

## 5. RATIO AND PROPORTION

### Introduction to the Topic

The concept of ratio & proportion is fundamental to many other topics in quant. The chapter needs to be done thoroughly to get a grip on the problem solving techniques used in aptitude and answer questions quickly in the exam. The basics of the topic could very well prove to be the differentiator between an average and a good percentile in the exam.

### Relevance in CAT

Ratio & Proportion is seen by many as one of the most important topics for CAT exam. Besides its relevance in quant, the topic finds its application in Data Interpretation (DI), with the questions on DI forming a big chunk in the exam, the topic is a must for all to score well in CAT and almost all competitive exams.

Ratio is a comparison between two or more quantities of the same kind by division.

If  $a$  and  $b$  are two quantities of the same kind (in same units), then  $a : b$  is called the ratio of  $a$  and  $b$ . It can also be written as a fraction

$\frac{a}{b}$ . Thus, the ratio of  $a$  to  $b = a : b$  or  $\frac{a}{b}$ . The quantities  $a$  and  $b$  are called the terms of the ratio, where  $a$  is called the first term or

**antecedent** and  $b$  is called the **second term or consequent**.

#### Important points about Ratio

- Usually a ratio is expressed in lowest terms (or simplest form).  
*i.e.* the simplest form of  $6 : 4$  is  $3 : 2$ .
- Ratio exists only between quantities of the same kind.
- The order of the terms in a ratio is important.
- Quantities to be compared (by division) must be in the same units.
- Both terms of a ratio can be multiplied or divided by the same (non-zero) number.
- The ratio is only a number, *i.e.*, without any unit of measurement.

**Example 1:** Ram and Shyam got 175 and 225 marks respectively. What is the ratio of their marks?

**Solution:**

$$\frac{175}{225} = \frac{7}{9} = 7 : 9$$

**Example 2:** Rita scored 105 marks out of 150 and Geeta scored 175 marks out of 200. What is the ratio of the percentage marks scored by Rita and Geeta?

**Solution:**

$$\text{Percentage marks scored by Rita} = \frac{105}{150} \times 100 = \frac{7}{10} \times 100$$

$$\text{Percentage marks scored by Geeta} = \frac{175}{200} \times 100 = \frac{7}{8} \times 100$$

$$\text{Required Ratio} = \frac{\frac{7}{10} \times 100}{\frac{7}{8} \times 100} = \frac{4}{5} = 4 : 5$$

**Example 3:** What same value must be subtracted from both the numerator and the denominator of the fraction  $\frac{6}{7}$  to give a fraction equal to  $\frac{4}{5}$ ?

**Solution:**

Let  $x$  be subtracted from both the numerator and the denominator.

$$\frac{6-x}{7-x} = \frac{4}{5}$$

On solving, we get  $x = 2$

**Example 4:** What is the least integer, which when added to both terms of the ratio  $3 : 7$  will give a ratio greater than  $\frac{5}{9}$ ?

**Solution:**

If  $k$  be an integer,

$$\frac{3+k}{7+k} > \frac{5}{9} \Rightarrow 27 + 9k > 35 + 5k \Rightarrow 4k > 8 \Rightarrow k > 2$$

$\therefore$  The least integer greater than 2 is 3.

Hence the required integer is 3.

## Type of Ratios

### 1. Equivalent Ratio

If we multiply or divide both the antecedent and consequent of the ratio by the same number, we will get the equivalent ratio.

e.g. Equivalent ratios of  $a : b$  are  $2a : 2b$ ,  $3a : 3b$ ,  $5a : 5b$  etc.

### 2. Compound Ratio

If we combine two different ratios having same units into one single ratio, then we call it as a compound ratio.

e.g. If  $a : b$  and  $c : d$  are two ratios, the compound ratio will be  $ac : bd$ .

### 3. Inverse Ratio

For any ratio  $a : b$ , its inverse ratio is written as  $\frac{1}{a} : \frac{1}{b}$  or  $b : a$  and for any ratio  $a : b : c$ , its inverse ratio is written

as  $\frac{1}{a} : \frac{1}{b} : \frac{1}{c}$  or  $bc : ca : ab$ . This is also called as **Reciprocal** ratio.

### 4. Duplicate Ratio

For any ratio  $a : b$ , its duplicate ratio is written as  $a^2 : b^2$ .

### 5. Sub-Duplicate Ratio

For any ratio  $a : b$ , its sub-duplicate ratio is written as  $\sqrt{a} : \sqrt{b}$ .

### 6. Triplicate Ratio

For any ratio  $a : b$ , its triplicate ratio is written as  $a^3 : b^3$ .

### 7. Sub-Triplicate Ratio

For any ratio  $a : b$ , its sub-triplicate ratio is written as  $\sqrt[3]{a} : \sqrt[3]{b}$ .

**Example 5:** If it is given that  $x : y = 2 : 3$ , then what is the value of  $(3x + 2y) : (2x + 5y)$ .

**Solution:**

$$(3x + 2y) : (2x + 5y) = \frac{3x + 2y}{2x + 5y}$$

By dividing both the numerator and denominator by  $y$ ; we get

$$\frac{\left[3\left(\frac{x}{y}\right) + 2\right]}{\left[2\left(\frac{x}{y}\right) + 5\right]} = \frac{\left[3\left(\frac{2}{3}\right) + 2\right]}{\left[2\left(\frac{2}{3}\right) + 5\right]} = \frac{12}{19}$$

**Alternate Method:**

Assume values of  $x$  and  $y$  and find the value of the expression

For example; Let  $x = 2$  and  $y = 3$

$$= \frac{3x + 2y}{2x + 5y} = \frac{3(2) + 2(3)}{2(2) + 5(3)} = \frac{12}{19}$$

**Example 6:** If  $2x + 3y : 3x + 5y = 18 : 29$ , find  $x : y$ .

**Solution:**

$$\frac{2x + 3y}{3x + 5y} = \frac{18}{29}$$

Divide both the numerator and denominator by  $y$

$$\frac{2\left(\frac{x}{y}\right) + 3}{3\left(\frac{x}{y}\right) + 5} = \frac{2k + 3}{3k + 5} = \frac{18}{29}$$

$$\text{Let } \frac{x}{y} = k$$

$$\text{Expression} = \frac{2k + 3}{3k + 5} = \frac{18}{29}$$

$$58k + 87 = 54k + 90$$

$$\Rightarrow 3 = 4k$$

$$k = \frac{3}{4} = \frac{x}{y}$$

**Example 7:** I have one rupee coins, fifty paise coins and twenty five paise coins. The number of coins are in the ratio  $5 : 3 : 2$ . If the total amount with me is Rs. 210, find the number of one rupee coins.

**Solution:**

Ratio of number of one rupee, fifty paise, twenty five paise coins =  $5 : 3 : 2$

Let amount of Re. 1 coins, 50 paise coins and 25 paise coins be  $5x$ ,  $3x$  and  $2x$  respectively.

$$\text{So, } 5x + \frac{3x}{2} + \frac{2x}{4} = 210$$

$$\Rightarrow x = 30$$

Now, Value of one rupee coins = number of one rupee coins  
 $= 30 \times 5 = 150$ .

**Example 8:** From each of two given numbers, half the smaller number is subtracted. Of the resulting numbers the larger one is three times as large as the smaller. What is the ratio of the two numbers?

**Solution:**

Let the two numbers be  $x$  and  $y$  and  $x < y$ .

$$\text{Then, } \left(y - \frac{x}{2}\right) = 3 \left(x - \frac{x}{2}\right) \Rightarrow \left(y - \frac{x}{2}\right) = \frac{3x}{2}$$

$$\text{or } y = \frac{4x}{2} \Rightarrow y = 2x$$

$$\therefore y : x = 2 : 1$$

**Example 9:** Divide Rs. 784 into four parts such that 4 times the first part, three times the second part, twice the third part are each equal to twelve times the fourth part. Find the values of first three parts.

**Solution:**

Let  $p$ ,  $q$ ,  $r$  and  $s$  be the four parts, then we have

$$4p = 3q = 2r = 12s \quad \dots(1)$$

$$p + q + r + s = 784 \quad \dots(2)$$

Substituting values of  $p$ ,  $q$  and  $r$  in terms of  $s$  in (2)

we get,

$$3s + 4s + 6s + s = 784$$

$$\Rightarrow 14s = 784$$

$$s = 56$$

From this we can calculate the values of  $p$ ,  $q$  and  $r$  as Rs. 168, Rs. 224 and Rs. 336 respectively.

**Example 10:** The ratio of the prices of two types of tabs was 16 : 23. Two years later when the price of the first has increased by 10% and that of the second increases by Rs. 477, the ratio of the new prices becomes 11 : 20. Find the original prices of the two tabs.

**Solution:**

Let the original prices of two tabs be Rs.  $16x$  and Rs.  $23x$  respectively. Then according to the given conditions,

$$\frac{16x + 10\% \text{ of } 16x}{23x + 477} = \frac{11}{20}$$

$$\text{or, } \frac{16x + 1.6x}{23x + 477} = \frac{11}{20}, \text{ or } 320x + 32x = 253x + 5247$$

$$\text{or, } 352x - 253x = 5247, \text{ or } 99x = 5247;$$

$$\therefore x = 53$$

Hence, the original prices of two tabs are Rs.  $16 \times 53$  and Rs.  $23 \times 53$  i.e. Rs. 848 and Rs. 1219.

