模式识别与机器学习作业–第三章

2019.9.22

1. 在一个 10 类的模式识别问题中，有 3 类单独满足多类情况 1，其余的类别满足多类情况 2。问该模式识别问题所需判别函数的最少数目是多少？

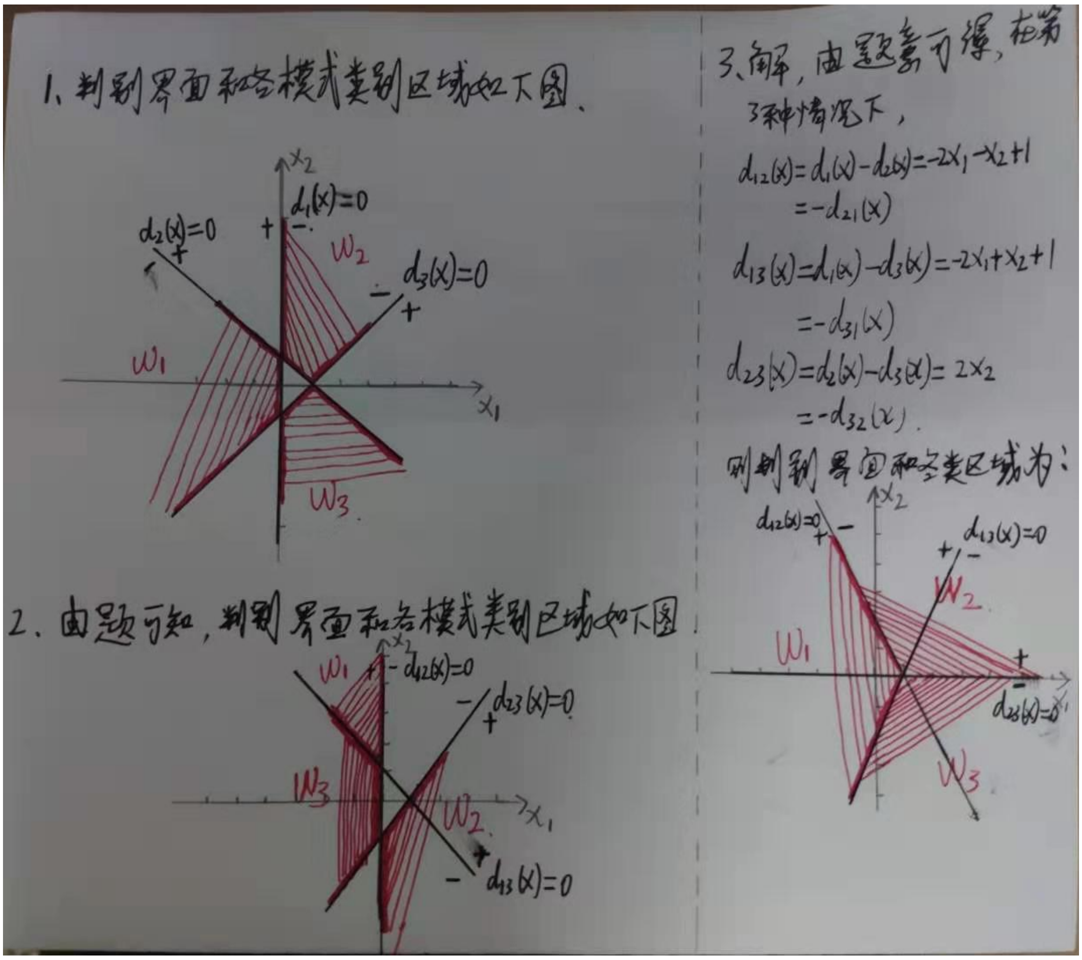
答：由题意，首先由于该模式种有三类单独满足多类情况 1，所以可以先把这一部分看成是 4 分类问题，此处需要 4 个判别函数；之后剩下的七个模式类别可以看作第四类中再进行二次划分的情况，这里需要 M(m-1)/2=21 中判别函数。所以整个问题一共最少需要判别函数为 21+4=25 个。

