

$$T_1 \quad f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

$$L(\mu) = \prod f(x_i) = \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^n \exp\left(-\frac{\sum (x_i - \mu)^2}{2\sigma^2}\right)$$

$$\begin{aligned} \Lambda &= \frac{L(\mu)}{L(\mu^*)} = \exp\left[-\frac{1}{2\sigma^2} (\sum (x_i - \mu)^2 - \sum (x_i - \mu^*)^2)\right] \\ &= \exp\left[-\frac{1}{2\sigma^2} \sum (2x_i - \mu - \mu^*)(\mu^* - \mu)\right] \end{aligned}$$

$$\Rightarrow -2\ln \Lambda = \frac{n}{\sigma^2} (\mu^* - \mu)^2 \sim \chi^2(1)$$

$$\alpha \quad \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1) \text{ 故与 } z \text{ 检验一致}$$

$$T_2 \quad H_0: \text{符合理论}$$

$$\Lambda = \frac{\sup_{\theta \in H_0} L(\theta)}{L(\theta^*)} = \prod \left(\frac{p_i}{p_i^*}\right)^{n_i}$$

$$\begin{aligned} \Rightarrow -2\ln \Lambda &= 2 \left(\sum n_i \ln \frac{n_i}{p_i n} \right) \\ &= \sum \frac{(O_i - E_i)^2}{E_i} \\ &= 0.9103 \end{aligned}$$

$$P = P(\chi^2(3) \geq 0.9103) = 0.63 > 0.05.$$

(符合理论)

$$T_3 \quad H_0: \sigma_1^2 = \sigma_2^2 \quad \text{vs.} \quad H_1: \sigma_1^2 \neq \sigma_2^2$$

$$H_0 \text{ 成立时 } F = \frac{S_X^2}{S_Y^2} \sim F(4, 6)$$

$$\text{而 } F_{0.025}(4, 6) = 6.227 \quad F_{0.975}(4, 6) = 0.108$$

$$\text{又 } F = 0.086 < 0.108 \quad \text{故拒绝 } H_0. \quad \therefore \text{不相等.}$$

$$T_4 \quad X \sim N(\mu_1, \sigma^2) \quad \bar{Y} \sim N(\mu_2, \sigma^2)$$

$$\bar{X} - \bar{Y} \sim N(\mu_1 - \mu_2, (\frac{1}{n} + \frac{1}{m}) \sigma^2)$$

$$\text{又 } S^2 = \frac{(n-1)S_X^2 + (m-1)S_Y^2}{n+m-2} = 7.22 \times 10^{-6}$$

$$\Rightarrow T = \frac{\bar{X} - \bar{Y}}{S \sqrt{\frac{1}{n} + \frac{1}{m}}} = 3.328$$

$$t_{0.025}(18) = 2.101 < T \quad \Rightarrow \text{拒绝 } H_0. \quad \Rightarrow \text{不相等.}$$

$$F = \frac{S_X^2}{S_Y^2} = 1.9545 \in (0.248, 4.026)$$

$$\Rightarrow \text{不拒绝 } H_0. \quad \Rightarrow \text{支持}$$

T_5 小样本 $H_0: \mu = 0$ $H_1: \mu \neq 0$

$D \sim N(0, \sigma^2)$

用 s^2 代替 σ^2 .

D_i : 17 -2 11 13 18 15 7 -1 13 -2

$$T = \frac{\bar{D}}{s/\sqrt{n}} = 3.55 > t_{0.025}(9) = 2.26$$

拒绝 H_0 . \Rightarrow 有差异

T_6 . $\hookrightarrow P(H_0) = P(H_1)$

$$\Rightarrow P(X=x | H_1) > P(X=x | H_0)$$

$$\Rightarrow 1.4^x \cdot 0.6^{10-x} > 1$$

$$\Rightarrow x \geq 7$$

$$\hookrightarrow P(H_0) = 10 P(H_1)$$

$$\Rightarrow P(X=x | H_1) > 10 P(X=x | H_0)$$

$$\Rightarrow 1.4^x \cdot 0.6^{10-x} > 10$$

$$\Rightarrow x \geq 9$$

$$T_1 \rightarrow A: T = \frac{\bar{x} - 100}{s/\sqrt{n}} = -2.45 < -1.83 = t_{1-\alpha} (9)$$

⇒ 拒绝。

$$B: T = \frac{\bar{x} - 100}{s/\sqrt{n}} = -5 > -6.314 = t_{1-\alpha} (1)$$

⇒ 不拒绝。

⇒ 样本大小影响显著 不合直觉。

T8

```

1 import math
2
3 def normal_cdf(x):
4     neg = x < 0
5     if neg:
6         x = -x
7         k = 1.0 / (1.0 + 0.2316419 * x)
8         y = (((1.330274429 * k + 1.821255978) * k + 1.781477937) * k - 0.356563782) * k + 0.319381530
9         y = 1.0 - 0.398942280401 * math.exp(-0.5 * x * x) * y
10    return 1.0 - y if neg else y
11
12 def main():
13     print("A/B Test Analysis")
14     print("")
15
16     n_a = 2000
17     x_a = 320
18     n_b = 2100
19     x_b = 385
20
21     p_a = x_a / n_a
22     p_b = x_b / n_b
23
24     print(f"Control Group (A): {x_a} purchases out of {n_a} users")
25     print(f"Control Group Conversion Rate: {p_a:.4f} ({p_a*100:.2f}%")
26
27     print(f"Test Group (B): {x_b} purchases out of {n_b} users")
28     print(f"Test Group Conversion Rate: {p_b:.4f} ({p_b*100:.2f}%")
29
30     print(f"Conversion Rate Difference: {(p_b-p_a)*100:.2f}%")
31
32     p_pooled = (x_a + x_b) / (n_a + n_b)
33     print(f"Pooled Proportion: {p_pooled:.4f}")
34
35     se = math.sqrt(p_pooled * (1 - p_pooled) * (1/n_a + 1/n_b))
36     print(f"Standard Error: {se:.4f}")
37
38     z_stat = (p_b - p_a) / se
39     print(f"Z-statistic: {z_stat:.4f}")
40
41     p_value = 2 * (1 - normal_cdf(abs(z_stat)))
42     print(f"P-value: {p_value:.4f}")
43
44     alpha = 0.05
45     print(f"Significance Level (alpha): {alpha}")
46
47     print("\nConclusion:")
48     if p_value < alpha:
49         print(f"Since p-value ({p_value:.4f}) < alpha ({alpha}), we reject the null hypothesis.")
50         print("The new page design significantly improves the conversion rate.")
51     else:
52         print(f"Since p-value ({p_value:.4f}) >= alpha ({alpha}), we fail to reject the null hypothesis.")
53         print("We cannot conclude that the new page design significantly improves the conversion rate.")
54
55 if __name__ == "__main__":
56     main()

```

A/B Test Analysis

Control Group (A): 320 purchases out of 2000 users
 Control Group Conversion Rate: 0.1600 (16.00%)
 Test Group (B): 385 purchases out of 2100 users
 Test Group Conversion Rate: 0.1833 (18.33%)
 Conversion Rate Difference: 2.33%
 Pooled Proportion: 0.1720
 Standard Error: 0.0118
 Z-statistic: 1.9792
 P-value: 0.0478
 Significance Level (alpha): 0.05

Conclusion:

Since p-value (0.0478) < alpha (0.05), we reject the null hypothesis.
 The new page design significantly improves the conversion rate.
 Press any key to continue . . .