# Senior Project Proposal 2102490 Year 2017

## Analysis and Design of Planar Phased Array Antenna for 5 GHz Applications

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## Contents

# 1    Introduction

Nowadays, antennas have been using in every wireless communication systems. It has been found out that microstrip patch is one of the most popular[1] in the world. Also, the patch antenna is cost-effective and easy to fabricate[2]. Many design approaches have been revealed for antenna engineers to follow the procedure. In order to obtain the precise dimension, a deep analysis might've been engaged.

A phased array is the set of antennas that could be a formation as a line or a planar which could steer its beam electrically[3] which means that the direction of the antenna can be controlled without moving any of the mechanic parts. Therefore, this proposal is aiming at two of very interesting technologies in this era.

# 2    Objectives

The propose of this work is mainly focused on analyzing and designing the 5.6 GHz planar phased array antenna based on methods which have been analyzed, formulated or derived in the past.

# 3    Methodology

This section will provide a full analysis of designing a microstrip antenna and a phased array antenna.

## 3.1    Basic Dimension Design of The Microstrip Antenna

a dimension of microstrip antenna can be designed easily with this expressions[4].

$$L = \frac{c}{2f_0\sqrt{\epsilon_r \mu_r}} - 2\Delta l W = \frac{c}{2f_0} \tag{1}$$

where $L$ and $W$ are the dimensions of an antenna

$f_0$ is the resonance frequency

$\epsilon_r$ is the relative dielectric constant

$\mu_r$ is the relative magnetic constant

$c$ is the speed of light in free space

$\Delta l$ is the fringing effect at the edge of the antenna

However from [3] the best width should be

$$W = \frac{1}{2f_r\sqrt{\mu_0\epsilon_0}}\sqrt{\frac{2}{\epsilon_r + 1}} = \frac{v_0}{2f_r}\sqrt{\frac{2}{\epsilon_r + 1}} \tag{2}$$

where $v_0$ is the free-space velocity of light

## 3.2    Transmission Line Model

Transmission Line Model is considered as the easiest way to analyze the description of the rectangular microstrip patch.
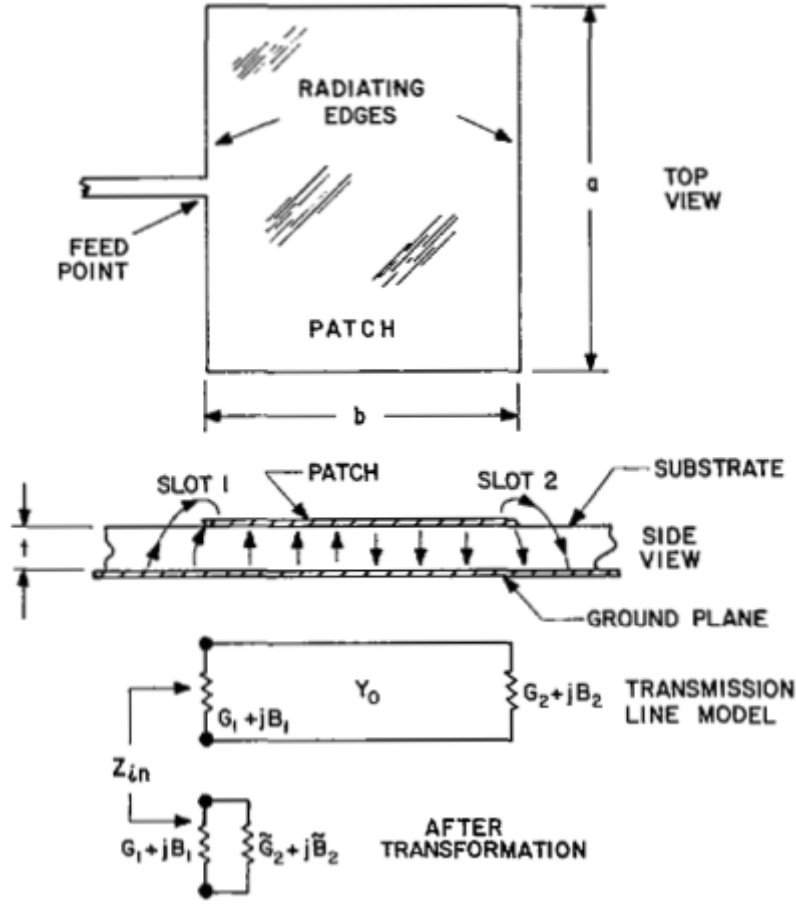