2. 一个三类问题，其判别函数为： d1(x)=-x1, d2(x)=x1+x2-1, d3(x)=x1-x2-1

1). 设这些函数是在多类情况 1 条件下确定的，绘出其判别界面和每一个模式类别的区域。

2). 设为多类情况 2，并使：d12(x)= d1(x), d13(x)= d2(x), d23(x)= d3(x)。绘出其判别界面和多类情况 2 的区域。

3). 设 d1(x), d2(x) 和 d3(x) 是在多类情况 3 的条件下确定的，绘出其判别界面和每类的区域。



3. 两类模式，每类包括 5 个 3 维不同的模式，且良好分布。如果它们是线性可分的，问权向量至少需要几个系数分量？假如要建立二次的多项式判别函数，又至少需要几个系数分量？（设模式的良好分布不因模式变化而改变。）

答：由题意，当两类模式线性可分且每类模式为 3 维时，权向量的分量至少为 $(n+1)=4$ 个；当需要建立二次多项式判别函数时，此时需要的系数分量至少为 $\frac{(n+1)(n+2)}{2}=(4\times 5)/2=10.$

4. (1) 用感知器算法求下列模式分类的解向量 w:

$$
\begin{aligned}
w_1:&\ \left\{(0,0,0)^T,(1,0,0)^T,(1,0,1)^T,(1,1,0)^T\right\}\\
w_2:&\ \left\{(0,0,1)^T,(0,1,1)^T,(0,1,0)^T,(1,1,1)^T\right\}
\end{aligned}
$$

答：首先将 w_2 类的样本乘（-1），并写成增广向量形式，得到新的样本如下

$$
\begin{aligned}
w_1:&\ \left\{(0,0,0,1)^T,(1,0,0,1)^T,(1,0,1,1)^T,(1,1,0,1)^T\right\}\\
w_2:&\ \left\{(0,0,-1,-1)^T,(0,-1,-1,-1)^T,(0,-1,0,-1)^T,(-1,-1,-1,-1)^T\right\}
\end{aligned}
$$

取 $C=1,\ \ w(1)=(0,0,0,0)^T$，则迭代过程如下：

$episode1:$

$$
\begin{aligned}
&w^T(1)x_1=(0,0,0,0)(0,0,0,1)^T=0\not>0, && w(2)=w(1)+x_1=(0,0,0,1)^T\\
&w^T(2)x_2=(0,0,0,1)(1,0,0,1)^T=1>0, && w(3)=w(2)=(0,0,0,1)^T\\
&w^T(3)x_3=(0,0,0,1)(1,0,1,1)^T=1>0, && w(4)=w(3)=(0,0,0,1)^T\\
&w^T(4)x_4=(0,0,0,1)(1,1,0,1)^T=1>0, && w(5)=w(4)=(0,0,0,1)^T\\
&w^T(5)x_5=(0,0,0,1)(0,0,-1,-1)^T=-1\not>0, && w(6)=w(5)+x_5=(0,0,-1,0)^T\\
&w^T(6)x_6=(0,0,-1,0)(0,-1,-1,-1)^T=1>0, && w(7)=w(6)=(0,0,-1,0)^T\\
&w^T(7)x_7=(0,0,-1,0)(0,-1,0,-1)^T=0\not>0, && w(8)=w(7)+x_7=(0,-1,-1,-1)^T\\
&w^T(8)x_8=(0,-1,-1,-1)(-1,-1,-1,-1)^T=3>0, && w(9)=w(8)=(0,-1,-1,-1)^T
\end{aligned}
$$

$episode2:$

$$
\begin{aligned}
&w^T(9)x_1=(0,-1,-1,-1)(0,0,0,1)^T=-1\not>0, && w(10)=w(9)+x_1=(0,-1,-1,0)^T\\
&w^T(10)x_2=(0,-1,-1,0)(1,0,0,1)^T=0\not>0, && w(11)=w(10)+x_2=(1,-1,-1,1)^T\\
&w^T(11)x_3=(1,-1,-1,1)(1,0,1,1)^T=1>0, && w(12)=w(11)=(1,-1,-1,1)^T\\
&w^T(12)x_4=(1,-1,-1,1)(1,1,0,1)^T=1>0, && w(13)=w(12)=(1,-1,-1,1)^T\\
&w^T(13)x_5=(1,-1,-1,1)(0,0,-1,-1)^T=0\not>0, && w(14)=w(13)+x_5=(1,-1,-2,0)^T\\
&w^T(14)x_6=(1,-1,-2,0)(0,-1,-1,-1)^T=3>0, && w(15)=w(14)=(1,-1,-2,0)^T\\
&w^T(15)x_7=(1,-1,-2,0)(0,-1,0,-1)^T=1>0, && w(16)=w(15)=(1,-1,-2,0)^T\\
&w^T(16)x_8=(1,-1,-2,0)(-1,-1,-1,-1)^T=2>0, && w(17)=w(16)+x_8=(1,-1,-2,0)^T
\end{aligned}
$$