**Example 11:** A sum of Rs. 2,25,000 is available out of which Rs. 5000 is to be divided between  $P$  and  $Q$  in the ratio 2 : 3. The rest of the money is to be divided between  $P$ ,  $Q$  and  $R$  in the ratio 5 : 3 : 3. How much money did  $P$  and  $Q$  get, respectively?

**Solution:**

Rs. 5,000 is to be divided between  $P$  and  $Q$  in the ratio of 2 : 3.

$$P\text{'s share } \frac{2}{5} \times 5000 = \text{Rs. } 2,000$$

$$Q\text{'s share } \frac{3}{5} \times 5000 = 3,000$$

Amount left is 2,20,000 and it is to be divided in the ratio of 5 : 3 : 3.

$$P's \text{ share } \frac{5}{11} \times 220000 = \text{Rs. } 100,000$$

$$Q's \text{ share } \frac{3}{11} \times 220000 = \text{Rs. } 60,000$$

$$R's \text{ share } \frac{3}{11} \times 220000 = \text{Rs. } 60,000$$

$$P's \text{ total share} = \text{Rs. } 1,02,000$$

$$Q's \text{ total share} = \text{Rs. } 63,000$$

**Example 12:** If  $a : b = 2 : 3$  and  $b : c = 4 : 3$ , then find the ratio of  $a : b : c$ .

**Solution:**

$b$  is common to both the ratios. Hence we rewrite the two ratios such that the term  $b$  will be same in both the ratios. This can be easily done by taking  $b$  to be the LCM of the two values of  $b$  pertaining in the two ratios. These two values are 3 and 4 & their LCM is 12.

So, we rewrite the two ratios such that  $b$  is 12 in both of them.

(If  $b$  is made 12 by multiplying with 4 then  $a$  is also multiplied by 4 to give 8)

Similarly  $b : c = 4 : 3 = 12 : 9$ . Hence,  $a : b : c = 8 : 12 : 9$ .

### Proportions

When two ratios are equal, the four quantities composing them are said to be in *proportion*. In of the notations, if  $\frac{A}{B} = \frac{C}{D}$  then  $A, B, C$

and  $D$  are in a proportion. We use the symbols " : " to express it mathematically. So whenever we write  $A : B : : C : D$ , it means that  $A, B, C$  and  $D$  are in a proportional. The terms  $A$  and  $D$  are called the *extremes* and the terms  $B$  and  $C$  are called the *means*. It is to note that  $A \times D = B \times C$ .

This result is more commonly expressed as:

"*Product of the means = Product of the extremes*".

### Continued Proportion:

$a, b, c, d, e, f, \dots$  are said to be in continued proportion if

$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} = \frac{e}{f} = \dots$$

When three quantities  $a, b$  and  $c$  are in continued proportion then  $b$  is called the *mean proportional* and  $c$  is called the *third proportional*,

$$\frac{a}{b} = \frac{b}{c} \Rightarrow b^2 = a \times c$$

When four quantities  $a, b, c$  and  $d$  are proportional then  $d$  is called the *fourth proportional*.

**Example 13:** Rs. 1380 is to be divided between  $P, Q$  and  $R$  such that the ratio of share of  $P$  to that of  $Q$  is equal to 3 : 2 and share of  $Q$  to share of  $R$  is equal to 3 : 4. Find their individual share.

**Solution:**

$$\text{Here } P : Q = 3 : 2 = 9 : 6 \text{ and}$$

$$Q : R = 3 : 4 = 6 : 8$$

$$\text{Therefore, } P : Q : R = 9 : 6 : 8$$

1380 can be divided between them as follows:

$$P's \text{ share} = \frac{9}{23} \times 1380 = 540$$

$$Q's \text{ share} = \frac{6}{23} \times 1380 = 360$$

$$R's \text{ share} = \frac{8}{23} \times 1380 = 480$$

## Operations on Ratio

If four quantities  $a, b, c, d$  form a proportion, many other proportions may be deduced by the properties of fractions. The results of these operations are very useful.

### Invertendo

If  $a : b = c : d$ , then  $b : a = d : c$ .

For  $\frac{a}{b} = \frac{c}{d}$ ; therefore  $1 \div \frac{a}{b} = 1 \div \frac{c}{d}$ ;

that is,  $\frac{b}{a} = \frac{d}{c}$ ;

or  $b : a = d : c$ ,

### Alternendo

If  $a : b = c : d$ , then  $a : c = b : d$ .

For  $ad = bc$ ; therefore  $\frac{ad}{cd} = \frac{bc}{cd}$ ;

that is,  $\frac{a}{c} = \frac{b}{d}$ ;

or  $a : c = b : d$ .

### Componendo

If  $a : b = c : d$ , then  $a + b : b = c + d : d$ .

For  $\frac{a}{b} = \frac{c}{d}$ ; therefore  $\frac{a}{b} + 1 = \frac{c}{d} + 1$ ;

that is,  $\frac{a + b}{b} = \frac{c + d}{d}$ ;

or  $a + b : b = c + d : d$ .

### Dividendo

If  $a : b = c : d$ , then  $a - b : b = c - d : d$

For  $\frac{a}{b} = \frac{c}{d}$ ; therefore  $\frac{a}{b} - 1 = \frac{c}{d} - 1$ ;

that is,  $\frac{a - b}{b} = \frac{c - d}{d}$ ;

or  $a - b : b = c - d : d$ .

### Componendo and Dividendo

If  $a : b = c : d$ , then  $a + b : a - b = c + d : c - d$

Or By (Componendo)  $\frac{a + b}{b} = \frac{c + d}{d}$ ;

and By (Dividendo)  $\frac{a - b}{b} = \frac{c - d}{d}$ ;

$\therefore$  By division,  $\frac{a + b}{a - b} = \frac{c + d}{c - d}$ ;

or  $a + b : a - b = c + d : c - d$ .

This proposition is usually quoted as Componendo and Dividendo.

**Example 14:** If  $a : b :: c : d$  then prove that  $4a + 5b : 4a - 5b :: 4c + 5d : 4c - 5d$

**Solution:**

We have  $\frac{a}{b} = \frac{c}{d}$

Multiplying both the sides by  $\frac{4}{5}$  we get

$$\frac{4}{5} \times \frac{a}{b} = \frac{4}{5} \times \frac{c}{d}$$

$$\text{Or } \frac{4a}{5b} = \frac{4c}{5d}$$

Using the componendo and dividendo property we get,

$$\frac{4a + 5b}{4a - 5b} = \frac{4c + 5d}{4c - 5d}$$

**Example 15:** Solve for  $y$ ,  $\frac{\sqrt{1+y} + \sqrt{1-y}}{\sqrt{1+y} - \sqrt{1-y}} = \frac{2}{1}$

**Solution:**

$$\frac{\sqrt{1+y} + \sqrt{1-y}}{\sqrt{1+y} - \sqrt{1-y}}$$

Applying Componendo and Dividendo:

$$\frac{(\sqrt{1+y} + \sqrt{1-y}) + (\sqrt{1+y} - \sqrt{1-y})}{(\sqrt{1+y} + \sqrt{1-y}) - (\sqrt{1+y} - \sqrt{1-y})} = \frac{2+1}{2-1}$$

$$\text{Or } \frac{2\sqrt{1+y}}{2\sqrt{1-y}} = \frac{3}{1}$$

Simplifying and then squaring on both sides, we get:

$$\frac{1+y}{1-y} = 9$$

You can proceed to get  $1+y = 9(1-y)$  and solve for  $y$ . Another way of proceeding from here is to apply componendo and dividendo, one more time. So that we get:

$$\frac{(1+y) + (1-y)}{(1+y) - (1-y)} = \frac{9+1}{9-1} = \frac{5}{4}$$

$$\text{So, } y = \frac{4}{5}.$$

## Direct Proportion and Inverse Proportion

### Direct Proportional

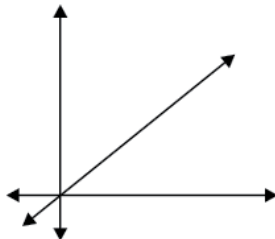
If two quantities  $A$  and  $B$  are directly proportional to each other, then if  $A$  increases there will be a proportional increase in the value of  $B$ . We can denote this relation as below

$A \propto B$  ( $\propto$  is the sign of Proportionality)

or  $A = KB$  ( $K$  is constant of Proportionality)

or  $K = \frac{A}{B}$

In other words two variables  $A$  &  $B$  are said to be in direct proportion if the ratio of  $A$  &  $B$  i.e.  $\frac{A}{B}$  is constant for all the possible values of  $A$  and the corresponding values of  $B$ .



#### Examples of Direct Proportion

- I. "A" is Speed and "B" is Distance. (For a constant time).
- II. "A" is Radius of the circle and "B" is the Perimeter.

