PERMUTATIONS AND COMBINATIONS

Factorial Notation, Fundamental principle of counting, Permutation as an arrangement, Counting formulasfor permutations, Combination as selection, Difference between permutation and combination,Counting formulas for combination, Counting formulas for combination, Division and Distribution of Objects, Derangement Theorem, Divisibility of Numbers, Sum of Number, Important results abount points, Solved Examples, CBSE Section, AIEEE Section, IIT – JEE Section

FACTORIAL NOTATION

Factorial n : The product of first n natural numbers is denoted by $\underline{|n}$ or by n! and is read as 'factorial n'

$$n! = 1.2.3.4...\ (n-1).\ n = n\ (n-1)\ (n-2)...3.2.1$$

$$5! = 1.2.3.4.5 = 120$$

Some results related to factorial n : (i) $n! = n.(n-1)(n-2)...3.2.1$

(ii) $\dfrac{n!}{r!} = n(n-1)(n-2)...(r+1)$

Note : 0! is 1 and 1! is 1

Factorial of negative numbers or proper fractions is not defined.

FUNDAMENTAL PRINCIPLE OF COUNTING

(1) Multiplication Principle : If an operation can be performed in 'm' different ways; following which a second operation can be performed in 'n' different ways, then the two operations in succession can be performed in m × n ways. This can be extended to any finite number of operations.

SOLVED EXAMPLES

Example. A person wants to go from station P to station R via station Q. There are 4 routes from P to Q and 5 routes from Q to R. In how many ways can he travel from P to R-

(a) 9 (b) 1
(c) 20 (d) 12

Solution. He can go from P to Q in 4 ways

and Q to R in 5 ways

so number of ways of travel from P to R is 4 × 5 = 20

Example. A college offers 6 courses in the morning and 4 in the evening. The possible number of choices with the student if he wants to study one course in the morning and one in the evening is-

(a) 24 (b) 2

(c) 12 (d) 10

Solution. The student has 6 choices from the morning courses out of which he can select one course in 6 ways. For the evening course, he has 4 choices out of which he can select one in 4 ways.

Hence the total number of ways = $6 \times 4 = 24$.

(2) Addition Principle : If an operation can be performed in 'm' different ways and another operation, which is independent of the first operation, can be performed in 'n' different ways. Then either of the two operations can be performed in (m + n) ways. This can be extended to any finite number of mutually exlusive operations.

SOLVED EXAMPLES

Example. A person wants to leave station Q. There are 4 routes from station Q to P and 5 routes from Q to R. In how many ways can he travel the station Q-

(a) 9 (b) 1

(c) 20 (d) 12

Solution. He can go from Q to P in 4 ways

and Q to R in 5 ways

he can leave station Q in 4 + 5 = 9 ways.

Example. A college offers 6 courses in the morning and 4 in the evening. The number of ways a student can select exactly one course, either in the morning or in the evening-

(a) 6 (b) 4

(c) 10 (d) 24

Solution. The student has 6 choices from the morning courses out of which he can select one course in 6 ways. For the evening course, he has 4 choices out of which he can select one in 4 ways.

Hence the total number of ways = $6 + 4 = 10$.

PERMUTATION AS AN ARRANGEMENT

Each of the arrangements which can be made by taking some or all of a number of things is called a permutation.

Let *n* and *r* be non-negative integers such that $o \le r \le n$. Then

COUNTING FORMULAS FOR PERMUTATIONS

(1) Without Repetition :

(i) The number of permutations of n different things, taking r at a time is denoted by ${}^nP_r$ or P(n, r)

then $${}^nP_r = \frac{n!}{(n-r)!} \quad (0 £ r £ n)$$

$= n(n-1)(n-2) \ldots\ldots (n-r+1)$, n Î N and r Î W

(ii) The number of arrangements of n different objects taken all at a time is ${}^nP_n = n\,!$

Note : ${}^nP_1 = n$, ${}^nP_r = n.\,{}^{n-1}P_{r-1}$, ${}^nP_r = (n-r+1).\,{}^nP_{r-1}$, ${}^nP_n = {}^nP_{n-1}$

SOLVED EXAMPLES

Example. The number of ways in which four persons can sit on six chairs is-

(a) 24 (b) 10

(c) 48 (d) 360

Solution. ${}^6P_4 = 6.5.4.3 = 360$

(2) With Repetition :

(i) The number of permutations of n things taken all at a time, p are alike of one kind, q are alike of second kind and r are alike of a third kind and the rest n – (p + q + r) are all different is

$$\frac{n!}{p!\ q!\ r!}$$

(ii) The number of permutations of n different things taken r at a time when each thing may be repeated any number of times is n^r.

SOLVED EXAMPLES

Example. The number of words that can be formed out of the letters of the word COMMITTEE is

(a) $\frac{\lfloor 9}{(\lfloor 2)^3}$ (b) $\frac{\lfloor 9}{(\lfloor 2)^2}$

(c) $\frac{\lfloor 9}{\lfloor 2}$ (d) $\lfloor 9$

Solution. **(a)**

There are 9 letters in the given word in which two T's, two M's and two E's are identical. Hence

the required number of words $= \frac{\lfloor 9}{\lfloor 2\,\lfloor 2\,\lfloor 2} = \frac{\lfloor 9}{(\lfloor 2)^3}$

Example. In how many ways can 5 prizes be distributed among 4 boys when every boy can take one or more prizes?

(a) 1024 (b) 625

(c) 120 (d) 600

Solution. **(a)**

First prize may be given to any one of the 4 boys, hence first prize can be distributed in 4 ways.

Similarly every one of second, third, fourth and fifth prizes can also be given in 4 ways.

$\therefore$ the number of ways of their distribution

$= 4 \times 4 \times 4 \times 4 \times 4 = 4^5 = 1024$

(3) Number of permutations under certain conditions :

- The number of permutation of n different things taken all together when r particular things are to be place at some r given places

$$= {}^{n-r}P_{n-r} = \lfloor n-r$$

- The number of permutations of n different things taken r at a time when m particular things are to be placed at m given places = ${}^{n-m}P_{r-m}$.
- Number of permutations of n different things, taken r at a time, when a particular thing is to be always included in each arrangement, is.

  $r \cdot {}^{n-1}P_{r-1}$
- Number of permutation of n different things, taken r at a time, when a particular thing is never taken in each arrangement is

  ${}^{n-1}P_r$
- Number of permutations of n different things, taken all at a time, when m specified things always come together is

  $m! \times (n - m + 1)!$
- Number of permutations of n different things, taken all at a time, when m specified things never come together is

  $n! - m! \times (n - m + 1)!$

SOLVED EXAMPLES

Example. How many different words can be formed with the letters of the word 'JAIPUR' which start with 'A' and end with 'I'.