$episode3:$

$$
\begin{aligned}
&w^T(17)x_1=(1,-1,-2,0)(0,0,0,1)^T=0\not>0, && w(18)=w(17)+x_1=(1,-1,-2,1)^T
\end{aligned}
$$

$$w^T(18)x_2=(1,-1,-2,1)(1,0,0,1)^T=2>0, \quad w(19)=w(18)=(1,-1,-2,1)^T$$
$$w^T(19)x_3=(1,-1,-2,1)(1,0,1,1)^T=0\not>0, \quad w(20)=w(19)+x_3=(2,-1,-1,2)^T$$
$$w^T(20)x_4=(2,-1,-1,2)(1,1,0,1)^T=3\not>0, \quad w(21)=w(20)=(2,-1,-1,2)^T$$
$$w^T(21)x_5=(2,-1,-1,2)(0,0,-1,-1)^T=-1\not>0, \quad w(22)=w(21)+x_5=(2,-1,-2,1)^T$$
$$w^T(22)x_6=(2,-1,-2,1)(0,-1,-1,-1)^T=2>0, \quad w(23)=w(22)=(2,-1,-2,1)^T$$
$$w^T(23)x_7=(2,-1,-2,1)(0,-1,0,-1)^T=0\not>0, \quad w(24)=w(23)+x_7=(2,-2,-2,0)^T$$
$$w^T(24)x_8=(2,-2,-2,0)(-1,-1,-1,-1)^T=2>0, \quad w(25)=w(24)+x_8=(2,-2,-2,0)^T$$

$episode4:$

$$w^T(25)x_1=(2,-2,-2,0)(0,0,0,1)^T=0\not>0, \quad w(26)=w(25)+x_1=(2,-2,-2,1)^T$$
$$w^T(26)x_2=(2,-2,-2,1)(1,0,0,1)^T=3>0, \quad w(27)=w(26)=(2,-2,-2,1)^T$$
$$w^T(27)x_3=(2,-2,-2,1)(1,0,1,1)^T=1>0, \quad w(28)=w(27)=(2,-2,-2,1)^T$$
$$w^T(28)x_4=(2,-2,-2,1)(1,1,0,1)^T=1>0, \quad w(29)=w(28)=(2,-2,-2,1)^T$$
$$w^T(29)x_5=(2,-2,-2,1)(0,0,-1,-1)^T=1>0, \quad w(30)=w(29)=(2,-2,-2,1)^T$$
$$w^T(30)x_6=(2,-2,-2,1)(0,-1,-1,-1)^T=3>0, \quad w(31)=w(30)=(2,-2,-2,1)^T$$
$$w^T(31)x_7=(2,-2,-2,1)(0,-1,0,-1)^T=1>0, \quad w(32)=w(31)=(2,-2,-2,1)^T$$
$$w^T(32)x_8=(2,-2,-2,1)(-1,-1,-1,-1)^T=1>0, \quad w(33)=w(32)=(2,-2,-2,1)^T$$

$episode5:$

$$w^T(33)x_1=(2,-2,-2,1)(0,0,0,1)^T=1>0, \quad w(34)=w(33)=(2,-2,-2,1)^T$$
$$w^T(34)x_2=(2,-2,-2,1)(1,0,0,1)^T=3>0, \quad w(35)=w(34)=(2,-2,-2,1)^T$$
$$w^T(35)x_3=(2,-2,-2,1)(1,0,1,1)^T=1>0, \quad w(36)=w(35)=(2,-2,-2,1)^T$$
$$w^T(36)x_4=(2,-2,-2,1)(1,1,0,1)^T=1>0, \quad w(37)=w(36)=(2,-2,-2,1)^T$$
$$w^T(37)x_5=(2,-2,-2,1)(0,0,-1,-1)^T=1>0, \quad w(38)=w(37)=(2,-2,-2,1)^T$$
$$w^T(38)x_6=(2,-2,-2,1)(0,-1,-1,-1)^T=3>0, \quad w(39)=w(38)=(2,-2,-2,1)^T$$
$$w^T(39)x_7=(2,-2,-2,1)(0,-1,0,-1)^T=1>0, \quad w(40)=w(39)=(2,-2,-2,1)^T$$
$$w^T(40)x_8=(2,-2,-2,1)(-1,-1,-1,-1)^T=1>0, \quad w(41)=w(40)=(2,-2,-2,1)^T$$