**Note:** (when  $A$  is in a direct proportion to  $B$  then  $B$  is in a direct proportion to  $A$  as well.)

#### Inverse Proportional

If two quantities  $A$  and  $B$  are inversely proportional to each other, then if  $A$  increase there will be a proportional decrease in the value of  $B$ .

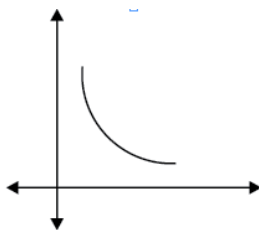
We can denote this relation as below

$$A \propto \frac{1}{B}$$

$$A = \frac{K}{B}$$

$$AB = K$$

In other words, two variables  $A$  and  $B$  are said to be in *inverse proportion* if the product  $A \times B$  is constant for all the possible values of  $A$  and the corresponding values of  $B$ .



#### Examples of Inverse Proportion

- I. 'A' is Speed and "B" is Time. (for a fixed distance)
- II. 'A' is the number of people working on a given project and "B" is the time taken in the completion of the project.

**Note:** (when  $A$  is in inverse proportion to  $B$  then  $B$  is in inverse proportion to  $A$  as well.)

**Example 16:** Speed of car 'A' is twice of that of truck 'B'. Both started from a point  $X$  and reached a point  $Y$ . If 'B' took 4 hours more than 'A', what is the time taken by the truck 'B'?

**Solution:**



Since distance is constant, speed & time are inversely proportional. Let  $V_A$  = speed of A,

$V_B$  = speed of B,

$T_A$  = time taken

by A;  $T_B$  = time taken by B then,

$$\frac{V_A}{V_B} = \frac{2}{1} \Rightarrow \frac{T_A}{T_B} = \frac{1}{2}. \text{ If A takes } x \text{ hours then,}$$

$$\frac{x}{x+4} = \frac{1}{2} \Rightarrow x = 4 \Rightarrow B \text{ takes 8 hours.}$$

**Example 17:** The cube root of  $a$  varies inversely as the square

of  $b$ ; if  $a = 8$  when  $b = 3$ , find  $a$  when  $b = 1\frac{1}{2}$ .

**Solution:**

By supposition  $\sqrt[3]{a} = \frac{m}{b^2}$ , where  $m$  is constant.

Putting  $a = 8$ ,  $b = 3$ , we have  $2 = \frac{m}{9}$ ,

$$\therefore m = 18,$$

$$\text{and } \sqrt[3]{a} = \frac{18}{b^2};$$

hence, by putting  $b = \frac{3}{2}$ , we obtain  $a = 512$ .

**Example 18:** The square of the time of a planet's revolution varies as the cube of its distance from the Sun; find the time of Venus' revolution, assuming the distance of Earth and Venus from the Sun to be  $91\frac{1}{4}$  and 66 millions of miles respectively.

**Solution:**

Let  $P$  be the time of revolution in days,  $D$  the distance in millions of miles; we have  $P^2 \propto D^3$ ,  
or  $P^2 = kD^3$ ,

Where  $k$  is some constant.

$$\text{For the Earth, } 365 \times 365 = k \times 91\frac{1}{4} \times 91\frac{1}{4} \times 91\frac{1}{4},$$

$$\text{hence } k = \frac{4 \times 4 \times 4}{365};$$

$$\therefore P^2 = \frac{4 \times 4 \times 4}{365} D^3$$

$$\text{For Venus, } P^2 = \frac{4 \times 4 \times 4}{365} \times 66 \times 66 \times 66;$$

$$\text{hence } P = 4 \times 66 \times \sqrt{\frac{264}{365}}$$

$$= 264 \times \sqrt{.7233}, \text{ approximately,}$$

$$= 264 \times .85$$

$$= 224.5$$

Hence, the time of revolution is nearly  $224\frac{1}{2}$  days.

**Example 19:** The volume of a right circular cone varies as square of the radius of the base when the height is constant, and it varies to the height when the base is constant. If the radius of the base is 7 cm. and the height 15 cm., the volume is 770 cubic cm., find the height of a cone whose volume is 132 cubic cm. Which stands on a base whose radius is 3 cm.

**Solution:**

Let  $h$  and  $r$  denote respectively the height and radius of the cone measured in cm.; also let  $v$  be the volume in cubic cm.

By supposition,  $770 = m \times 7^2 \times 15$ ;

$$\text{hence } m = \frac{22}{21};$$

$$\therefore V = \frac{22}{21} r^2 h.$$

$\therefore$  By substituting  $V = 132$ ,  $r = 3$ , we get

$$132 = \frac{22}{21} \times 9 \times h;$$

$\Rightarrow h = 14$ ; and therefore the height is 14 cm.

**Example 20:** The cost of a diamond varies as the square of its weight. A diamond weighing 10 gms. costs Rs. 800. Find the loss incurred when it breaks in the ratio 1 : 4.

**Solution;**

Let the cost be  $C$  & weight be  $W$ .

$$\therefore C \propto W^2 \text{ or } C = KW^2$$

$$800 = K \times (10)^2 \Rightarrow K = 8$$

$$\text{Weight of 1}^{\text{st}} \text{ broken piece} = \frac{1}{5} \text{ of } 10 \text{ gms.} = 2 \text{ gms.}$$

$$\text{Weight of 2}^{\text{nd}} \text{ broken piece} = \frac{4}{5} \text{ of } 10 \text{ gms.}$$

$$= 8 \text{ gms}$$

$$\text{Cost of 1}^{\text{st}} \text{ broken piece, } C_1 = 8 \times 2^2 = \text{Rs. } 32$$

$$\text{Cost of the 2}^{\text{nd}} \text{ broken piece} = C_2 = 8 \times 8^2 = \text{Rs. } 512$$

$$\therefore \text{Total cost} = 512 + 32 = \text{Rs. } 544$$

$$\therefore \text{Loss incurred} = 800 - 544 = \text{Rs. } 256$$

**Direct Relation:**

If  $x$  is directly related to or varies directly as  $y$ , then as  $y$  increases,  $x$  also increases but not proportionally.

The mathematical expression for this relationship is  $x = k_1y + k_2$ , where  $k_1$  and  $k_2$  are any constants.

Let us understand this with the help of an example:

The cost of a picnic is directly related to the number of people going on the picnic. The cost per person going on a picnic decreases from Rs. 100 to 90 when the number of people increases from 100 to 120. What will be the cost per person when the number of people going for the picnic is 150?

Since, this is a case of direct relation, we have  $C = k_1N + k_2$ , where  $C$  is total cost of the picnic,  $N$  is the number of people going for the picnic and  $k_1$  and  $k_2$  are constants. Thus we have  $10000 = 100k_1 + k_2$  and  $10800 = 120k_1 + k_2$

(Solving these two equations simultaneously we get  $k_1 = 40$  and  $k_2 = 6000$ ) and [the relation between cost of picnic and number of people is  $C = 40N + 6000$ .]

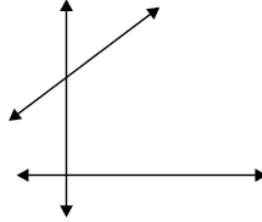
Substituting  $N = 150$ , we have  $C = 12000$  and the cost per person is  $\frac{12000}{150} = 80$ .

In direct proportion the line passes through origin *i.e.* when the value of  $x$  is zero,  $y$  is also zero. But in the case of direct relation, even though  $x = 0$ ,  $y$  is not zero and takes some value in real life. Even if no unit is produced, in a manufacturing plant yet there would still be some cost incurred, usually called the fixed cost. In real life, total cost of production

$$= \text{fixed cost} + (\text{cost per unit} \times \text{number of units produced})$$

Cost per unit is called as variable cost and will be incurred only if any unit is produced, e.g. raw material. Fixed cost is the cost incurred even if there is no production.

e.g. rent of place, cost of machinery.



### Inverse Relation:

If  $y$  is inversely related to  $x$ , or varies inversely with  $x$ , then as  $x$  increases (or decreases),  $y$  decreases (or increase) but not proportionally.

The mathematical expression for Inverse Relation is

$$y = \frac{k_1}{x} + k_2,$$

where  $k_1$  and  $k_2$  are constants.

**Example 21:** The expenses for a picnic party are partly fixed and partly variable with the number of people going for the picnic. The charge comes to Rs. 50 per head, when there are 20 people, and comes to Rs. 40 per head when there are 35 people. Find the charge per head when there are 150 people.

### Solution:

Total expenses ( $E$ )

= Fixed cost ( $f$ ) + Variable cost ( $kv$ )

$E = f + kv$ , where  $f$  and  $k$  are constants.

$$50 \times 20 = f + 20k$$

$$\Rightarrow 1000 = f + 20k$$

$$\text{and } 40 \times 35 = f + 35k$$

$$\Rightarrow 1400 = f + 35k$$

$$\Rightarrow 400 = 15k$$

$$\Rightarrow k = \frac{80}{3}$$

$$\text{So we have, } 50 \times 20 = f + 20 \times \frac{80}{3}$$

$$f = 1000 - \frac{1600}{3} = \frac{1400}{3}$$

Total expenses when total number of people are

150 will be  $= f + 150k$

$$= \frac{1400}{3} + 150 \times \frac{80}{3} = \frac{1400 + 12000}{3} = \frac{13400}{3}$$

So, the charge per head will be

$$= \frac{13400}{3 \times 150} = \text{Rs. } 29.78 \text{ approximately.}$$

## Partnerships

When two or more persons invest money in a common business, they are called *partners* and the business relation is called Partnership.

