An Exploration of Shell Sort Gap Intervals

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August 23, 2017

Contents

Abstract

Searches for an optimal gap sequence to use in the Shell sort algorithm using arrays of four distinct sizes ranging from 524,288 to 4,194,304. Some literature states that factor of 2.5 is appropriate. The data here find that this factor is sub-optimal, and makes a suggestion to improve Shell sort performance for large arrays.

1 Introduction

I recently had a situation where I wanted to use a Shell sort, but I was unsure how to set the gaps. I did a little research on the web and found this page on Stack Overflow. The page states that the optimum gap sequence is 1, 4, 10, 23, 57, 132, 301, 701, 1750. After 1750, the next number in the sequence is $\frac{5}{2}$ the previous number. $n_i = \lfloor \frac{5n_{i-1}}{2} \rfloor$ Being a skeptic, I decided to do an experiment.

Wikipedia offers a fine explanation of Shell's sorting algorithm, and its history. There is no compelling reason duplicate that information here. Suffice it to say that, when recursion is limited, the Shell sort remains a viable alternative. This is particularly true when the data are partially sequenced, already. Another advantage is that this algorithm is easily implemented.

Theoretical analysis of Shell's algorithm is extremely challenging. Therefore, the empirical method was employed.

All source code and raw data are available here, in a repository on Github.

2 Procedure

All observations were made using a MacBook Pro, model 11,5 with a 2.8 GHz i7 processor and 16 GB of memory. The operating system used was MacOS 10.12.6. Programs were written for Python 2.7.10 with NumPy. The graphics were generated using the powerful, open source program Gnuplot. This document was prepared using SageTEX.

The programs written for this experiment are named shell.py and plot.py. The sorting and data collection is done in shell. Plot organizes the raw data for Gnuplot.

Test table G on page 30.

Test table 22 on page 30.

Array sizes were randomly selected to be tested. Possible sizes range `522,240` to `8,421,376`. The upper limit is due to performance considerations. The computer used is too slow to sort larger arrays in a reasonable amount of time. Five randomized arrays of each size were sequenced using these fifteen gap sequence factors: 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, and 2.9. I arrived at these factors through several preliminary runs. The those runs employed a range of gap factors ranging from 2.0 to 4.0. Some, such as whole numbers, yielded incredibly slow execution times. One factor that showed promise was $\phi + \frac{1}{\phi}$. If time permits, I may do another paper on gap factors that use ϕ.

Every combination of array size and gap factor is tested five times, using the same set of five unique, randomly generated arrays for each array size. The time used to execute each sort is saved in the observations file and summarized in the observations section. In the analysis, only the median time for each combination of array size and gap factor is used.

3 Observations

All gap sequences begin with 1, 4 10, 23, 57, 132, 301, 701, 1750. For gaps greater than 1750, the gap factor becomes active. Each value after 1750 is the product of the current gap factor and the previous gap value. The process continues until $n_i > \lfloor \frac{N}{3} \rfloor$.

Figure 1 plots the relationship between gap factor and median time for each combination of array size and gap factor. The gap factors have there own shape and color. Axes are labeled, with time on the y-axis, and array length on the x-axis.

Figure 2 provides four box and whiskers plots indicating the distribution of the fifteen gap sequences for each array size.

4 Conclusion

Finally, it is important to consider the environment in which this experiment was performed. MacOS Sierra adds a random element to the measurements. The execution of other tasks in background may have influenced the observations.

Clearly, an algorithm such as Hoare's Quicksort will be substantially faster. However, there are times when resources are limited, or recursion is inappropriate. In such times, Shell's algorithm, with the right set of gaps, is a worthy alternative.

A Source Code

A.1 shell

```
from collections import OrderedDict
import copy
from my_obs import data
import numpy as np
import pickle
import time


def disp_results(d):
    """
    d: dictionary of observations
    returns: None
    """
    e = ' {:0.4e}'
    print '\nfact       min       mean        max        std     median'
    for k in OrderedDict(sorted(d.items())):
        ln = '{:3s}'.format(k)
        a = d[k]['times']
        ln += e.format(np.amin(a))
        ln += e.format(np.mean(a))
        ln += e.format(np.amax(a))
        ln += e.format(np.std(a))
        ln += e.format(np.median(a))
        print ln
```

```
def make_seq(n, fact):
    """
    n: int (size of list to be sorted)
    fact: float (gap size)
    returns: list of ints
    """
    s = [1, 4, 10, 23, 57, 132, 301, 701, 1750]
    my_limit = n // 3
    while s[-1] <= my_limit:
        s.append(int(s[-1] * fact))
    s.reverse()
    return s


def main():
    """
    builds & serializes results of sort timings in obs
    """
    n = 1 << 18
    all_obs = {}
    r = 6
    for i in range(r):
        t = n >> 4
        t = np.random.randint(n-t, n+t)
        z = 7
        print '\nt = {}, t // 3 = {}, {} of {}'.format(t, t // 3, i + 1, r)
        test_arrays = np.random.randint(-(n << 4), n << 4, size=(z, t))

        obs = {'2.10': {}, '2.15': {}, '2.20': {}, '2.05': {}, '2.85': {},
               '2.25': {}, '2.30': {}, '2.35': {}, '2.70': {}, '2.90': {},
               '2.40': {}, '2.45': {}, '2.50': {}, '2.75': {}, '2.95': {},
               '2.55': {}, '2.60': {}, '2.65': {}, '2.80': {}, '2.00': {},
               '3.00': {}}

        for k in OrderedDict(sorted(obs.items())):
            gap_factor = float(k)
            s = make_seq(n, gap_factor)
            print '{} {:4s} {}'.format(time.strftime('%H:%M:%S',
                                                     time.localtime()), k, s)
            obs[k]['seq'] = s
            obs[k]['times'] = []
            obs[k]['inner'] = []
            obs[k]['outer'] = []
            for test_array in test_arrays:
                a = shell_sort(copy.deepcopy(test_array), s)
                if not verify(a.a):
                    print '{} not in sequence.'.format(k)
                obs[k]['times'].append(a.t)
                obs[k]['inner'].append(a.inner)
                obs[k]['outer'].append(a.outer)

        disp_results(obs)
        all_obs[t] = obs
        with open('observations.pickle', 'wb') as handle:
            pickle.dump(all_obs, handle, protocol=pickle.HIGHEST_PROTOCOL)

        n <<= 1

    print 'Finished at {}'.format(time.strftime('%H:%M:%S', time.localtime()))


def shell_sort(lst, gaps):
    """
```

```
    lst: list to be ordered
    gaps: liat of ints gaps[-1] must be 1
    returns: names tuple
    """
    swaps = 0
    compares = 0
    t = time.time()
    for gap in gaps:
        for i in range(gap, len(lst)):
            j = i
            temp = lst[j]
            swaps += 1
            while j >= gap and lst[j - gap] > temp:
                compares += 1
                lst[j] = lst[j - gap]
                j -= gap
            lst[j] = temp
    return data(time.time() - t, swaps, compares, lst)


def verify(lst):
    """
    lst: list of comparables
    returns: bool
    """
    for i in range(1, len(lst)):
        if lst[i-1] > lst[i]:
            return False
        return True


if __name__ == '__main__':
    main()
```

A.2 plot

```
from collections import OrderedDict
import numpy as np
import pickle


def display_ordered_results(d):
    """
    d: observations from shell.py
    returns: None
    Displays sequences fastest to slowest for each list size
    """
    results = {}
    for k0 in OrderedDict(sorted(d.items())):
        tmp = []
        for k1 in OrderedDict(sorted(d[k0].items())):
            tmp.append((k1, np.median(d[k0][k1]['times'])))
        tmp.sort(key=lambda tup: tup[1])
        results[k0] = tmp

    for k0 in OrderedDict(sorted(results.items())):
        print '{}\t{} - {}'.format(k0, results[k0][1][1], results[k0][-1][1])
        for t in results[k0]:
            print '{:4s}'.format(t[0]),
        print '\n'
```

```
def extract_box_data(d):
    results = []
    for k0 in OrderedDict(sorted(d.items())):
        results.append([k0])
        for k1 in OrderedDict(sorted(d[k0].items())):
            results[-1].append(np.median(d[k0][k1]['times']))

    dset = np.asarray(results)
    dset = dset.transpose()
    np.savetxt('/Users/philiphuffman/shell21/box.dat', dset)


def extract_size_times(d):
    """
    d: observations from shell.py
    returns: None
    Writes a numpy array to be used by Gnuplot
    """
    xy_lst = []
    for k0 in OrderedDict(sorted(d.items())):
        xy_lst.append([float(k0)])
        for k1 in OrderedDict(sorted(d[k0].items())):
            xy_lst[-1].append(np.median(d[k0][k1]['times']))

    dset = np.asarray(xy_lst)
    np.savetxt('/Users/philiphuffman/shell21/TimevSize.dat', dset)


def main():
    """
    reads observations from shell.py Also prints all sequences ordered by
    median run time, for each list
    """
    with open('observations.pickle', 'rb') as handle:
        d = pickle.load(handle)

    display_ordered_results(d)

    extract_size_times(d)

    extract_box_data(d)


if __name__ == '__main__':
    main()
```