(a) 10 (b) 20

(c) 4 (d) 24

Solution. **(d)**

After putting A and I at their respective places (only in one way) we shall arrange the remaining 4 different letters at 4 places in $\underline{|4}$ ways. Hence the required number

$= 1 \times \underline{|4} = 24$

Example How many different 3-letter words can be formed with the letter of word 'JAIPUR' when A and I are always to be excluded ?

(a) 24 (b) 4

(c) 20 (d) 10

Solution. **(a)**

After leaving A and I, we are remained with 4 different letters which are to be used for forming 3-letter words. Hence the required number

$= {}^4P_3 = 4.3.2 = 24.$

Example How many different 4-letter words can be formed with the letters of the word 'JAIPUR' when A and I are always to be included?

(a) 24 (b) 4

(c) 20 (d) 144

Solution. **(d)**

Since A and I are always to be included, so first we select 2 letters from the remaining 4, which can be done in $^4C_2 = 6$ ways. Now these 4 letters can arranged $\underline{|4}$ = 24 ways, so the required number = 6 × 24 = 144.

(4) Circular Permutations :

(i) Arrangement round a circular table :

The number of circular permutations of n different things taken all at a time is $(n-1)!$, if clockwise and anticlockwise orders are taken as different.

(ii) Arrangement of beads or flowers (all defferent) around a circular necklace or garland :

The number of circular permutations of n different things taken all at a time is $\frac{1}{2}(n-1)!$, if clockwise and anticlockwise orders are taken as not different.

(iii) Number of circular permutations of n different things taken r at a time :

- **Case I :** If clockwise and anticlockwise orders are taken as different, then the required number of circular permutations = $(^nP_r)/r$.

- **Case II :** If clockwise and anticlockwise orders are taken as not different, then the required number of circular permutations = $(^nP_r)/(2r)$.

(iv) Restricted Circular Permutations

When there is a restriction in a circular permutation then first of all we shall perform the restricted part of the operation and then perform the remaining part treating it similar to a linear permutation.

SOLVED EXAMPLES

Example. In how many ways can 4 beads out of 6 different beads be strung into a ring-

(a) 45 (b) 24

(c) 360 (d) 180

Solution. **(a)**

In this case a clockwise and corresponding anticlockwise ordered will give the same circular permutation. So the required number

$$= \frac{{}^6P_4}{4.2} = \frac{6.5.4.3}{4.2} = 45$$

Example. The number of ways in which 10 persons can sit round a circular table so that none of them has the same neighbours in any two arrangements, is

(a) $\underline{|9}$ (b) $\frac{1}{2}\underline{|9}$

(c) $\underline{|10}$ (d) $\frac{1}{2}\underline{|10}$

Solution. **(b)**

10 persons can sit round a circular table in $\underline{|9}$ ways. But here clockwise and anticlockwise orders will give the same neighbours. Hence the required number of ways = $\frac{1}{2}\underline{|9}$

Example. In how many ways can 6 ladies and 6 gentlemen be seated at a round table so that no two ladies are next to each other?

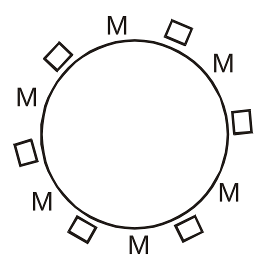
(a) 3500 (b) 86400

(c) 90000 (d) 74000

Solution **(b)**

Let us arrange the men first. 6 men can be arranged in a circle in (6 – 1) ! = 5 ! ways. We now have 6 possible places for the six ladies. The six ladies can be made to occupy six possible places in 6 ! ways.

$\therefore$ Total no. of arrangements is $5! \times 6! = 86400$.



COMBINATION AS SELECTION

Combination is the selection of a group which can be made by some or all of number of things without reference to the order.

Hence just the selections is combination where as selection cum arrangement is permutation.

Generally, number of permutations is larger than number of selections.

The number of combinations of n different things taken r at a time is given by ${}^nC_r$ or $C(n, r)$ or $\binom{n}{r}$.

DIFFERENCE BETWEEN PERMUTATION AND COMBINATION

	Problems of Permutations	Problems of Combinations
1.	Arrangements	Selections, choose
2.	Standing in a line, seated in a row	Distributed group is formed
3.	Problems on digits	Committee
4.	Problems on letters from a word	Geometrical problems

COUNTING FORMULAS FOR COMBINATION

(1) Selection of objects without Repetition

The number of combinations of n differnt things taken r at a time is denoted by ^nC_r or C (n, r) or $\binom{n}{r}$

Then $$^nC_r = \frac{n!}{r!(n-r)!} \;;\; o \le r \le n$$

$$= \frac{^nP_r}{r!} = \frac{n(n-1)(n-2)....(n-r+1)}{r(r-1)(r-2).....2.1}$$

If r > n, then $^nC_r = 0$.

KEY RESULTS ON ^nC_r

(i) $^nC_r = \frac{n!}{r!(n-r)!} = \frac{^nP_r}{r!}$ (if $r > n$, then $^nC_r = 0$)

(ii) $^nC_r \in N$

(iii) $^nC_0 = {}^nC_n = 1$

(iv) $^nC_1 = n$

(v) $^nC_r = {}^nC_{n-r}$

(vi) $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

(vii) $n.{}^{n-1}C_{r-1} = (n-r+1).{}^nC_{r-1}$

(viii) greatest value of $^nC_r = {}^nC_{n/2}$ when n is even

$$= {}^nC_{\frac{n+1}{2}} \text{ or } {}^nC_{\frac{n-1}{2}} \text{ when } n \text{ is odd}$$

(ix) $^nC_r = \frac{n}{r}\,{}^{n-1}C_{r-1}$

(x) $\frac{^nC_r}{^nC_{r-1}} = \frac{n-r+1}{r}$

(xi) $C_0 + C_1 ....+ C_n = 2^n$

(xii) $C_0 + C_2 + C_4 + ... = C_1 + C_3 + ... = 2^{n-1}$

(xiii) ${}^{2n+1}C_0 + {}^{2n+1}C_1 + ... + {}^{2n+1}C_n = 2^{2n}$

(xiv) ${}^{n}C_n + {}^{n+1}C_n + {}^{n+2}C_n + ... + {}^{2n-1}C_n = {}^{2n}C_{n+1}$

Note : Product of r consecutive positive integers is divisible by r!.

SOLVED EXAMPLES

Example. How many combination of 4 letters can be made out of the letters of the word 'JAIPUR'?