In a partnership business, every partner invests two entities:

- I. Time
- II. Money

## Types of Partners

Working Partner

Sleeping Partner

### Working Partner

A partner who participates in day-to-day functioning of the business is a Working Partner. These are also known as "Active Partner".

### Sleeping Partner

A partner who is limited to provide capital to the business and do not actively participate is a Sleeping Partner. These are also known as Dormant Partner.

So, the profits left after paying the working partner's remuneration or salary are shared by the partners.

Sometimes, partners also want interest on their capitals and in that case only remaining profits are shared.

Profit sharing among partners depends on understanding between the partners. However, if there is no agreement, then profits are shared based on the investments. There are three different possibilities, which are as follows:

- I. If the partners invest different amount as capital for the same time period throughout the year, then profits of all the partners are shared in the ratio of their investments.

If two partners  $A$  and  $B$  have invested capital  $C_A$  and  $C_B$  for time period  $t$ , then profit will be shared as follows:

$$= \frac{C_A}{C_B} = \frac{P_A}{P_B}$$

- II. If the partners invest same amount as capital for the different time periods, then profits of all the partners are shared in the ratio of their respective time periods for which their capitals were invested.

If  $A$  and  $B$  have invested capital  $C$  for time  $T_A$  and  $T_B$  respectively, then profit will be shared as follows:

$$= \frac{T_A}{T_B} = \frac{P_A}{P_B}$$

- III. If two partners  $A$  and  $B$  have invested capitals  $C_A$  and  $C_B$  for time periods  $T_A$  and  $T_B$  respectively, then profit will be shared as follows:

$$= \frac{C_A \times T_A}{C_B \times T_B} = \frac{P_A}{P_B}$$

where  $P_A$  and  $P_B$  are share of profits of  $A$  and  $B$  respectively.

**Example 22:**  $A$ ,  $B$  and  $C$  invested Rs. 1,000, Rs. 600 and Rs. 400 respectively to start a business. The profit is Rs. 200 which is to be divided among  $A$ ,  $B$  and  $C$  in the ratio of their capital invested. Calculate the share of profit for each of them.

**Solution:**

As nothing specific is given about the time periods of their investments, we will assume that each of them invested for an equal period of time. Now, the ratio of the investments of  $A$ ,  $B$  &  $C$  is the same as the ratio of the money that each of them invested or  $P_A : P_B : P_C = 1000 : 600 : 400 = 5 : 3 : 2$

$$A's \text{ share of profit} = \frac{5}{10} \times 200 = \text{Rs. } 100$$

$$B's \text{ share of profit} = \frac{3}{10} \times 200 = \text{Rs. } 60$$

$$C's \text{ share of profit} = \frac{2}{10} \times 200 = \text{Rs. } 40$$

**Example 23:**  $A$ ,  $B$  and  $C$  enter into a partnership with an amount of Rs. 10,000 each. After 4 months,  $A$  invests an additional amount of Rs. 2,000. Three months later,  $B$  invests Rs. 4,000, and  $C$  at the same time withdraws Rs. 2000 from business. Calculate their respective shares, also  $C$  is allowed Rs. 2,000 as monthly salary from profits at the end?

If Profit for the year is Rs. 2,17,000.

**Solution:**

$A$ 's capital investment in that year  
 $= 10000 \times 4 + 12000 \times 8 = 1,36,000$

$B$ 's capital investment in that year  
 $= 10000 \times 7 + 14000 \times 5 = 1,40,000$

$C$ 's capital investment in that year  
 $= 10000 \times 7 + 8000 \times 5 = 1,10,000$

Ratio in which profits are to be shared  $= 68 : 70 : 55$

Annual Salary of  $C = 2000 \times 12 = \text{Rs. } 24,000$

Available profits at the end after  $C$ 's salary  $= 217000 - 24000 = \text{Rs. } 1,93,000$

$$A's \text{ share} = \frac{68}{193} \times 193000 = \text{Rs. } 68000$$

$$B's \text{ share} = \frac{70}{193} \times 193000 = \text{Rs. } 70000$$

$$C's \text{ share} = \frac{55}{193} \times 193000 = \text{Rs. } 55000$$

**Example 24:** 64 men can dig a trench 50 yards long, 2 yards wide and 2 yards deep in 5 days of 12 hours each. How many days of 8 hours each will 80 men take to dig another trench 75 yards long, 6 yards wide & 2 yards deep?

**Solution:**

We should first see whether the other variables are proportional or inversely proportional to the variable that is to be found.

| Men | Work                   | Days | Hrs./Day |
|-----|------------------------|------|----------|
| 64  | $50 \times 2 \times 2$ | 5    | 12       |
| 80  | $75 \times 6 \times 2$ | $x$  | 8        |

More men means less days (multiply by  $\frac{64}{80}$ )

(Since men and number of days are inversely proportional)

More work means more days (multiply by  $\frac{75 \times 6 \times 2}{50 \times 2 \times 2}$ )

(Since work and number of days are directly proportional)

Less hours means more days (multiply by  $\frac{12}{8}$ )

(Since number of hours and number of days are inversely proportional)

$$\therefore x = 5 \times \frac{64}{80} \times \frac{75 \times 4 \times 3}{50 \times 2 \times 2} \times \frac{12}{8} \Rightarrow x = 27 \text{ days.}$$

**Example 25:** The concentration of an acid solution is inversely proportional to the volume of the solution if the amount of acid is not changed. A 40% hydrochloric acid solution becomes 30% solution when 30 L. of water is added to it. Find the original volume of the solution.

**Solution:**

The original concentration = 40%

The final concentration = 30%

$$I_C \times I_V = F_C \times F_V$$

where  $I_C$  and  $F_C$  indicate initial and final concentrations respectively.

$I_V$  and  $F_V$  indicate initial and final volumes respectively.

$$\therefore \frac{30}{40} = \frac{I_V}{F_V} \Rightarrow F_V = \frac{4}{3} I_V$$

There is an increase of  $\frac{1}{3} I_V$ , which is 30 L.

Hence, initial volume =  $30 \times 3 = 90$  L.

**Example 26:** In a camp of 100 soldiers, there is food which lasts for 8 days. After the first 2 days, 50 more soldiers join them. How long will the food last now? (Assume all the soldiers have equal eating capacity.)

**Solution:**

Suppose, no more soldiers had joined, then the remaining food would have lasted for 6 days. But since 50 more soldiers are joining, it now lasts for fewer than 6 days. There are 2 ways of approaching this problem.

This is the method where we use proportions. If there are more people, then the ratio should last for lesser time. The number of soldiers now is  $\frac{3}{2}$  times the original number.

Hence, the number of days it would last for would be  $\frac{3}{2}$  times the original number of days *i.e.*  $\left(\frac{2}{3}\right) \times 6 = 4$  days.

**Alternate Method:**

Let each soldiers consume  $x$  kilograms per day. Hence, if the food lasts for 8 days when there are 100 soldiers, the amount of food left at the end of 2 days =  $(100)(6)(x) = 600x$  kg. This has to be now consumed by 150 soldiers.

Hence, the number of days this would last =  $\frac{600x}{150x} = 4$  days.