所以最终的解向量为 $w=(2,-2,-2,1)^T$，相应的判别函数为 $d(x)=2x_1-2x_2-2x_3+1$

5. 用多类感知器算法求下列模式的判别函数：

$$w_1: \ (-1,-1)^T$$
$$w_2: \ (0,0)^T$$
$$w_3: \ (1,1)^T$$

答：首先将模式样本写成增广向量形式，得到新的样本如下

$$x_1=(-1,-1,1)^T, \quad x_2=(0,0,1)^T, \quad x_3=(1,1,1)^T$$

取 $C=1,\ w_1(1)=w_2(1)=w_3(1)=(0,0,0)^T$，则迭代过程如下：

$$
\begin{aligned}
episode1 : & k=1, x_1=(-1,-1,1)^T, \\
& d_1(1)=w_1^T(1)x_1=(0,0,0)(-1,-1,1)^T=0 \\
& d_2(1)=w_2^T(1)x_1=(0,0,0)(-1,-1,1)^T=0 \\
& d_3(1)=w_3^T(1)x_1=(0,0,0)(-1,-1,1)^T=0 \\
& d_1(1)\not> d_2(1),\quad d_1(1)\not> d_3(1) \\
& w_1(2)=w_1(1)+x_1=(0,0,0)^T+(-1,-1,1)^T=(-1,-1,1)^T \\
& w_2(2)=w_2(1)-x_1=(0,0,0)^T-(-1,-1,1)^T=(1,1,-1)^T \\
& w_3(2)=w_3(1)-x_1=(0,0,0)^T-(-1,-1,1)^T=(1,1,-1)^T \\
episode2 : & k=2, x_2=(0,0,1)^T, \\
& d_1(2)=w_1^T(2)x_2=(-1,-1,1)(0,0,1)^T=1 \\
& d_2(2)=w_2^T(2)x_2=(1,1,-1)(0,0,1)^T=-1 \\
& d_3(2)=w_3^T(2)x_2=(1,1,-1)(0,0,1)^T=-1 \\
& d_2(2)\not> d_1(2),\quad d_2(2)\not> d_3(2) \\
& w_1(3)=w_1(2)-x_2=(-1,-1,1)^T-(0,0,1)^T=(-1,-1,0)^T \\
& w_2(3)=w_2(2)+x_2=(1,1,-1)^T+(0,0,1)^T=(1,1,0)^T \\
& w_3(3)=w_3(2)-x_2=(1,1,-1)^T-(0,0,1)^T=(1,1,-2)^T \\
episode3 : & k=3, x_3=(1,1,1)^T, \\
& d_1(3)=w_1^T(3)x_3=(-1,-1,0)(1,1,1)^T=-2 \\
& d_2(3)=w_2^T(3)x_3=(1,1,0)(1,1,1)^T=2 \\
& d_3(3)=w_3^T(3)x_3=(1,1,-2)(1,1,1)^T=0 \\
& d_3(3)> d_1(3),\quad d_3(3)\not> d_2(3) \\
& w_1(4)=w_1(3)=(-1,-1,0)^T \\
& w_2(4)=w_2(3)-x_3=(1,1,0)^T-(1,1,1)^T=(0,0,-1)^T \\
& w_3(4)=w_3(3)+x_3=(1,1,-2)^T+(1,1,1)^T=(2,2,-1)^T \\
episode4 : & k=4, x_1=(-1,-1,1)^T, \\
& d_1(4)=w_1^T(4)x_1=(-1,-1,0)(-1,-1,1)^T=2 \\
& d_2(4)=w_2^T(4)x_1=(0,0,-1)(-1,-1,1)^T=-1 \\
& d_3(4)=w_3^T(4)x_1=(2,2,-1)(-1,-1,1)^T=-5 \\
& d_1(4)> d_2(4),\quad d_1(4)> d_3(4) \\
& w_1(5)=w_1(4)=(0,0,1)^T \\
& w_2(5)=w_2(4)=(0,0,-1)^T \\
& w_3(5)=w_3(4)=(2,2,-1)^T
\end{aligned}
$$

