Some computational results for generalized pressure Schur complement eigenvalues of the surface Stokes problem

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1 Bilinear forms and matrices

$$\langle \mathbf{A}\,\bar{\mathbf{u}}, \bar{\mathbf{v}} \rangle = \int_\Gamma \big(E_s(\mathbf{u}_h) : E_s(\mathbf{v}_h) + \mathbf{u}_h \cdot \mathbf{v}_h + \tau\, u_{h,N}\, v_{h,N} \big)\, \mathrm{d}s + \rho_u \int_{\Omega_h^\Gamma} ([\nabla \mathbf{u}_h]\, \hat{\mathbf{n}}) \cdot ([\nabla \mathbf{v}_h]\, \hat{\mathbf{n}})\, \mathrm{d}\mathbf{x}, \tag{1}$$

$$\langle \mathbf{B}\,\bar{\mathbf{u}}, \bar{\mathbf{q}} \rangle = \tag{2}$$

We set

2 Solution description

$n_{\mathbf{A}}$ is the number of velocity d.o.f, $n_{\mathbf{S}}$ is the number of pressure d.o.f., $0 = \lambda_1 < \lambda_2 \leq \cdots \leq \lambda_{n_{\mathbf{S}}}$ is the (approximate) spectrum of $\mathbf{S}$, and r_i are the residual norm, i.e.

$$r_i \coloneqq \|\mathbf{S}\,\mathbf{x} - \lambda_i\, \mathbf{M}\,\mathbf{x}\|_2$$

Note: $h = 2.6 \times 10^{-2}$ for $\mathbf{P}_2 - P_1$ (the last mesh level) was computed w/ $\epsilon = 10^{-5}$, and everything else w/ $\epsilon = 10^{-4}$. Apparently $\epsilon = 10^{-4}$ did not work for the finest mesh level: λ_2 turned out to be negative for full and normal stabs.

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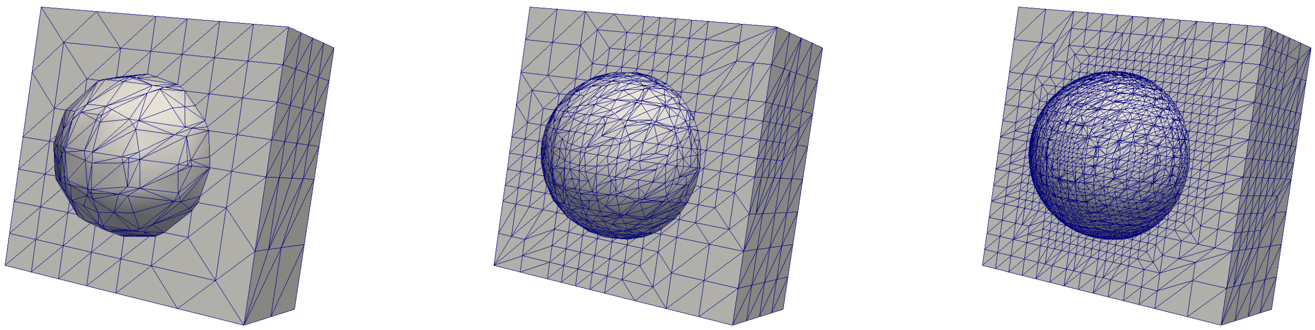

(a) $h = 8.33 \times 10^{-1}$ (b) $h = 4.17 \times 10^{-1}$ (c) $h = 2.08 \times 10^{-1}$