(a) 15 (b) 30

(c) 360 (d) 24

Solution. **(a)**

Here 4 things are to be selected out of 6 different things. So the number of combinations

$= {}^{6}C_4 = \dfrac{6\cdot 5\cdot 4\cdot 3}{4\cdot 3\cdot 2\cdot 1} = 15.$

Example. If ${}^{20}C_r = {}^{20}C_{r-10}$, then ${}^{18}C_r$ is equal to

(a) 4896 (b) 816

(c) 1632 (d) None of these

Solution. **(b)**

${}^{20}C_r = {}^{20}C_{r-10} \Rightarrow r + (r-10) = 20 \qquad \Rightarrow r = 15$

$\therefore\ {}^{18}C_r = {}^{18}C_{15} = {}^{18}C_3 = \dfrac{18.17.16}{1.2.3} = 816.$

COUNTING FORMULAS FOR COMBINATION

(2) Selection of Objects with Repetition :

The total number of selections of r things from n differents things when each thing may be repeated any number of times is

${}^{n+r-1}C_r$

SOLVED EXAMPLES

Example. 8 pens are to be selected from pens of 3 colours (pens of each colour being available any number of times), then total number of selections.

(a) 45 (b) 90

(c) 180 (d) 28

Solution. **(a)**

${}^{3+8-1}C_8 = {}^{10}C_8 = \dfrac{10.9}{2} = 45$

(3) Restricted Selection / Arrangement :

(i) The number of combinations of n different things taken r at a time,

- when k particular objects occur is ${}^{n-k}C_{r-k}$.
- If k particular objects never occur is ${}^{n-k}C_r$.

(ii) The number of arrangements of n distinct objects taken r at a time so that k particular object are

- always included = ${}^{n-k}C_{r-k} \, . \, r\,!$
- never included = ${}^{n-k}C_r \, .r\,!$

(iii) The number of combinations of n objects, of which p are indentical, taken r at a time is

- ${}^{n-p}C_r + {}^{n-p}C_{r-1} + {}^{n-p}C_{r-2} + \ldots\ldots\ldots + {}^{n-p}C_0$ if $r \le p$.
- ${}^{n-p}C_r + {}^{n-p}C_{r-1} + {}^{n-p}C_{r-2} + \ldots\ldots\ldots + {}^{n-p}C_{r-p}$ if $r > p$.

(a) To arrange *n* different things into *r* different groups.

If blank groups are allowed, then ${}^{n-r+1}P_n$

If blank groups are not allowed, then $n!.{}^{n-1}C_{r-1}$

(b) To distribute *n* different things into *r* different groups

$r^n - {}^rC_1(r-1)^n + {}^rC_2(r-2)^n \ldots\ldots\ldots\ldots (-1)^{r-1}\, {}^rC_{-r}$.

OR

Coefficient of x^n in $n!.(e^x-1)^r$

(Blank groups are not allowed here)

Note : Coefficient of x^r in $e^{px} = \dfrac{p^r}{r!}$

(c) To distribute *n* identical things in *r* groups so that no group contains less than *l* and more than *m* things (*m* > *l*) is coefficient of x^n in the expansion of $(x^l + x^{l+1} \ldots\ldots + x^m)^r$

(d) To select *r* things from a group of *n* things having *p* things identical is

$$\sum_{r=0}^{r} {}^{n-p}C_r \quad \text{if } r \le p$$

and $$\sum_{r=r-p}^{r} {}^{n-p}C_r \quad \text{if } r > p$$

SOLVED EXAMPLES

Example. From 4 gentlemen and 6 ladies a committee of five is to be selected. The number of ways in which the committee can be formed so that gentlemen are in majority is

(a) 66 (b) 156

(c) 60 (d) None of these

Solution. **(a)**

The committee will consist of 4 gentlemen and 1 lady or 3 gentlemen and 2 ladies

$\therefore$ the number of committees $= {}^4C_4 \times {}^6C_1 + {}^4C_3 \times {}^6C_2 = 66$

(4) Selection from distinct objects :

The number of ways (or combinations) of n different things selecting at least one of them is ${}^nC_1 + {}^nC_2 + {}^nC_3 + ... + {}^nC_n = 2^n - 1$. This can also be stated as the total number of combination of n different things.

SOLVED EXAMPLES

Example. Ramesh has 6 friends. In how many ways can he invite one or more of them at a dinner?

(a) 61 (b) 62

(c) 63 (d) 64

Solution. **(c)**

He can invite one, two, three, four, five or six friends at the dinner. So total number of ways of his invitation

$= {}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6$

$= 2^6 - 1 = 63$

(5) Selection from identical objects

· The number of combination of n identical things taking r(r £ n) at a time is 1.

· The number of ways of selecting r things out of n alike things is n + 1 (where r = 0, 1, 2, ..., n).

· The numner of ways to select some or all out of (p + q + r) things where p are alike of first kind, q are alike of second kind and r are alike of third kind is

= (p + 1) (q + 1) (r + 1) – 1

SOLVED EXAMPLES

Example. A bag contains 3 one rupee coins, 4 fifty paise coins and 5 ten paise coins. How many selections of money can be formed by taking atleast one coin from the bag-

(a) 120 (b) 60

(c) 119 (d) 59

Solution. **(c)**

Here are 3 things of first kind, 4 things of second kind and 4 things of third kind so the total number of selections

= (3 + 1) (4 + 1)(5 + 1) –1 = 119

Example. There are n different books and p copies of each in a library. The number of ways in which one or more than one book can be selected is

(a) $p^n + 1$ (b) $(p + 1)^n - 1$

(c) $(p + 1)^n - P$ (d) p^n

Solution. **(b)**

Total cases = p + 1 (if selected or not)

Required number of ways

$$=(p+1)(p+1)......\text{n terms} -1$$

$$=(p+1)^n -1$$

(6) Selection when both identical and distinct objects are present :

If out of (p + q + r + t) things, p are alike one kind, q are alike of second kind, r are alike of third kind and t are different, then the total number of combinations is

$$(p + 1)(q + 1)(r + 1)2^t - 1$$

DIVISION AND DISTRIBUTION OF OBJECTS

(i) The number of ways in which (m + n) different things can be divided into two groups which contain m and n things respectively is

$$^{m+n}C_m\ ^nC_n = \frac{(m+n)!}{m!\,n!},\ m \neq n$$

Particular case :

When m = n, then total number of combination is

- $\dfrac{(2m)!}{(m!)^2}$ when order of groups is considered.
- $\dfrac{(2m)!}{2!(m!)^2}$ when order of groups is not considered.

(ii) The number of ways in which (m + n + p) different things can be divided into three groups which contain m, n and p things respectively is

$$^{m+n+p}C_m.\ ^{n+p}C_n.\ ^pC_p = \frac{(m+n+p)!}{m!\,n!\,p!},\ m \neq n \neq p$$

Particular case :

When m = n = p, then total number of combination is

- $\dfrac{(3m)!}{(m!)^3}$ when order of groups is considered.

- $\dfrac{(3m)!}{3!\ (m\ !)^3}$ when order of groups is not considered.

(iii) Total number of ways to divide n identical things among r person is

$^{n+r-1}C_{r-1}$

Also total number of ways to divide n identical things among r persons so that each gets atleast one is

$^{n-1}C_{r-1}$

SOLVED EXAMPLES

Example. In how many ways can a pack of 52 cards be divided in 4 sets, three of them having 17 cards each and fourth just one card?