$episode5:k = 5, x_2 = (0,0,1)^T,$

$$d_1(5) = w_1^T(5)x_2 = (-1,-1,0)(0,0,1)^T = 0$$

$$d_2(5) = w_2^T(5)x_2 = (0,0,-1)(0,0,1)^T = -1$$

$$d_3(5) = w_3^T(5)x_2 = (2,2,-1)(0,0,1)^T = -1$$

$$d_2(5) \ngtr d_1(5), \quad d_2(5) \ngtr d_3(5)$$

$$w_1(6) = w_1(5) - x_2 = (-1,-1,0)^T - (0,0,1)^T = (-1,-1,-1)^T$$

$$w_2(6) = w_2(5) + x_2 = (0,0,-1)^T + (0,0,1)^T = (0,0,0)^T$$

$$w_3(6) = w_3(5) - x_2 = (2,2,-1)^T - (0,0,1)^T = (2,2,-2)^T$$

$episode6:k = 6, x_3 = (1,1,1)^T,$

$$d_1(6) = w_1^T(6)x_3 = (-1,-1,-1)(1,1,1)^T = -3$$

$$d_2(6) = w_2^T(6)x_3 = (0,0,0)(1,1,1)^T = 0$$

$$d_3(6) = w_3^T(6)x_3 = (2,2,-2)(1,1,1)^T = 2$$

$$d_3(6) > d_1(6), \quad d_3(3) > d_2(3)$$

$$w_1(7) = w_1(6) = (-1,-1,-1)^T$$

$$w_2(7) = w_2(6) = (0,0,0)^T$$

$$w_3(7) = w_3(6) = (2,2,-2)^T$$

$episode7:k = 7, x_1 = (-1,-1,1)^T,$

$$d_1(7) = w_1^T(7)x_1 = (-1,-1,-1)(-1,-1,1)^T = 1$$

$$d_2(7) = w_2^T(7)x_1 = (0,0,0)(-1,-1,1)^T = 0$$

$$d_3(7) = w_3^T(7)x_1 = (2,2,-2)(-1,-1,1)^T = -6$$

$$d_1(7) > d_2(7), \quad d_1(7) > d_3(7)$$

$$w_1(8) = w_1(7) = (-1,-1,-1)^T$$

$$w_2(8) = w_2(7) = (0,0,0)^T$$

$$w_3(8) = w_3(7) = (2,2,-2)^T$$

$episode8:k = 7, x_2 = (0,0,1)^T,$

$$d_1(8) = w_1^T(8)x_2 = (-1,-1,-1)(0,0,1)^T = -1$$

$$d_2(8) = w_2^T(8)x_2 = (0,0,0)(0,0,1)^T = 0$$

$$d_3(8) = w_3^T(8)x_2 = (2,2,-2)(0,0,1)^T = -2$$

$$d_2(8) > d_1(8), \quad d_2(5) > d_3(5)$$

$$w_1(9) = w_1(8) - x_2 = (-1,-1,-1)^T$$

$$w_2(9) = w_2(8) + x_2 = (0,0,0)^T$$

$$w_3(9) = w_3(8) - x_2 = (2,2,-2)^T$$

因此，权向量的解为

$$w_1 = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}, \quad w_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad w_3 = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}$$

相应的判别函数为

$$\begin{cases} d_1 = -x_1 - x_2 - 1 \\ d_2 = 0 \\ d_3 = 2x_1 + 2x_2 - 2 \end{cases}$$

6. 采用梯度法和如下准测函数，式中实数 b>0，试导出两类模式的分类算法。

$$J(w,x,b) = \frac{1}{8\|x\|^2}\left[(w^T x - b) - |w^T x - b|\right]^2$$

答：由题意可知，对准则函数 J 进行微分，得：

$$\frac{\partial J}{\partial w} = \frac{1}{4\parallel x \parallel^2}\left[\left(w^T x - b\right) - |w^T x - b|\right] \times [x - x \times sign(\, w^T x - b)]$$

$$where, \quad sign(w^T - b) = \begin{cases} 1, & w^T x - b > 0 \\ -1, & w^T x - b \le 0 \end{cases}$$