Figure 1: The antenna at top view, side view, and its transmission line model[5]

### 3.2.1    Slot admittance

The slot admittance is given by [5]

$$G_1 + jB_1 \cong \frac{\pi a}{\lambda_0 z_0}[1 + j(1 - 0.636 \ln k_0 w)] \tag{3}$$

where $a$ is the length of the patch antenna, $\lambda_0$ is free space wavelength, $k_0 = frac2\pi\lambda_0$, and $w$ is the slot width which is approximately equals to the thickness of the substrate $t$

### 3.2.2    Characteristic admittance

Assume that there is no field variation along the edge of plate, so the characteristic admittance is given by [5]

$$Y_0 = \frac{a\sqrt{\epsilon_r}}{tz_0} \tag{4}$$

where $t$ is the substrate thickness and the impedance of free space which is $\sqrt{\frac{\mu_0}{\epsilon_0}}$

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### 3.2.3 Total Admittance & Resistance at Resonance frequency

By using Smith chart to get the length that will reflute out the imaginary part. Then, the total impedance would be

$$Y_{in} = 2G_1 \tag{5}$$

Typically, $b$ should be at $0.48\lambda_d$ to $0.49\lambda_d$ because of the imaginary part reflution and the compensation of the fringing effect that cause an extra effective length. Also, $a$ should be around $0.5\lambda_0$ as well to get the best power radiation.

After complete the calculation of the admittance from the above parameters, so that the admittance $G_1 = 0.00417$ mhos. Then the input impedance of the antenna would be

$$Z_{in} = \frac{1}{2G_1} = 120 \quad \Omega \tag{6}$$

### 3.2.4 Resonance frequency

The resonant frequency is found from

$$f_r = \frac{c}{\lambda_d\sqrt{\epsilon_r}} = q\frac{c}{2b\sqrt{\epsilon_r}} \tag{7}$$

where $q$ is the accurary of the resonant frequency and could easily determined by measuring $f_r$ [5]

### 3.2.5 Summary

The transmission line model is very easy to design and calculate parameters. However, the transmission line model is hardly to adapt with the other shape of the patch or. Also, this model is lack of accurate data. In order to find more precise infomation about the antenna, the Cavity model will be introduced in the next section which is much more accurate than this one.[5, 4]

## 3.3 Cavity Line Model

The cavity model has more accurate formulation for the input impedance, resonance with a little increase of mathematical complexity.[5]
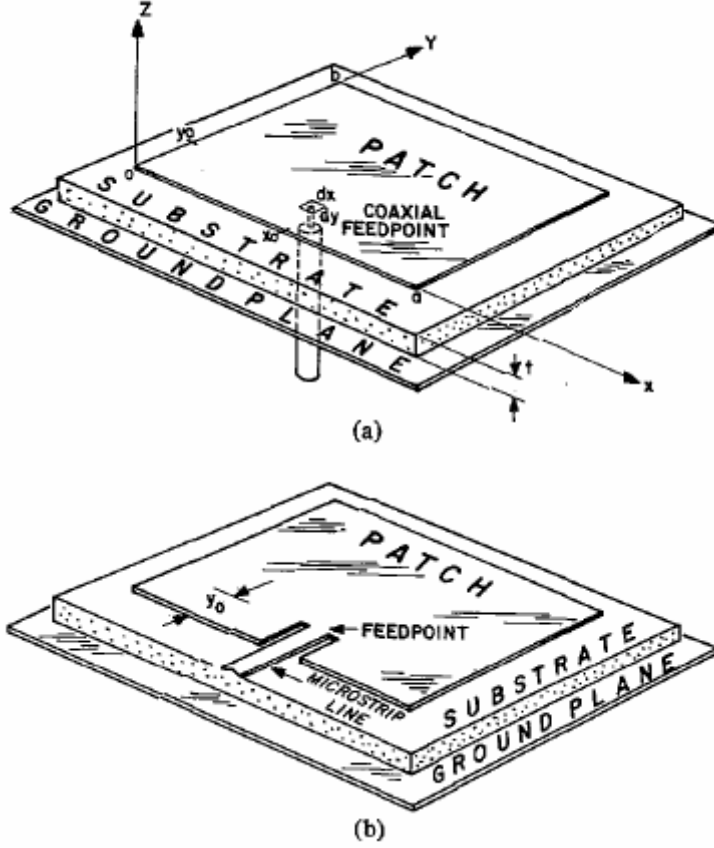


