number of odd days become 20 + 1 (from the leap year), so total days are 21, which is three weeks and 0 odd days. In 400 years there are 0 odd days.

Example : What was the day on 25^{th} January, 1975 ?

Solution:

Counting the years 1600 + 300 + 74

In 1600 years, there are zero odd days.

In 300 years, there is one odd day

In 74 years, there are 18 leap years and 56 normal years, so the odd days are :

$(18 \times 2 + 56 \times 1) = \frac{92}{7} = 1$ odd day

Which is 13 weeks and 1 odd day

In 25 days of January, 1975, there are 3 weeks and 4 odd days.

Total odd days = 0 + 1 + 1 + 4 = 6

This final vale which is coming 6 in the is question decides which day it was.

Final Value	**Day**
1	Monday
2	Tuesday
3	Wednesday
4	Thursday
5	Friday
6	Saturday
0	Sunday

So, it was a Saturday.

Example 23: What day of the week was on 20th June 1837?

Solution:

20th June, 1837 means 1836 complete years + first 5 months of the year 1837 + 20 days of June.

1600 years give no odd days 200 years give 3 odd days 36 years give (36 + 9) or 3 odd days.

Thus 1836 years give 6 odd days

From 1st January to 20th June there are 3 odd days

Odd days : January 3, February 0, March 3, April 2, May 3, June 6 = 17.

Thus the total number of odd days = 6 + 3 or 2 odd days.

This means that the 20th of June fell on 2nd day commencing from Monday.

The required day was Tuesday.

Example 24: Today is 3rd November. The day of the week is Monday. This is a leap year. What will be the day of the week on this date after 3 years?

Solution:

This is a leap year. So none of the next 3 years will be leap years. Each year will give one odd day so the day of the week will be 3 odd days after Monday *i.e.* it will be Thursday.

Example 25: The calender of year 1982 is same as which year?

a. 1987 b. 1988 c. 1990 d. 1993

Solution:

We need to have 0 odd days, counting from 1982.

Year	Odd days	Total
1982	1	1
1983	1	2
1984	2	4
1985	1	5
1986	1	6
1987	1	7
1988	2	9
1989	1	10
1990	1	11
1991	1	12
1992	2	14
1993	1	

Therefore 1988 could be the year with the same calender as 1982, buts it’s a leap year and 1982 is not. Therefore next is 1993, where it fits, so calender of 1982 is same as 1993.

Example 73: What day was 12th February, 2001?

a. Monday b. Tuesday

c. Sunday d. Thursday

Solution: (a)

For almost all questions of this type, the best approach is to calculate the number of odds days.

Every 400 years gives us zero odds days. So, the first 2000 years give us 0 odd days. January, 2001, gives us 31 odd days or 3 odd days. 12 more days in February takes us to 15 odd days or 1 odd day.

⇒ **February 12th should have been a Monday.**

Example 73: 29th of February of a certain year was a Monday. What day was 20th of August in the same year?

a.	Thursday	b.	Friday
c.	Saturday	d.	Monday

Solution: (c)

In March we have 31 days, or 3 odd days.

April gives us 2 odd days, May gives 3, June 2, July 3. So, till the end of the July we have 13 odd days or 6 odd days . + 20 more in August takes us to 26 odd days, or 5 odd days. If February 29th was a Monday, August 19th should be Saturday.

Example 74: There were five Mondays and five Tuesdays in the month of April in a certain non-leap year. What day was January 1st of that year?

a.	Sunday	b.	Monday
c.	Tuesday	d.	Wednesday

Solution: (c)

There were five Mondays and five Tuesdays in the month of April. The month of April has 30 days, or, 4 full weeks and 2 days. So, if there are 5 Mondays and 5 Tuesdays, this tells us that 29th and 30th of the month were Monday and Tuesday.

⇒ The 1st of the month should have been a Monday.

April 1st = Monday. This was a non-leap year.

January – 31 days, February 28 days, March 31 days. The 91st days of the year is a Monday. So, the first day should have been Tuesday.

Example 75: In a leap year, there were 53 Sundays. What day was February 29th of that year?

a.	Saturday	b.	Sunday
c.	Monday	d.	None of these

Solution: (d)

In a leap year, there will be 53 occurrences of 2 days. So if there were 53 Sundays, there could have been 53 Saturdays and 53 Sundays or, 53 Sundays and 53 Mondays.

Scenario 1: 53 Saturdays and 53 Sundays Þ January 1st was a Saturday Þ January 29th was a Saturday Þ February 5th was a Saturday. Þ February 26th was a Saturday Þ February 29th was a Tuesday.

Scenario 2: 53 Sundays and 53 Mondays Þ January 1st was Sunday Þ February 29th was a Wednesday.

Base System

The ordinary numbers with which we are acquainted in general are expressed by means of multiples of powers of 10.

For example:

$25 = 2 \times 10 + 5$

$4705 = 4 \times 10^3 + 7 \times 10^2 + 0 \times 10 + 5.$

This method of representing numbers is called the base system. The symbols employed in this system of notation are the nine digits and zero.

If 7 is the base system, a number expressed by 2453 represents $2 \times 7^3 + 4 \times 7^2 + 5 \times 7 + 3$; and in scale no digit higher than 6 can occur.

Again in a scale whose base system is denoted by r the above number 2453 stands for $2r^2 + 4r^2 + 5r + 3$.

More generally, if in the scale whose base system is r we denote the digits, beginning with that in the unit's place, by a_0, a_1, a_2, ... a_n; then the number so formed will be represented by

$$a_n r^n + a_{n-1} r^{n-1} + a_{n-2} r^{n-2} + ... + a_2 r^2 + a_1 r + a_0$$

Decimal	Binary	Octal	Hexadecimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
15	1111	17	F
16	10000	20	10
17	10001	21	11
18	10010	22	12

Base system works like a odometer of a vehicle. (that which record cumulative mileage) Whenever a wheel starts repeating the cycle, it increases the wheel on the left by 1.

Conversion from base 10 to any other base

Example 76: Convert $(132)_{10}$ to base 8 system.
a. 164 b. 184 c. 204 d. 214
Solution:

8	132	
8	16	4
8	2	0
	0	2

The number in decimal is consecutively divided by the number of the base to which we are converting the decimal number. Then list down all the remainders in the reverse sequence to get the number in that base.
So, here $(132)_{10} = (204)_8$

Conversion from any other base to decimal base system

Example 78: Convert $(136)_8$ into decimal system.

a. 146 b. 92 c. 83 d. 89

Solution:

In $(136)_8$, the value of the position of each of the numbers (as in decimal system is):

$6 = 8^0 \times 6$

$3 = 8^1 \times 3$

$1 = 8^2 \times 1$

[This is equivalent to 10^0 (unit's); 10^1 (ten's); 10^2 (hundred's) places in the decimal system.]

Hence, $(136)_8 = (8^0 \times 6 + 8^1 \times 3 + 8^2 \times 1)_{10}$

$= (1 + 24 + 64)_{10} = (89)_{10}$.

Miscellaneous Topics

- Fermat's Little Theorem

If p be a prime number, then $a^p - a$ is divisible by p, where a is a natural number greater than 1.

- Wilson's Theorem

If p is a prime number, then $(p - 1)!$ will leave a remainder $(p - 1)$ when divided by p.

Also, if p is a prime number, then $(p - 2)!$ will leave a remainder of 1 when divided by p.

- **Euler's Totient Function**

Euler's totient function, E[n] denotes number of positive integers that are coprime to and less than a certain positive integer 'n'. Such a count is not difficult to deduce. Let's take a simple example and find out Euler's totient for 100, *i.e.*, number of positive integers less than 100 and coprime to 100.

Prime divisors of 100 are 2 and 5.