### Practice Exercise – Easy

- The ratio of the age of  $A$  and  $B$  is  $2 : 5$  and ratio of the age of  $B$  and  $C$  is  $3 : 4$ . What is the ratio of the age of  $A$ ,  $B$  and  $C$ ?
  - $3 : 5 : 9$
  - $15 : 6 : 20$
  - $20 : 6 : 15$
  - $6 : 15 : 20$
- If the angles  $A$ ,  $B$ , and  $C$  of a triangle are in the ratio  $5 : 12 : 13$ , find the value of angle  $A$ .
  - $45^\circ$
  - $60^\circ$
  - $30^\circ$
  - $15^\circ$
- What number must be subtracted from both the numerator and the denominator  $27 : 35$ , so that it becomes equal to  $2 : 3$ ?
  - 5
  - 8
  - 12
  - None of these
- $x$  is directly proportional to  $y$ , and  $y = 7$  when  $x = 15$ , find  $x$  when  $y = 14$ .
  - 23
  - 30
  - 33
  - 39
- Find the 4 the proportional to 5, 20, and 7.
  - 28
  - 29
  - 22
  - 25
- If  $a : b = b : c$ , then ratio  $a^4 : b^4 = ?$ 
  - $ac : b^2$
  - $a^2 : c^2$
  - $c^2 : a^2$
  - $b^2 : ac$
- If  $4a = 5b$  and  $7b = 9c$ , then  $a : b : c = ?$ 
  - $45 : 36 : 28$
  - $44 : 33 : 28$
  - $28 : 36 : 45$
  - $36 : 28 : 45$
- If  $P = \frac{1}{4} Q$  and  $= \frac{1}{2} R$ , then find the value of  $P : Q : R$ .
  - $8 : 4 : 1$
  - $4 : 2 : 1$
  - $1 : 4 : 8$
  - $1 : 2 : 4$
- If  $(p + q) : (p - q) = 5 : 3$ , then  $(p^2 + q^2) : (p^2 - q^2) = ?$ 
  - $17 : 15$
  - $25 : 9$
  - $4 : 1$
  - $16 : 1$
- The ratio of angles of a triangle is  $3 : 5 : 1$ . Find the largest angle.
  - $100^\circ$
  - $45^\circ$
  - $60^\circ$
  - $50^\circ$
- If 70% of a number is equal to three-fifth of another number, what is the ratio between the 1<sup>st</sup> and the 2<sup>nd</sup> numbers, respectively?
  - $7 : 6$
  - $6 : 7$
  - $3 : 7$
  - $7 : 3$
- Mean proportional between 8 and 98 is:
  - 28
  - 24
  - 20
  - 35
- A man left one - half of the property for his wife. One- third to his son and the remainder to his daughter and her share was worth Rs. 45000, how much money did the man leave?
  - Rs. 245000
  - Rs. 260000
  - Rs. 270000
  - Rs. 275000
- Ratio of Ashok's age to Pradeep's age is  $4 : 3$ . Ashok will be 26 years old after 6 years. How old is Pradeep now?
  - 18 years
  - 21 years
  - 15 years
  - 24 years
- The incomes of Chanda and Kim are in the ratio  $9 : 4$  and their expenditures are in the ratio  $7 : 3$ . If each saves Rs. 2000, then Chanda's expenditure is:
  - Rs. 60000
  - Rs. 80000
  - Rs. 70000
  - Rs. None of these
- Find the sum of three numbers in the ratio of  $2 : 3 : 5$ , such that the sum of their squares is equal to 1862.
  - 70
  - 75
  - 69
  - 60
- Divide 94 into two parts in such a way that fifth part of the first and eighth part of the second are in the ratio  $3 : 4$ .
  - 30, 64
  - 30, 65
  - 35, 69
  - 20, 44

18. If  $P$  is  $\frac{1}{3}$  of  $Q$  and  $Q$  is  $\frac{1}{2}$  of  $R$ , then  $P : Q : R$  is:
- a.  $1 : 3 : 6$     b.  $2 : 3 : 6$     c.  $3 : 2 : 6$     d.  $3 : 1 : 2$
19. When 30 per cent of a number is added to another number the second number increases to its 140 per cent. What is the ratio between the first and the second number?
- a.  $4 : 1$     b.  $4 : 3$   
c.  $3 : 4$     d. Data inadequate
20. When 50% of one number is added to a second number, the second number increases to its four - thirds. What is the ratio between the first number and the second number?
- a.  $2 : 5$     b.  $4 : 3$   
c.  $2 : 3$     d. Data inadequate
21. A sum of Rs. 782 has been divided between  $P$ ,  $Q$  and  $R$  in the ratio of  $\frac{1}{2} : \frac{2}{3} : \frac{3}{4}$ . Find the share of  $R$ .
- a. 153    b. 305    c. 453    d. 306
22. A bag contains Rs. 1, 50 paise and 25 paise coins in the ratio of  $8 : 9 : 11$ . If the total money in the bag is Rs. 366, find the number of 25 paise coins.
- a. 264    b. 364    c. 241    d. 245
23. What must be added to each term of the fraction  $\frac{4}{7}$ , so that it becomes  $\frac{2}{3}$ ?
- a. 5    b. 6    c. 4    d. 2
24.  $P$ ,  $Q$  and  $R$  join a partnership contributing Rs. 2000, Rs. 1500 and Rs. 1250 respectively. What is  $P$ 's share if total profit is Rs. 3610?
- a. 1500    b. 2290    c. 1870    d. 1520
25. If  $A$  and  $B$  shared in the ratio of  $2 : 9$ , what is the ratio of  $A$ 's share to the difference between  $B$ 's &  $A$ 's share?
- a.  $2 : 3$     b.  $4 : 10$     c.  $2 : 7$     d.  $2 : 9$
26. What must be subtracted from the ratio  $68 : 49$  so that it becomes  $3 : 2$ ?
- a. 7    b. 0    c. 5    d. 11
27. If  $4a = 3b = 5c$ , then  $a : b : c$  is:
- a.  $4 : 3 : 2$     b.  $2 : 3 : 4$   
c.  $5 : 4 : 3$     d.  $15 : 20 : 12$
28. In a ratio equal to  $3 : 4$ , the antecedent is 36, the consequent is:
- a. 36    b. 16    c. 12    d. 48
29. Two whole numbers whose sum is 84 cannot be in the ratio of:
- a.  $9 : 3$     b.  $3 : 5$     c.  $19 : 2$     d.  $2 : 19$
30. In a school having rool strength 286, the ratio of boys and girls is  $8 : 5$ . If 22 more girls get admitted into the school, the ratio of boys and girls becomes:
- a.  $12 : 7$     b.  $10 : 7$     c.  $8 : 7$     d.  $4 : 3$
31. The number of students in three classes are in the ratio  $2 : 3 : 5$ . If the number of students in each class is increased by 20 the ratio becomes  $4 : 5 : 7$ . The total number of students before the increase was:
- a. 80    b. 90    c. 100    d. 120
32. If  $\frac{p}{q} = \frac{3}{4}$ , then find the value of the expression  $\frac{(5p - 3q)}{(7p + 2q)}$
- a.  $\frac{3}{29}$     b.  $\frac{5}{27}$     c.  $\frac{7}{31}$     d.  $\frac{5}{58}$



33. According to the Boyle's law, at a constant temperature, pressure of a definite mass of gas is inversely proportional to the volume. If the pressure is reduced by 20%, find the respective change in volume.
- a. - 33.33%                      b. + 25%  
c. - 25%                          d. + 33.33%
34. The side of a square is increased by half of its original measure. Find the ratio of the areas of the original square to the new square.
- a. 4 : 7              b. 2 : 3              c. 4 : 9              d. 9 : 4
35.  $P : Q = 3 : 7$  and the sum of  $P$  and  $Q$  is 50. Find the value of  $Q$ .
- a. 28              b. 33.5              c. 35              d. 36
36. A fraction bears the same ratio to  $\frac{3}{7}$  as  $\frac{1}{24}$  does to  $\frac{1}{28}$ . Find the fraction.
- a.  $\frac{4}{9}$               b.  $\frac{1}{3}$               c.  $\frac{1}{4}$               d.  $\frac{1}{2}$
37. In a mixture of milk and water of volume 30 litres, the ratio of milk and water is 7 : 3. Find the quantity of water to be added to the mixture to make the ratio of milk and water 1 : 2.
- a. 30 litres                      b. 33 litres  
c. 35 litres                      d. 39 litres
38. In a mixture of milk and water of volume 100 litres, the ratio of milk and water is 14 : 11. Find the quantity of water to be added to the mixture to make the ratio of milk and water 1 : 2.
- a. 60              b. 62              c. 68              d. 66
39. A and B invested Rs. 12,000 and Rs. 18,000 respectively in a business for the whole year. At the year - end, there was a total profit of Rs. 2,000. What is the share of A in the profit?
- a. Rs. 800                      b. Rs. 1,200  
c. Rs. 1,600                      d. None of these
40. A bottle contains 40 litres of a mixture of hydrochloric acid and sulphuric acid in the ratio 3 : 1. How much sulphuric acid should be added to the mixture so that the ratio of hydrochloric acid and sulphuric acid be 1 : 3?
- a. 65              b. 70              c. 80              d. 90
41. A mixture contains alcohol and water in the ratio of 4 : 3. If 14 litres of water is added to the mixture, the ratio of alcohol and water becomes 3 : 4. Find the quantity of alcohol in the mixture.
- a. 35 litres    b. 18 litres    c. 24 litres    d. 29 litres
42. The speed of three buses are in the ratio 2 : 3 : 4. The ratio between the time taken by these buses to travel the same distance is
- a. 2 : 3 : 4    b. 4 : 3 : 2    c. 4 : 3 : 6    d. 6 : 4 : 3
43. The difference between the two positive numbers is 10 and the ratio between them is 5 : 3. Find the product of the numbers.
- a. 375              b. 325              c. 275              d. 125
44. Four numbers in the ratio of 1 : 3 : 4 : 7 add up to give a sum of 75. Find the value of the biggest number.
- a. 42              b. 35              c. 49              d. 63
45. When we increase both the numerator and the denominator by 7, a fraction changes to  $\frac{3}{4}$ . Find the original fraction.
- a.  $\frac{5}{12}$               b.  $\frac{7}{9}$               c.  $\frac{2}{5}$               d.  $\frac{3}{8}$
46. A person distributes his pens among four friends A, B, C and D in the ratio  $\frac{1}{3} : \frac{1}{4} : \frac{1}{5} : \frac{1}{6}$ . What is the minimum number of pens that the person should have?
- a. 65              b. 55              c. 23              d. 57
47. In a mixture of 60 L, the ratio of milk and water is 2 : 1. If the ratio of milk and water is to be 1 : 2, then the amount of water to be further added must be:

- a. 40 L      b. 30 L      c. 20 L      d. 60 L
48. A and B started a business by investing Rs. 50000 and Rs. 25000. What is the share of B if yearly profit is Rs. 2000.  
a. 400      b. 666.67      c. 1333.34      d. 1600
49. Given that the ratio of A's money to that of B's money is 4 : 5 and B's money to C's is 2 : 3. If A has Rs. 1,000, then total amount of money among A, B and C is:  
a. Rs. 3,500      b. Rs. 4,125  
c. Rs. 3,000      d. Rs. 4,000
50. A, B and C partnered in a business for an year. A contribute Rs. 12000 for 6 months, B Rs. 10000 for 8 months and C Rs. 8000 for the entire year and their profit was Rs. 2728. What is the share of C?  
a. 968      b. 792      c. 1056      d. 880

### Practice Exercise – Medium

- The ratio of the incomes of Sapna and Kavita is 3 : 5 and the ratio of their expenses is 1 : 3. Who is saving more?  
a. Sapna      b. Kavita  
c. Both save equal      d. Cannot be determined
- The sum of the ages of the 4 members of Chintamani family is 140 years. 5 years ago the ages of the 4 members Dipa, Nisha, Mrs. Chintamani and Mr. Chintamani were years ratio of 2 : 3 : 7 : 8. After how many years would Dipa be as old as the present age of her mother?  
a. 15 years      b. 22 years  
c. 27 years      d. 30 years
- X works twice as fast as Y, whereas X and Y together can work three times as fast as Z. If X, Y and Z together work on a job, in what ratio should they share the earnings?  
a. 2 : 1 : 1      b. 1 : 2 : 1  
c. 1 : 1 : 2      d. None of these
- A, B and C are three partners. They altogether invested Rs. 14000 in business. At the end of the year, A got Rs. 337.50, B Rs. 1125 and C Rs. 637.50 as profit. The difference between the investments of B and A was:  
a. Rs. 2200      b. Rs. 3200      c. Rs. 4200      d. Rs. 5250
- A bag contains rupee, 50-paise and 25-paise coins in the ratio 5 : 6 : 7. If the total amount is Rs. 390, find the number of 25-paise coins.  
a. 280      b. 240      c. 200      d. 120
- P and Q rent a pasture for 10 months. P puts in 100 cows for 8 months. How many can Q put in for the remaining 2 months, if he pays 1.5 times as much again as P?  
a. 400      b. 600      c. 520      d. 480
- A sum of money is to be divided equally among P, Q and R in the respective ratio of 5 : 6 : 7 and another sum of money is to be divided between S and T equally. If S got Rs. 2100 less than P, how much amount did Q receive?  
a. Rs. 25000  
b. Rs. 2000  
c. Rs. 1500  
d. Cannot be determined
- Four years ago, the ratio of the ages of A and B was 13 : 9 and eight years hence it would be 4 : 3. The difference of their present ages is:  
a. 56 years      b. 40 years      c. 16 years      d. 24 years
- What is the angle in degrees made by a sector, the ratio of whose area with the area of the semicircle is equal to 1 : 10?  
a. 36      b. 18      c. 24      d. 9
- The value of a diamond varies directly as the square of its weight. If a diamond worth Rs. 10,000 is divided into 2 pieces in the ratio of 4 : 6, what is the loss in value?  
a. 52%      b. 48%  
c. 36%      d. None of these

11. A watermelon is cut into two pieces in the ratio of 3 : 5 by weight. The biggest of the two is further cut in the ratio of 5 : 7 by weight. Find the ratio of each of the three pieces.
- a. 3 : 5 : 7                      b. 36 : 25 : 35  
c. 15 : 25 : 56                  d. None of these
12. Anil, Pawan and Rajan join a partnership. Anil invested Rs. 16,000 for 6 months, Pawan invested Rs. 12,000 for rd year and Rajan invested Rs. 1,000 for 12 months. In which ratio will they share the future profits?
- a. 8 : 8 : 1    b. 10 : 8 : 7    c. 6 : 8 : 12    d. 8 : 7 : 10
13. Rs. 9,700 has been divided among  $a$ ,  $b$  and  $c$  such that if their shares are reduced respectively by Rs. 30, Rs. 20 and Rs. 50, the balances are in the ratio of 3 : 4 : 5. What is  $b$ 's share?
- a. Rs. 3,180                      b. Rs. 3,220  
c. Rs. 3,253.33                  d. Rs. 3,200
14. An amount of money is distributed amongst  $A$ ,  $B$  and  $C$  such that  $A$  gets half that of  $B$  and  $B$  gets twice that of  $C$ . What is the ratio of the share of  $B$  to that of the sum of the shares of  $A$  and  $B$ .
- a. 3 : 2        b. 2 : 3        c. 2 : 2        d. 4 : 3
15. Four years ago, a man's age was 6 times that of his son. 12 years from now, his age will be twice that of the son. What is the ratio of their present ages?
- a. 5 : 1        b. 4 : 7        c. 7 : 1        d. 7 : 2
16. A sum of money is divided among  $X$ ,  $Y$  and  $Z$  in such a manner that for each one rupee that  $X$  gets,  $Y$  gets 65 paise and  $Z$  gets 35 paise. If  $Z$ 's share is Rs. 560, the sum is:
- a. Rs. 2,900                      b. Rs. 3,000  
c. Rs. 3,200                      d. Rs.3,800
17. The income of Sunil and Rahul are in the ratio of 3 : 2 and their expenses are in the ratio of 5 : 3. If each of them saves Rs. 3000, then Rahul's income is :
- a. Rs. 9,000                      b. 6,000  
c. Rs. 11,000                      d. 12,000
18. In a partnership business,  $P$ ,  $Q$  and  $R$  invest money in the ratio 8 : 7 : 5.  $P$  withdraws half her money after 5 months. If the profit is Rs. 26,500 for the year, find  $Q$ 's share.
- a. Rs. 10,200                      b. Rs. 9,500  
c. Rs. 10,500                      d. Rs. 12,600
19. The total salary of Ravi, Ajay and Bhuvan is Rs. 350000. If they spend 70%, 75% and 80% of their salaries respectively, their savings are in ratio of 15 : 10 : 25. Find Ajay salary.
- a. 76542    b. 97638    c. 89474    d. 63158
20. A chemist has 6 litres of a 25% alcohol solution. To make a solution containing 50% alcohol how much alcohol should he add?
- a. 4.5        b. 3        c. 2.5        d. 6
21. Total expenses at a hostel is partly fixed and partly variable. When the number of students is 20, total expense is Rs. 15000 and when the number of students is 30, total expense is Rs. 20000. What will be the expense when the number of students are 40?
- a. Rs. 25000                      b. Rs. 22000  
c. Rs. 21500                      d. Rs. 20800
22. A farmer decided to distribute gold coins to his daughters in the following way: 2<sup>nd</sup> daughter would get  $\frac{5}{7}$ <sup>th</sup> of what the 1<sup>st</sup> daughter would get and the 3<sup>rd</sup> daughter would get  $\frac{3}{5}$ <sup>th</sup> of what the 2<sup>nd</sup> daughter would get. 1<sup>st</sup> daughter got 60 gold coins more than the 3<sup>rd</sup> daughter. How many gold coins were distributed to the three daughters?
- a. 165        b. 195        c. 225        d. 235
23. Men, women and children are employed to do a work in the proportion of 1 : 2 : 3 and their wages are in the proportion of 6 : 3 : 2. When 50 men are employed, total wages of all amount to Rs. 4500. What is the weekly wages paid to a man, a woman and a child, in rupees?
- a. 210, 105, 80                      b. 210, 105, 70