Figure 2: (a) Microstrip patch with inset coaxial feed (b) Microstrip patch with inset transmission-line feed

### 3.3.1 General form of electric field

Considering a rectangular patch with width $a$ and length $b$ over a ground plane with $t$ substrate thickness and a dielectric constant $\epsilon_r$. With the relatively short thickness of the substrate, the electric field will be in z-direction with $TM_{mn}$ interior modes so that[5]

$$E_z = \sum_m \sum_n A_{mn} e_{mn}(x, y) \tag{8}$$

where $A_{mn}$ are the mode amplitude coefficients and $e_{mn}$ are z direction electric mode vectors. However, the mode that is used in the antenna is just only $TM_{10}$ so the equation can easily write as

$$E_z = A_{10} e_{10}(x, y) \tag{9}$$

### 3.3.2 Nonradiating cavity with perfect open-circuit wall equation of electric field

With a calculation from boundary condition, it could be found that

$$e_{mn}(x, y) = \frac{\chi_{mn}}{\sqrt{\epsilon a b t}} \cos k_n x \cos k_m y \tag{10}$$

with

$$\chi_{mn} = \begin{cases} 1, & m = 0 \quad \text{and} \quad n = 0 \\ \sqrt{2}, & m = 0 \quad \text{or} \quad n = 0 \\ 2, & m \neq 0 \quad \text{and} \quad n \neq 0 \end{cases} \tag{11}$$

### 3.3.3  Wavenumber

The homogenous wave function and the eigenvalues must be complied with the seperation equation

$$k_{mn}^2 = \omega_{mn}^2 \mu\epsilon = k_n^2 + k_m^2 \tag{12}$$

in the case of nonradiating cavity

$$k_n = \frac{n\pi}{a} k_m = \frac{m\pi}{b} \tag{13}$$

### 3.3.4  Nonradiating cavity with perfect open-circuit wall equation of magnetic field

From Maxwell-Faraday equation

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{14}$$

or in phasor form,

$$\nabla \times \vec{E} = -j\omega \vec{B} \tag{15}$$

after substitute the electric field equation, the magnetic field will be

$$\vec{h}_{mn} = \frac{1}{j\omega\mu} \frac{\chi_{mn}}{\sqrt{\epsilon abt}} (\vec{x} k_m \cos k_n x \sin k_m y - \vec{y} k_n \sin k_n x \cos k_m y) \tag{16}$$

For nonradiating case, the boundary condition $\vec{n} \times \vec{h}_{mn} = 0$ is satisfied on every walls. But, in radiating case, $\vec{h}_{mn}$ will not have a zero tangential on the cavity sidewall anymore. However, those pertubation from radiating effect cause just a little error on $e_{mn}$[5]

### 3.3.5  Mode Coefficient

From z-direction current, with the current probe $I_0$ at the location $(x_0, y_0)$ as in the figures illustrates in 3.3. The coefficients from each mode can be obtained from

$$A_{mn} = \frac{j\sqrt{\mu\epsilon}k}{k^2 - k_{mn}^2} \int \int \int \vec{J} \cdot \vec{e}_{mn} dv \tag{17}$$

$$A_{mn} = jI_0 \sqrt{\frac{\mu t}{ab}} \frac{k\chi_{mn}}{k^2 - k_{mn}^2} G_{mn} \cos k_m y_0 \cos k_n x_0 \tag{18}$$

where

$$G_{mn} = \frac{\sin(n\pi d_x/2a)}{n\pi d_x/2a} \frac{\sin(m\pi d_y/2b)}{m\pi d_y/2b} \tag{19}$$

and

$$k_{mn} = \widetilde{\omega}\sqrt{\mu\epsilon} \tag{20}$$

$\widetilde{\omega}$ is the complex resonance frequency of the $mn$th mode which could found by 12