This value can be written in simpler manner as:

$$100\left(1-\frac{1}{2}\right)\left(1-\frac{1}{5}\right) = 40$$

And number of integers which are divisible by both 2 and

$$5 \ (i.e.,\ 10) \text{ are } = \frac{100}{10} = 10$$

Now the number of integers which are divisible by either 2 or 5 = 50 + 20 − 10 = 60

So, the number of integers less than 100 and not divisible by 2 or 5 = 100 − 60 = 40

Example 68: What will be the remainder when 12^{31} is divided by 31?

a. 6 b. 19 c. 17 d. 12

Solution: (d)

As per **Fermat's Little Theorem,** $a^p - a$ is divisible by p, where a is any natural number greater and 1 and p is a prime number.

$\therefore$ $12^{31} - 12$ is divisible by 31.

Therefore 12^{31} will leave the remainder 12 when divided by 31.

Example 69: What will be the remainder when 18! is divided by 19?

a. 0 b. 1 c. 18 d. 2

Solution: (c)

According to **Wilson's Theorem,** if p is a prime then $(p - 1)!$ will give the remainder $p - 1$ when divided by p.

So when 18! is divided by 19, it will give a remainder of 18.

Last 2 digits of a Number

- **Last two digits of numbers which end in one**

The unit digits is always 1. to get the tens digits, multiply the tens digit of the number with the last digit of the exponent to get the tens digit.

Example : Find the last two digits of 41^{2789}.

a. 41 b. 21 c. 61 d. 81

Solution: (d)

Tens digit will be $4 \times 9 = 6$

Unit digit is 1. Therefore, 61 is the answer.

Example : Find the last two digits of $51^{456} \times 61^{567}$.

a. 21 b. 61 c. 41 d. None of these

Solution: (a)

The last two digits of 51^{456} will be 01 and the last two digits of 61^{567} will be 21.

Therefore, the last two digits of $51^{456} \times 61^{567}$ will be the last two digits of $01 \times 21 = 21$

Last two digits of numbers which end in 3, 7 and 9 Convert the number till the number gives 1 and then find the last two digits according to the above send method.

Example : Find the last two digits of 29^{266}.

a. 21 b. 41 c. 61 d. 81

Solution: (d)

$29^{266} = (29^2)^{133}$. Now, 29^2 ends in 41. Thus, we need to find the last two digits of $(41)^{133}$.

From the above method, we know that the last two digits of 41^{133} will be 21.

Example : Find the last two digits of 13^{256}.

a. 21 b. 41 c. 61 d. 81

Solution: (b)

$13^{256} = (13^4)^{64}$. Now, 13^4 ends on 61 ($169 \times 169 = 28561$). Thus, we need to find the last two digits of $(61)^{64}$.

From the above method, the last two digits of 61^{64} will be 41.

Example : Find the last two digits of 57^{474}.

a. 39 b. 19 c. 49 d. 69

Solution: (c)

$57^{474} = 57^{472} \times 57^2 = (57^4)^{118} \times 57^2 = (49 \times 49)^{118} \times 49$ (The last two digits of 57^2 are 49) $= 01^{118} \times 49$

$= 01 \times 49 = 49$

Last two digits of numbers which end in 2, 4, 6 and 8. There is only one even two-digit number which always ends in itself (last two digits) – 76 i.e. 76 raised to any power gives the last two digits as 76. Also, 24^2 ends in 76 and 2^{10} ends in 24.

24 raised to an even power always ends with 76 and 24 raised to an odd power always ends with 24. Therefore, 24^{24} will end in 76 and 24^{23} will end in 24.

Example : Find the last two digits of 2^{343}.

a. 18 b. 08 c. 38 d. 78

Solution: (b)

$2^{343} = (2^{10})^{34} \times 2^3 = (24)^{34}$

$(\therefore\ 2^{10} = 1024) \times 2^3 = 76 \times 8 = 08$

Note: When 76 is multiplied with 2^n, the last two digits remain the same as the last two digits of 2^n. Therefore, the last two digits of 76×2^7 will be the last two digits of $2^7 = 28$

Example : Find the last two digits of 78^{379}.

a. 94 b. 82 c. 76 d. 92

Solution: (d)

$78^{379} = (2 \times 39)^{379} = 2^{379} \times 39^{379} = (2^{10})^{37} \times 2^9 \times (39^2)^{189} \times 39 = 24 \times 12 \times 81 \times 39 = 92$

Example : Find the last two digits of 56^{293}.

a. b. c. d.

Solution: (b)

$56^{293} = (2^3 \times 7)^{293} = 2^{879} \times 7^{293} = (2^{10})^{87} \times 2^9 \times (7^4)^{73} \times 7^1$

$= 24 \times 12 \times (01)^{73} \times 7 = 16$

Some Important Conversion

1 trillion = 10^{12} = 1000000000000

1 billion = 10^9 = 1000000000

1 million = 10^6 = 1000000

1 crore = 10^7 = 100 lakh

10 lakh = 10^6 = 1 million

1 lakh = 10^5 = 100000 = 100 thousand

Practice Exercise – Easy

1. Which of the following numbers can't be written as the sum of two prime numbers?
   a. 23 b. 16 c. 9 d. 12

2. What is the remainder if the square of any prime number greater than 5 when divided by 24 is:
   a. Always 1 b. Always 3
   c. Always 5 d. Either 1 or 3

3. Let p be a prime number greater than 3. Then what will be the remainder when $p^2 + 17$ will be divided by 12?
   a. 1 b. 5 c. 6 d. 7

4. Four prime numbers are in ascending order of their magnitudes. The product of the first three is 385 and that of last three is 1001. The largest given prime number is:
   a. 11 b. 13 c. 17 d. 19

5. n^3 is odd. Which of the following statement(s) is/are true?
   I. n is odd
   II. n^2 is odd
   III. n^2 is even
   a. I only b. II only
   c. I and II only d. I and III only

6. x, y and z are distinct integers, If x and y are both odd and z is even, then which of the following expressions is/are always false?
   I. $(x + y)\,x$ is odd
   II. $(x + z)\,z$ is even
   III. $(x + z)\,x^3$ is odd
   IV. $(x + z)^3\,x$ is odd
   a. I b. II c. III d. IV

7. If the sum of a number of two digits and a number formed by reversing the digits is 99, then what is the sum of the digits of the original number?
   a. 18 b. 81 c. 9 d. 8

8. If the number in the units place exceeds the number in the tens by 2 in a two-digit number and the product of the number and the sum of its digits is equal to 280, what would be the sum of its digits?
   a. 4 b. 8 c. 9 d. 10

9. Half way through the journey from Delhi to Chennai, Krishnan began to look out of the window of Anna Express and continued it until the distance which was remained to cover was half of what he has covered. Now at this time, how much distance he has to cover?
   a. 2/2 b. 1/4 c. 1/3 d. 1/6

10. If a + 1/a = b, then for $a > 0$:
    a. $b = 0$ b. $-2 < b < 2$
    c. $b \geq 2$ d. Does not exist

11. The greatest fraction among $\frac{2}{5}, \frac{3}{5}, \frac{1}{5}, \frac{7}{15}$ and $\frac{4}{5}$ is:
    a. 4/5 b. 3/5 c. 2/5 d. 7/15

12. Rambha was trying to find 5/8th of a number. Unfortunately, she found out 8/5th of the number and realized that the difference between the answer she got and the correct answer is 39. What is the number?
    a. 39 b. 38 c. 40 d. 52

13. Anandita had to do a multiplication. Instead of taking 35 as one of the multipliers, she took 53. As a result, the product went up by 540. What is the correct product?
    a. 1050 b. 540 c. 1040 d. 1590

14. If n^2 = 12345678987654321, what is n?
    a. 12344321 b. 1235789

c. 111111111 d. 11111111

15. The least number of 4 digits which is a perfect square is:
a. 1064 b. 1040 c. 1024 d. 1012

16. Find the least number, which must be added to 7147 to make it a perfect square.
a. 76 b. 77 c. 78 d. 79

17. The value of $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\ldots\ldots\left(1-\frac{1}{n}\right)$ is:

a. 1 b. $\left(1-\frac{1}{n}\right)^n$

c. $\frac{1}{n}$ d. $\left(\frac{1}{n}\right)^2$

18. Kaveri Degree College has a waiting list of 5005 applicants. The list shows that there are at least 5 males between any two females. The largest number of females in the list could be:
a. 920 b. 835 c. 721 d. 1005