(a) $\dfrac{\underline{|52}}{(\underline{|17})^3}$ (b) $\dfrac{\underline{|52}}{(\underline{|17})^3\,\underline{|3}}$

(c) $\dfrac{\underline{|52}}{(\underline{|17})^3\,\underline{|4}}$ (d) None of these

Solution. **(b)**

Since the cards are to be divided into 4 sets, 3 of them having 17 cards each and 4th just one card, so number of ways

$$= \frac{\underline{|52}}{(\underline{|17})^3\,\underline{|3}\,\underline{|1}} = \frac{\underline{|52}}{(\underline{|17})^3\,\underline{|3}}$$

Example. In how many ways 20 identical mangoes may be divided among 4 persons and if each person is to be given atleast one mango, then numbrt of ways will be

(a) 1771, 969 (b) 1717, 969

(c) 1777, 696 (d) none of these

Solution. **(a)**

20 identical mangoes may be divided among 4 persons in

$^{20+4-1}C_{4-1} = {}^{19}C_3 = 1771$ ways.

If each person is to be given atleast one mango, then numbrt of ways will be

$^{20-1}C_{4-1} = {}^{19}C_3 = 969.$

DERANGEMENT THEOREM

Any change in the given order of the thing is called a **Derangement.**

(i) If n items are arranged in a row, then the number of ways in which they can be rearranged so that no one of them occupies the place assigned to it is

$$n!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}+....+(-1)^{n}\frac{1}{n!}\right]$$

(ii) If n things are arranged at n places then the number of ways to rearrange exactly r things at right places is

$$\frac{n!}{r!}\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}+....+(-1)^{n-r}\frac{1}{(n-r)!}\right]$$

SOLVED EXAMPLES

Example. There are 4 balls of different colour and 4 boxes of colours same as those of the balls. Then the number of ways to place two balls in boxes with respect to their colour is :

(a) 6 (b) 4
(c) 2 (d) none of these

Solution. **(a)**

The required number of ways

$$\frac{4!}{2!}\left[1-\frac{1}{1!}+\frac{1}{2!}\right] = 4.3\left[1-1+\frac{1}{2}\right] = 6$$

Example. There are 3 letters and 3 envelopes The number of ways in which all letters are put in the wrong envelopes is :

(a) 2 (b) 4
(c) 6 (d) 8

Solution. **(a)**

The required number of ways $= \underline{|3}\left[1-\frac{1}{\underline{|1}}+\frac{1}{\underline{|2}}-\frac{1}{\underline{|3}}\right] = 3 - 1 = 2$

DIVISIBILITY OF NUMBERS

The following table shows the conditions of divisibility of some numbers

Divisible by	Condition
2	whose last digit is even
3	sum of whose digits is divisible by 3
4	whose last two digits number is divisible by 4
5	whose last digit is either 0 or 5
6	which is divisible by both 2 and 3
8	whose last three digits number is divisible by 8
9	sum of whose digits is divisible by 9
25	whose last two digits are divisible by 25

SUM OF NUMBER

(i) For given n different digits a_1, a_2, a_3 ...., a_n the sum of the digits in the unit place of all numbers fromed (if numbers are not repeated) is

$(a_1 + a_2 + a_3 + ... + a_n)(n - 1)!$
i.e. (sum of the digits) $(n - 1)!$

(ii) Sum of the total numbers which can be formed with given n different digits $a_1, a_2, a_3 \ldots\ldots a_n$ is

$$(a_1 + a_2 + a_3 + \ldots + a_n)\,(n-1)!\,.\,(111 \ldots\ldots\ n \text{ times})$$

SOLVED EXAMPLES

Example. The sum of all 4 digit numbers formed with the digits 1, 2, 4 and 6 is

(a) 86650 (b) 86658

(c) 86660 (d) None of these

Solution. **(b)**

Sum = $(a_1 + a_2 + a_3 + \ldots\ldots + a_n)\ \underline{|n-1}\ (111\ldots\ldots n \text{ times})$

Using formula Sum = $(1 + 2 + 4 + 6).\underline{|3}.(1111) = 13 \times 6 \times 1111 = 86658$

Second Method : Here total 4-digit numbers will be $\underline{|4} = 24$. So every digit will occur 6 times at every one of the four places. Now since the sum of the given digits $= 1 + 2 + 4 + 6 = 13$. So the sum of all the digits at every place of all 24 numbers = 13 × 6 = 78.

So the sum of the values of all digits

at first place = 78

at ten place = 780

at hundred place = 7800

at thousand place = 78000

$\therefore$ the required sum $= 78 + 780 + 7800 + 78000 = 86658$

IMPORTANT RESULTS ABOUT POINTS

If there are n points in a plane of which m(< n) are collinear, then

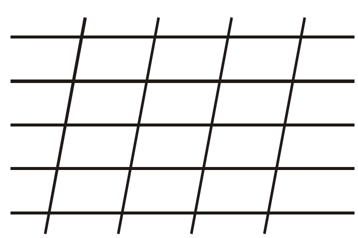
- Total number of different straight lines obtained by joining these n points is ${}^nC_2 - {}^mC_2 + 1$
- Total number of different triangles formed by joining these n points is ${}^nC_3 - {}^mC_3$
- Number of diagonals in polygon of n sides is ${}^nC_2 - n$ i.e. $\frac{n(n-3)}{2}$
- If m parallel lines in a plane are intersected by a family of other n parallel lines. Then total number of parallelograms so formed is ${}^mC_2 \times {}^nC_2$ i.e. $\frac{mn(m-1)(n-1)}{4}$

Example. If 5 parallel straight lines are intersected by 4 parallel straight lines, then the number of parallelograms thus formed, is

(a) 20 (b) 60

(c) 101 (d) 126

Solution. **(b)**



Number of parallelograms $= {}^5C_2 \times {}^4C_2 = 60$

Example. There are 10 points in a plane and 4 of them are collinear. The number of straight lines joining any two of them is

(a) 45 (b) 40

(c) 39 (d) 38

Solution. **(b)**

A straight line can be drawn joining two points, so there will be ${}^{10}C_2$ straight lines joining 10 points. But 4 of them are collinear, so we shall get only one line joining any two of these 4 points. Hence the total number of lines

$= {}^{10}C_2 - {}^{4}C_2 + 1 = 40$

Example. There are 8 points in a plane, out of these 4 are collinear. The number of triangle obtained by joining these is :

(a) 56 (b) 52

(c) 64 (d) 48

Solution. **(b)** ${}^{8}C_4 - {}^{4}C_3 = 52$

SOLVED EXAMPLES

Example 1. The total number of eight digit numbers in which all digits are different is

(i) $9\underline{|9}$ (ii) $\frac{9\underline{|9}}{2}$ (iii) $\underline{|9}$ (iv) None of these

Solution. **[ii]**

There are ten digits 0, 1, 2, ..., 9. Permutations of these digits taken eight at a time $= {}^{10}P_8$ which include permutations having 0 at the first place. When 0 is fixed at the first place, then number of such permutations $= {}^{9}P_7$. So required number

$$= {}^{10}P_8 - {}^{9}P_7 = \frac{\underline{|10}}{\underline{|2}} - \frac{\underline{|9}}{\underline{|2}} = \frac{9\underline{|9}}{2}$$

Example 2. The number of ways in which 20 different pearls of two colours can be set alternately on a necklace, there being 10 pearls of each colour, is

(i) $9! \times 10!$ (ii) $5(9!)^2$ (iii) $(9!)^2$ (iv) None of these

Solution. **[ii]**

Ten pearls of one colour can be arranged in $\frac{1}{2}.(10-1)!$ ways. The number of arrangements of 10 pearls of the other colour in 10 places between the pearls of the first colour = 10!