### 3.3.6 Complete Electric Field form

After combining 8 10 18 altogether, the complete electric field form will be

$$E_z(x,y) = jI_0Z_0k \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\psi_{mn}(x,y)\psi_{mn}(x_0,y_0)}{k^2 - k_{mn}^2} G_{mn} \tag{21}$$

where $Z_0 = \sqrt{\mu\epsilon}$, $k = \omega\sqrt{\mu\epsilon}$, $k_{mn}^2 = k_m^2 + k_n^2$ and

$$\begin{aligned} \psi_{mn} &= \frac{\chi_{mn}}{\sqrt{ab}} \cos k_n x \cos k_m y \\ &= \frac{\chi_{mn}}{\sqrt{ab}} \cos \frac{n\pi x}{a} \cos \frac{m\pi y}{b} \end{aligned} \tag{22}$$

### 3.3.7 Voltage at the feeding point

From $E = -\nabla V$, or rewrite as $V = -Ed$, the voltage at the feeding point will be

$$\begin{aligned} V_{in} &= -tE_z(x_0,y_0) \\ &= -jI_0Z_0k \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\psi_{mn}^2(x_0,y_0)}{k^2 - k_{mn}^2} G_{mn} \end{aligned} \tag{23}$$

### 3.3.8 Input Resistance

From Ohm's law, $V = IR$, the input impedance is

$$Z_{in} = \frac{V_{in}}{I_0} = -jZ_0k \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\psi_{mn}^2(x_0,y_0)}{k^2 - k_{mn}^2} G_{mn} \tag{24}$$

### 3.3.9 Summary

From this model, cavity model, it's found that this has more accurary than the transmission line because it has less approximation and it has more calculation complexity

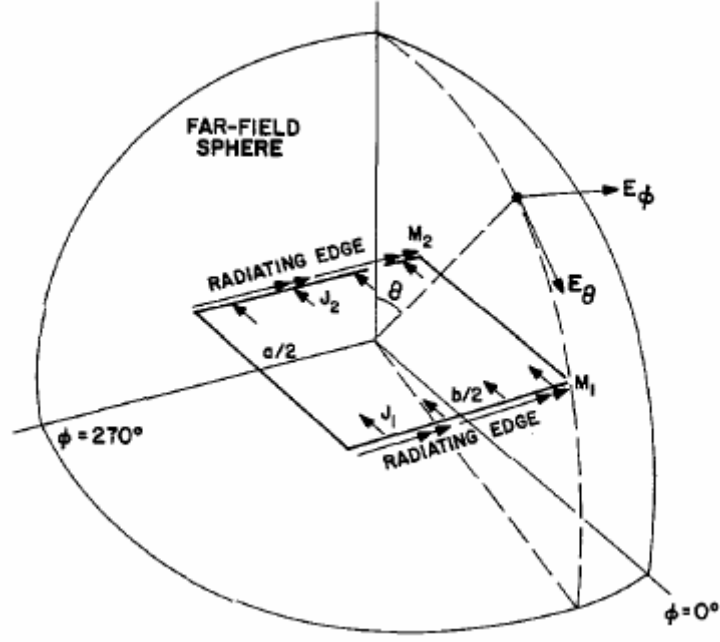## 3.4  Radiation Pattern of Rectangular Patch



Figure 3: Geometry Far-field pattern of a rectangular microstrip [5]