19. Which of the following is true?
a. $7^{3^2} = (7^3)^2$ b. $7^{3^2} > (7^3)^2$
c. $7^{3^2} < (7^3)^2$ d. None of these

20. The expression $(a + b)^{-1} \times (a^{-1} + b^{-1})$ is equivalent to:
a. 1 b. $(ab)^{-1}$ c. a^b d. $ab^{-1} + a^{-1}b$

21. $5^{x-1} + 5^x + 5^{x+1} = 775$, then the value of x, where x is a positive integer:
a. 1 b. 3 c. 2 d. 4

22. Evaluate: $11^2 + 11^4 \div 11^3 - 11 + (0.5) \times 11^2$.
a. 180.5 b. 181.5 c. 121 d. 484

23. If $x = \sqrt{2} + 1$, then value of $x + \frac{1}{x}$ is:
a. $\sqrt{\frac{3}{2}}$ b. $\frac{\sqrt{3}}{2}$ c. $\frac{\sqrt{2}}{2}$ d. $2\sqrt{2}$

24. If $A = \left(\frac{-3}{4}\right)^3$, $B =$, $\left(\frac{-2}{5}\right)^2$ $C = (0.3)^2$, $D = (-1.2)^2$ then
a. $A > B > C > D$ b. $D > C > A > B$
c. $D > B > C > A$ d. $D > A > B > C$

25. If $x = (6 - \sqrt{35})$, then the reciprocal of x is:
a. $\frac{1}{(6+\sqrt{35})}$ b. $(6 + \sqrt{35})$
c. $\sqrt{35}$ d. 12

26. Find the value of $\left(\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}\right)$

a. 1/2 b. 2/3 c. $\frac{(n-1)}{(n+1)}$ d. 3/2

27. The value of is : $\dfrac{55^3 + 45^3}{55^2 - 55 \times 45 + 45^2}$

a. 100 b. 105 c. 125 d. 75

28. The Positive square root of $31 + 4\sqrt{57}$?

a. $\sqrt{19} + 4\sqrt{3}$ b. $\sqrt{19} + 2\sqrt{3}$

c. $\sqrt{17} + \sqrt{14}$ d. $\sqrt{17} + 3\sqrt{14}$

29. In Anna Nagar the building were numbered from 1 to100. Then how many 4's will be present in the numbers ?

a. 18 b. 19 c. 20 d. 21

30. In all the number from 501 to 700 are written, what is the total number of times does the digit 6 appear ?

a. 138 b. 139 c. 140 d. 141

31. If $\log_{12} 27 = a$, then $\log_6 16$ is:

a. $\dfrac{4(3-a)}{3+a}$ b. $\dfrac{4(3+a)}{3-a}$

c. $\dfrac{3+a}{4(3-a)}$ d. $\dfrac{3-a}{4(3+a)}$

32. If $a^x = b$, $b^y = c$, $c^z = a$, then the value of xyz is:

a. 0 b. 1 c. 2 d. 4

33. Find the number of digits in 8^{10}. (Given that $\log_{10} 2 = 0.3010$)

a. 7 b. 8 c. 12 d. 10

34. $\log_6 \dfrac{3-a}{4(3+a)}$ is:

a. 3 b. 3/2

c. 7/2 d. None of these

35. If $\log_7 \log_5 \left(\sqrt{x} + 5 + \sqrt{x}\right) = 0$, what is the value of x?

a. 1 b. 0

c. 2 d. None of these

36. Which of the following numbers is divisible by 21 but not divisible by 9?

a. 1371804 b. 247212

c. 531441 d. 2534178

37. 5a79 is divisible by 33. What is the value of a?

a. 3 b. 4 c. 5 d. 6

38. What least value must be assigned to * so that the number 451 * 603 becomes exactly divisible by 9?

a. 2 b. 7 c 8 d. 5

39. The sum of 3 consecutive even numbers is always divisible by:

a. 24 b. 48

c. 6 d. None of these

40. $(2^{19} + 1)$ is divisible by:

a. 3 b. 4 c. 6 d. both 3 & 6

41. For a number to be divisible by 88, it should be:

a. Divisible by 22 and 8 b. Divisible by 11 and 8

c. Divisible by 2 and 44 d. All of these

42. The number $(10^n - 1)$ is always divisible by 11 for

a. even values of n b. odd values of n c. all values of n d. n as multiple of 11

43. The number which is formed by writing any digit 6 times (e.g. 111111, 444444, etc.) is always divisible by:
a. 7 b. 11
c. 13 d. (a), (b) and (c)

44. Let $N = 55^3 + 17^3 - 72^3$. N is divisible by:
a. both 7 and 13 b. both 3 and 13
c. both 16 and 7 d. both 3 and 17

45. By what largest number the product of any five consecutive natural numbers is divisible:
a. 120 b. 160
c. 100 d. None of these

46. A number when divided by 84 leaves a remainder of 57. What is the remainder when the same number is divided by 28?
a. 2 b. 7
c. 1 d. Cannot be determined

47. When 'n' is divided by 5 the remainder is 2. What is the remainder when n^2 is divided by 5?
a. 2 b. 1 c. 3 d. 4

48. When 2^{256} is divided by 17, the remainder would be: a. 1 b. 16
c. 14 d. None of these

49. What is the remainder when 90^{91} is divided by 13?
a. 0 b. 7 c. 12 d. 1

50. What will be the remainder if $2^{34} + 67$ is divided by 7?
a. 6 b. 0 c. 4 d. 5

51. The remainder, when $(15^{23} + 23^{23})$ is divided by 19, is:
a. 4 b. 15 c. 0 d. 18

[CAT 2004]

52. Find the unit digit of $27^{15^{16}}$?
a. 9 b. 3 c. 7 d. 1

53. Find the units digits of the expression $11^1 \times 12^2 \times 13^3 \times 14^4 \times 15^5 \times 16^6$.
a. 4 b. 3 c. 7 d. 0

54. Find the last digit of $2^{22} \times 3^{33} \times 4^{44} \times 5^{55}$?
a. 2 b. 4 c. 8 d. 0

55. What will be the last digit of the multiplication $3^{153} \times 7^{162}$?
a. 5 b. 9 c. 7 d. 6

56. Find the unit digit of $3^{200} \times 4^{500}$?
a. 2 b. 4 c. 8 d. 6

57. $4^{11} + 4^{12} + 4^{13} + 4^{14} + 4^{15}$ is divisible by:
a. 11 b. 31
c. 341 d. All of the above

Directions for questions 58 and 59: Amit buys a toffee, a chocolate and a packet of chips for Rs. 41. If the least cost of any of the three items is Rs. 12 and it is known that a toffee costs less than a chocolate and a packet of chips costs more than a chocolate, answer the following questions:

58. What is the cost of the toffee?
a. 12 b. 13 c. 14 d. 15

59. If it is known that the packet of chips cost is not divisible by 4, the cost of the chocolate could be:
a. 12 b. 13 c. 14 d. 15

Directions for questions 60 to 62: Given two distinct prime number A and B, find the number of divisors of the following.