A.3 Time v. Size

```
#!/usr/local/Cellar/gnuplot/5.0.6_3/bin/gnuplot
#
#
#            G N U P L O T
#            Version 5.0 patchlevel 6    last modified 2017-03-18
#
#            Copyright (C) 1986-1993, 1998, 2004, 2007-2017
#            Thomas Williams, Colin Kelley and many others
#
#            gnuplot home:     http://www.gnuplot.info
#            faq, bugs, etc:   type "help FAQ"
#            immediate help:   type "help"  (plot window: hit 'h')
```

```
reset
set terminal qt 0 font "Sans,22"
set output
unset clip points
set clip one
unset clip two
set bar 1.000000 front
set border 31 front lt black linewidth 0.500 dashtype solid
set zdata
set ydata
set xdata
set y2data
set x2data
set boxwidth
set style fill  empty border
set style rectangle back fc  bgnd fillstyle   solid 1.00 border lt -1
set style circle radius graph 0.02, first 0.00000, 0.00000
set style ellipse size graph 0.05, 0.03, first 0.00000 angle 0 units xy
set dummy x, y
set format x "% h"
set format y "% h"
set format x2 "% h"
set format y2 "% h"
set format z "% h"
set format cb "% h"
set format r "% h"
set timefmt "%d/%m/%y,%H:%M"
set angles radians
set tics back
unset grid
set raxis
set style parallel front  lt black linewidth 2.000 dashtype solid
set key title "Gap Factors" center
set key inside left top vertical Left noreverse enhanced autotitle nobox font ",9"
set key noinvert samplen 3.2 spacing 2.0 width 2 height 1.1
set key maxcolumns 4 maxrows 7
set key noopaque
unset label
unset arrow
set style increment default
unset style line
unset style arrow
set style histogram clustered gap 2 title textcolor lt -1
unset object
set style textbox transparent margins  1.0,  1.0 border
set offsets 0, 0, 0, 0
set pointsize 1
set pointintervalbox 1
set encoding default
unset polar
unset parametric
unset decimalsign
unset micro
unset minussign
set view 60, 30, 1, 1
set samples 100, 100
set isosamples 10, 10
set surface
unset contour
set cntrlabel  format '%8.3g' font '' start 5 interval 20
set mapping cartesian
set datafile separator whitespace
unset hidden3d
```

```
set cntrparam order 4
set cntrparam linear
set cntrparam levels auto 5
set cntrparam points 5
set size ratio 0 1,1
set origin 0,0
set style data points
set style function lines
unset xzeroaxis
unset yzeroaxis
unset zzeroaxis
unset x2zeroaxis
unset y2zeroaxis
set xyplane relative 0.5
#set tics scale  1, 0.5, 1, 1, 1
set mxtics default
set mytics default
set mztics default
set mx2tics default
set my2tics default
set mcbtics default
set mrtics default
set xtics border in scale 1,0.5 nomirror norotate  offset character 0, -2.25, 0 autojustify
set xtics  norangelimit autofreq  font ", 9" format "%0.0f"
set ytics border in scale 1,0.5 nomirror norotate  autojustify
set ytics  norangelimit autofreq  font ", 9"
set ztics border in scale 1,0.5 nomirror norotate  autojustify
set ztics  norangelimit autofreq  font ", 9"
set logscale x 2
unset x2tics
unset y2tics
set cbtics border in scale 1,0.5 mirror norotate  autojustify
set cbtics  norangelimit autofreq  font ", 9"
set rtics axis in scale 1,0.5 nomirror norotate  autojustify
set rtics  norangelimit autofreq  font ", 9"
unset paxis 1 tics
unset paxis 2 tics
unset paxis 3 tics
unset paxis 4 tics
unset paxis 5 tics
unset paxis 6 tics
unset paxis 7 tics
set title ""
set title  font "" norotate
set timestamp bottom
set timestamp ""
set timestamp  font "" norotate
set rrange [ * : * ] noreverse nowriteback
set trange [ * : * ] noreverse nowriteback
set urange [ * : * ] noreverse nowriteback
set vrange [ * : * ] noreverse nowriteback
set xrange [ * : * ] noreverse nowriteback
set xlabel "Number of Elements"
set xlabel  offset character 0, -3.25556, 0 font "" textcolor lt -1 norotate
set x2label ""
set x2label  font "" textcolor lt -1 norotate
set x2range [ * : * ] noreverse nowriteback
set ylabel "Time (Seconds)"
set ylabel  offset character -0.225, 0, 0 font "" textcolor lt -1 rotate by -270
set y2label ""
set y2label  font "" textcolor lt -1 rotate by -270
set yrange [ * : * ] noreverse nowriteback
set y2range [ * : * ] noreverse nowriteback
```

```
set zlabel ""
set zlabel  font "" textcolor lt -1 norotate
set zrange [ * : * ] noreverse nowriteback
set cblabel ""
set cblabel  font "" textcolor lt -1 rotate by -270
set cbrange [ * : * ] noreverse nowriteback
set paxis 1 range [ * : * ] noreverse nowriteback
set paxis 2 range [ * : * ] noreverse nowriteback
set paxis 3 range [ * : * ] noreverse nowriteback
set paxis 4 range [ * : * ] noreverse nowriteback
set paxis 5 range [ * : * ] noreverse nowriteback
set paxis 6 range [ * : * ] noreverse nowriteback
set paxis 7 range [ * : * ] noreverse nowriteback
set zero 1e-08
set lmargin  -1
set bmargin  -1
set rmargin  -1
set tmargin  -1
set locale "en_US.UTF-8"
set pm3d explicit at s
set pm3d scansautomatic
set pm3d interpolate 1,1 flush begin noftriangles noborder corners2color mean
set palette positive nops_allcF maxcolors 0 gamma 1.5 color model RGB
set palette rgbformulae 7, 5, 15
set colorbox default
set colorbox vertical origin screen 0.9, 0.2, 0 size screen 0.05, 0.6, 0 front  noinvert bdefault
set style boxplot candles range  1.50 outliers pt 7 separation 1 labels auto unsorted
set loadpath
set fontpath
set psdir
set fit brief errorvariables nocovariancevariables errorscaling prescale nowrap v5
GNUTERM = "qt"
x = 0.0
fng = "TimevSize.gp"
fnt = "TimevSize.tex"
t = 0.0
## Last datafile plotted: "TimevSize.dat"
plot "TimevSize.dat" \
   using 1:2  title "2.00" w lp lc rgb '#006490' lw 1.00 pt  1 ps 1,\
"" using 1:3  title "2.05" w lp lc rgb '#006400' lw 1.00 pt  2 ps 1,\
"" using 1:4  title "2.10" w lp lc rgb '#0064ff' lw 1.00 pt  3 ps 1,\
"" using 1:5  title "2.15" w lp lc rgb '#806400' lw 1.00 pt  4 ps 1,\
"" using 1:6  title "2.20" w lp lc rgb '#44aa00' lw 1.00 pt  5 ps 1,\
"" using 1:7  title "2.25" w lp lc rgb '#4B0082' lw 1.00 pt  6 ps 1,\
"" using 1:8  title "2.30" w lp lc rgb '#000000' lw 1.00 pt  7 ps 1,\
"" using 1:9  title "2.35" w lp lc rgb '#55cc55' lw 1.00 pt  8 ps 1,\
"" using 1:10 title "2.40" w lp lc rgb '#777700' lw 1.00 pt  9 ps 1,\
"" using 1:11 title "2.45" w lp lc rgb '#337777' lw 1.00 pt 10 ps 1,\
"" using 1:12 title "2.50" w lp lc rgb '#3377f7' lw 1.00 pt 11 ps 1,\
"" using 1:13 title "2.55" w lp lc rgb '#330777' lw 1.00 pt 12 ps 1,\
"" using 1:14 title "2.60" w lp lc rgb '#33c777' lw 1.00 pt 13 ps 1,\
"" using 1:15 title "2.65" w lp lc rgb '#3377c7' lw 1.00 pt 14 ps 1,\
"" using 1:16 title "2.70" w lp lc rgb '#c37777' lw 1.00 pt 15 ps 1,\
"" using 1:17 title "2.75" w lp lc rgb '#337700' lw 1.00 pt 16 ps 1,\
"" using 1:18 title "2.80" w lp lc rgb '#6307c0' lw 1.00 pt 17 ps 1,\
"" using 1:19 title "2.85" w lp lc rgb '#63a740' lw 1.00 pt 18 ps 1,\
"" using 1:20 title "2.90" w lp lc rgb '#662200' lw 1.00 pt 19 ps 1,\
"" using 1:21 title "2.95" w lp lc rgb '#558800' lw 1.00 pt 20 ps 1,\
"" using 1:22 title "3.00" w lp lc rgb '#558890' lw 1.00 pt 21 ps 1

#    EOF
```

B Summary

N	← Slower							Gap Factor							Faster →					
247,959	2.00	3.00	2.90	2.75	2.70	2.25	2.95	2.30	2.85	2.60	2.20	2.65	2.50	2.45	2.55	2.80	2.35	2.10	2.15	2.05	2.40
556,034	3.00	2.00	2.90	2.65	2.75	2.85	2.95	2.80	2.50	2.55	2.70	2.60	2.45	2.40	2.35	2.05	2.15	2.20	2.10	2.30	2.25
1,037,375	2.00	3.00	2.50	2.35	2.60	2.65	2.10	2.20	2.15	2.75	2.55	2.25	2.45	2.40	2.05	2.70	2.80	2.30	2.85	2.90	2.95
1,970,626	2.00	3.00	2.15	2.05	2.10	2.35	2.50	2.90	2.85	2.25	2.70	2.80	2.95	2.40	2.20	2.60	2.75	2.65	2.30	2.45	2.55
4,365,249	2.00	3.00	2.50	2.95	2.90	2.85	2.60	2.75	2.05	2.80	2.10	2.35	2.65	2.15	2.20	2.70	2.55	2.45	2.25	2.40	2.30