This section will show the far-field radiation pattern of a rectangular microstrip patch on $TM_{10}$ mode at the radiating edge[5]

$$E_\theta = -\frac{jV_0k_0ae^{-jk_0r}}{\pi r}\cos(kt\cos\theta)\frac{\sin[k_0\frac{a}{2}\sin\theta\sin\phi]}{k_0\frac{a}{2}\sin\theta\sin\phi}\cos(k_0\frac{b}{2}\sin\theta\cos\phi)\cos\phi,(0\leq\theta\leq\frac{\pi}{2}) \quad (25)$$

$$E_\phi = \frac{jV_0k_0ae^{-jk_0r}}{\pi r}\cos(kt\cos\theta)\frac{\sin[k_0\frac{a}{2}\sin\theta\sin\phi]}{k_0\frac{a}{2}\sin\theta\sin\phi}\cos(k_0\frac{b}{2}\sin\theta\cos\phi)\cos\theta\sin\phi,(0\leq\theta\leq\frac{\pi}{2}) \quad (26)$$

## 3.5   Q Factor

There are 4 main loss that should be considered[4]

- $Q_{rad}$ is radiation loss due to a loss which propagates into a space

$$Q_{rad} = \frac{3}{16} \frac{\epsilon_r}{p} \frac{a_e}{b_e} \frac{\lambda_0}{h} \frac{1}{1 - \frac{1}{\epsilon_r \mu_r} + \frac{2}{5\epsilon_r^2 \mu_r^2}} \tag{27}$$

where $a_e, b_e, h$ is the effective length, width and thickness of the antenna respectively, $p$ is the ratio of the power that radiated by the patch antenna to the power radiated by an equivalent dipole[4]

- $Q_{sw}$ is surface-wave loss which represents the amount of power coupled into space waves

$$Q_{sw} = Q_{rad}(\frac{\eta_r^0}{1 - \eta_r^0}) \tag{28}$$

where $\eta_r^0$ is the radiation efficiency without dielectric or conductor loss

- $Q_d$ is dielectric loss which defined as the ratio (or angle in a complex plane) of the lossy reaction to the electric field E in the curl equation to the lossless reaction

$$Q_d = \frac{1}{\tan \delta} \tag{29}$$

- $Q_c$ is metalization loss

$$Q_c = h\sqrt{\mu \pi f \sigma} \tag{30}$$

The total quality factor of the antenna can be given by using this formula.

$$\frac{1}{Q} = \frac{1}{Q_{rad}} + \frac{1}{Q_{sw}} + \frac{1}{Q_d} + \frac{1}{Q_c} \tag{31}$$

## 3.6 Fringing Effect of the Microstrip Antenna

Because of fringing effect, this will cause many phenomenas such as Effective length extension, dielectric constant distortion and resonance frequency distortion[3]

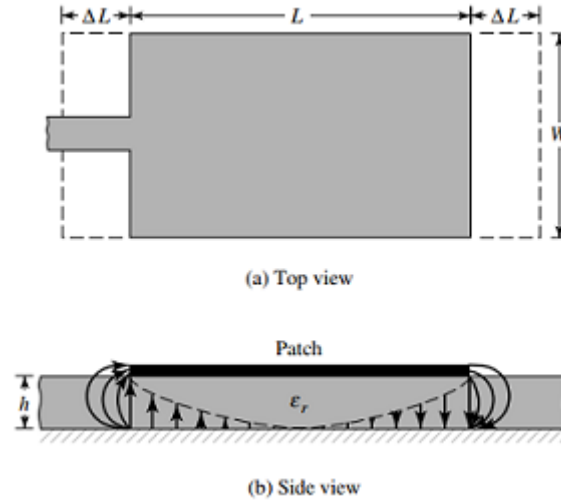### 3.6.1 Effective Length Extension



(a) Top view

(b) Side view

Figure 4: The fringing effect that cause to length extension

The length is extended because of fringing effect, that spreading at the edge of all sides so $L_{eff} = L + 2\Delta L$[3]

$$\frac{\Delta l}{h} = 0.412 \frac{(\epsilon_{r(eff)} + 0.3)(\frac{W}{h} + 0.264)}{(\epsilon_{r(eff)} - 0.258)(\frac{W}{h} + 0.8)} \tag{32}$$