60. $A \times B$

a. 2 b. 4 c. 6 d. 8

61. $A^2 \times B$

a. 2 b. 4 c. 6 d. 8

62. $A^3 \times B^2$

a. 2 b. 4 c. 6 d. 12

63. A and B are two positive integers such that $AB = 64$. Which of the following cannot be the value of $A + B$?

a. 20 b. 65 c. 16 d. 35

64. Find the number of ways in which 48 can be written as a product of two different factors.

a. 2 b. 3 c. 4 d. 5

65. How many times is the HCF of 32, 36, 72 and 24 contained in their LCM?

a. 16 b. 64 c. 72 d. 84

66. Two numbers $P = 2^3 \times 3^{10} \times 5$ and $Q = 2^5 \times 3^1 \times 7^1$ are given. Find the GCD of P and Q.

a. $2 \times 3 \times 5 \times 7$ b. 3×2^2
c. $2^2 \times 3^2$ d. $2^3 \times 3$

67. The smallest number (> 2) that leaves a remainder of 2 when divided by 4, 6 or 7 is:

a. 44 b. 62 c. 80 d. 86

68. A bunch of flowers, when sorted into groups of 32, 40, 72, leave remainders 10, 18 and 50 respectively. Find the least number of flowers in the bunch.

a. 1418 b. 1628 c. 1291 d. 1440

69. Two numbers are in the ratio 3 : 4 and the product of their L.C.M & H.C.F is 10,800. Find the numbers.

a. 90 & 120 b. 30 & 150
c. 120 & 160 d. 108 & 144

70. What is the least 3 digit number which when divided by 2, 3, 4, 5 or 6 leaves a remainder of 1?

a. 131 b. 161
c. 121 d. None of these

71. Three bells ring at intervals of 12, 18 and 36 minutes respectively. If they ring together at 11.25 AM, when will they ring together for the first time after that?

a. 11:56 AM b. 12:04 PM
c. 12:01 PM d. 12:50 PM

72. When Santa distributes some cookies among 40 kids, four cookies are left. If he distributes the same number of cookies to the 40 kids and the father, eight cookies are left. Find the minimum number of cookies the Santa has?

a. 1443 b. 1476 c. 1478 d. 1480

73. The sides of a pentagonal field (not regular) are 1737 metres, 2160 metres, 2358 metres, 1422 metres and 2214 metres respectively. Find the greatest length of the tape by which the five sides may be measured completely.

a. 7 b. 13 c. 11 d. 9

74. There are 576 boys and 448 girls in a school that are to be divided into equal sections of either boys or girls alone. Find the minimum number of sections formed.

a. 24 b. 32 c. 16 d. 20

75. The HCF of two numbers P and Q is 26. The HCF of two other numbers R and S is 39. What is the HCF of P, Q, R and S?

a. 7 b. 13 c. 26 d. 28

76. Find the number of zeros at the end of 1090!

a. 270 b. 268 c. 269 d. 271

77. N has 37 zeros at its end. How many values of N is/are possible?

a. 0 b. 1 c. 5 d. Infinite

78. Find the highest power of 10^p that divides 50! completely.

a. 13 b. 11 c. 5 d. 12

79. $\frac{155!}{20^n}$ is an integer. Find the highest possible value of n for this to be true.

a. 77 b. 38 c. 75 d. 37

80. Which day of the week was on 15th February, 1763?

a. Monday b. Tuesday
c. Wednesday d. Friday

81. Which day of the week was on 3rd October, 1979?

a. Sunday b. Tuesday
c. Wednesday d. Monday

82. If 11th October, 1962 was Sunday, what will be the day on 11th October, 1991?

a. Sunday b. Monday
c. Friday d. Saturday

83. If 1st June, 1857 was Sunday, what will be the day on 31st August, 1919?

a. Thursday b. Saturday
c. Friday d. Sunday

84. What can be the minimum number of days in any 7 consecutive years?

a. 2557 b. 2555 c. 2558 d. 2556

85. What is the hexadecimal equivalent of $(32312)_4$?

a. $(2126)_{16}$ b. $(3126)_{16}$ c. $(2B6)_{16}$ d. $(3B6)_{16}$

86. Convert the number 1982 from base 10 to base 12. The result is:

a. 1182 b. 1912
c. 1192 d. 1292

87. The symbol 25_b represents *a* two-digit number in base *b*. If the number 52_b is double the number 25_b, then b is:

a. 7 b. 8 c. 9 d. 11

Practice Exercise – Medium

1. There are 3 consecutive odd integers such that the product is a prime number. Find the product?

a. 1 b. 3 c. 5 d. 7

2. x is a prime number and $(x^2 + 3)$ is also a prime number. The number of values that x can assume is:

a. 3 b. 2 c. 1 d. Can't say

3. When you reverse the digits of the number 14, the number increases by 27. How many other two-digit numbers increases by 27 when their digits are reversed?

a. 5 b. 6 c. 7 d. 8

4. Number *S* is obtained by squaring the sum of digits of a two digit number *D*. If difference between *S* and *D* is 27, then the two digit number *D* is:

a. 24 b. 54 c. 34 d. 45

[CAT 2002]

5. How many numbers from 287 to 803 will contain 2 as one of its digit?

a. 108 b. 107
c. 109 d. None of these

6. *A* is a three-digit natural number. If you strike out extreme left digit of *A*, remaining number is a perfect square. If you strike out extreme right digit of *A*, remaining number is still a perfect square. How many different values of *A* are possible ?

a. 2 b. 3 c. 4 d. 5

7. $(BE)^2 = MPB$, where B, E, M and P are distinct integers, then M = ?
   a. 2 b. 3
   c. 9 d. None of these

8. You are selecting 10 numbers randomly out of the first 100 odd numbers. Sum of these 10 odd numbers is A. How many different values of A are possible ?
   a. $^{100}C_{10}$ b. 1801 c. 1800 d. 901

9. A boy added all natural number from 1 to n, but however he missed one number due to which the sum becomes 177. Find the number which the boy missed ?
   a. 11 b. 12 c. 13 d. 14

10. The sum of 10 consecutive natural numbers cannot be :
   a. 785 b. 755
   c. 385 d. None of these

11. The sum of 100 terms of the series:
   $1 - 3 + 5 - 7 + 9 - 11 + 13 - 15 + ...$
   a. – 100 b. – 50
   c. – 200 d. None of these

12. If $1 + 2 + 3 + ... + k = N^2$ and N is less than 100 then the value of k can be:
   a. 1 b. 8
   c. 49 d. All of these

13. $1^2 - 2^2 + 3^2 - 4^2 + ... - 198^2 + 199^2$:
   a. 19900 b. 12321
   c. 19998 d. None of these

14. Consider a sequence where the nth term, $t_n = \frac{n}{(n+2)}$, $n = 1, 2,....$ The value of $t_3 \times t_4 \times t_5 \times .... \times t_{53}$ equals.
   a. $\frac{2}{495}$ b. $\frac{2}{477}$
   c. $\frac{12}{55}$ d. $\frac{1}{1485}$

15. If the product of three consecutive integers is 720, then their sum is:
   a. 54 b. 45 c. 18 d. 27

16. Three numbers are such that the second is as much lesser than the third as the first is lesser than the second. The product of the two smaller numbers is 85 and the product of two larger numbers is 115. What is the middle number.
   a. 9 b. 8 c. 12 d. 10

17. On January 1st, 2015, Lakshmi saved Re. 1. Everyday starting from January 2nd, 2015, he saved Re. 1 more than the previous day. Find the first date after January 1st, 2015 at the end of which his total savings will be a perfect square.
   a. 17th January, 2015 b. 18th February, 2015
   c. 26th January, 2015 d. None of these

18. Once I met two persons of the same parents namely Aatif and Fatima. Aatif told me that he has twice the number of sisters as the number of brothers. Further Fatima told me that she has twice the number of brothers as the number of sisters. Actually it was very confusing for me, so do you know that how many brothers and sisters are in their family?
   a. 4 b. 5
   c. 6 d. Cannot be determined

19. Ram and Laxman, the two bird hunters went to woods. Laxman fires 5 shots when Ram fires 7 shots. But Laxman kills 2 out of 5 while Ram kills 3 out of 7. When Ram has missed 32 shots, then how many birds has Laxman killed?
   a. 25 b. 24 c. 16 d. 12