Table 1: Summary of effect of gap factors on performance

C Statistics

Factor	Minimum	Mean	Maximum	Std. Dev.	Median
2.00	2.6628e+00	2.7340e+00	2.8337e+00	6.4053e-02	2.7139e+00
2.05	2.0581e+00	2.0916e+00	2.1448e+00	2.8076e-02	2.0878e+00
2.10	2.0515e+00	2.1012e+00	2.2311e+00	5.7103e-02	2.0940e+00
2.15	2.0242e+00	2.1340e+00	2.3044e+00	9.0712e-02	2.0929e+00
2.20	2.0244e+00	2.1474e+00	2.2670e+00	8.7102e-02	2.1800e+00
2.25	2.1126e+00	2.2077e+00	2.3674e+00	8.7959e-02	2.2089e+00
2.30	2.0834e+00	2.2136e+00	2.3427e+00	8.6098e-02	2.1948e+00
2.35	2.0407e+00	2.1649e+00	2.4120e+00	1.2349e-01	2.1264e+00
2.40	2.0227e+00	2.0925e+00	2.1845e+00	6.1880e-02	2.0756e+00
2.45	2.0398e+00	2.1384e+00	2.2517e+00	7.6768e-02	2.1532e+00
2.50	2.0665e+00	2.1672e+00	2.2364e+00	5.2895e-02	2.1553e+00
2.55	2.0659e+00	2.1712e+00	2.4376e+00	1.1841e-01	2.1408e+00
2.60	2.0916e+00	2.1773e+00	2.2494e+00	5.4643e-02	2.1844e+00
2.65	2.0739e+00	2.1543e+00	2.2777e+00	6.5536e-02	2.1588e+00
2.70	2.0530e+00	2.2207e+00	2.3563e+00	8.9052e-02	2.2130e+00
2.75	2.1287e+00	2.2477e+00	2.3609e+00	8.3530e-02	2.2290e+00
2.80	2.0376e+00	2.1463e+00	2.2305e+00	6.0922e-02	2.1382e+00
2.85	2.0571e+00	2.1820e+00	2.2971e+00	7.8392e-02	2.1891e+00
2.90	2.2156e+00	2.3405e+00	2.4277e+00	6.5512e-02	2.3327e+00
2.95	2.0494e+00	2.2289e+00	2.4361e+00	1.2515e-01	2.2021e+00
3.00	2.5915e+00	2.6583e+00	2.7355e+00	5.0480e-02	2.6525e+00

Table 2: Statistics for array of size 247,959

Factor	Minimum	Mean	Maximum	Std. Dev.	Median
2.00	7.4176e+00	7.6493e+00	7.9302e+00	1.7137e-01	7.6945e+00
2.05	4.9680e+00	5.2049e+00	5.3932e+00	1.4060e-01	5.2369e+00
2.10	4.9981e+00	5.1972e+00	5.5040e+00	1.7911e-01	5.1245e+00
2.15	4.9997e+00	5.1747e+00	5.4086e+00	1.2380e-01	5.1658e+00
2.20	4.9535e+00	5.1587e+00	5.4265e+00	1.5754e-01	5.1409e+00
2.25	4.9421e+00	5.0789e+00	5.3201e+00	1.2355e-01	5.0196e+00
2.30	4.9514e+00	5.1378e+00	5.4401e+00	1.4517e-01	5.1057e+00
2.35	4.9727e+00	5.2113e+00	5.3785e+00	1.2973e-01	5.2657e+00
2.40	5.2229e+00	5.5031e+00	5.8314e+00	2.2107e-01	5.4642e+00
2.45	5.7561e+00	5.8157e+00	5.9157e+00	4.7373e-02	5.8184e+00
2.50	5.8086e+00	5.9453e+00	6.0808e+00	9.1786e-02	5.9418e+00
2.55	5.8075e+00	5.9254e+00	6.0503e+00	1.0013e-01	5.8898e+00
2.60	5.8109e+00	5.9124e+00	6.0894e+00	9.5441e-02	5.8849e+00
2.65	5.8861e+00	6.0161e+00	6.1382e+00	7.5954e-02	6.0188e+00
2.70	5.7750e+00	5.8759e+00	5.9829e+00	6.2867e-02	5.8863e+00
2.75	5.8282e+00	5.9940e+00	6.1048e+00	1.0275e-01	6.0003e+00
2.80	5.8181e+00	5.9697e+00	6.0798e+00	8.9654e-02	5.9694e+00
2.85	5.8752e+00	5.9816e+00	6.0901e+00	6.3400e-02	5.9899e+00
2.90	5.8677e+00	6.0226e+00	6.1027e+00	8.0670e-02	6.0714e+00
2.95	5.9053e+00	6.0036e+00	6.1695e+00	9.5862e-02	5.9777e+00
3.00	8.3176e+00	8.5199e+00	8.6780e+00	1.3278e-01	8.4860e+00

Table 3: Statistics for array of size 556,034

Factor	Minimum	Mean	Maximum	Std. Dev.	Median
2.00	1.8958e+01	1.9406e+01	1.9652e+01	2.2678e-01	1.9431e+01
2.05	1.1656e+01	1.1820e+01	1.2144e+01	1.6393e-01	1.1719e+01
2.10	1.1632e+01	1.1883e+01	1.2195e+01	2.0792e-01	1.1848e+01
2.15	1.1522e+01	1.1851e+01	1.2102e+01	1.8319e-01	1.1818e+01
2.20	1.1647e+01	1.1919e+01	1.2154e+01	1.9416e-01	1.1835e+01
2.25	1.1651e+01	1.1798e+01	1.1941e+01	8.9150e-02	1.1797e+01
2.30	1.1611e+01	1.1740e+01	1.1874e+01	1.0056e-01	1.1678e+01
2.35	1.1754e+01	1.1945e+01	1.2094e+01	9.3726e-02	1.1957e+01
2.40	1.1541e+01	1.1748e+01	1.2149e+01	1.8957e-01	1.1748e+01
2.45	1.1597e+01	1.1786e+01	1.2020e+01	1.2376e-01	1.1753e+01
2.50	1.1856e+01	1.2087e+01	1.2393e+01	1.5262e-01	1.2045e+01
2.55	1.1581e+01	1.1853e+01	1.2154e+01	1.8601e-01	1.1803e+01
2.60	1.1557e+01	1.1861e+01	1.1991e+01	1.4237e-01	1.1933e+01
2.65	1.1656e+01	1.1869e+01	1.2087e+01	1.5516e-01	1.1873e+01
2.70	1.1589e+01	1.1761e+01	1.2042e+01	1.4819e-01	1.1716e+01
2.75	1.1713e+01	1.1869e+01	1.2102e+01	1.4168e-01	1.1811e+01
2.80	1.1610e+01	1.1736e+01	1.2009e+01	1.2333e-01	1.1709e+01
2.85	1.0147e+01	1.0982e+01	1.1793e+01	5.5992e-01	1.1025e+01
2.90	1.0087e+01	1.0560e+01	1.0776e+01	2.3365e-01	1.0591e+01
2.95	9.9654e+00	1.0510e+01	1.0799e+01	2.8819e-01	1.0534e+01
3.00	1.6472e+01	1.6735e+01	1.7057e+01	2.2067e-01	1.6597e+01

Table 4: Statistics for array of size 1,037,375

Factor	Minimum	Mean	Maximum	Std. Dev.	Median
2.00	3.8266e+01	3.8917e+01	3.9561e+01	4.4969e-01	3.8934e+01
2.05	2.1155e+01	2.1978e+01	2.3337e+01	7.2387e-01	2.1751e+01
2.10	2.0605e+01	2.1556e+01	2.2769e+01	6.1239e-01	2.1366e+01
2.15	2.1065e+01	2.1735e+01	2.2542e+01	4.4104e-01	2.1754e+01
2.20	2.0158e+01	2.1003e+01	2.2458e+01	7.9409e-01	2.0766e+01
2.25	2.0564e+01	2.1040e+01	2.1784e+01	3.5764e-01	2.0972e+01
2.30	1.9966e+01	2.0566e+01	2.1056e+01	3.5569e-01	2.0556e+01
2.35	2.0491e+01	2.1325e+01	2.2727e+01	7.2327e-01	2.1200e+01
2.40	2.0401e+01	2.1048e+01	2.2102e+01	5.5405e-01	2.0793e+01
2.45	2.0210e+01	2.0569e+01	2.0972e+01	2.8320e-01	2.0464e+01
2.50	2.0661e+01	2.1479e+01	2.2582e+01	7.1201e-01	2.1131e+01
2.55	2.0193e+01	2.0568e+01	2.1158e+01	3.6604e-01	2.0386e+01
2.60	2.0174e+01	2.0662e+01	2.0890e+01	2.2771e-01	2.0741e+01
2.65	2.0372e+01	2.0766e+01	2.1303e+01	2.9540e-01	2.0720e+01
2.70	2.0318e+01	2.0822e+01	2.1415e+01	3.5510e-01	2.0906e+01
2.75	2.0562e+01	2.0731e+01	2.0894e+01	1.2160e-01	2.0726e+01
2.80	2.0488e+01	2.0864e+01	2.1545e+01	3.3482e-01	2.0890e+01
2.85	2.0351e+01	2.0952e+01	2.1540e+01	3.6358e-01	2.0987e+01
2.90	2.0905e+01	2.1086e+01	2.1336e+01	1.4495e-01	2.1071e+01
2.95	2.0150e+01	2.0957e+01	2.1737e+01	4.5730e-01	2.0889e+01
3.00	3.7338e+01	3.8484e+01	3.9278e+01	6.6263e-01	3.8808e+01