- c. 210, 105, 90                      d. 200, 105, 70

24. One year ago, the ratio between Mahesh's and Suresh's salaries was 3 : 5. The ratio of their individual salaries of last year and present year are 2 : 3 and 4 : 5 respectively. If their total salaries for the present year are Rs. 43000, what is the present salary of Mahesh?
- a. Rs. 19000                      b. Rs. 18000  
c. Rs. 16000                      d. Rs. 15500
25. The prime cost of an article is three times the value of the raw material used. The cost of raw materials increases in the ratio of 5 : 12 and manufacturing expenses in the ratio 4 : 5. The article, which originally cost Rs. 6, will new cost
- a. Rs. 10                      b. Rs. 17  
c. Rs. 20.50                      d. None of these
26. Anu is a working partner and Bimla is a sleeping partner in a business. Anu puts in Rs. 5000 and Bimla puts in Rs. 6000. Anu receives 12.5% of the profit for managing the business and the rest is divided in proportion to their capital. What does each get out of a profit of Rs. 880?
- a. Rs. 400 and Rs. 480                      b. Rs. 460 and Rs. 420  
c. Rs. 450 and Rs. 430                      d. Rs. 470 and Rs. 410
27. A man ordered 4 pairs of black socks and some pairs of brown socks. The price of a black pair is double that of a brown pair. While preparing the bill, the clerk did a mistake and interchanged the number of black and brown pairs. This increased the bill by 50%. The ratio of the number of black and brown pairs of socks in the original order was
- a. 4 : 1                      b. 2 : 1                      c. 1 : 4                      d. 1 : 2
28. X and Y entered into partnership with Rs. 700 and Rs. 600 respectively. After 3 months, X withdrew  $\frac{2}{7}$  of his stock but after 3 months, he puts back  $\frac{3}{5}$  of what he had withdrawn. The profit at the end of the year is Rs. 726. How much of this should X receive?
- a. Rs. 336                      b. Rs. 366                      c. Rs. 633                      d. Rs. 663
29. Divide Rs. 671 among A, B and C such that if their shares be increased by Rs. 3, Rs. 7 and Rs. 9 respectively, the remainder shall be in the ratio 1 : 2 : 3.
- a. Rs. 110, Rs. 220 and Rs. 336  
b. Rs. 112, Rs. 223 and Rs. 336  
c. Rs. 105, Rs. 223 and Rs. 330  
d. None of the above
30. An amount of money is to be distributed among A, B and C in the ratio of 6 : 19 : 7 respectively. If C gives Rs. 200 of his share to B the ratio among A, B and C becomes 3 : 10 : 3 respectively. What was the total amount?
- a. Rs. 6400                      b. Rs. 12800  
c. Rs. 3200                      d. Data inadequate
31. X and Y enter into a partnership with their capitals in the ratio 7 : 9. At the end of 8 months, X withdraws his capital. If they receive the profits in the ratio 8 : 9, find how long Y's capital was used.
- a. 8                      b. 6                      c. 7                      d. 11
32. Anmol started a business by investing Rs. 2700. After sometime Vijay joined him by investing Rs. 2025. At the end of one year, the profit was divided in the ratio 2 : 1. After how many months did Vijay join the business?
- a. 4 months                      b. 6 months  
c. 3 months                      d. 2 months
33. A cat takes 5 leaps for every 4 leaps of a dog, but 3 leaps of the dog are equal to 4 leaps of the cat. What is the ratio of the speeds of the cat to that of the dog?
- a. 11 : 15                      b. 15 : 11                      c. 16 : 15                      d. 15 : 16
34. The ratio of 1<sup>st</sup> and 2<sup>nd</sup> classes train fares between two stations is 3 : 1 and that of the number of passengers travelling between these stations by 1<sup>st</sup> and 2<sup>nd</sup> classes is 1 : 50. If on a particular day, Rs. 2650 be collected from the passengers travelling between these stations, then find the amount collected from 2<sup>nd</sup> class passengers.

- a. Rs. 3000                      b. Rs. 3500
- c. Rs. 2800                      d. Rs. 2500

35. There are two triangles  $A$  and  $B$ . The angles of triangle  $A$  are in the ratio of  $3 : 4 : 5$  and the angles of triangle  $B$  are in the ratio of  $5 : 6 : 7$ . What is the difference between largest angle of triangle  $A$  and the smallest angle of triangle  $B$ ?
- a.  $28^\circ$       b.  $35^\circ$       c.  $25^\circ$       d.  $40$
36. In a factory, the ratio of the numbers of employees of three types  $P$ ,  $Q$  and  $R$  is  $9 : 13 : 18$  and their wages are in the ratio of  $10 : 7 : 4$ . If number of employees of type  $R$  is 54 and wages of every employee of type  $Q$  is Rs. 1400, then find the total wages of all the employees of type  $P$ .
- a. Rs. 51000                      b. Rs. 54000
  - c. Rs. 56000                      d. Rs. 57000
37. King Dashrath decided to distribute gold coins to his three queens in the following way: 2<sup>nd</sup> queen would get  $\frac{5}{7}$ th of what the 1<sup>st</sup> queen would get and the 3<sup>rd</sup> queen would get  $\frac{3}{5}$ th of what the 2<sup>nd</sup> queen would get. 1<sup>st</sup> queen got 60 gold coins more than the 3<sup>rd</sup> queen. How many gold coins were distributed to the three queens?
- a. 120      b. 175      c. 225      d. 250
38. The sum of the ages of the 4 members of Sinha family is 140 years. 5 years ago the ages of the 4 members Nishu, Vicky, Mrs Sinha and Mr Sinha were in the ratio of  $2 : 3 : 7 : 8$ . After how many years would Nishu be as old as the present age of his mother?
- a. 10 years    b. 17 years    c. 30 years    d. 32 years
39. Two-fifth of Anil's salary is equal of Bhuvan's salary and seven - ninth of Bhuvan's salary is equal to Chandra's salary. The sum of the salary of all of them is Rs. 770. Which of the following is the salary of each?
- a. 300, 225, 250                      b. 500, 425, 375
  - c. 450, 180, 140                      d. 520, 610, 475
40. Inspector Vijay Shankar spends one-fourth of his salary on the house rent, one-third on food and one-sixth on travel. After spending one-tenth of the remaining amount on clothes, he is left with Rs. 13, 500. What is the difference between the amounts spent on the house rent and food?
- a. Rs. 7,500                      b. Rs. 10,000
  - c. Rs. 4,000                      d. Rs. 5,000
41. On the eve of the foundation day party of Due North Inc., the ratio of males to females was  $5 : 3$ . However, when 10 males left the party, this ratio got changed to  $1 : 1$ . How many people were there originally at the party?
- a. 48      b. 32      c. 64      d. 40
42. In a forest reserve, the ratio of the number of deers, bear and fox is  $3 : 7 : 5$ . If the difference between the number of deers and bears is a multiple of 3 as well as 7, what is the minimum number of animals in the park?
- a. 315                      b. 310
  - c. 45                      d. Cannot be determined
43. A diamond falls down and breaks into three pieces whose weights are in the ratio  $1 : 3 : 6$ . If the value of a diamond is proportional to the square of its weight and the original value of the diamond was Rs. 30,000, then what is the loss in the value due to the breakage?
- a. 15,200    b. 17000    c. 16800    d. 16200
44.  $P$  starts a business with Rs. 4,000.  $Q$  joins him after 3 months with Rs. 8,000.  $R$  puts a sum of Rs. 12,000 in the business for 2 months only in the same year. At the end of the year, the business generated a profit of Rs. 5,200. Find the share of  $Q$ .
- a. Rs. 1,500                      b. Rs. 1,800
  - c. Rs. 2,600                      d. Rs. 4,000
45. The manufacturing cost of a product involves only the expenses on Material ( $M$ ), the Labour ( $L$ ), and the overheads cost ( $O$ ). If  $M : L : O$  is  $5 : 7 : 3$  and the product is sold at 20% profit then what is the ratio of the material costs to the profit?
- a.  $3 : 4$       b.  $7 : 5$       c.  $3 : 5$       d.  $7 : 3$

46. A common foodstuff is found to contain 2.5% iron. The serving size is 90.0 grams. If the recommended daily allowance is 18 gm of iron, how many servings would a person have to get 100% of the daily allowance of iron?  
a. 8              b. 12              c. 10              d. 15
47. The expenses per month of Sumit's car partly constant and partly vary with the number of kilometres he travels. When he travels 150 km in a month, the total expenses come to Rs. 3400. If he travels 200 km, it is Rs. 4000. Find the total expenses, if he travels 300 km in a month?  
a. 5000          b. 4800          c. 5200          d. 3600
48. Four milkmen rented a pasture. Ram grazed his 12 goats for 6 months, Rahim 10 goats for 5 months and Mohan 14 goats for 10 months and Shyam 9 goats for 7 months. If the total rent of the field is Rs. 325 then find the total amount paid as rent by Mohan and Shyam?  
a. 212          b. 205          c. 190          d. 225
49. In two alloys the ratio of Iron and Copper is 4 : 3 and 6 : 1 respectively. If 14 kg of the first alloy and 35 kg of the second alloy are mixed together to form a new alloy, then what will be the ratio of copper to iron in the new alloy:  
a. 38 : 11      b. 11 : 38      c. 3 : 8          d. 11 : 8
50. In a zoo, there are goats and pigeons. If heads are counted there are 340 heads and if legs are counted there are 1060 legs. How many pigeons are there?  
a. 120          b. 150          c. 180          d. 170

### Practice Exercise – Difficult

1. If  $\frac{x}{3a+2b} = \frac{y}{3b+2c} = \frac{z}{3c+2a} = 5$  and  $a, b$  and  $c$  are in continued proportion and  $b, c$ , and  $a$  are in continued proportion, then find  $\frac{x}{a} + \frac{y}{2b} + \frac{z}{3c}$ .  
a.  $55\frac{1}{5}$           b. 25          c.  $4\frac{1}{6}$           d.  $45\frac{5}{6}$
2. A hare and a tortoise were talking about their ages. When the tortoise said "I am thrice as old as what you will be when you are twice as old as you are". The hare then said "you will be eleven times as old as me when I will be as old as you are". What is the sum of the present ages of the hare and the tortoise in years?  
a. 49                      b. 490  
c. 91                      d. Cannot be determined
3. Instead of walking along two adjacent sides of a rectangular field, a boy took a short cut along the diagonal and saved a distance equal to half the longer side. Then, the ratio of the shorter side to the longer side is:  
a.  $\frac{1}{2}$           b.  $\frac{2}{3}$           c.  $\frac{1}{4}$           d.  $\frac{3}{4}$

**Directions for questions 4 to 6: Answer the questions based on the following information.**

Alphonso, on his death bed, keeps half his property for his wife and divide the rest equally among his three sons Ben, Carl and Dave. Some years later Ben dies leaving half his property to his widow and half to his brothers Carl and Dave together, shared equally. When Carl makes his will he keeps half his property for his widow and the rest he bequeaths to his younger brother Dave. When Dave dies some years later, he keeps half his property for his widow and the remaining for his mother. The mother now has Rs. 1,575,000.