20. How many integers A in the set of integers {1, 2, 3, ..., 100} are there such that $A^2 + A^3$ is a perfect square?
   a. 5 b. 7 c. 9 d. 11

21. The product of the digits of a three digit number which is both a perfect square and a perfect cube is:
   a. 126 b. 256
   c. 18 d. None of these

22. Raju purchased a ticket for the cricket match between India and Pakistan in the World Cup. The number on the ticket was a 5 digit perfect square such that the first and the last digit were the same and the 2^{nd} and 4^{th} digit were the same. If the 3^{rd} digit was 3, then what was the ticket number?
   a. 24342 b. 12321
   c. 21312 d. None of these

23. A cigarette pack is $5/6^{th}$ full of its capacity, then 5 cigarettes were taken out of 2 another cigarettes were put inside the pack. Now it is $4/5^{th}$ full. How many cigarettes can this pack contain when it is full?
   a. 90 b. 80
   c. 72 d. Cannot be determined

24. If $N = 1 + m$, where m is the product of four consecutive positive integers, then which of the following is/are true?
   I. N is odd
   II. N is not a multiple of 3
   III. N is a perfect square
   a. All three b. I and II only
   c. I and III only d. None of these

25. The value of 'm' when $3^m = 9^n$ and $4^{(m+n+2)n} = 16^{mn}$ is:
   a. 2 b. 1
   c. 4 d. None of these

26. A natural number A is such that $A = x^2 = y^4 = z^8$, where x, y, z are distinct positive integers, then the least possible value of A is:
   a. 729 b. 1000
   c. 256 d. None of these

27. Which among $2^{1/2}$, $3^{1/3}$, $4^{1/4}$, $6^{1/6}$ and $12^{1/12}$ is the largest ?
   a. $2^{1/2}$ b. $3^{1/3}$ c. $4^{1/4}$ d. $6^{1/6}$

28. Which one of the following is greatest one?
   a. 3^{3322} b. 33^{322} c. 333^{22} d. 22^{333}

29. The mean of 1, 2, 2^2 ... 2^{31} lies in between
   a. 2^{24} to 2^{25} b. 2^{25} to 2^{26}
   c. 2^{26} to 2^{27} d. 2^{29} to 2^{30}

30. $\sqrt{a}\sqrt{b} = \sqrt{ab}$ is true only when
   a. a > 0, b > 0 b. a > 0 and b < 0
   c. a < 0 and b > 0 d. All of these

31. How many numbers are there from 1 to 500 in which the digit '4' comes at any place ?
   a. 180 b. 176
   c. 100 d. None of these

32. How many numbers lie between 300 to 500 in which the digit 4 comes only one time ?
   a. 99 b. 100 c. 110 d. 120

33. If $\log_b a = 3$ and $\log_b 8b = 4$, then find a.
   a. 128 b. 1024 c. 256 d. 512

34. If log 15 = p and log 20 = q, then find the log 12 in terms of p and q.
   a. $p + 3q - 3$ b. $p + 3q - 4$
   c. $3p + q - 4$ d. $3p + q - 3$

35. If $|\log_{(a-1)}(a + 1)| = 1$, then how many values a can take?
   a. 4 b. 1 c. 3 d. 0

36. A number $X = 897324A64B$ is divisible by both 8 and 9. Which of the following is the value of $A + B$?

I. 2
II. 11
III. 9

a. Either I or II b. Either II or III
c. Either I or II or III d. None of these

37. 122*p*214*q*; $p > q$
Both p and q are natural numbers. Find the value of p for which the number is completely divisible by 33.
a. 4 b. 5
c. 4 or 5 or 6 d. 4 or 5

38. A six digit number *abcabc* such that $a, b, c \in N$, then which is the most correct statement is:
a. It is divisible by 91 b. It is divided by 143
c. It is divisible by 6 d. Only *a* and *b* are correct

39. The number 444444... (999 times) is definitely divisible by:
a. 22 b. 44
c. 222 d. All of these

40. Satya has forgotten his 6 digit password but he can only remember that it was of the form *A*515*A*0 and was divisible by 36. What is the value of *A*?
a. 4 b. 7 c. 8 d. 9

41. Let P, Q and R be digits of a three digit number such that $(100P + 10Q + R)(P + Q + R) = 2005$. What is the value of P?
a. 4 b. 2 c. 3 d. 1

42. If $2^{3a} + 3^{a}$ is always divisible by 11, then a must be
a. a multiple of 11
b. a prime number
c. an odd number
d. an even number other than 2

43. If n is positive integer, then $(3^{4n} - 4^{3n})$ is always divisible by:
a. 145 b. 17 c. 112 d. 7

44. If $4^{n+1} + x$ and $4^{2n} - x$ are divisible by 5, n being an positive even integer, find the least value of x.
a. 1 b. 2 c. 3 d. 0

45. $S = 20 \times 21 \times 22 \times 23 \times 24 \times .... \times 40$
If S is divisible by 10^{a}, then find the maximum value of a.
a. 2 b. 3 c. 6 d. 5

46. Given a number 12345678901234567890..... upto 500 digits. Find the smallest number n that should be added to the number such that the sum is exactly divisible by 11.
a. 6 b. 5 c. 8 d. 1

47. For a positive integer n greater than 1, $2^{4n} - 2^{n}(7n + 1)$ is perfectly divisible by:
a. 2 b. 4
c. 7 d. All of them

48. The expression $2222^{7777} + 7777^{2222}$ is divisible by:
a. 99 b. 101
c. 13 d. All of these

49. $7^{6n} - 6^{6n}$, where n is integer > 0, is divisible by:
a. 13 b. 127
c. 559 d. All of these

50. The remainder when the number 123456789101112..... 484950 is divided by 16 is:
a. 3 b. 4 c. 5 d. 6

51. For any positive integer n, which of the following is necessarily false?

a. $n(n + 1)(2n + 1)$ is always even.
b. $n(n + 1)(2n + 1)$ is always divisible by 3.
c. $n(n + 1)(2n + 1)$ is always divisible by the sum of squares of first n integer.
d. $n(n + 1)(2n + 1)$ is never divisible by 237.

52. Find the smallest positive integer n such that every digit of $15n$ is 0 or 8.
a. 88 b. 296 c. 888 d. 592

53. If x and q are natural numbers and $x < 10$, then for how many values of x will $x^q + x^{q+2}$ is divisible by 10?
a. 0 b. 3 c. 4 d. 5

54. What is the remainder when 4^{96} is divided by 6?
a. 0 b. 2 c. 3 d. 4

[CAT]

55. If $N = (16^3 + 17^3 + 18^3 + 19^3)$, then N divided by 70 leaves a remainder of:
a. 0 b. 1 c. 69 d. 35

[CAT]

56. What is the remainder when $9 + 9^2 + 9^3 + ... 9^{2n+1}$ is divided by 6?
a. 1 b. 2 c. 3 d. 4

57. P is a positive integer. When P is divided by any one of the numbers 2, 11, 13, 71 and 89 the remainder is 1. What is the remainder when P^{256} is divided by 16?
a. 1 b. 15 c. 9 d. 0

58. What is the remainder when (1! + 2! + 3! + ... 1000!) is divided by 5?
a. 1 b. 2 c. 3 d. 4

59. What is the remainder when 7 + 77 + 777+ 7777 + ... (till 100 terms) is divided by 8?
a. 0 b. 2 c. 4 d. 6

60. Let a (> 1) be a positive integer. Then the largest integer b, such that $(a^b + 1)$ divides $(1 + a + a^2 + ... a^{127})$, is:
a. 127 b. 63 c. 64 d. 32

61. Let $n! = 1 \times 2 \times 3 \times ... \times n$ for integer $n > 1$. If $p = 1! + (2 \times 2!) + (3 \times 3!) + ... + (10 \times 10!)$, then $p + 2$ when divided by 11! leaves a remainder of:
a. 10 b. 0 c. 7 d. 1

[CAT 2005]