Table 5: Statistics for array of size 1,970,626

Factor	Minimum	Mean	Maximum	Std. Dev.	Median
2.00	1.1212e+02	1.1300e+02	1.1412e+02	6.6266e-01	1.1321e+02
2.05	4.8239e+01	4.9247e+01	5.0182e+01	5.8293e-01	4.9119e+01
2.10	4.8462e+01	4.9094e+01	4.9644e+01	3.4955e-01	4.9091e+01
2.15	4.8349e+01	4.8811e+01	4.9242e+01	3.1394e-01	4.8785e+01
2.20	4.7895e+01	4.8634e+01	4.9505e+01	4.8859e-01	4.8703e+01
2.25	4.7864e+01	4.8617e+01	4.9256e+01	4.6985e-01	4.8600e+01
2.30	4.7471e+01	4.8231e+01	4.8627e+01	4.0321e-01	4.8326e+01
2.35	4.8434e+01	4.8967e+01	4.9816e+01	4.6581e-01	4.9036e+01
2.40	4.8002e+01	4.8605e+01	4.9310e+01	4.6877e-01	4.8569e+01
2.45	4.7875e+01	4.8581e+01	4.9364e+01	5.3506e-01	4.8631e+01
2.50	4.9409e+01	5.0305e+01	5.0922e+01	4.9658e-01	5.0241e+01
2.55	4.7784e+01	4.8592e+01	4.9208e+01	4.7940e-01	4.8671e+01
2.60	4.8191e+01	4.8984e+01	4.9302e+01	3.7259e-01	4.9209e+01
2.65	4.8526e+01	4.9057e+01	4.9739e+01	3.9813e-01	4.8880e+01
2.70	4.8465e+01	4.8807e+01	4.9128e+01	2.4521e-01	4.8678e+01
2.75	4.9148e+01	4.9371e+01	4.9936e+01	2.8390e-01	4.9174e+01
2.80	4.8499e+01	4.9122e+01	4.9665e+01	3.5084e-01	4.9115e+01
2.85	4.8994e+01	4.9605e+01	5.0503e+01	4.7899e-01	4.9530e+01
2.90	4.8585e+01	4.9606e+01	5.0881e+01	6.8762e-01	4.9612e+01
2.95	4.9234e+01	4.9663e+01	5.0167e+01	2.6462e-01	4.9667e+01
3.00	1.1103e+02	1.1489e+02	1.2320e+02	5.1907e+00	1.1214e+02

Table 6: Statistics for array of size 4,365,249

D Median Run Times

Factor	Run Times						
2.00	2.68574	2.71395	2.73002	2.68532	2.66285	2.82671	2.83372
2.05	2.14485	2.11681	2.07856	2.08778	2.05813	2.06436	2.09055
2.10	2.23106	2.10665	2.05284	2.05151	2.10409	2.09397	2.06858
2.15	2.06857	2.02425	2.06975	2.09290	2.20183	2.30444	2.17650
2.20	2.08509	2.23728	2.18001	2.02439	2.18626	2.26697	2.05178
2.25	2.20886	2.11937	2.24816	2.12811	2.11259	2.36742	2.26957
2.30	2.34268	2.17862	2.19476	2.08336	2.28546	2.28047	2.12963
2.35	2.10314	2.04073	2.14190	2.27573	2.12640	2.05471	2.41203
2.40	2.18451	2.02269	2.05272	2.02996	2.07559	2.10244	2.17960
2.45	2.17818	2.03982	2.08720	2.04546	2.15322	2.25170	2.21349
2.50	2.22600	2.15497	2.23638	2.18710	2.15533	2.06655	2.14430
2.55	2.06594	2.14081	2.16365	2.43760	2.08137	2.09745	2.21133
2.60	2.20744	2.18442	2.24938	2.23883	2.13735	2.13231	2.09162
2.65	2.10890	2.20646	2.09475	2.07390	2.15950	2.15877	2.27769
2.70	2.35628	2.27995	2.21042	2.26755	2.21305	2.05303	2.16457
2.75	2.32501	2.36089	2.32083	2.22904	2.15113	2.21845	2.12865
2.80	2.10534	2.18438	2.13816	2.20430	2.12364	2.03756	2.23047
2.85	2.23518	2.17657	2.05709	2.08795	2.18910	2.29708	2.23115
2.90	2.33271	2.32185	2.21564	2.33365	2.42005	2.42767	2.33211
2.95	2.10896	2.43606	2.16967	2.32097	2.31502	2.04939	2.20214
3.00	2.73554	2.66280	2.65252	2.62105	2.59155	2.72525	2.61967

Table 7: Run times for array of size 247,959
cdf(0) = 0.50

Factor	Run Times						
2.00	7.79135	7.71495	7.41761	7.69446	7.47680	7.93016	7.51997
2.05	5.20382	5.23691	5.03664	4.96797	5.39322	5.29131	5.30473
2.10	5.50401	5.04045	5.39545	5.25581	5.12448	5.06233	4.99809
2.15	5.11093	5.21499	4.99968	5.25030	5.40855	5.07231	5.16582
2.20	4.99233	5.28418	4.95351	5.24665	5.42649	5.14088	5.06683
2.25	5.07708	5.32013	5.00786	5.01964	4.94207	4.99033	5.19524
2.30	5.04015	5.06492	4.95139	5.10565	5.21063	5.15182	5.44010
2.35	5.37846	4.97270	5.30499	5.26574	5.28107	5.19064	5.08518
2.40	5.22291	5.46424	5.37024	5.52293	5.29908	5.83137	5.81076
2.45	5.81886	5.81837	5.91566	5.75613	5.80013	5.77348	5.82701
2.50	5.80861	5.88267	6.06548	5.88612	5.94178	6.08083	5.95163
2.55	5.80747	5.88977	5.83085	6.04934	6.01179	6.05029	5.83795
2.60	5.83192	5.81093	5.94105	5.99832	5.88493	5.83051	6.08939
2.65	5.88611	5.99720	6.13819	6.04551	5.95294	6.01881	6.07407
2.70	5.92044	5.84288	5.77496	5.88631	5.82683	5.89663	5.98294
2.75	6.00033	6.09861	5.94959	6.10477	6.09099	5.82821	5.88548
2.80	5.96940	5.92026	6.06647	5.81812	6.03530	5.89868	6.07985
2.85	5.93021	5.99041	5.98989	5.87517	6.09013	5.96831	6.02706
2.90	5.86767	5.96920	5.97720	6.08090	6.08919	6.10272	6.07145
2.95	5.91651	5.97769	6.16951	5.90526	6.11460	5.92391	6.01786
3.00	8.67795	8.39369	8.63829	8.66954	8.45638	8.31761	8.48605

Table 8: Run times for array of size 556,034
cdf(0) = 0.50

Factor	Run Times						
2.00	19.25139	19.65175	19.52589	19.64955	19.37781	18.95809	19.43052
2.05	12.14401	11.70850	11.71947	11.69107	11.89917	11.65555	11.92322
2.10	11.63359	12.04306	11.75428	12.19470	12.07531	11.63234	11.84752
2.15	11.52209	11.97701	12.02158	11.74337	12.10203	11.77296	11.81839
2.20	12.13422	12.15373	12.11596	11.78236	11.76390	11.83543	11.64703
2.25	11.71330	11.79674	11.65092	11.78172	11.84441	11.94086	11.86124
2.30	11.86347	11.61136	11.66668	11.67831	11.81954	11.66532	11.87382
2.35	11.91868	12.09375	11.94043	11.96807	11.98186	11.95723	11.75366
2.40	11.57587	11.79629	11.79204	11.54087	11.63059	12.14921	11.74813
2.45	11.74038	11.75287	11.88554	11.59726	12.01966	11.78440	11.72355
2.50	12.00859	12.16508	12.39272	12.04476	11.85589	12.09896	12.04231
2.55	11.58087	11.80286	11.67923	11.78076	12.15391	11.93920	12.03251
2.60	11.97912	11.99108	11.84020	11.94484	11.78317	11.93254	11.55707
2.65	11.82312	11.87325	11.68572	11.89168	12.08694	11.65552	12.06704
2.70	11.58909	12.04215	11.67001	11.62067	11.86686	11.82390	11.71642
2.75	11.71276	12.03750	11.72266	11.81090	11.78605	12.10167	11.91265
2.80	11.62655	11.74929	12.00928	11.60990	11.70931	11.68706	11.76097
2.85	11.79259	11.10819	11.58860	11.02489	10.90184	10.31296	10.14671
2.90	10.57930	10.59111	10.74636	10.76371	10.77598	10.08741	10.37403
2.95	10.79915	10.75926	9.96541	10.76486	10.23883	10.50539	10.53382
3.00	16.90298	16.47198	16.58811	16.98387	16.54193	16.59710	17.05697

Table 9: Run times for array of size 1,037,375
cdf(0) = 0.50

Factor	Run Times						
2.00	39.38279	39.19163	38.53710	39.56146	38.93386	38.26565	38.54424
2.05	22.62315	21.28924	23.33690	21.55685	21.15504	22.13422	21.75070
2.10	21.30959	21.36436	21.36594	21.57960	21.89666	22.76869	20.60501
2.15	22.00835	21.70452	22.54162	21.75448	21.30550	21.06463	21.76741
2.20	20.41813	20.49716	20.76557	21.94115	20.15832	20.78437	22.45846
2.25	20.96729	20.97200	21.20307	21.78421	20.75965	20.56422	21.03291
2.30	21.01918	21.05601	20.41713	20.61532	19.96620	20.33012	20.55570
2.35	20.61453	21.20023	21.64370	20.49107	21.73384	20.86344	22.72654
2.40	21.47436	22.10195	21.27226	20.79331	20.62477	20.66876	20.40139
2.45	20.76231	20.90324	20.32024	20.46415	20.34982	20.97203	20.20983
2.50	21.13097	20.77314	20.66091	21.72695	21.08990	22.58229	22.39002
2.55	20.30675	21.11277	20.38649	20.38114	21.15756	20.43833	20.19302
2.60	20.89036	20.73147	20.17433	20.81219	20.78200	20.50383	20.74070
2.65	20.61010	20.37160	21.07381	20.72036	21.30277	20.73007	20.55452
2.70	20.66481	20.31767	20.90581	20.41565	21.41485	21.05447	20.98307
2.75	20.72573	20.88316	20.79213	20.65476	20.89425	20.60679	20.56156
2.80	20.48807	21.00047	21.54523	20.94379	20.89038	20.54535	20.63577
2.85	21.24749	20.35109	21.06426	20.84699	20.62513	20.98661	21.53956
2.90	20.97037	21.33573	20.98479	21.25388	20.90527	21.08087	21.07078
2.95	21.23222	20.88506	20.66464	20.88943	21.14017	20.15037	21.73718
3.00	38.85648	38.37497	39.27818	38.80821	37.33777	37.71689	39.01771