4. What was the worth of the total property?  
a. Rs. 30 lakh              b. Rs. 8 lakh  
c. Rs. 18 lakh              d. Rs. 24 lakh
5. What was Carl's original share?  
a. Rs. 4 lakh              b. Rs. 12 lakh  
c. Rs. 6 lakh              d. Rs. 5 lakh
6. What was the ratio of the property owned by the widows of the three sons, in the end?  
a. 7 : 9 : 13              b. 8 : 10 : 15

c. 5 : 7 : 9

d. 9 : 12 : 13

7. In an election in which each elector may vote for two candidates, half of the electors vote for  $P$ , but divide their other votes between  $Q$ ,  $R$  and  $S$  in the ratio 3 : 2 : 1. Of the remainder, half vote for  $Q$  and divide their other votes between  $R$  and  $S$  in the ratio of 2 : 1. Of the remainder, half vote for  $R$  and  $S$  and the remainder 1680 in number do not vote. How many votes does  $R$  get?
- a. 3980      b. 6720      c. 5960      d. 6160
8. Distance covered by a train is directly proportional to the time taken and also it varies directly as the square root of fuel used and varies inversely as the number of wagons attached to it. A train covers 384 km journey in 40 hours when there are 10 wagons attached to it and total fuel consumption was 256 litres of diesel. Find the consumption of fuel per km when a train goes 400 km in 50 hours with 15 wagons attached to it:
- a. 4 l/km      b. 1 l/km      c. 2.2 l/km      d. 2.5 l/km
9. Three vessels having volumes in the ratio 3 : 2 : 4 are full of a mixture of water and milk. In the first vessel ratio of water and milk is 1 : 3, in second 2 : 3 and in third vessel, 2 : 5. If all the three vessels were poured out in a large container, what is the resulting ratio of milk and water?
- a. 85 : 36                      b. 883 : 377  
c. 386 : 883                      d. 377 : 883
10.  $A$  and  $B$  are two chemical fertilizers.  $A$  consists of  $N$ ,  $P$  and  $K$  and  $B$  consists of only  $N$  and  $P$ . A mixture  $A$  and  $B$  is prepared in which the ratio of  $N$ ,  $P$  and  $K$  is 26%, 68% and 6% respectively. The ratio of  $N$ ,  $P$  and  $K$  in  $A$  is 20%, 70% and 10% respectively. What is the ratio of  $N$  and  $P$  in the  $B$ ?
- a. 22% and 78%              b. 35% and 65%  
c. 30% and 70%              d. 25% and 75%

**Directions for questions 11 and 12: Answer questions on the basis of the information given below.**

An airline has a certain free luggage allowance and charges for excess luggage at a fixed rate per kg. Two passengers, Raja and Praja have 60 kg of luggage between them, and are charged Rs 1200 and Rs 2400 respectively for excess luggage. Had the entire luggage belonged to one of them, the excess luggage charge would have been Rs 5400.

11. What is the weight of Praja's luggage?
- a. 20 kg      b. 25 kg      c. 30 kg      d. 35 kg
12. What is the free luggage allowance?
- a. 10 kg      b. 15 kg      c. 20 kg      d. 25 kg
13. Let  $a$ ,  $b$ ,  $c$ ,  $d$  and  $e$  such that  $a = 6b = 12c$ , and  $2b = 9d = 12e$ . Then which of the following pairs contains a number that is not an integer?
- a.  $\left[ \frac{a}{27}, \frac{b}{e} \right]$                       b.  $\left[ \frac{a}{36}, \frac{c}{e} \right]$   
c.  $\left[ \frac{a}{12}, \frac{bd}{18} \right]$                       d.  $\left[ \frac{a}{6}, \frac{c}{d} \right]$

[CAT 2003]

**Directions for questions 14 and 15: Answer questions on the basis of the information given below.**

The batting average (BA) of a test batsman is computed from runs scored and innings played completed and incomplete innings (not out) in the following manner:

$r_1$  = number of runs scored in completed innings;

$n_1$  = number of completed innings

$r_2$  = number of runs scored in incomplete innings;

$n_2$  = number of incomplete innings

$$\text{Batting Average (BA)} = \frac{r_1 + r_2}{n_1}$$

To better assess batsman's accomplishments, the ICC considering two other measures  $MBA_1$  and  $MBA_2$  defined as follows:

$$MBA_1 = \frac{r_1}{n_1} + \frac{n_2}{n_1} \max \left[ 0, \left( \frac{r_2}{n_2} - \frac{r_1}{n_1} \right) \right]$$

$$MBA_2 = \frac{r_1 + r_2}{n_1 + n_2}$$

14. Based on the information provided which of the following is true?

- a.  $MBA_1 \leq BA \leq MBA_2$
- b.  $BA \leq MBA_2 \leq MBA_1$
- c.  $MBA_2 \leq BA \leq MBA_1$
- d. None of these

[CAT 2004]

15. An experienced cricketer with no incomplete innings has a BA of 50. the next time he bats, the innings is incomplete and the scores 45 runs. It can be inferred that:

- a. BA and  $MBA_1$  will both increase
- b. BA will increase and  $MBA_2$  will decrease
- c. BA will increase and not enough data is available to assess change in  $MBA_1$  and  $MBA_2$
- d. None of these

[CAT 2004]

16. A truck travelling at 70 km/hr uses 30% more diesel to travel a certain distance than it does when it travels at a speed of 50 km/hr. If the truck can travel 19.5 km/L of diesel at 50 km/hr, how far can the truck travel on 10 L of diesel at a speed of 70 km/hr?

- a. 130 km    b. 140 km    c. 150 km    d. 175 km

[CAT 2000]

17. Utsav goes from A to B, via C, everyday by car. The distance between A and C is four times the distance between C and B when he travels by car. However, when he travels by the train, the ratio of the distances is the inverse of the ratio of the distances when travelled in the car. The average speed from A to C is double that of the speed from C to B for both the car and the train. If the average speed for the entire route is 45 km/hr for both, then what is the difference between the average speed of the car and the train from C to B?

- a. 27 km/hr                      b. 40.5 km/hr
- c. 13.5 km/hr                  d. 54 km/hr

**Directions for questions 18 to 20: Answer questions on the basis of the information given below.**

Three person A, B and C, started playing a game. They had money in the ratio of 3 : 2 : 1. It is decided that if A lost the game he would distribute  $\frac{1}{4}$ <sup>th</sup> of the money that he had prior to the round, equally between B and C. If B lost game he would distribute half of his money equally between A and C and if C lost the game he would distribute  $\frac{2}{3}$ <sup>rd</sup> of his money prior to that round equally between A and B. B lost the game first, and then C lost the next game followed by A and then followed by B again. In the process A gained Rs. 10.

18. What was the total sum of money they had initially?

- a. Rs. 210    b. Rs. 120    c. Rs. 180    d. Rs. 250

19. After which round the game would have been stopped for A to gain the maximum profit?

- a. Round 4    b. Round 2    c. Round 3    d. Round 1

20. Which of these did not have any chance to making profit in any of the rounds?

- a. Both B and C                      b. Both A and C
- c. Only B                                d. Only C

**Directions for questions 21 to 23: Answer questions on the basis of the information given below.**

There are N numbers of gold biscuits in the house, in which four people are lived. If the first man woke up and divided the biscuits into 5 equal piles and found one extra biscuit and hide them. He then gathered the remaining four piles into a big piles, woke up the second person and went to sleep. Each of the other 3 persons did the same one by one i.e. divided the big pile into 5 equal piles and found one extra biscuit. Each hide one of the piles along with the extra biscuit and gathered the remaining 4 piles into a big pile.



21. If  $N > 1000$ , what could be the least value of  $N$ ?  
a. 1249      b. 1023      c. 1202      d. 1246
22. If  $N < 1000$ , how many biscuits were left after the fourth man took his share?  
a. 624      b. 252      c. 621      d. 257
23. If the previous question, what is the sum of the numbers of biscuits hidden by the last 2 men?  
a. 142      b. 144      c. 180      d. 178
24. Ram, Rajeev and Rahul have some stones with each of them. Five times the number of stones with Rajeev. Equals seven times the number of stones with Ram while five times the number of stones with Ram equals seven times the number of stones with Rahul. What is the minimum number of stones that can be there with all three of them put together?  
a. 113      b. 109      c. 93      d. 97
25. Rajesh bought two varieties of rice, costing 5 cent per ounce and 6 cents per ounce each, and mixed them in some ratio. Then he sold the mixture at 7 cents per ounce, making a profit of 20 percent. What was the ratio of the mixture?  
a. 1 : 10      b. 1 : 5      c. 2 : 7      d. 9 : 8