Assignment1

2019.9.16

Exercise.4(a)

由题非齐次方程可得增广矩阵，并化为行阶梯形式，

$$A = \left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 3 \\ 2 & 4 & 1 & 3 & 4 \\ 3 & 6 & 1 & 4 & 5 \end{array}\right) \rightarrow \left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 3 \\ 0 & 0 & 1 & 1 & 2 \\ 3 & 6 & 1 & 4 & 5 \end{array}\right) \rightarrow \left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 3 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 & 2 \end{array}\right) \rightarrow \left(\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$= E_{[A|b]}$$

由上式可知，x_2 和 x_4 是自由变量, 将其带入原方程可得

$$x_1 = 1 - 2x_2 - x_4$$

$$x_2 \ is \ "free"$$

$$x_3 = 2 - x_4$$

$$x_4 \ is \ "free"$$

由此可得此非齐次线性方程组的通解是：

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 - 2x_2 - x_4 \\ x_2 \\ 2 - x_4 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -1 \\ 0 \\ -1 \\ 1 \end{pmatrix}$$

Exercise.4(b)

由题非齐次方程可得增广矩阵，并化为行阶梯形式，

$$A = \left(\begin{array}{ccc|c} 2 & 1 & 1 & 4 \\ 4 & 2 & 1 & 6 \\ 6 & 3 & 1 & 8 \\ 8 & 4 & 1 & 10 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 2 & 1 & 1 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 3 & 6 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 2 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) = E_{[A|b]}$$

由上式可知，y 是自由变量, 将其带入原方程可得

$$x = \frac{1}{2}(2 - y)$$
$$y \ is \ "free"$$
$$z = 2$$

由此可得此非齐次线性方程组的通解是：

$$x = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 - \frac{1}{2}y \\ y \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + y \begin{pmatrix} -\frac{1}{2} \\ 1 \\ 0 \end{pmatrix}$$