Table 10: Run times for array of size 1,970,626
cdf(0) = 0.50

Factor	Run Times						
2.00	113.21041	113.26105	112.12870	112.79090	114.12121	113.34831	112.11675
2.05	49.30749	48.23878	49.09939	49.83975	49.11896	48.94008	50.18229
2.10	48.82444	49.64399	49.31216	48.46250	49.27243	49.09054	49.05045
2.15	48.66179	48.78452	49.24244	49.24228	48.53146	48.34859	48.86433
2.20	48.33168	47.89452	48.78042	49.50495	48.96384	48.70316	48.26087
2.25	48.84767	48.41086	49.15341	49.25571	47.86379	48.60021	48.18536
2.30	48.62687	48.62473	47.81690	48.32565	48.26525	47.47081	48.48945
2.35	48.62491	49.17226	49.24081	48.43357	49.03645	48.44276	49.81583
2.40	49.12705	49.31038	48.56904	48.83559	48.00164	48.28379	48.10859
2.45	47.97090	48.73544	49.36356	48.27642	48.63115	49.21518	47.87525
2.50	50.67344	49.91113	50.78838	50.19264	49.40856	50.92242	50.24060
2.55	48.37801	48.67117	49.18117	48.72960	47.78437	48.19296	49.20774
2.60	49.20860	49.22058	49.00645	48.19100	49.23894	48.71932	49.30174
2.65	49.17013	48.87964	48.83620	48.76230	49.48472	48.52606	49.73881
2.70	48.99466	48.67768	49.10948	48.63164	49.12793	48.64291	48.46528
2.75	49.16035	49.64286	49.17422	49.17446	49.14777	49.93551	49.36334
2.80	49.16746	49.66479	48.89939	49.03565	48.49939	49.11505	49.46878
2.85	48.99367	49.52959	49.85554	49.88556	49.26784	50.50302	49.19876
2.90	49.61247	48.58484	49.82016	49.26752	50.88134	49.05363	50.02417
2.95	49.47462	49.62340	49.66697	49.78688	49.23448	49.68674	50.16653
3.00	111.55887	111.02547	112.34665	111.03842	122.93085	123.20042	112.14226

Table 11: Run times for array of size 4,365,249
cdf(0) = 0.50

E Outer Iteration Counts

Factor	Iterations						
2.00	5,886,939	5,927,100	5,884,591	5,855,908	5,807,357	5,870,924	5,890,405
2.05	3,658,083	3,679,360	3,672,512	3,689,245	3,674,839	3,680,257	3,669,514
2.10	3,676,992	3,705,415	3,707,518	3,683,610	3,692,724	3,718,065	3,722,777
2.15	3,706,282	3,703,027	3,704,085	3,683,126	3,678,036	3,678,244	3,685,364
2.20	3,736,180	3,753,556	3,736,299	3,730,740	3,752,548	3,739,715	3,737,983
2.25	3,788,139	3,778,039	3,778,672	3,784,043	3,786,981	3,787,767	3,802,552
2.30	3,777,920	3,776,658	3,783,441	3,767,129	3,785,279	3,785,034	3,782,196
2.35	3,868,600	3,879,475	3,896,203	3,868,920	3,905,224	3,877,712	3,910,494
2.40	3,850,960	3,850,304	3,845,937	3,844,710	3,846,835	3,832,968	3,849,164
2.45	3,849,616	3,872,971	3,875,250	3,885,911	3,871,232	3,869,428	3,886,322
2.50	4,002,489	4,006,455	3,997,955	4,002,725	3,981,263	3,988,363	4,069,318
2.55	3,908,033	3,921,385	3,925,320	3,896,159	3,910,811	3,905,131	3,908,320
2.60	3,945,909	3,944,843	3,956,166	3,951,019	3,956,209	3,955,155	3,935,459
2.65	4,006,007	4,000,958	3,966,851	3,981,717	3,994,006	3,974,724	3,998,031
2.70	4,019,460	4,030,050	4,033,405	4,023,059	4,015,563	4,032,394	4,012,239
2.75	4,060,029	4,048,476	4,048,142	4,055,205	4,050,629	4,047,593	4,057,562
2.80	4,045,997	4,069,670	4,059,505	4,044,310	4,056,061	4,045,619	4,054,382
2.85	4,111,037	4,147,616	4,135,392	4,131,050	4,130,399	4,097,312	4,115,202
2.90	4,147,544	4,139,367	4,125,549	4,098,068	4,117,201	4,136,432	4,116,273
2.95	4,133,170	4,134,538	4,135,987	4,140,146	4,121,292	4,133,248	4,126,175
3.00	6,063,440	6,112,338	6,067,050	6,012,097	5,990,888	6,037,446	6,069,273

Table 12: Outer loop iterations for array of size 247,959

Factor	Iterations						
2.00	17,208,548	17,247,930	17,158,978	17,194,458	17,148,897	17,200,015	17,418,659
2.05	8,870,793	8,876,793	8,901,473	8,882,213	8,873,872	8,862,601	8,864,635
2.10	8,950,157	8,978,325	8,933,961	8,983,713	9,011,222	9,066,353	8,937,772
2.15	8,940,312	8,925,974	8,931,527	8,932,160	8,938,752	8,940,050	8,961,225
2.20	9,065,073	9,077,043	9,077,971	9,085,133	9,065,602	9,075,255	9,063,113
2.25	9,189,249	9,208,818	9,231,829	9,217,726	9,209,068	9,215,055	9,179,250
2.30	9,197,117	9,212,825	9,201,688	9,180,188	9,210,604	9,201,597	9,174,791
2.35	9,588,875	9,408,382	9,491,825	9,430,966	9,534,867	9,525,362	9,474,802
2.40	9,385,487	9,395,721	9,405,054	9,363,151	9,370,344	9,360,506	9,353,318
2.45	9,468,074	9,438,434	9,443,178	9,448,202	9,437,934	9,446,239	9,458,984
2.50	9,808,084	9,875,122	9,982,295	9,860,467	9,927,916	9,869,448	9,879,983
2.55	9,730,568	9,695,301	9,687,804	9,680,808	9,681,940	9,707,849	9,663,706
2.60	9,739,191	9,748,998	9,749,905	9,750,393	9,718,954	9,717,309	9,779,080
2.65	9,831,286	9,788,178	9,782,641	9,800,745	9,763,406	9,774,191	9,815,227
2.70	9,889,578	9,876,193	9,829,596	9,852,263	9,846,747	9,852,765	9,864,140
2.75	9,964,077	10,015,837	9,949,884	9,917,256	9,954,673	9,924,704	9,947,381
2.80	10,007,329	10,012,792	9,981,199	10,009,471	9,967,232	9,952,902	10,002,936
2.85	10,058,468	10,149,411	10,111,856	10,085,644	10,073,050	10,093,829	10,101,237
2.90	10,168,433	10,188,104	10,121,574	10,185,618	10,141,534	10,137,961	10,148,469
2.95	10,224,800	10,220,939	10,198,982	10,196,315	10,219,708	10,177,441	10,208,185
3.00	17,841,674	17,865,409	17,792,861	17,792,586	17,755,480	17,814,888	18,022,832

Table 13: Outer loop iterations for array of size 556,034

Factor	Iterations						
2.00	40,010,699	40,371,792	40,295,828	40,325,309	40,106,484	40,267,081	39,977,543
2.05	17,421,509	17,431,487	17,444,051	17,436,443	17,435,725	17,456,804	17,431,984
2.10	17,600,146	17,614,730	17,604,740	17,740,714	17,622,040	17,566,628	17,648,580
2.15	17,756,205	17,755,547	17,755,912	17,755,363	17,750,229	17,797,088	17,768,803
2.20	17,846,476	17,845,243	17,888,195	17,870,989	17,866,034	17,853,925	17,845,726
2.25	18,125,543	18,138,166	18,155,693	18,163,501	18,176,969	18,136,418	18,124,450
2.30	18,175,176	18,185,202	18,183,847	18,157,669	18,146,767	18,129,057	18,135,677
2.35	18,616,391	18,799,366	18,712,320	18,686,646	18,726,952	18,694,125	18,571,142
2.40	18,511,281	18,572,210	18,558,347	18,600,944	18,651,357	18,516,249	18,573,319
2.45	18,948,694	18,906,003	18,986,073	18,872,816	18,956,082	18,960,508	18,925,246
2.50	19,839,988	19,986,006	19,877,402	19,820,914	19,744,105	19,827,273	19,877,222
2.55	19,097,682	19,155,440	19,112,577	19,056,794	19,130,541	19,147,793	19,160,986
2.60	19,297,822	19,311,605	19,309,409	19,410,555	19,344,047	19,341,844	19,295,629
2.65	19,430,269	19,446,006	19,427,012	19,468,578	19,457,155	19,408,971	19,509,787
2.70	19,619,516	19,591,684	19,581,641	19,561,076	19,485,239	19,593,786	19,586,487
2.75	19,874,471	19,803,441	19,811,169	19,797,811	19,912,078	19,869,269	19,862,058
2.80	19,903,270	19,885,068	19,944,505	19,903,843	19,896,469	19,885,984	19,849,881
2.85	20,197,659	20,424,438	20,284,702	20,316,270	20,268,198	20,345,487	20,324,727
2.90	20,520,043	20,489,447	20,472,727	20,419,009	20,446,142	20,484,581	20,501,594
2.95	20,544,609	20,502,322	20,567,257	20,670,827	20,463,417	20,516,938	20,601,086
3.00	41,608,385	41,915,393	42,024,528	42,089,909	41,646,032	42,024,282	41,609,800