62. $10^{26} - 23$ is divisible by:
a. 3 b. 9 & 3 c. 11 d. 3, 9 & 11

63. Find the last digit of $4^1 \times 4^2 \times 4^3 \times ... \times 4^{1234}$.
a. 8 b. 6 c. 2 d. 4

64. The last digit of the expression
$4 \times 9^2 \times 4^3 \times 9^4 \times 4^5 \times 9^6 \times ..... \times 4^{99} \times 9^{100}$ is:
a. 4 b. 6 c. 9 d. 1

65. Find the unit digit of the given product: $(6 + 1)(6^2 + 1)(6^3 + 1).....(6^{102} + 1)$.
a. 9 b. 1 c. 3 d. 7

66. If the unit digit in the product $(47a \times 49b \times 729 \times 345 \times 343)$ is 5, what is the maximum number of values that (a, b) can take?
a. 5 b. 25 c. 20 d. 40

67. The rightmost non-zero digit of the number 30^{2720} is:
a. 1 b. 3 c. 7 d. 9

[CAT]

68. The unit digit of the expression $(1!)^{1!} + (2!)^{2!} + (3!)^{3!} + ... + (100!)^{100!}$:
a. 0 b. 1 c. 2 d. 7

69. A man took a 5 digit number ending in 9 and raised it to an even power greater than 50. He then multiplied it with 17 raised to a multiple of 4. What is the last digit of the resulting number?

a. 1 b. 9 c. 3 d. 7

70. $\frac{12^{55}}{3^{11}} + \frac{8^{48}}{16^{18}}$ will give the digit at the unit place as:

a. 4 b. 6 c. 8 d. 0

71. A positive number p is such that $(p + 4)$ is divisible by 7. 'N' being a smallest possible number larger than first prime number, which can make $(p + N^2)$ divisible by 7. The value of N is:

a. 3 b. 9 c. 5 d. 7

72. The remainder when n is divided by 3 is 1 and the remainder when $(n + 1)$ is divided by 2 is 1. The remainder when $(n - 1)$ is divided by 6 is:

a. 2 b. 3

c. 5 d. None of these

73. A naughty boy Anil watches an innings of M.S. Dhoni and acts according to the number of runs he sees Dhoni scoring. The details of these are given below.

1 run	Place an apple in the basket
2 runs	Place a banana in the basket
3 runs	Place a mango in the basket
4 runs	Remove a mango and a banana from the basket

One fine day, at the start of the match, the basket is empty. The sequence of runs scored by Dhoni in that innings are given as 1123241123423234112131114. At the end of the above innings, how many more apples were there compared to bananas inside the basket? (The Basket was empty initially).

a. 4 b. 5 c. 6 d. 7

74. $N = 84^3 - 72^3 - 12^3$. How many factors does N have?

a. 192 b. 384 c. 98 d. 96

75. How many distinct factors of 3600 are perfect squares?

a. 12 b. 10 c. 9 d. 7

76. What is the difference between the number of even divisors and the number of prime divisors of 630?

a. 12 b. 24 c. 8 d. 3

77. Find the sum of all the factors of 52920?

a. 205200 b. 41040 c. 420400 d. 235200

78. Find the product of all the odd factors of 10584?

a. $3^6 \times 7^{10}$ b. $3^4 \times 7^8$ c. $3^{12} \times 7^{16}$ d. $3^{18} \times 7^{12}$

79. Two friends Jo and Harry were discussing about 2 numbers. They found the two numbers to be such that one was twice the other. However, both had the same number of prime factors while the larger one had 4 more factors than the smaller one. What are the numbers?

a. 40, 80 b. 20, 40 c. 30, 60 d. 50, 100

80. Find the number of such numbers that are less than 250 and co-prime to 250.

a. 100 b. 200 c. 105 d. 205

81. How many numbers are there from 1 to 100 which are co-prime to 300?

a. 25 b. 22 c. 80 d. 26

82. LCM of two natural numbers is 590 and their HCF is 59. How many sets of the two numbers are possible?

a. 1 b. 2 c. 5 d. 10

83. The product of two numbers is 7168 and their HCF is 16. How many pairs of numbers are possible such that the above conditions are satisfied?
    a. 2   b. 3   c. 4   d. 6

84. By what least number 25930800 should be divided to get a perfect square?
    a. 2   b. 3
    c. 5   d. Cannot be determined

85. Find the greatest number of six digits which, when divided by 6, 7, 8, 9 & 10, gives 4, 5, 6, 7 & 8 respectively as remainders.
    a. 997920   b. 995398
    c. 997918   d. 995400

86. Students from the SMR School are writing their exams in Kendriya Vidyalaya. There are 60 students writing their Mathematics exams, 72 students writng their Science exam and 96 students writing their Hindi exam. The authorities of the Kendriya Vidyalaya have to make arrangments such that each classroom contains equal number of students. What is the minimum number of classrooms required to accommodate all students of SMR School?
    a. 19   b. 38   c. 13   d. 6

87. A milkman has 3 jars containing 57 litres, 129 litres and 177 litres of pure milk respectively. A measuring can leaves the same amount of milk unmeasured in each jar after a different number of exact measurement of milk in each jar. What is the volume of largest such can?
    a. 12 litres   b. 16 litres   c. 24 litres   d. 48 litres

88. (15, 3)! is defined as the product of 3 consecutive numbers starting from 15. If H is the HCF of (15, 3)! and 3!, then what can be said about H?
    a. H = 15!   b. H = 3!   c. $H^3 = 3!$ d. $H^3 = 45$

89. A rectangular floor is fully covered with square tiles of identical size. The tiles on the edges are white and the tiles in the interior are red. The number of white tiles is the same as the number of red tiles. A possible value of the number of tiles along one edge of the floor is:
    a. 10   b. 12   c. 14   d. 16

90. In the morning batch at "IIMA" we have observed that when five students took seat per bench and 4 students remained unseated. But when eleven students took seat per bench, 4 benches remained vacant. The number of students in our morning batch were?
    a. 55   b. 48
    c. 26   d. None of these

91. Find the HCF of $(3^{100} – 1)$ and $(3^{120} – 1)$.
    a. $(3^{10} – 1)$ b. $(3^{30} – 1)$ c. $(3^{20} – 1)$ d. 1

92. The number of digits in the product of $5^{72} \times 8^{27}$ is:
    a. 77   b. 75
    c. 99   d. None of these

93. $N = 204 \times 221 \times 238 \times 255 \times .... \times 850$. How many consecutive zeros will be there at the end of this number N?
    a. 8   b. 10   c. 11   d. 12

94. How many zeros will be there at the end of $N = 18! + 19!$?
    a. 3   b. 4
    c. 5   d. 6

95. Find the number of zeros in the product: $1^1 \times 2^2 \times 3^3 \times 4^4 \times \ .... \times 98^{98} \times 99^{99} \times 100^{100}$
    a. 1200   b. 1300   c. 1050   d. 1225

96. The highest power on 990 that will exactly divide 1090! is:
    a. 101   b. 100   c. 108   d. 109

97. If $(3\,A\,3)_6 + (44\,A)_6 = (1242)_6$, what is the value of A?
    a. 4   b. 2   c. 5   d. 3

98. In a number system the product of 44 and 11 is 1034. The number 3111 of this system, when converted to the decimal number system becomes:

a. 406 b. 1086 c. 213 d. 691

99. A positive whole number M less than 100 is represented in base 2 notation, base 3 notation, and base 5 notation. It is found that in all the cases the last digit is 1, while in exactly two out of three cases the leading digit is 1. Then M equals:
a. 31 b. 63 c. 75 d. 91

100. What is the value of $(1)_2 + (2)_3 + (3)_4 + (4)_5 + (5)_6 + (6)_7 +$ .... $(9)_{10}$?
a. 45 b. 90 c. 36 d. 55

Practice Exercise – Difficult

1. Let a_n = 1111111 ..... 1, where 1 occurs n number of times. Then,
I. a_{741} is not a prime.
II. a_{534} is not a prime.
III. a_{123} is not a prime.
IV. a_{77} is not a prime.
a. I is correct b. I and II are correct
c. II and III are correct d. All of them are correct

2. Let p be a prime number greater than 5. Then $(p^2 - 1)$ is:
a. Never divisible by 6
b. Always divisible by 6, and may or may not be divisible by 12.
c. Always divisible by 12, and may or may not be divisible by 24.
d. Always divisible by 24.