由此得到迭代式并化简后得：

$$w(k+1) = w(k) + \frac{C}{4\parallel x \parallel^2}\left[\left(w^T(k)x - b\right) - |w^T(k)x - b|\right] \times [x - x \times sign(\, w^T(k)x - b)]$$

$==>,$

$$w(k+1) = w(k) + C\begin{cases} 0, & w^T(k)x - b > 0 \\ \dfrac{(b - w^T(k)x)}{\parallel x \parallel^2}x, & w^T(k)x - b \le 0 \end{cases}$$

7. 用二次埃尔米特多项式的势函数算法求解以下模式的分类问题

$$w_1: \left\{(0,1)^T, (0,-1)^T\right\}, \quad w_2: \left\{(1,0)^T, (-1,0)^T\right\}$$

解：二次~~埃~~埃尔米特多项式的表达式为

$$H_0(x)=1, \quad H_1(x)=2x, \quad H_2(x)=4x^2-2.$$

因此其正交函数集合为：

$$\varphi_1(x)=\varphi_1(x_1,x_2)=H_0(x_1)H_0(x_2)=1$$
$$\varphi_2(x)=\varphi_2(x_1,x_2)=H_0(x_1)H_1(x_2)=2x_2$$
$$\varphi_3(x)=\varphi_3(x_1,x_2)=H_0(x_1)H_2(x_2)=4x_2^2-2$$
$$\varphi_4(x)=\varphi_4(x_1,x_2)=H_1(x_1)H_0(x_2)=2x_1$$
$$\varphi_5(x)=\varphi_5(x_1,x_2)=H_1(x_1)H_1(x_2)=4x_1x_2$$
$$\varphi_6(x)=\varphi_6(x_1,x_2)=H_1(x_1)H_2(x_2)=2x_1(4x_2^2-2)$$
$$\varphi_7(x)=\varphi_7(x_1,x_2)=H_2(x_1)H_0(x_2)=4x_1^2-2$$
$$\varphi_8(x)=\varphi_8(x_1,x_2)=H_2(x_1)H_1(x_2)=2x_2(4x_1^2-2)$$
$$\varphi_9(x)=\varphi_9(x_1,x_2)=H_2(x_1)H_2(x_2)=(4x_1^2-2)(4x_2^2-2)$$

则势函数为

$$K(x,x_k)=\sum_{i=1}^{9}\varphi_i(x)\varphi_i(x_k)=1+4x_2x_{k2}+(4x_2^2-2)(4x_{k2}^2-2)+4x_1x_{k1}+16x_1x_2x_{k1}x_{k2}$$
$$+4x_1(4x_2^2-2)x_{k1}(4x_{k2}^2-2)+(4x_1^2-2)(4x_{k1}^2-2)+4x_2(4x_1^2-2)x_{k2}(4x_{k1}^2-2)$$
$$+(4x_1^2-2)(4x_2^2-2)(4x_{k1}^2-2)(4x_{k2}^2-2)$$

第1步，取 $x_1=\binom{0}{1}\in w_1$，故 $K_1(x)=-15+20x_2+40x_2^2+24x_1^2-64x_1^2x_2^2-32x_1^2x_2$

第2步，取 $x_2=\binom{0}{-1}\in w_1$，则 $K_1(x_2)=5>0$，故 $K_2(x)=K_1(x)$.

第3步，取 $x_3=\binom{1}{0}\in w_2$，则 $K_2(x_3)=9>0$，故.