Table 14: Outer loop iterations for array of size 1,037,375

Factor	Iterations						
2.00	98,496,050	97,216,082	98,188,778	98,699,302	96,769,122	97,584,640	97,955,623
2.05	34,911,632	34,841,697	34,903,827	34,859,777	34,872,311	34,869,806	34,848,293
2.10	35,408,929	35,588,289	35,612,586	35,524,303	35,455,765	35,442,592	35,773,507
2.15	35,593,789	35,573,283	35,546,979	35,541,052	35,594,161	35,593,196	35,588,427
2.20	35,800,980	35,814,048	35,812,767	35,796,107	35,816,197	35,855,529	35,779,272
2.25	36,365,506	36,409,191	36,358,513	36,315,345	36,447,274	36,358,253	36,367,898
2.30	36,507,368	36,520,939	36,450,979	36,476,680	36,489,324	36,521,086	36,502,350
2.35	37,594,974	37,608,049	37,821,695	37,481,871	37,757,779	38,255,064	37,646,378
2.40	37,612,535	37,581,805	37,623,151	37,750,389	37,772,357	37,679,353	37,628,428
2.45	37,961,192	37,984,196	38,035,539	38,066,159	38,083,166	38,069,199	38,222,939
2.50	40,642,910	40,324,311	40,362,227	40,372,469	40,624,265	40,351,213	40,367,814
2.55	38,518,065	38,589,648	38,605,540	38,433,088	38,470,872	38,471,605	38,552,316
2.60	38,934,783	38,978,412	39,002,634	38,982,426	38,951,340	38,920,711	38,979,183
2.65	39,364,983	39,362,928	39,379,835	39,255,700	39,370,365	39,366,675	39,299,385
2.70	39,776,733	39,852,245	39,878,813	39,854,205	40,008,088	39,787,636	39,747,942
2.75	40,389,260	40,570,338	40,582,862	40,456,281	40,372,065	40,499,383	40,447,533
2.80	40,469,088	40,492,088	40,472,525	40,441,534	40,415,094	40,492,180	40,462,656
2.85	41,299,466	41,333,327	41,247,462	41,123,955	41,248,343	41,249,153	41,264,889
2.90	41,272,469	41,438,060	41,419,282	41,587,604	41,424,559	41,281,581	41,320,325
2.95	41,827,405	41,714,426	41,713,655	41,694,931	41,595,881	41,645,790	41,735,552
3.00	102,827,349	101,939,682	102,709,657	103,205,445	101,137,205	101,756,722	102,265,037

Table 15: Outer loop iterations for array of size 1,970,626

Factor	Iterations						
2.00	298,990,003	298,084,594	303,001,059	298,529,652	305,965,496	304,645,504	296,880,228
2.05	82,221,080	82,102,738	82,289,300	82,238,130	82,188,186	82,282,797	82,212,510
2.10	84,258,462	83,846,004	83,954,721	83,943,236	84,501,673	84,388,664	83,764,916
2.15	83,956,109	83,948,241	83,846,421	83,957,968	83,936,202	83,912,202	83,909,344
2.20	84,659,894	84,690,525	84,620,579	84,837,219	84,596,518	84,650,922	84,593,582
2.25	85,946,611	85,977,972	85,959,967	86,043,671	85,913,160	86,048,999	86,090,903
2.30	86,394,242	86,457,745	86,380,709	86,550,401	86,368,068	86,605,277	86,435,030
2.35	90,101,573	90,768,065	90,035,703	90,220,216	89,694,128	89,882,181	90,151,642
2.40	89,319,158	89,120,385	89,076,061	89,189,474	89,147,936	89,071,205	89,463,645
2.45	90,224,427	90,417,898	90,320,176	90,455,579	90,237,092	90,316,310	90,393,880
2.50	96,292,642	96,931,375	96,728,515	96,318,611	96,368,190	97,177,519	96,686,798
2.55	91,729,923	91,658,685	91,687,136	91,605,754	91,619,342	91,729,693	91,799,418
2.60	93,614,932	93,730,113	93,671,312	93,873,360	93,782,485	93,632,099	93,743,206
2.65	94,470,006	94,345,613	94,498,722	94,389,768	94,366,570	94,367,420	94,576,049
2.70	95,137,631	95,432,436	95,107,430	95,084,176	95,097,484	95,090,257	95,055,235
2.75	96,673,195	96,275,398	96,309,557	96,652,637	96,518,043	96,922,975	96,645,815
2.80	96,587,295	96,777,945	96,689,571	96,617,418	96,575,690	96,715,513	96,485,621
2.85	98,678,601	98,626,733	98,727,318	98,410,446	98,289,368	98,259,435	98,603,600
2.90	99,240,722	98,931,435	98,984,528	99,288,731	99,216,664	99,334,712	99,192,686
2.95	99,624,756	100,429,608	99,600,822	100,083,094	99,935,207	99,600,353	99,944,685
3.00	314,044,064	313,102,291	317,372,203	313,985,885	320,210,847	319,545,173	311,304,199

Table 16: Outer loop iterations for array of size 4,365,249

F Inner Iteration Counts

Factor	Iterations						
2.00	3,495,906	3,495,906	3,495,906	3,495,906	3,495,906	3,495,906	3,495,906
2.05	3,466,301	3,466,301	3,466,301	3,466,301	3,466,301	3,466,301	3,466,301
2.10	3,433,244	3,433,244	3,433,244	3,433,244	3,433,244	3,433,244	3,433,244
2.15	3,396,588	3,396,588	3,396,588	3,396,588	3,396,588	3,396,588	3,396,588
2.20	3,306,313	3,306,313	3,306,313	3,306,313	3,306,313	3,306,313	3,306,313
2.25	3,289,986	3,289,986	3,289,986	3,289,986	3,289,986	3,289,986	3,289,986
2.30	3,272,324	3,272,324	3,272,324	3,272,324	3,272,324	3,272,324	3,272,324
2.35	3,253,198	3,253,198	3,253,198	3,253,198	3,253,198	3,253,198	3,253,198
2.40	3,232,571	3,232,571	3,232,571	3,232,571	3,232,571	3,232,571	3,232,571
2.45	3,210,427	3,210,427	3,210,427	3,210,427	3,210,427	3,210,427	3,210,427
2.50	3,186,551	3,186,551	3,186,551	3,186,551	3,186,551	3,186,551	3,186,551
2.55	3,160,955	3,160,955	3,160,955	3,160,955	3,160,955	3,160,955	3,160,955
2.60	3,133,417	3,133,417	3,133,417	3,133,417	3,133,417	3,133,417	3,133,417
2.65	3,103,997	3,103,997	3,103,997	3,103,997	3,103,997	3,103,997	3,103,997
2.70	3,075,567	3,075,567	3,075,567	3,075,567	3,075,567	3,075,567	3,075,567
2.75	3,065,981	3,065,981	3,065,981	3,065,981	3,065,981	3,065,981	3,065,981
2.80	3,055,888	3,055,888	3,055,888	3,055,888	3,055,888	3,055,888	3,055,888
2.85	3,045,349	3,045,349	3,045,349	3,045,349	3,045,349	3,045,349	3,045,349
2.90	3,034,248	3,034,248	3,034,248	3,034,248	3,034,248	3,034,248	3,034,248
2.95	3,022,669	3,022,669	3,022,669	3,022,669	3,022,669	3,022,669	3,022,669
3.00	3,010,488	3,010,488	3,010,488	3,010,488	3,010,488	3,010,488	3,010,488

Table 17: Inner loop iterations for array of size 247,959

Factor	Iterations						
2.00	8,449,065	8,449,065	8,449,065	8,449,065	8,449,065	8,449,065	8,449,065
2.05	8,377,266	8,377,266	8,377,266	8,377,266	8,377,266	8,377,266	8,377,266
2.10	8,295,252	8,295,252	8,295,252	8,295,252	8,295,252	8,295,252	8,295,252
2.15	8,202,197	8,202,197	8,202,197	8,202,197	8,202,197	8,202,197	8,202,197
2.20	7,976,988	7,976,988	7,976,988	7,976,988	7,976,988	7,976,988	7,976,988
2.25	7,932,059	7,932,059	7,932,059	7,932,059	7,932,059	7,932,059	7,932,059
2.30	7,882,426	7,882,426	7,882,426	7,882,426	7,882,426	7,882,426	7,882,426
2.35	7,827,585	7,827,585	7,827,585	7,827,585	7,827,585	7,827,585	7,827,585
2.40	7,767,230	7,767,230	7,767,230	7,767,230	7,767,230	7,767,230	7,767,230
2.45	7,701,097	7,701,097	7,701,097	7,701,097	7,701,097	7,701,097	7,701,097
2.50	7,628,418	7,628,418	7,628,418	7,628,418	7,628,418	7,628,418	7,628,418
2.55	7,474,005	7,474,005	7,474,005	7,474,005	7,474,005	7,474,005	7,474,005
2.60	7,446,467	7,446,467	7,446,467	7,446,467	7,446,467	7,446,467	7,446,467
2.65	7,417,047	7,417,047	7,417,047	7,417,047	7,417,047	7,417,047	7,417,047
2.70	7,385,487	7,385,487	7,385,487	7,385,487	7,385,487	7,385,487	7,385,487
2.75	7,351,792	7,351,792	7,351,792	7,351,792	7,351,792	7,351,792	7,351,792
2.80	7,315,718	7,315,718	7,315,718	7,315,718	7,315,718	7,315,718	7,315,718
2.85	7,277,366	7,277,366	7,277,366	7,277,366	7,277,366	7,277,366	7,277,366
2.90	7,236,327	7,236,327	7,236,327	7,236,327	7,236,327	7,236,327	7,236,327
2.95	7,192,771	7,192,771	7,192,771	7,192,771	7,192,771	7,192,771	7,192,771
3.00	7,146,247	7,146,247	7,146,247	7,146,247	7,146,247	7,146,247	7,146,247