$\therefore$ the required number of ways $= \frac{1}{2} \times 9! \times 10! = 5\ (9\ !)^2$

Example 3. How many numbers can be formed with the digits 1, 2, 3, 4, 3, 2, 1 so that the odd digits always occupy the odd places?

(i) 17 (ii) 18 (iii) 19 (iv) 20

Solution. **[ii]**

There are 4 odd digits (1, 1, 3, 3) and 4 odd places (first, third, fifth and seventh). At these places the odd digits can be arranged in $\frac{\underline{|4}}{\underline{|2}\,\underline{|2}} = 6$ ways

Then at the remaining 3 places, the remaining three digits (2, 2, 4) can be arranged in $\frac{\underline{|3}}{\underline{|2}} = 3$ ways

$\therefore$ The required number of numbers = $6 \times 3 = 18$

Example 4. The number of different seven digit numbers that can be written using only the three digits 1, 2 and 3 with the condition that the digit 2 occurs twice in each number is

(i) ${}^7P_2 2^5$ (ii) ${}^7C_2 2^5$ (iii) ${}^7C_2 5^2$ (iv) none of these

Solution. **[ii]**

Other than 2, remaining five places by 1 and 3 for each place two conditions

no. of ways for five places $= 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

For 2, selecting 2 places out of $7 = {}^7C_2$

$\therefore$ Required no. of ways ${}^7C_2.2^5$

Example 5. To fill up 12 vacancies, there are 25 candidates of which 5 are from SC. If 3 of these vacancies are reserved for SC candidates while the remaining are open to all then the number of ways in which the selection can be made is

(i) ${}^{5}C_{3}\times{}^{15}C_{9}$ (ii) ${}^{5}C_{3}\times{}^{22}C_{9}$ (iii) ${}^{5}C_{3}\times{}^{20}C_{9}$ (iv) None of these

Solution. **[ii]**

3 vacancies for SC candidates can be filled up from 5 candidates in ${}^{5}C_{3}$ ways.

After this for remaining 12–3=9 vacancies, there will be 25–3=22 candidates. These vacancies can be filled up in ${}^{22}C_{9}$ ways.

Hence required number of ways = ${}^{5}C_{3}\times{}^{22}C_{9}$

Example 6. Total number of ways in which six '+' and four '–' signs can be arranged in a line such that no two '–' signs occur together, is

(i) 35 (ii) 18 (iii) 15 (iv) 42

Solution. **[i]**

First we write six '+' signs at alternate places i.e., by leaving one place vacant between two successive '+' signs. Now there are 5 places vacant between these signs and there are two places vacant at the ends. If we write 4 '–' signs at these 7 places then no two '–' will come together. Hence total number of ways

$={}^{7}C_{4}=35$

Example 7. If a denotes the number of permutations of x + 2 things taken all at a time, b the number of permutations of x things taken 11 at a time and c the number of permutations of x – 11 things taken all at a time such that

a = 182 bc, then the value of x is

(i) 15 (ii) 12 (iii) 10 (iv) 18

Solution. **[ii]**

$${}^{x+2}P_{x+2}=a\Rightarrow a=(x+2)!$$

$${}^{x}P_{11}=b\Rightarrow b=\frac{x!}{(x-11)!}$$

and ${}^{x-11}P_{x-11}=c\Rightarrow c=(x-11)!$

$$\therefore a=182bc$$

$$(x+2)!=182\frac{x!}{(x-11)!}(x-11)!$$

$$\Rightarrow (x+2)(x+1)=182=14\times13$$

$$\therefore x+1=13 \quad \therefore x=12$$

Example 8. In a college examination, a candidate is required to answer 6 out of 10 questions which are divided into two sections each containing 5 questions. Further the condidate is not permitted to attempt more than 4 questions from either of the section. The number of ways in which he can make up a choice of 6 questions is

(i) 200 (ii) 150 (iii) 100 (iv) 50

Solution. **[i]**

The required number of ways

$= {}^5C_4.{}^5C_2 + {}^5C_3.{}^5C_3 + {}^5C_2.{}^5C_4$

$= 50 + 100 + 50 = 200$

Example 9. If the letters of the word RACHIT are arranged in all possible ways and these words are written out as in a dictionary, then the rank of this word is

(i) 365 (ii) 702 (iii) 481 (iv) None of these

Solution. **[iii]**

The number of words beginning with A (i.e., in which A comes in first place) is ${}^5P_5 = \underline{|5}$. Similarly number of words beginning with C is $\underline{|5}$, beginning with H is $\underline{|5}$ and beginning with I is also $\underline{|5}$.

Now letters of RACHIT in alphabetic order are as ACHIRT

So before R, four letters A, C, H, I can occur in $4(\underline{|5}) = 480$ ways. Now word RACHIT happens to be the first word beginning with R. Therefore the rank of this word = 480 + 1 = 481.

Example 10. There are three coplanar parallel lines. If any p points are taken on each of the lines, the maximum number of triangles with vertices at these points is

(i) $3p^2(p-1)+1$ (ii) $3p^2(p-1)$ (iii) $p^2(4p-3)$ (iv) None of these

Solution. **[iii]**

The number of triangles with vertices on different lines

$= {}^pC_1 \times {}^pC_1 \times {}^pC_1 = p^3$

The number of triangles with 2 vertices on one line and the third vertex on any one of the other two lines

$= {}^3C_1\left\{{}^pC_2 \times {}^{2p}C_1\right\} = 6p.\frac{p(p-1)}{2}$

$\therefore$ the required number of triangles $= p^3 + 3p^2(p-1)$ $= p^2(4p-3)$

Note: The word "maximum" ensures that no selection of points from each of the three lines are collinear.