### 3.6.2 Dielectric Constant Distortion



(a) Microstrip line

(b) Electric field lines

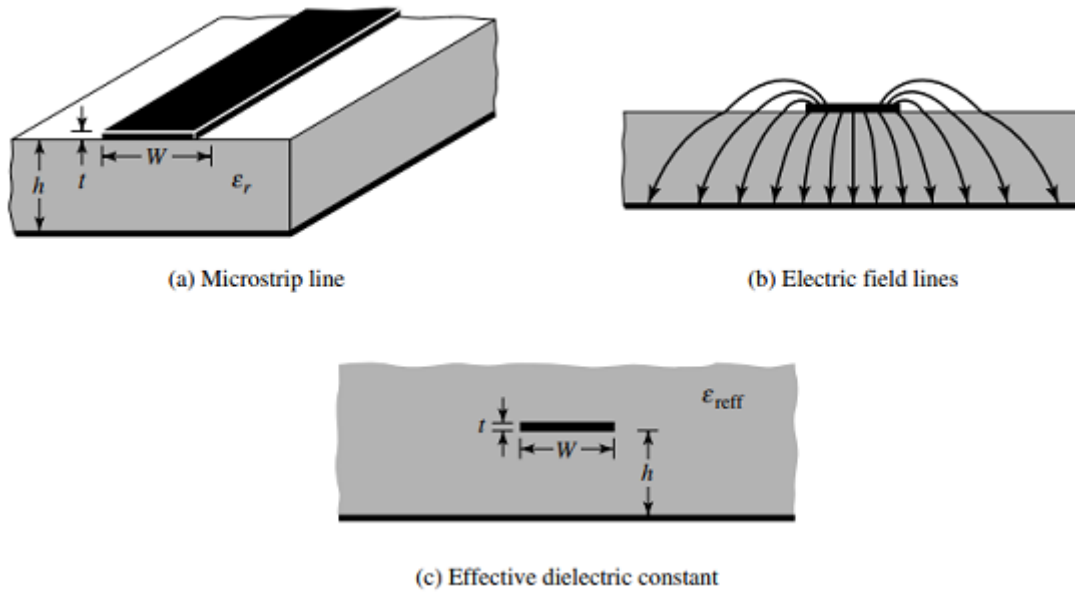(c) Effective dielectric constant

Figure 5: The fringing effect that cause to length extension

Due to the electric field is not only happened inside the microstrip line, there's a leakage electric

field in the air, so the dielectric that will be used for calculating will need to be reconsidered as follow[3]

$$\epsilon_{reff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}[1 + 12\frac{h}{W}]^{-0.5} \tag{33}$$

### 3.6.3 Resonance frequency distortion

In $TM_{010}$ mode, the resonant frequency is given by

$$f_{r(010)} = \frac{1}{2L\sqrt{\epsilon_r}\sqrt{\epsilon_{\mu_0\epsilon_0}}} = \frac{v_0}{2L\sqrt{\epsilon_r}} \tag{34}$$

With fringing effect, the equation will be given by

$$f_{r(010)} = \frac{1}{2L_{eff}\sqrt{\epsilon_{r(eff)}}\sqrt{\epsilon_{\mu_0\epsilon_0}}} = \frac{1}{2(L + 2\Delta L)\sqrt{\epsilon_{r(eff)}}\sqrt{\epsilon_{\mu_0\epsilon_0}}} \tag{35}$$

## 3.7 Phased Array Antena

In phased array antenna principle, there are 2 main factor in the equations. First one is an element factor which is the factor that come from only one antenna itself and the another factor is called array factor which is come from the effect of multiple antennas combining together.

$$S(\vartheta) = S_e(\vartheta)S_a(\vartheta) \tag{36}$$

Whereas

$S_e$ is a radiation pattern from only one element
$S_a$ is a element factor with

$$S_a(\vartheta) = \sum_{i=1}^{K} a_i e^{jk(K-i)dsin(\vartheta)} \tag{37}$$

$a_i$ is an amplitude taper.
$K$ is a number of array antenna.
$d$ is a distance of each antenna.
$\vartheta$ is a wavefront angle