Table 18: Inner loop iterations for array of size 556,034

Factor	Iterations						
2.00	16,739,896	16,739,896	16,739,896	16,739,896	16,739,896	16,739,896	16,739,896
2.05	16,570,400	16,570,400	16,570,400	16,570,400	16,570,400	16,570,400	16,570,400
2.10	16,372,266	16,372,266	16,372,266	16,372,266	16,372,266	16,372,266	16,372,266
2.15	15,903,653	15,903,653	15,903,653	15,903,653	15,903,653	15,903,653	15,903,653
2.20	15,797,979	15,797,979	15,797,979	15,797,979	15,797,979	15,797,979	15,797,979
2.25	15,678,775	15,678,775	15,678,775	15,678,775	15,678,775	15,678,775	15,678,775
2.30	15,544,258	15,544,258	15,544,258	15,544,258	15,544,258	15,544,258	15,544,258
2.35	15,392,538	15,392,538	15,392,538	15,392,538	15,392,538	15,392,538	15,392,538
2.40	15,222,100	15,222,100	15,222,100	15,222,100	15,222,100	15,222,100	15,222,100
2.45	14,921,212	14,921,212	14,921,212	14,921,212	14,921,212	14,921,212	14,921,212
2.50	14,848,533	14,848,533	14,848,533	14,848,533	14,848,533	14,848,533	14,848,533
2.55	14,769,082	14,769,082	14,769,082	14,769,082	14,769,082	14,769,082	14,769,082
2.60	14,682,019	14,682,019	14,682,019	14,682,019	14,682,019	14,682,019	14,682,019
2.65	14,587,218	14,587,218	14,587,218	14,587,218	14,587,218	14,587,218	14,587,218
2.70	14,483,696	14,483,696	14,483,696	14,483,696	14,483,696	14,483,696	14,483,696
2.75	14,371,147	14,371,147	14,371,147	14,371,147	14,371,147	14,371,147	14,371,147
2.80	14,248,566	14,248,566	14,248,566	14,248,566	14,248,566	14,248,566	14,248,566
2.85	14,115,888	14,115,888	14,115,888	14,115,888	14,115,888	14,115,888	14,115,888
2.90	13,975,101	13,975,101	13,975,101	13,975,101	13,975,101	13,975,101	13,975,101
2.95	13,931,545	13,931,545	13,931,545	13,931,545	13,931,545	13,931,545	13,931,545
3.00	13,885,021	13,885,021	13,885,021	13,885,021	13,885,021	13,885,021	13,885,021

Table 19: Inner loop iterations for array of size 1,037,375

Factor	Iterations						
2.00	33,679,789	33,679,789	33,679,789	33,679,789	33,679,789	33,679,789	33,679,789
2.05	33,287,615	33,287,615	33,287,615	33,287,615	33,287,615	33,287,615	33,287,615
2.10	32,818,344	32,818,344	32,818,344	32,818,344	32,818,344	32,818,344	32,818,344
2.15	32,007,463	32,007,463	32,007,463	32,007,463	32,007,463	32,007,463	32,007,463
2.20	31,740,324	31,740,324	31,740,324	31,740,324	31,740,324	31,740,324	31,740,324
2.25	31,432,176	31,432,176	31,432,176	31,432,176	31,432,176	31,432,176	31,432,176
2.30	31,076,887	31,076,887	31,076,887	31,076,887	31,076,887	31,076,887	31,076,887
2.35	30,667,719	30,667,719	30,667,719	30,667,719	30,667,719	30,667,719	30,667,719
2.40	30,154,116	30,154,116	30,154,116	30,154,116	30,154,116	30,154,116	30,154,116
2.45	29,963,489	29,963,489	29,963,489	29,963,489	29,963,489	29,963,489	29,963,489
2.50	29,749,882	29,749,882	29,749,882	29,749,882	29,749,882	29,749,882	29,749,882
2.55	29,511,740	29,511,740	29,511,740	29,511,740	29,511,740	29,511,740	29,511,740
2.60	29,245,858	29,245,858	29,245,858	29,245,858	29,245,858	29,245,858	29,245,858
2.65	28,950,768	28,950,768	28,950,768	28,950,768	28,950,768	28,950,768	28,950,768
2.70	28,622,649	28,622,649	28,622,649	28,622,649	28,622,649	28,622,649	28,622,649
2.75	28,369,912	28,369,912	28,369,912	28,369,912	28,369,912	28,369,912	28,369,912
2.80	28,247,331	28,247,331	28,247,331	28,247,331	28,247,331	28,247,331	28,247,331
2.85	28,114,653	28,114,653	28,114,653	28,114,653	28,114,653	28,114,653	28,114,653
2.90	27,970,344	27,970,344	27,970,344	27,970,344	27,970,344	27,970,344	27,970,344
2.95	27,814,510	27,814,510	27,814,510	27,814,510	27,814,510	27,814,510	27,814,510
3.00	27,645,411	27,645,411	27,645,411	27,645,411	27,645,411	27,645,411	27,645,411

Table 20: Inner loop iterations for array of size 1,970,626

Factor	Iterations						
2.00	79,356,252	79,356,252	79,356,252	79,356,252	79,356,252	79,356,252	79,356,252
2.05	78,462,789	78,462,789	78,462,789	78,462,789	78,462,789	78,462,789	78,462,789
2.10	77,368,196	77,368,196	77,368,196	77,368,196	77,368,196	77,368,196	77,368,196
2.15	75,363,815	75,363,815	75,363,815	75,363,815	75,363,815	75,363,815	75,363,815
2.20	74,701,511	74,701,511	74,701,511	74,701,511	74,701,511	74,701,511	74,701,511
2.25	73,920,224	73,920,224	73,920,224	73,920,224	73,920,224	73,920,224	73,920,224
2.30	72,999,698	72,999,698	72,999,698	72,999,698	72,999,698	72,999,698	72,999,698
2.35	71,376,310	71,376,310	71,376,310	71,376,310	71,376,310	71,376,310	71,376,310
2.40	70,907,045	70,907,045	70,907,045	70,907,045	70,907,045	70,907,045	70,907,045
2.45	70,371,277	70,371,277	70,371,277	70,371,277	70,371,277	70,371,277	70,371,277
2.50	69,758,994	69,758,994	69,758,994	69,758,994	69,758,994	69,758,994	69,758,994
2.55	69,062,788	69,062,788	69,062,788	69,062,788	69,062,788	69,062,788	69,062,788
2.60	67,559,826	67,559,826	67,559,826	67,559,826	67,559,826	67,559,826	67,559,826
2.65	67,264,736	67,264,736	67,264,736	67,264,736	67,264,736	67,264,736	67,264,736
2.70	66,936,617	66,936,617	66,936,617	66,936,617	66,936,617	66,936,617	66,936,617
2.75	66,573,323	66,573,323	66,573,323	66,573,323	66,573,323	66,573,323	66,573,323
2.80	66,170,683	66,170,683	66,170,683	66,170,683	66,170,683	66,170,683	66,170,683
2.85	65,727,011	65,727,011	65,727,011	65,727,011	65,727,011	65,727,011	65,727,011
2.90	65,236,337	65,236,337	65,236,337	65,236,337	65,236,337	65,236,337	65,236,337
2.95	64,697,238	64,697,238	64,697,238	64,697,238	64,697,238	64,697,238	64,697,238
3.00	64,102,755	64,102,755	64,102,755	64,102,755	64,102,755	64,102,755	64,102,755

Table 21: Inner loop iterations for array of size 4,365,249

G Sequences

Factor	Gap
2.00	112000, 56000, 28000, 14000, 7000, 3500, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.05	129851, 63342, 30899, 15073, 7353, 3587, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.10	150072, 71463, 34030, 16205, 7717, 3675, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.15	172814, 80379, 37386, 17389, 8088, 3762, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.20	90186, 40994, 18634, 8470, 3850, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.25	100894, 44842, 19930, 8858, 3937, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.30	112601, 48957, 21286, 9255, 4024, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.35	125403, 53363, 22708, 9663, 4112, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.40	139344, 58060, 24192, 10080, 4200, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.45	154455, 63043, 25732, 10503, 4287, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.50	170887, 68355, 27342, 10937, 4375, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.55	188656, 73983, 29013, 11378, 4462, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.60	207922, 79970, 30758, 11830, 4550, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.65	228671, 86291, 32563, 12288, 4637, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.70	92996, 34443, 12757, 4725, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.75	100072, 36390, 13233, 4812, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.80	107564, 38416, 13720, 4900, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.85	115436, 40504, 14212, 4987, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.90	123769, 42679, 14717, 5075, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.95	132511, 44919, 15227, 5162, 1750, 701, 301, 132, 57, 23, 10, 4, 1
3.00	141750, 47250, 15750, 5250, 1750, 701, 301, 132, 57, 23, 10, 4, 1

Table 22: Sequences for array of size 247,959

Factor	Gap
2.00	224000, 112000, 56000, 28000, 14000, 7000, 3500, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.05	266194, 129851, 63342, 30899, 15073, 7353, 3587, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.10	315151, 150072, 71463, 34030, 16205, 7717, 3675, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.15	371550, 172814, 80379, 37386, 17389, 8088, 3762, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.20	198409, 90186, 40994, 18634, 8470, 3850, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.25	227011, 100894, 44842, 19930, 8858, 3937, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.30	258982, 112601, 48957, 21286, 9255, 4024, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.35	294697, 125403, 53363, 22708, 9663, 4112, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.40	334425, 139344, 58060, 24192, 10080, 4200, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.45	378414, 154455, 63043, 25732, 10503, 4287, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.50	427217, 170887, 68355, 27342, 10937, 4375, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.55	188656, 73983, 29013, 11378, 4462, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.60	207922, 79970, 30758, 11830, 4550, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.65	228671, 86291, 32563, 12288, 4637, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.70	251089, 92996, 34443, 12757, 4725, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.75	275198, 100072, 36390, 13233, 4812, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.80	301179, 107564, 38416, 13720, 4900, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.85	328992, 115436, 40504, 14212, 4987, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.90	358930, 123769, 42679, 14717, 5075, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.95	390907, 132511, 44919, 15227, 5162, 1750, 701, 301, 132, 57, 23, 10, 4, 1
3.00	425250, 141750, 47250, 15750, 5250, 1750, 701, 301, 132, 57, 23, 10, 4, 1