Example 11. In a club election the number contestants is one more than the number of maximum candidates for which a voter can vote. If the total number of ways in which a voter can vote be 62 then the number of candidates is

(i) 7 (ii) 5 (iii) 6 (iv) None of these

Solution. **[iii]**

$62 = {}^nC_1 + {}^nC_2 + {}^nC_3 + ... + {}^nC_{n-1} = 2^n - {}^0C_0 - {}^nC_n = 2^n - 2$

$\therefore 2^n = 64 = 2^6$ Þ $n = 6$

Example 12. The number of ways of arranging six persons (having A, B, C and D among them) in a row so that A, B, C and D are always in order ABCD (not necessarily together) is

(i) 4 (ii) 10 (iii) 30 (iv) 720

Solution. **[iii]**

The number of ways of arranging ABCD is 4!. For each arrangement of ABCD, the number of ways of arranging six persons is same. Hence required number is $\frac{6!}{4!} = 30$

Example 13. Five balls of different colours are to be placed in three boxes of different sizes. Each box can hold all five balls. The number of ways in which we can place the balls in the boxes (order is not considered in the box) so that no box remains empty is

(i) 150 (ii) 300 (iii) 200 (iv) None of these

Solution. **[i]**

One possible arrangement is

2	2	1

Three such arrangements are possible. Therefore, the number of ways is

$\left({}^5C_2\right)\left({}^3C_2\right)\left({}^1C_1\right)(3) = 90$

The other possible arrangements

1	1	3

Three such arrangements are possible. In this case, the number of ways is

$\left({}^5C_1\right)\left({}^4C_1\right)\left({}^3C_3\right)(3) = 60$

Hence, the total number of ways is 90 + 60 = 150.

Example 14. How many different numbers can be formed by using all the digits 1, 2, 3, 4, 3, 2, 1 so that odd digits always occupy odd places ?

(i) 9 ways (ii) 11 ways (iii) 5 ways (iv) 18 ways

Solution.

1	2	3	4	5	6	7

There are 7 digits ; we require 7 digit numbers, which is same as filling 7 places in a row with the 7 digits 1, 2, 3, 4, 3, 2, 1.

The odd places are place numbers 1, 3, 5, 7.

Odd digits available are 1, 3, 3, 1 ; there are two 1's and two 3's.

The 4 odd digits can be arranged at the 4 odd places in $\frac{{}^4P_4}{2!\ 2!} = \frac{4!}{4} = 6$ ways

Now, there are 3 even places, *i.e.*, place numbers 2, 4, 6 and three digits available, *viz.*, 2, 4, 2. These 3 digits can be arranged at the 3 even places in $\frac{^3P_3}{2\,!} = 3$ ways.

By fundamental theorem, the required no. of ways is

$6 \times 3 = 18$ ways.

Example 15. Given 5 different green dyes, four different blue dyes and 3 different red dyes, how many combinations of dyes can be chosen taking at least one green and one blue dye ?

(i) 4732 (ii) 3720 (iii) 9245 (iv) 8325

Solution. At least one green dye can be selected out of 5 different green dyes in

$^5C_1 + {^5C_2} + ... + {^5C_5}$ ways, i.e.

$2^5 - 1$ ways.

After selecting one or more green dyes, we can select at least one blue dye out of 4 different blue dyes in $2^4 - 1$ ways.

Subsequently at least one red dye or no red dye can be chosen in $^3C_0 + {^3C_1} + {^3C_2} + ... + {^3C_3}$ ways $= 2^3$ ways

\ Total no. of combinations of dyes

$$= (2^5 - 1) \cdot (2^4 - 1) \cdot 2^3$$

$$= 31 \times 15 \times 8 = 3720.$$

Example 16. Five balls of different colours are to be placed in three boxes of different sizes. Each box can hold all the 5 balls. In how many different ways can we place the balls so that no box remains empty?

(i) 150 (ii) 300 (iii) 450 (iv) 100

Solution. Each box must contain at least one ball since no box remains empty. Two cases arise :

Case I.

	Box I	Box II	Box III
No. of balls	1	1	3

This can be done in $^5C_1 \times {^4C_1} \times {^3C_3} = 20$ ways

But the box which contains 3 balls can be chosen in $^3C_1 = 3$ ways.

\ Required no. of ways = $20 \times 3 = 60$

Case II.

	Box I	Box II	Box III
No. of balls	1	2	2

This can be done in $^5C_1 \times {^4C_2} \times {^2C_2} = 5 \times 6 \times 1 = 30$ ways

But the two boxes containing two balls each can be chosen in $^3C_2$ ways.

\ Required number $= 30 \times 3 = 90$

Since case I and case II are non overlapping the total number of ways $= 60 + 90 = 150$.

Example 17. A condolence meeting is being held in a hall which has 7 doors, by which mourners enter the hall. One can use any of the 7 doors to enter and can come at any time during the meeting. At each door, a register is kept in which a mourner has to affix his signature while entering the hall. If 200 people attend the meeting, how many different sequences of 7 lists of signatures can arise ?

(i) $\frac{|\underline{206}}{|\underline{6}}$ (ii) $\frac{|\underline{205}}{|\underline{5}}$ (iii) $\frac{|\underline{150}}{|\underline{3}}$ (iv) $\frac{|\underline{100}}{|\underline{4}}$

Solution. There are 7 lists, say 1, 2, ...7. Suppose that list i has x_i names ; then,

$$x_1 + ... + x_7 = 200 \quad \text{where } x_i \geq 0 \text{ is an integer}$$

We need to first find the no. of solutions of this equation.

(Note that this does not complete the solution to the question as, list 1 may contain 7 names which no. would remain the same in 7! arrangements of the names)

The number of solutions is ${}^{200+7-1}C_{7-1} = {}^{206}C_6$

But corresponding to any one solution, $(x_1 \text{ .... } x_7)$ (i.e. list j contains x_j names) we can have 200 ! arrangements consistent with distribution of x_j names to j^{th} list

\ No. of different sequences of 7 lists

$$= {}^{206}C_6 \times 200\,! = \frac{206\,!}{6\,!}.$$

Example 18. There are p intermediate railway stations on a railway line from one terminal to another. In how many ways can a train stop at three of these intermediate stations if no two of these stations (where it stops) are to be consecutive?

(i) $p-2_{C_3}$ (ii) $p-3_{C_2}$ (iii) $p-1_{C_3}$ (iv) $p-5_{C_6}$

Solution. Since the train does not stop at $(p - 3)$ stations, the problem reduces to the following :

In how many ways can three objects be placed among $(p - 3)$ objects in a row such that no two of them are next to each other (at most 1 object is to be placed between any two of these $(p - 3)$ objects).

Since there are $(p - 2)$ positions to place the three objects, the required number of ways $= {}^{p-2}C_3$.

CBSE SECTION

SECTION - I

1. (i) If $^{20}P_r = 6840$, find r.

   (ii) If $^{m+n}P_2 = 90$ and $^{m-n}P_2 = 30$, find m and n

   (iii) If $^{12}P_r = 11880$, find r.