# 4 Preliminary results

This project results will consist of 1) a simulation program that have ability to simulate a microstrip from the given parameters. 2) an phased antenna that could steer its direction to the desired point

## 4.1 Simulation Application

By using all the parameters that required to design the antenna, this simulation application will provide all the data from the algorithm that was proposed from the past. There are many techniques to simulate the antenna as Finite Element(FE), Method of Moments(MoM), or Numerical Method. However, the technique that will be used is one of those. In order to inspect on its correctness, the standard simulation application will be required to compare. Consequently, the antenna results is expected to have similar outcomes as well.

## 4.2 Microstrip Antenna

It's expected that a patch antenna should be have an impedance of zero Ohm at 5.6 GHz frequency
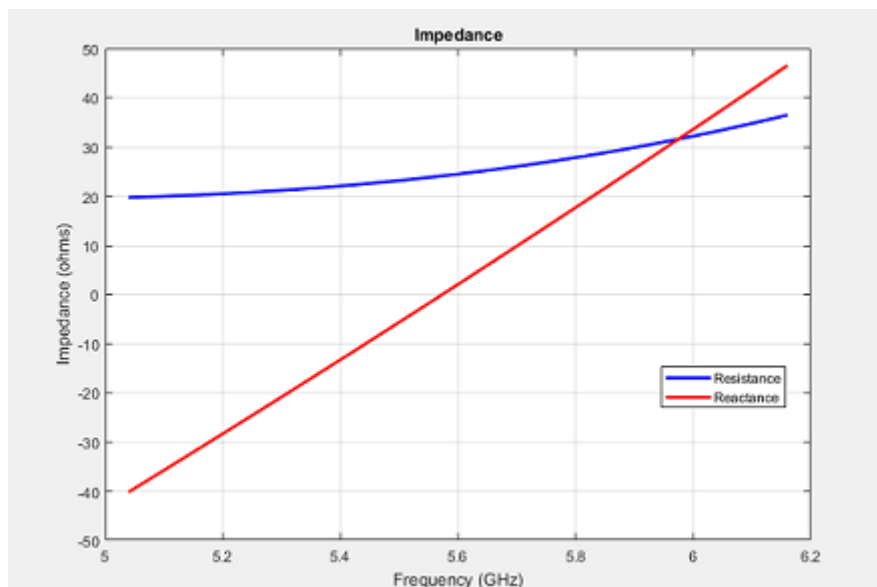


