Notes for MAT102

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Contents

1 11. Functions of two variables

$$\text{Gradient: } \Delta f(a,b) = (f_x(a,b), f_y(a,b))$$

$$\text{Linear approximation: } f(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(x-b)$$

$$\text{Hessian Matrix: } \begin{pmatrix} f_{xx} & f_{yx} \\ f_{xy} & f_{yy} \end{pmatrix}$$

Second order approximation:

$$f(x,y) = f(a,b)+\Delta f(a,b) \begin{pmatrix} x-a \\ y-b \end{pmatrix}+\frac{1}{2} \begin{pmatrix} x-a & y-b \end{pmatrix} \begin{pmatrix} f_{xx}(a,b) & f_{yx}(a,b) \\ f_{xy}(a,b) & f_{yy}(a,b) \end{pmatrix} \begin{pmatrix} x-a \\ y-b \end{pmatrix}$$

For classifying statinary points, you look at the Hessian matrix.

$$\Delta = \begin{vmatrix} f_{xx}(a,b) & f_{yx}(a,b) \\ f_{xy}(a,b) & f_{yy}(a,b) \end{vmatrix} = f_{xx}(a,b)f_{yy} - f_{yxx}(a,b)^2$$

a) if $\Delta > 0$ and $f_{xx}(a,b) > 0$, (a,b) is a local minimum.
b) if $\Delta > 0$ and $f_{xx}(a,b) < 0$, (a,b) is a local maximum.
c) if $\Delta < 0$, (a,b) is a saddle point
d) if $\Delta = 0$, all possibilities are still open.

2 14. Sampling from discrete distributions

$$\text{Mean value: } \overline{X}\frac{1}{n}\sum_{i=1}^{n} X_i$$

$$\text{Empirical variance: } S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$$

The standard deviation is $S = \sqrt{S^2}$

The expectation E(X) of X is $E(X) = \sum_x x \cdot P(X = x)$

The variation of a stochastic variable X is

$$Var(X) = \sum_x (x - E(X))^2 \cdot P(X = x)$$

The standard deviation of X is $\sqrt{Var(X)}$

Binomial coefficient:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

On the calculator you will find this as nCr, where n is n and k is r.

Let $X \sim bin(n, p)$. Then:

$$\text{Binomial distribution: } P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

You can find the expectation of a binomial distrubution $E(X)$ using the formula:

$$E(X) = np$$

You can find the variance of a binomial distrubution $Var(X)$ using the formula:

$$Var(X) = np(1-p)$$

Let $X \sim hypergeom(N, M, n)$. Then:

$$P(X = x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$$

The veriance of a hypergeometrical distribution:

$$n\frac{K}{N}\frac{(N-K)}{N}\frac{N-n}{N-1}$$

- N is the population size
- K is the number of success states in the population
- n is the number of draws

- k is the number of observed successes
- $\binom{a}{b}$ is a binomial coefficient, (nCr)

Poisson distribution:

$$P(X = x) = \frac{(\lambda t)^x}{x!} e^{-\lambda t}$$

Let $X \sim Po(\lambda t)$

1. X is probability distribution
2. $E(X) = \lambda t$
3. $Var(X) = \lambda t$

3 16. Probability

Let A and B be two sets of possible outcomes in the same experiment (or subsets of a given measure space). Then we define the conditional probability of A given B by the formula:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

This formula makes sense as long as $P(B) \neq 0$, and we require this to define the conditional probability.