   (iv) If $^{56}P_{r+6} : {}^{54}P_{r+3} = 308000 : 1$, find r

2. How many numbers each lying between 100 and 1000 can be formed with the digits 2, 3, 4, 0, 8, 9 ; no digit being repeated?

3. Find the number of numbers lying between 300 and 4000 that can be formed with the digits 0, 1, 2, 3, 4, 5 ; no digit being repeated.

4. How many numbers less than 1000 and divisible by 5 can be formed in which no digit occurs more than once in the same number ?

5. Find how many numbers between 100 and 999 can be formed with the digits 0, 4, 5, 6, 7, 8; no digit being used more than once. How many of them are odd?

6. Find the number of numbers of 4 digits formed with the digits 1, 2, 3, 4, 5 in which 3 occurs in the thousands place and 5 occurs in the units place.

7. A number of four different digits is formed by using the digits 1, 2, 3,4, 5,6, 7. Find (i) how many such numbers can be formed? (ii) how many of them are greater than 3400?

8. Find the sum of all the four digit numbers that can be formed with the digits 3, 2, 3, 4.

9. Find the sum of all numbers greater than 10,000 formed with the digits 0, 2, 4, 6 and 8 ; no digit being repeated in any number.

10. Find the sum of the 5 digit numbers which can be formed with the digits 3, 4, 5, 6, 7 using each digit only once in each arrangement.

11. In how many ways can n things be given to p persons, when each person can get any number of things (n > p)

12. There are m men and n monkeys (m < n). If a man may have any number of monkeys, in how many ways every monkey have a master?

13. there are stalls for 12 animals in a ship. In how many ways the shipload can be made if there are cows, calves and horses to be transported, animals of each kind being not less than 12 ?

14. In how many ways can a ten question multiple choice examination with one correct answer be answered if there are four choices a, b, c and d to each question ? If no two consecutive questions are answered the same way, how many ways are there?

15. A library has 5 copies of one book, 4 copies of each of 2 books, 6 copies of each of 3 books and single copies of 8 books, In how many ways can all books be arranged so that copies of the same book are always together?

16. In a dinner party there are 10 Indian, 5 Americans and 5 Englishmen. In how many ways can they be arranged in a row so that all persons of the same nationality sit together?

17. Show that the number of ways in which n books may be arranged on a shelf so that two particular books shall not be together is (n – 2)(n – 1) !

18. You are given 6 balls of different colours (black, white, red, green, violet, yellow); in how many ways can you arrange them in a row so that black and white balls may never come together?

19. In how many ways can 16 rupees and 12 paise coins be arranged in a line so that no two paise coins may occupy consecutive positions?

20. m men and n women are to be seated in a row so that no two women sit together. If m > n, then show that the number of ways in which they can be seated is $\dfrac{m!(m+1)!}{(m-n+1)!}$

21. Find the number of different arrangements (permutations) of the letters of the word 'BANANA'.

22. Find the number of permutations of the letters of the word "INDEPENDENCE". Find also the number of re-arrangements.

23. How many different words can be formed with the letters of the word "MATHEMATICS" ? In how many of them the vowels are together and consonants are together ?

24. Find the number of words that can be made by arranging the letters of the word 'INTERMEDIATE' so that

    (i) The relative order of vowels and consonants do not change.

    (ii) The order of vowels do not change.

25. In how many different ways can the letters of the word 'SALOON' be arranged if the consonants and vowels must occupy alternate places?

26. How many words can be formed by using the letters of the word 'BHARAT'? How many of these words will not contain B and H together. How many of these start with B and end withT?

27. How many numbers greater than for millions (4000000) can be formed with the digits 2, 3, 0, 3, 4, 2, 5?

28. How many signals can be made by hoisting 2 blue, 2 red and 5 yellow flags on a pole at the same time.

29. How many different signals can be made by hoisting 6 differently coloured flags one above the other when any number of them may be hoisted at once?

30. Find the number of arrangements of the letters of the word 'DELHI' if E always comes before I.

31. In how many ways 6 boys and 5 girls can sit at a round tabel when no two girls sit next to each other?

32. A round table conference is to be held bwtween 20 delegates of 20 countries. In how many ways can they and the host be seated if two particular delegates are always to sit one either side of the host.

33. Four gentlemen and four ladies are invited to a certain party. Find the number of ways of seating them around a table so that only ladies are seated on the two side of each gentleman.

34. In how many ways can 7 Englishmen and 6 Indians sit down around a table so that no two Indians are together.

35. In how many ways can 7 person sit around a table so that all shall not have same neighbours in any two arrangements.

SECTION - II

1. (i) If ${}^{18}C_r = {}^{18}C_{r+2}$, find ${}^{r}C_6$

   (ii) If ${}^{n}P_r = 2520$ and ${}^{n}C_r = 21$, find r

   (iii) Show that ${}^{20}C_{13} + {}^{20}C_{14} - {}^{20}C_6 - {}^{20}C_7 = 0$

   (iv) Prove that ${}^{n-1}C_3 + {}^{n-1}C_4 > {}^{n}C_3$, if n > 7

   (v) If ${}^{n}C_{r-1} = 36$, ${}^{n}C_r = 84$ and ${}^{n}C_{r+1} = 126$, find n and r

2. How many quadrilaterals can be formed joining the vertices of a polygon of n sides?
3. Prove that the number of combinations of n things taken r at a time in which p particular things always occur is ${}^{n-p}C_{r-p}$
4. There are 6 students A, B, C, D, E, F

   (a) In how many ways can they be seated in a line so that C and d do not sit together?

   (b) In how many ways can a committee of 4 be formed so as to always include C?

   (c) In how many ways can a committee of 4 be formed so as to always include C but exclude E?
5. There are n stations on a railways line. The number of kinds of tickets printed (no return tickets) is 105. Find the number of stations.
6. The sides AB, BC, CA of a triangle ABC have 3, 4 and 5 interior points respectively on them. Find the number of triangles that can be constructed using given interior points as vertices.
7. To fill 12 vacancies there are 25 candidates of which 5 are from scheduled castes. If 3 of the vacancies are reserved for scheduled caste candidates while the rest are open to all, find the number of ways in which the selections can be made.
8. At an election three wards of a town are convassed by 4, 5 and 8 men respectively. If there are 20 volunteers, in how many ways can they be alloted to different wards?
9. A candidate is required to answer six out of ten questions which are divided into groups, each containing fove questions and he is not permited to attempt more than 4 from any group. In how many ways can he make up his choice?
10. From a class of 25 students, 10 are to be chosen for an excursion party. There are three students who decide that either all of them will join or none of them will join. In how many ways can the 10 studetns be chosen?
11. A sports team of 11 students is to be constituted, choosing at least 5 from class XI and at least 5 from class XII. If there are 25 students in each of these classes, in how many ways can the teams be constituted.
12. A committee of 5 is to be selected from among 6 boys and 5 girls. Determine the number of ways of selecting the committee if it is to consist of at least one boy and one girl.
13. In a village, there are 87 families, of which 52 families have at most 2 children. In a rural development programme, 20 families are to be helped chosen for assistance, of which at least 18 families must have at most 2 children. In how many ways can the choice be made?
14. How many words, each of 3 vowels and 2 consonants, can be formed from the letters of the word INVOLUTE?
15. How many (i) straight lines (ii) triangles can be formed by joining n points, p of which are in the same line?
16. A man has 15 acquaintances of whom 10 are relatives. In how many ways can he invite 9 guests so that 7 of them may be relatives?
17. A question paper has two parts, Part a and Part B, each containing 10 questions. If a student has to choose 8 from part A and 5 from Part B, in how many ways can he choose the questions?
18. Out of 7 consonants and 4 vowels, how many words can be made each containing 3 consonants and 2 vowels?