Figure 6: The designed antenna's impedance should have approximately 0 Ohm reactance at 5.6 GHz
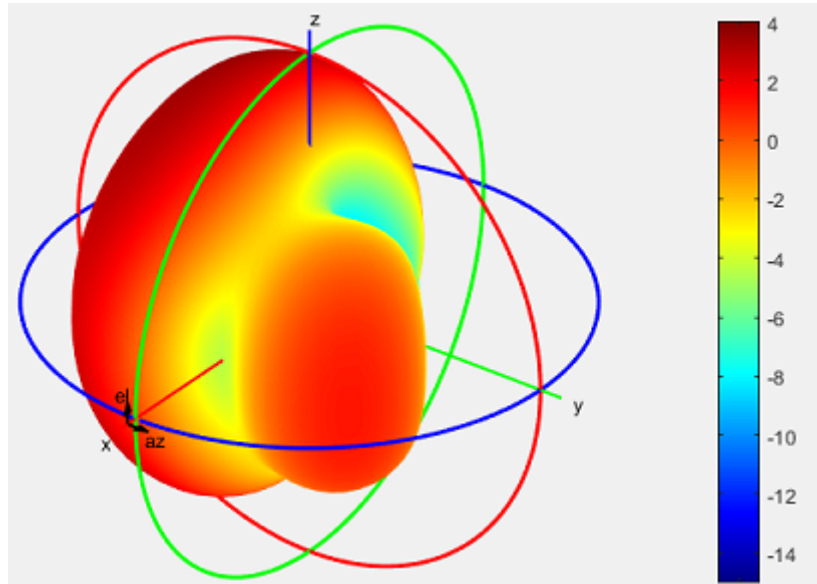
Figure 7: The designed antenna's radiation pattern like the analysis
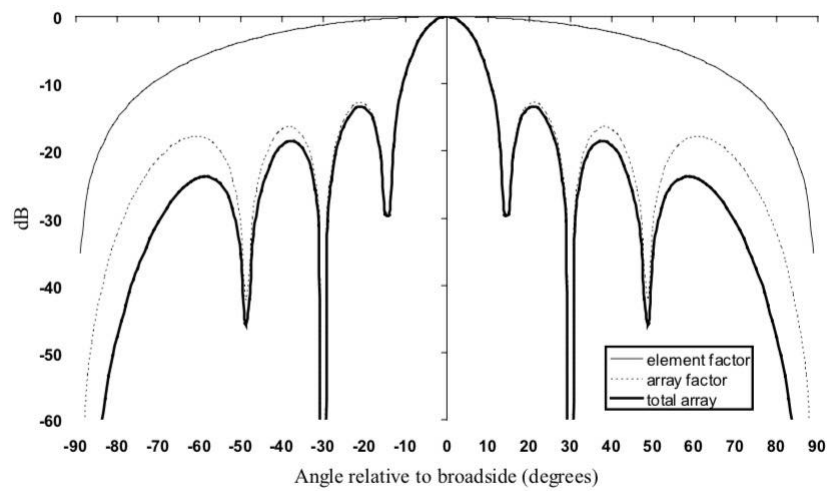
## 4.3 Array Antenna Patern



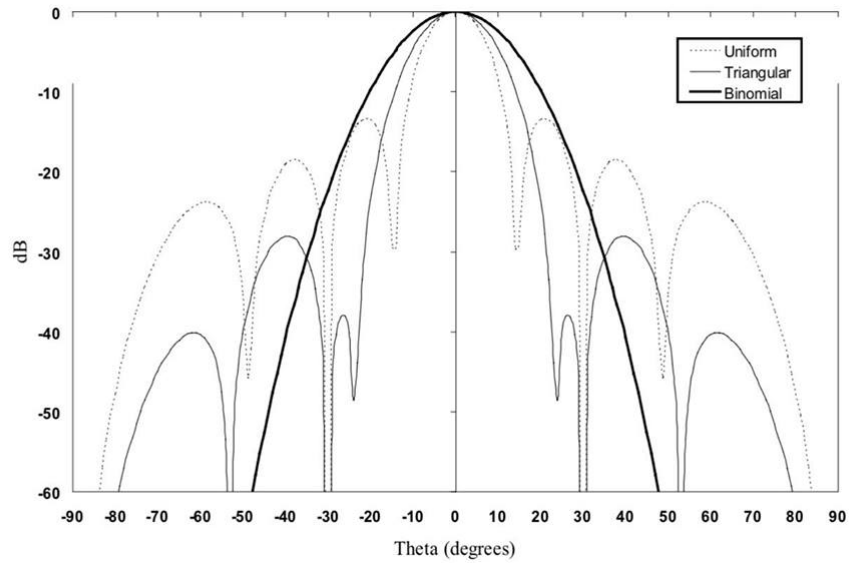Figure 8: Power radiation pattern without amplitude taper aka. $\forall a_i = 1$

Figure 9: Power radiation pattern with amplitude taper of each type

# 5   Project overview

## 5.1   Scope of work

- An analysis of a microstrip antenna and phased array antenna

- A basic simulation application for finding antennas' radiation pattern

- A physical antenna device with its test result.

## 5.2   Expected outcomes

It's expected that, the simulation application and the application that come from other author will have similar results. Also, the physical device should have a preliminary result like those simulation applications as well.

# 6    References

[1] P. M. J. Akhtar and P. K. V. Srivastava. Find the change in characteristics impedance and reflection coefficients of the transmission line by changing the dielectric properties of materials embedded between two conductors. [Online]. Available: http://www.iitk.ac.in/mimt_lab/vlab/ reflection_coefficients/theory1.html

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