3. The integers 1, 2 ..... 40 are written on a blackboard. The following operation is then repeated 39 times: In each repetition, any two numbers say a and b, currently on the blackboard are erased and a new number $a + b - 1$ is written. What will be the number left on the board at the end?
a. 820 b. 821 c. 781 d. 819

[CAT 2008]

4. Find the sum $\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}} + \sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}} + ... + \sqrt{1+\frac{1}{2007^2}+\frac{1}{2008^2}}$

a. $2008 - \frac{1}{2008}$ b. $2007 - \frac{1}{2008}$
c. $2008 - \frac{1}{2007}$ d. $2007 - \frac{1}{2007}$

[CAT 2008]

5. Let a, b, c be distinct digits. Consider a two digit number 'ab' and three digits 'ccb' both defined under the usual decimal number system. If $(ab)^2 = ccb$ and $ccb > 300$, then the value of b is:
a. 1 b. 0 c. 5 d. 6

6. If pq, rs, qp and sr are two digit numbers then the maximum value of $(pq \times rs) - (qp \times sr)$ is, where p, q, r, s are distinct non-zero positive integers.
a. 7938 b. 7128
c. 6930 d. None of these

7. Find the 28383rd term of the series: 123456789101112....
a. 3 b. 4 c. 9 d. 7

8. Consider four digit numbers for which the first two digits are equal and the last two digits are also equal to. How many such numbers are perfect squares?
a. 2 b. 4 c. 0 d. 1

9. An intelligence agency forms a code of two distinct digits selected from 0, 1, ..., 9 such that the first digit of the code is non-zero. The code, handwritten on a slip, however, can potentially create confusion when read upside down, e.g., the code 91 may appear as 16. How many codes are there for which no such confusion can arise?

   a. 80 b. 63
   c. 71 d. None of these

10. Each of A, B, C and D equals either 0 or 1. It is given that
    If $B = 0$, then $C = 1$
    If $C = 0$, then $A = D$
    If $D = 0$, then $A = 1$
    Assume $C = 0$, find the value of $(A + B + C + D)$?

    a. 0 b. 1 c. 2 d. 3

11. When asked about his date of birth in 1996, Mehul replied that "last two digits of my birth year stands for my age." When Sid was asked about his age, he also replied the same. But Sid is older to Mehul. What is the difference in their age?

    a. 46 b. 50
    c. 0 d. Cannot be determined

Directions for questions 12 and 13: A mock test is taken at 'IIM'. The test paper comprises of questions in three levels of difficulty – Easy, Medium and Difficult.

The following table gives the details of the positive and negative marks attached to each question type:

Difficulty Level	**Positive marks for answering the question correctly**	**Negative marks for answering the question wrongly**
Easy	4	2
Medium	3	1.5
Difficult	2	1

The test had 200 questions with 80 on Easy and 60 each on Medium and Difficult.

12. If a student has solved 100 questions exactly and scored 120 marks, the maximum number of incorrect questions that he/she might have marked is:

    a. 44 b. 56
    c. 60 d. None of these

13. If a man attempted the least number of questions and got a total of 130 marks, and if it is known that he attempted at least one of every type, then number of questions he must have attempted is:

    a. 34 b. 35
    c. 36 d. None of these

14. What is the largest value of x for which $2^{54} + 2^{2000} + 2^{2x}$ equals to the square of a whole number?

    a. 3944 b. 572 c. 1946 d. 1972

15. Three numbers a, b, c are such that $a^b = b^c$, where $a, b, c > 1$, then the correct relation between b and c is:

    a. $b/c = 1$ b. $b > c$
    c. $b < c$ d. Indeterminable

Directions for questions 16 and 17: Read the passage below and solve the questions based on it.

The multiplication of two numbers is shown below.

$$\begin{array}{r} PQ4 \\ \times\, R \\ \hline P206 \\ \hline \end{array}$$

where P, Q and R are all distinct digits.

16. The value of Q is:
    a. 3 b. 9 c. 2 d. 7

17. The value of P + R is:
    a. 6 b. 9 c. 11 d. 10

Directions for questions 18 and 19: Read the following information given below and answer the questions that follow.

In the diagram below, the seven letters correspond to seven unique digits chosen from 0 to 9. The relationship among the digits is such that:

$P \times Q \times R = X \times Y \times Z = Q \times A \times Y$

P		X
Q	A	Y
R		Z

18. The value of A is:
    a. 0 b. 2 c. 3 d. 6

19. The sum of digits which are not used is:
    a. 8 b. 10
    c. 14 d. None of the above

Directions for questions 20 to 22: Read the following information given below and answer the questions that follow.

Substitute different digits (0, 1, 2, ... 9) for different letters in the problem below, so that the corresponding addition is correct and it results in the maximum possible value of MONEY.

		P	A	Y
			M	E
	R	E	A	L
M	O	N	E	Y

20. The letter '*y*' should be:
    a. 0 b. 2
    c. 3 d. None of the above

21. There are nine letters and ten digits. The digit that remains unutilized is:
    a. 4 b. 3
    c. 2 d. None of the above

22. The resulting value of 'Money' is:
    a. 10364 b. 10563
    c. 10978 d. None of the above

23. abc is a three digit natural number such that $abc = a^3 + b^3 + c^3$. What is the value of c?
    a. 0 b. 1
    c. 3 d. Cannot be determined

Directions for questions 24 to 26: If a sequence is as given below:

1, 1, 2, 3, 5, 8, 13, 21, 34, ...

24. The unit digit of the 75th term of this sequence will be;
    a. 0 b. 5

c. 7 d. None of these

25. In the above sequence the 55th term will be:
 a. an even number b. an odd number
 c. either even or odd d. Cannot be determined

26. The unit digit of the sum of the 88th and 89th term of the same sequence will be:
 a. 5 b. 2
 c. 0 d. None of these

27. Which of the following is correct if $P = 3^{3^{3^{3}}}$, $Q = 3^{33^{3}}$, $R = 3^{3^{33}}$ and $S = 3^{333}$?
 a. $P > Q = R > S$ b. $R > P > Q > S$
 c. $P > R > S > Q$ d. $R > Q > S > P$

28. Given that a and b are integers, then $\dfrac{(15a^3 + 6a^2 + 5a + b)}{a}$ for what condition is not an integer?
 a. a is positive b. b is divisible by a
 c. b is not divisible by a d. (a) and (c)

29. Find the value of x in $\sqrt{x + 2\sqrt{x + 2\sqrt{x + 2\sqrt{3x}}}} = x$.
 a. 1 b. 3 c. 6 d. 12

30. In how many ways can 729 be expressed as a difference of the square of whole numbers?
 a. 4 b. 6
 c. 8 d. None of these

31. X is a three digit number such that the ratio of the number to the sum of its digits is least. What is the difference between the hundreds and the tens digits of X?
 a. 9 b. 8
 c. 7 d. None of these

32. For the above question, for how many values of X will the ratio be the highest?
 a. 9 b. 8
 c. 7 d. None of these

33. If the difference of $10^{27} – 4$ and $10^{25} + a$ is divisible by 9, then what is the minimum value of a?
 a. 2 b. 3 c. 4 d. 5

34. N is a 1001 digit number consisting of 1001 sevens. What is the remainder when N is divided by 1001?
 a. 7 b. 700
 c. 777 d. None of these