19. How many committees of 5 members each can be formed with 8 officials and 4 non-official members in the following cases:
    (i) each consists of 3 officials and 2 non-official members?
    (ii) each consists of at least two non-official members?
    (iii) a particular official member is never included?
    (iv) a particular non-official member is always included?
20. A student has to answer 10 questions, choosing at least 4 from each of Part A and Part B. If there are 6 questions in Part a and 7 in Part b, in how many ways can the student choose 10 questions?
21. What is the number of ways of choosing 4 cards from a pack of 52 playing cards? In how many of these:
    (i) four cards are of same suit?
    (ii) four cards belong to four different suits?
    (iii) four cards are face cards?
    (iv) two are red cards and two are black cards?
    (v) four cards are of the same colour?
22. How many straight lines may be formed by joining any two of the eight points when:
    (i) no three of them are collinear?
    (ii) four of them are collinear?
23. How may (i) straight line (ii) triangles can be formed by joining 12 points, 7 of which are collinear?
24. In how many ways can 10 things be divided into two groups of 5 each?
25. In how many ways can 12 things be equally divided among 4 persons?
26. In how many ways can a selection be made out of 2 mangoes, 3 apples and 3 oranges?
27. In how many ways can 52 playing cards be placed in 4 heaps of 13 cards each? In how many ways can these be dealt out to four players giving 13 cards each?
28. In how many ways can 20 students be divided into four equal groups? In how many ways can these be sent to four different schools?
29. In how many ways can a selection be made out of 4 red, 3 blue and 2 black indentical balls.
30. A man has 6 one rupee coins, 5 fifty paise coins and 3 twenty five paise coins in his pocket. In how many different amounts can he give for a subscription?
31. From a class of 12 boys and 10 girls, 10 students are to be chosen for a competition, including at least 4 boys and 4 girls. The 2 girls who won the prizes least year should be included. In how many ways can the selections be made?
32. A student has three library tickets and 8 books of his interest in the library. Of these 8, he does not want to borrow Chemistry part II, unless chemistry part I is also borrowed. In how many ways can he choose the three books to be borrowed?
33. In a small village, there are 87 families, of which 52 families have at most 2 children. In a Rural Development Programme, 20 families ar eto be chosen for assistance, of which at least 18 families must have at most 2 children. In how many ways can the choice be made?
34. A committee of 7 has to be formed from 9 bodys and 4 girls. In how many ways can this be done when the committee consists of :
    (i) exactly 3 girls   (ii) at least 3 girls.
35. We wish to select 6 persons from 8, but if the person A is chosen, then B must be chosen. In how many ways can the selection be made?
36. The English alphabet has 5 vowels and 21 consonants. How may words with two different vowels and 2 different consonants can be framed from the alphabet?

CBSE - SECTION (ANSWERS)

SECTION - I

1. (i) 3 (ii) m = 8 , n = 2 (iii) 4 (iv) 41
2. 100
3. 240
4. 154
5. 100, 32
6. 6
7. (i) 840, (ii) 560
8. 39996
9. 5199960
10. 6666600
11. p^n
12. m^n
13. 3^{12}
14. 4^{10}, 4×3^9
15. 14 !
16. 3!10!5!5!
18. 480
19. 6188 when all rupee coins are identical and all paise coins are identical; $16! \times {}^{17}P_{12}$, if coins of same denomination are different.
21. 69
22. $\frac{12!}{3!2!4!} = 1663200$; 1663199
23. $\frac{11!}{2!2!2!}; \frac{2!7!4!}{2!2!2!}$
24. (i) $\frac{6!}{2!3!} \times \frac{6!}{2!} = 21600$ (ii) $\frac{12!}{6!2!} = 332640$
25. 36
26. 360; 240; 12
27. 360
28. $\frac{9!}{2!2!5!}$
29. ${}^6P_1 + {}^6P_2 + {}^6P_3 + {}^6P_4 + {}^6P_5 + {}^6P_6 = 1956$
30. $\frac{5!}{2!} = 60$
31. $5! \times {}^5P_5$
32. 18! 2!
33. 144
34. $6! \times {}^7P_6$
35. $\frac{6!}{2} = 360$

SECTION - II

1. (i) 28 (ii) 5 (v) n = 9, r = 3
2. ${}^{n}C_4$
4. (a) 480 (b) 10 (c) 4
5. 15
6. 205
7. ${}^{5}C_3 \times {}^{20}C_9 + {}^{5}C_4 \times {}^{20}C_8 + {}^{5}C_5 \times {}^{20}C_7$
8. ${}^{20}C_4 \times {}^{16}C_5 \times {}^{11}C_8$
9. 200
10. $170544 + 480700 = 651244$
11. ${}^{25}C_5 \times {}^{25}C_6 + {}^{25}C_6 \times {}^{25}C_5 = 2\,{}^{25}C_5 \times {}^{25}C_6$
12. 455
13. $({}^{52}C_{18} \times {}^{35}C_2) + ({}^{52}C_{19} \times {}^{35}C_1) + {}^{52}C_{20}$
14. 2880
15. (i) ${}^{n}C_2 - {}^{p}C_2 + 1$ (ii) ${}^{n}C_3 - {}^{p}C_3$
16. 1200
17. 11340
18. 25200
19. (i) 336 (ii) 456 (iii) 462 (iv) 330
20. 226
21. 270725
22. (i) 28 (ii) 23
23. 46, 185
24. 126
25. 369600
26. 47
27. $\dfrac{52!}{4!(13!)^4}, \dfrac{52!}{(13!)^4}$
28. $\dfrac{20!}{4!(5!)^4}, \dfrac{20!}{(5!)^4}$
29. 59
30. 167
31. 104874
32. 41
33. ${}^{52}C_{18} \times {}^{35}C_2 + {}^{52}C_{19} \times {}^{35}C_1 + {}^{52}C_{20} \times {}^{35}C_0$
34. (i) 504 (ii) 588
35. 22
36. 50400