Table 23: Sequences for array of size 556,034

Factor	Gap
2.00	448000, 224000, 112000, 56000, 28000, 14000, 7000, 3500, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.05	545697, 266194, 129851, 63342, 30899, 15073, 7353, 3587, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.10	661817, 315151, 150072, 71463, 34030, 16205, 7717, 3675, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.15	371550, 172814, 80379, 37386, 17389, 8088, 3762, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.20	436499, 198409, 90186, 40994, 18634, 8470, 3850, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.25	510774, 227011, 100894, 44842, 19930, 8858, 3937, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.30	595658, 258982, 112601, 48957, 21286, 9255, 4024, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.35	692537, 294697, 125403, 53363, 22708, 9663, 4112, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.40	802620, 334425, 139344, 58060, 24192, 10080, 4200, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.45	378414, 154455, 63043, 25732, 10503, 4287, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.50	427217, 170887, 68355, 27342, 10937, 4375, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.55	481072, 188656, 73983, 29013, 11378, 4462, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.60	540597, 207922, 79970, 30758, 11830, 4550, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.65	605978, 228671, 86291, 32563, 12288, 4637, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.70	677940, 251089, 92996, 34443, 12757, 4725, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.75	756794, 275198, 100072, 36390, 13233, 4812, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.80	843301, 301179, 107564, 38416, 13720, 4900, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.85	937627, 328992, 115436, 40504, 14212, 4987, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.90	358930, 123769, 42679, 14717, 5075, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.95	390907, 132511, 44919, 15227, 5162, 1750, 701, 301, 132, 57, 23, 10, 4, 1
3.00	425250, 141750, 47250, 15750, 5250, 1750, 701, 301, 132, 57, 23, 10, 4, 1

Table 24: Sequences for array of size 1,037,375

Factor	Gap
2.00	896000, 448000, 224000, 112000, 56000, 28000, 14000, 7000, 3500, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.05	1118678, 545697, 266194, 129851, 63342, 30899, 15073, 7353, 3587, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.10	1389815, 661817, 315151, 150072, 71463, 34030, 16205, 7717, 3675, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.15	798832, 371550, 172814, 80379, 37386, 17389, 8088, 3762, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.20	960297, 436499, 198409, 90186, 40994, 18634, 8470, 3850, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.25	1149241, 510774, 227011, 100894, 44842, 19930, 8858, 3937, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.30	1370013, 595658, 258982, 112601, 48957, 21286, 9255, 4024, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.35	1627461, 692537, 294697, 125403, 53363, 22708, 9663, 4112, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.40	802620, 334425, 139344, 58060, 24192, 10080, 4200, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.45	927114, 378414, 154455, 63043, 25732, 10503, 4287, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.50	1068042, 427217, 170887, 68355, 27342, 10937, 4375, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.55	1226733, 481072, 188656, 73983, 29013, 11378, 4462, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.60	1405552, 540597, 207922, 79970, 30758, 11830, 4550, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.65	1605841, 605978, 228671, 86291, 32563, 12288, 4637, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.70	1830438, 677940, 251089, 92996, 34443, 12757, 4725, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.75	756794, 275198, 100072, 36390, 13233, 4812, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.80	843301, 301179, 107564, 38416, 13720, 4900, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.85	937627, 328992, 115436, 40504, 14212, 4987, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.90	1040897, 358930, 123769, 42679, 14717, 5075, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.95	1153175, 390907, 132511, 44919, 15227, 5162, 1750, 701, 301, 132, 57, 23, 10, 4, 1
3.00	1275750, 425250, 141750, 47250, 15750, 5250, 1750, 701, 301, 132, 57, 23, 10, 4, 1

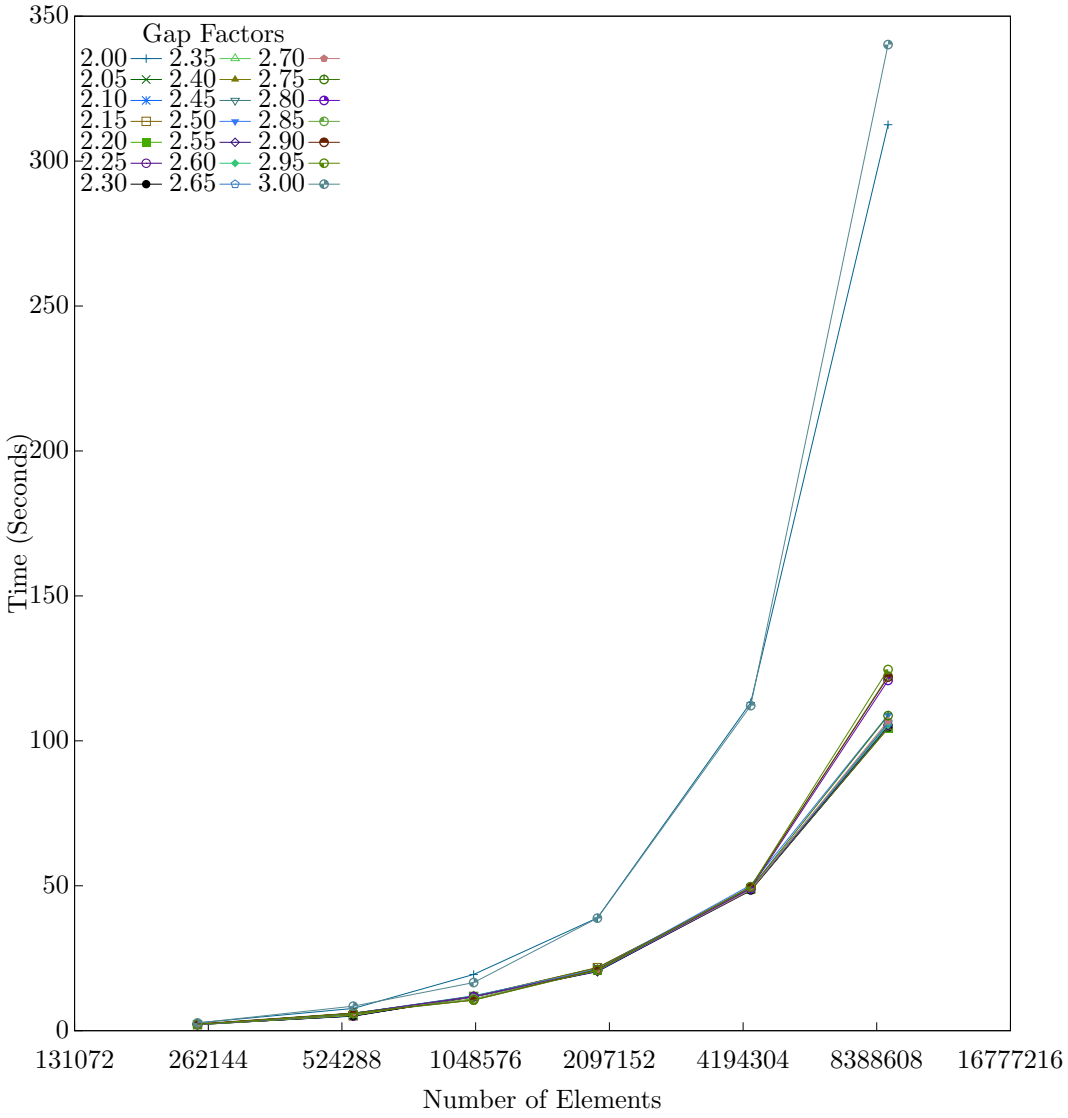
Table 25: Sequences for array of size 1,970,626

Factor	Gap
2.00	1792000, 896000, 448000, 224000, 112000, 56000, 28000, 14000, 7000, 3500, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.05	2293289, 1118678, 545697, 266194, 129851, 63342, 30899, 15073, 7353, 3587, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.10	2918611, 1389815, 661817, 315151, 150072, 71463, 34030, 16205, 7717, 3675, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.15	1717488, 798832, 371550, 172814, 80379, 37386, 17389, 8088, 3762, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.20	2112653, 960297, 436499, 198409, 90186, 40994, 18634, 8470, 3850, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.25	2585792, 1149241, 510774, 227011, 100894, 44842, 19930, 8858, 3937, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.30	3151029, 1370013, 595658, 258982, 112601, 48957, 21286, 9255, 4024, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.35	1627461, 692537, 294697, 125403, 53363, 22708, 9663, 4112, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.40	1926288, 802620, 334425, 139344, 58060, 24192, 10080, 4200, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.45	2271429, 927114, 378414, 154455, 63043, 25732, 10503, 4287, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.50	2670105, 1068042, 427217, 170887, 68355, 27342, 10937, 4375, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.55	3128169, 1226733, 481072, 188656, 73983, 29013, 11378, 4462, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.60	1405552, 540597, 207922, 79970, 30758, 11830, 4550, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.65	1605841, 605978, 228671, 86291, 32563, 12288, 4637, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.70	1830438, 677940, 251089, 92996, 34443, 12757, 4725, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.75	2081183, 756794, 275198, 100072, 36390, 13233, 4812, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.80	2361242, 843301, 301179, 107564, 38416, 13720, 4900, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.85	2672236, 937627, 328992, 115436, 40504, 14212, 4987, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.90	3018601, 1040897, 358930, 123769, 42679, 14717, 5075, 1750, 701, 301, 132, 57, 23, 10, 4, 1
2.95	3401866, 1153175, 390907, 132511, 44919, 15227, 5162, 1750, 701, 301, 132, 57, 23, 10, 4, 1
3.00	3827250, 1275750, 425250, 141750, 47250, 15750, 5250, 1750, 701, 301, 132, 57, 23, 10, 4, 1

Table 26: Sequences for array of size 4,365,249

H Run Times

Figure 1: Time v. Size



I Box & Whiskers

Figure 2: Box & Whiskers