Figure 1: First three mesh levels for sphere

Table 1: $\mathbf{P}_1 - P_1$

h	$n_{\mathbf{A}}$	$n_{\mathbf{S}}$	$\mathbf{S}_0$		$\mathbf{S}_n$		$\mathbf{S}_{\text{full}}$	
			λ_2	$\lambda_{n_{\mathbf{S}}}$	λ_2	$\lambda_{n_{\mathbf{S}}}$	λ_2	$\lambda_{n_{\mathbf{S}}}$
8.33×10^{-1}	153	51	1.32×10^{-2}	1.42	7.48×10^{-1}	1.13	9.58×10^{-1}	1.06
4.17×10^{-1}	570	190	5.12×10^{-3}	1.04	5.77×10^{-1}	1.	8.54×10^{-1}	1.
2.08×10^{-1}	1992	664	4.4×10^{-3}	7.93×10^{-1}	3.87×10^{-1}	1.	6.71×10^{-1}	1.
1.04×10^{-1}	8292	2764	2.01×10^{-3}	7.75×10^{-1}	2.19×10^{-1}	1.	5.82×10^{-1}	1.
5.21×10^{-2}	32736	10912	6.04×10^{-5}	9.81×10^{-1}	1.17×10^{-1}	1.	5.37×10^{-1}	1.
2.6×10^{-2}	131592	43864	3.53×10^{-5}	8.67×10^{-1}	5.72×10^{-2}	1.	5.16×10^{-1}	1.
1.3×10^{-2}	525864	175288	2.16×10^{-6}	7.34×10^{-1}	2.84×10^{-2}	1.	5.04×10^{-1}	1.
h	$n_{\mathbf{A}}$	$n_{\mathbf{S}}$	$\mathbf{S}_0$		$\mathbf{S}_n$		$\mathbf{S}_{\text{full}}$	
			r_2	$r_{n_{\mathbf{S}}}$	r_2	$r_{n_{\mathbf{S}}}$	r_2	$r_{n_{\mathbf{S}}}$
8.33×10^{-1}	153	51	$2. \times 10^{-17}$	$8. \times 10^{-10}$	$1. \times 10^{-7}$	$4. \times 10^{-8}$	$3. \times 10^{-7}$	$1. \times 10^{-7}$
4.17×10^{-1}	570	190	$3. \times 10^{-18}$	$3. \times 10^{-10}$	$6. \times 10^{-7}$	$1. \times 10^{-3}$	$1. \times 10^{-7}$	$8. \times 10^{-4}$
2.08×10^{-1}	1992	664	$2. \times 10^{-17}$	$6. \times 10^{-9}$	$6. \times 10^{-8}$	$9. \times 10^{-4}$	$2. \times 10^{-10}$	$8. \times 10^{-3}$
1.04×10^{-1}	8292	2764	$6. \times 10^{-16}$	$9. \times 10^{-10}$	$2. \times 10^{-8}$	$2. \times 10^{-3}$	$2. \times 10^{-8}$	$3. \times 10^{-3}$
5.21×10^{-2}	32736	10912	$8. \times 10^{-19}$	$1. \times 10^{-11}$	$1. \times 10^{-5}$	$7. \times 10^{-4}$	$1. \times 10^{-3}$	$7. \times 10^{-4}$
2.6×10^{-2}	131592	43864	$5. \times 10^{-18}$	$2. \times 10^{-12}$	$8. \times 10^{-9}$	$7. \times 10^{-4}$	$3. \times 10^{-8}$	$9. \times 10^{-4}$
1.3×10^{-2}	525864	175288	$5. \times 10^{-22}$	$8. \times 10^{-14}$	$8. \times 10^{-12}$	$2. \times 10^{-4}$	$3. \times 10^{-5}$	$4. \times 10^{-4}$

Table 2: $\mathbf{P}_2 - P_1$

h	$n_{\mathbf{A}}$	$n_{\mathbf{S}}$	$\mathbf{S}_0$		$\mathbf{S}_n$		$\mathbf{S}_{\text{full}}$	
			λ_2	$\lambda_{n_{\mathbf{S}}}$	λ_2	$\lambda_{n_{\mathbf{S}}}$	λ_2	$\lambda_{n_{\mathbf{S}}}$
8.33×10^{-1}	789	51	3.22×10^{-1}	1.73	8.27×10^{-1}	1.17	9.68×10^{-1}	1.07
4.17×10^{-1}	3240	190	9.17×10^{-2}	1.08	6.45×10^{-1}	1.	8.56×10^{-1}	1.
2.08×10^{-1}	11718	664	1.78×10^{-1}	8.31×10^{-1}	5.49×10^{-1}	1.	6.75×10^{-1}	1.
1.04×10^{-1}	48762	2764	1.04×10^{-1}	8.13×10^{-1}	5.14×10^{-1}	1.	5.82×10^{-1}	1.
5.21×10^{-2}	193014	10912	2.99×10^{-3}	9.89×10^{-1}	5.02×10^{-1}	1.	5.34×10^{-1}	1.
2.6×10^{-2}	775998	43864	1.17×10^{-3}	7.9×10^{-1}	4.96×10^{-1}	1.	5.17×10^{-1}	1.
h	$n_{\mathbf{A}}$	$n_{\mathbf{S}}$	$\mathbf{S}_0$		$\mathbf{S}_n$		$\mathbf{S}_{\text{full}}$	
			r_2	$r_{n_{\mathbf{S}}}$	r_2	$r_{n_{\mathbf{S}}}$	r_2	$r_{n_{\mathbf{S}}}$
8.33×10^{-1}	789	51	$4. \times 10^{-9}$	$4. \times 10^{-10}$	$2. \times 10^{-8}$	$2. \times 10^{-7}$	$2. \times 10^{-7}$	$3. \times 10^{-7}$
4.17×10^{-1}	3240	190	$6. \times 10^{-12}$	$4. \times 10^{-9}$	$7. \times 10^{-10}$	$4. \times 10^{-2}$	$3. \times 10^{-10}$	$4. \times 10^{-2}$
2.08×10^{-1}	11718	664	$1. \times 10^{-10}$	$3. \times 10^{-9}$	$2. \times 10^{-6}$	$7. \times 10^{-3}$	$2. \times 10^{-9}$	$1. \times 10^{-2}$
1.04×10^{-1}	48762	2764	$1. \times 10^{-11}$	$9. \times 10^{-10}$	$2. \times 10^{-5}$	$2. \times 10^{-3}$	$2. \times 10^{-7}$	$2. \times 10^{-3}$
5.21×10^{-2}	193014	10912	$2. \times 10^{-16}$	$3. \times 10^{-12}$	$7. \times 10^{-5}$	$1. \times 10^{-3}$	$5. \times 10^{-7}$	$2. \times 10^{-3}$
2.6×10^{-2}	775998	43864	$7. \times 10^{-18}$	$5. \times 10^{-12}$	$4. \times 10^{-8}$	$3. \times 10^{-4}$	$2. \times 10^{-8}$	$6. \times 10^{-4}$