35. What is the remainder when [100! + 1] is divided by 101?
 a. 100 b. 1 c. 0 d. 2

36. Sum of a three-digit number and its mirror image is a multiple of 111. What is the sum (given that 6 is one of the digits of the three-digit number)?
 a. 999 b. 666 c. 888 d. 777

37. What are the last two digits of 7^{2008}?
 a. 21 b. 61 c. 01 d. 41

[CAT 2008]

38. What will be the ten's place digit in the following expression: 1! + 2! + 3! + ... + 70!?
 a. 1 b. 3 c. 5 d. 6

39. A young girl counted in the following way on the fingers of her left hand. She started colling the thumb 1, the index finger 2, the middle finger 3, the ring finger 4, the little finger 5, then reversed direction calling the ring finger 6, the middle finger 7, the index finger 8, the thumb 9 then back to the index finger for 10, the middle finger for 11, and so on. She counted up to 1994. She ended on her

a. Thumb b. Index finger
c. Middle finger d. Ring finger

40. For what value of 'n' the number $2^{74} + 2^{2058} + 2^{2n}$ is a perfect square?
a. 491 b. 893 c. 1029 d. 2020

41. How many divisors of $N = 420$ will be of the form $4n + 1$, where n is a whole number?
a. 3 b. 4 c. 5 d. 8

42. How many divisors of 10^5 end with exactly one zero?
a. 1 b. 3 c. 9 d. 16

43. A number N has a total of 12 factors. What is the minimum & maximum number of prime factors that N can have?
a. 1, 2 b. 1, 4 c. 2, 5 d. 1, 3

Directions for questions 44 and 45: There are two natural numbers P_1 and P_2
A and B are the sum of the divisors of P_1 and P_2 respectively. Further, $A - B = P_1 - P_2$

44. Find the number of divisors of $P_1 \times P_2$.
a. 3 b. 4 c. 5 d. 9

45. Find the sum of the divisor of $P_1^{P_2}$.

a. $P_1^{P_2} + 1$ b. $\dfrac{P_1^{P_2} - 1}{P_2}$ c. $\dfrac{P_1^{P_2+1}}{P_2}$ d. $\dfrac{P_1^{P_2+1} - 1}{P_2 - 1}$

Directions for questions 46 and 48: Read the passage below and solve the questions based on it.
There are 100 prisoners in 100 cells. Cells are numbered from 1 to 100 and every cell is occupied by one prisoner only. One day jailer decides to release some of the prisoners and for this he defines an algorithm of 100 steps which is as follows:

Step 1: Reverse the position of all the cells which are divisible by 1.

Step 2: Reverse the position of all the cells which are divisible by 2.

Step 3: Reverse the position of all the cells which are divisible by 3.

....

....

Step 99: Reverse the position of all the cells which are divisible by 99.

Step 100: Reverse the position of all the cells which are divisible by 100.

Initially all the cells are closed. After executing all these steps, prisoners of all the cells which remain open are released.

46. How many prisoners are released?
a. 25 b. 10
c. 90 d. None of these

47. Which of the following cell number will be open at the end?
a. Cell number 56 b. Cell number 64
c. Cell number 72 d. Cell number 84

48. Which of the following is true about the family of cell numbers N that will be open at the end?
a. All the elements of N will be having only two factors inclusing the number itself.
b. All the elements of N will be having odd number of factors excluding the number itself.
c. All the elements of N will be having odd number of factors including the number itself.
d. None of these

Directions for questions 49 to 52: *The relation $R(p, q)$ can be defined for every positive integer p, q as $R(p, q) = p \times (p + 1) \times (p + 2) \times (p + 3) \times .... (p + q)$ and the relation $R(q)$ is equal to $q!$*

49. The value of $\frac{R(135)}{R(100,\ 35)}$ is:

a. 99! b. 100!

c. 270 d. None of these

50. The value of R (17) × R (19, 62) is:

a. $\frac{81!}{18}$ b. (81!) × 18

c. 36! d. 17 × (19 + 62)

51. The L.C.M of R (2, 995) and R (996, 1) is:

a. 1994 b. 996!

c. 997! d. None of these

52. The H.C.F of R (139, 2) and R (141):

a. 141 b. 2743860

c. 32, 16, 839 d. 19599

53. Little Pika who is five and half years old has just learnt addition. However, he does not know how to carry. For example, he can add 14 and 5, but he does not know how to add 14 and 7. How many pairs of consecutive integers between 1000 and 2000 (both 1000 and 2000 included) can Little Pika add?

a. 150 b. 155 c. 156 d. 258

54. a, b, c, d and e are integers such that $1 \le a < b < c < d < e$. If a, b, c, d and e are in geometric progression and lcm (m, n) is the least common multiple of m and n, then the maximum value of

$$\frac{1}{\text{lcm}(a,\ b)} + \frac{1}{\text{lcm}(b,\ c)} + \frac{1}{\text{lcm}(c,\ d)} + \frac{1}{\text{lcm}(d,\ e)}$$

a. 1 b. 15/16 c. 79/81 d. 7/8

55. If $N = \frac{38!}{19! \times 19!}$, where N is an integer. What the highest power of 23 in N?

a. 1 b. 3 c. 5 d. 6

56. A is a two digit number which has the property that: The product of factorials of its digits is greater than the sum of factorials of its digits. How many values of A exist?

a. 56 b. 64

c. 63 d. None of these

57. The value of $(n!)^n$ if $n + (n - 1) + (n - 2) = n(n - 1)(n - 2)$, where $n^3 > 9$, a positive number:

a. 27 b. 216 c. 256 d. 331776

58. n is an integer such that the sum of the digits of n is 2 & $10^{10} < n < 10^{11}$. The number of different values for n is:

a. 11 b. 10 c. 12 d. 5

Directions for questions 59 and 60: Read the passage below and solve the questions based on it.

Ronit purchased some pens, pencils and erasers for his young brothers and sisters for the CAT exams. He had to buy at least 11 pieces of each items in a manner that the number of pens purchased be more than the number of pencils, which is more than the number of erasers. He purchased a total of 38 pieces.

59. How many erasers did Ronit purchase?

a. 11 b. 10

c. 8 d. Cannot be determined

60. If each eraser costs Rs. 3, each pencil Rs. 2 each pen Rs. 10, what is the maximum amount that Ronit could have spent?

a. 207 b. 255 c. 288 d. 300

61. Suppose you have a currency, named Amen, in three denominations: 1 Amen, 10 Amen and 50 Amen. In how many ways can you pay a bill of 107 Amen?

a. 16 b. 18 c. 15 d. 9

62. Jayant was born on February 29th of 2012 which happened to be a Tuesday. If he lives to be 101 years old, how many birthdays would he celebrate on a Tuesday?
    a. 3 b. 4 c. 5 d. 1

63. How many of the following statements have to be true?
    I. No year can have 5 Sundays in the month of May and 5 Thursdays in the month of June.
    II. If February 14th of a certain year is a Tuesday, May 14th of the same year cannot be a Monday.
    III. If a year has 53 Sundays, it can have 5 Mondays in the month of May.
    a. 0 b. 1 c. 2 d. 3

64. There are many islands on earth for which we do not have much information about those islands. On such an island, which uses a system of S digits ($S \neq 10$) to write the mumbers on that island, selling price of a goat is Rs. 1,143. Mrunal bought a goat and paid the shopkeeper Rs. 1,150. Now shopkeeper returned Mrunal Rs. 5. How many digits are used to write the numbers in that system?
    a. 9 b. 8 c. 7 d. 6

65. If $(p)_{10}\ \Delta\ (q)_{10} = (2p + q - 2)_{10}$, then $(101)_2\ \Delta\ (100)_2$ is equivalent to
    a. $(100)_2$ b. $(1100)_2$ c. $(101)_2$ d. $(1001)_2$

66. If $f(a, b, c) = a + b - c$, then $f((15)_8, (15)_{10}, (15)_{16}) = ?$
    a. $(7)_{16}$ b. $(7)_8$
    c. $(7)_{10}$ d. All of the above