$$K_3(x)=K_2(x)-K(x,x_3)=20x_2+16x_2^2-20x_1-16x_1^2.$$

第4步，取 $x_4=\binom{-1}{0}\in w_2$，则 $K_3(x_4)=4>0$，故

$$K_4(x)=K_3(x)-K(x,x_4)=15+20x_2-56x_1^2-8x_2^2-32x_1^2x_2+64x_1^2x_2^2$$

第5步，取 $x_5=\binom{0}{1}\in w_1$，则 $K_4(x_5)=27>0$，故 $K_5(x)=K_4(x)$

第6步，取$x_6=\begin{pmatrix}0\\-1\end{pmatrix}\in w_1$，则$K_5(x_6)=-13<0$，故

$$K_6(x)=K_5(x)+K(x,x_6)=-32x_1^2+32x_2^2$$

第7步，取$x_7=\begin{pmatrix}1\\0\end{pmatrix}\in w_2$，则$K_6(x_7)=-32<0$，故

$$K_7(x)=K_6(x)$$

第8步，取$x_8=\begin{pmatrix}-1\\0\end{pmatrix}\in w_2$，则$K_7(x_8)=-32<0$，故

$$K_8(x)=K_7(x)$$

第9步，取$x_9=\begin{pmatrix}0\\1\end{pmatrix}\in w_1$，则$K_8(x_9)=32>0$，故

$$K_9(x)=K_8(x)$$

第10步，取$x_{10}=\begin{pmatrix}0\\-1\end{pmatrix}\in w_1$，则$K_9(x_{10})=32>0$，故

$$K_{10}(x)=K_9(x)$$

至此，全部样本分类正确，所以函数已收敛于判别函数，因此，判别函数为

$$d(x)=K_{10}(x)=-32x_1^2+32x_2^2.$$

8. 用下列势函数 $K(x,x_k)=e^{-\alpha\|x-x_k\|^2}$ 求解以下模式分类问题：

$$w_1:\ \left\{(0,1)^T,(0,-1)^T\right\},\quad w_2:\ \left\{(1,0)^T,(-1,0)^T\right\}$$

解，取 $\alpha=1$，则势函数为

$K(x,x_k)=e^{-\|x_k-x_k\|^2}=e^{-[(x_1-x_{k1})^2+(x_2-x_{k2})^2]}$.

则迭代过程如下：

①，取 $x_1=\binom{0}{1}\in w_1$，故 $K_1(x)=K(x,x_1)=e^{-[x_1^2+(x_2-1)^2]}$.

②，取 $x_2=\binom{0}{-1}\in w_1$，故 $K_1(x_2)=e^{-4}>0$，

则 $K_2(x)=K_1(x)$

③，取 $x_3=\binom{1}{0}\in w_2$，则 $K_2(x_3)=e^{-1}>0$，

则 $K_3(x)=K_2(x)-K(x,x_3)=e^{-[x_1^2+(x_2-1)^2]}-e^{-[(x_1-1)^2+x_2^2]}$

④，取 $x_4=\binom{-1}{0}\in w_2$，则 $K_3(x_4)=e^{-2}-e^{-4}>0$，

故 $K_4(x)=K_3(x)-K(x,x_4)=e^{-[x_1^2+(x_2-1)^2]}-e^{-[(x_1-1)^2+x_2^2]}-e^{-[(x_1+1)^2+x_2^2]}$.

⑤，取 $x_5=\binom{0}{1}\in w_1$，则 $K_4(x_5)=1-e^{-2}-e^{-2}>0$，

故 $K_5(x)=K_4(x)$.

⑥，取 $x_6=\binom{0}{-1}\in w_1$，则 $K_5(x_6)=e^{-4}-e^{-2}-e^{-2}<0$，

故 $K_6(x)=K_5(x)-K(x,x_6)=e^{-[x_1^2+(x_2+1)^2]}+e^{-[x_1^2+(x_2-1)^2]}-e^{-[(x_1-1)^2+x_2^2)}$

$-e^{-[(x_1+1)^2-x_2^2]}$.

⑦，取 $x_7=\binom{1}{0}\in w_2$，则 $K_6(x_7)=e^{-2}+e^{-2}-1-e^{-4}<0$，

故 $K_7(x)=K_6(x)$

⑧，取 $x_8=\binom{-1}{0}\in w_2$，则 $K_7(x_8)=e^{-2}+e^{-2}-e^{-4}-1<0$，

故 $K_8(x)=K_7(x)$.

⑨，取 $x_9=\binom{0}{1}\in w_1$，则 $K_8(x_9)=e^{-4}+1-e^{-2}-e^{-2}>0$，

故 $K_9(x)=K_8(x)$

⑩，取 $x_{10}=\binom{0}{-1}\in w_1$，则 $K_9(x_{10})=1+e^{-4}-e^{-2}-e^{-2}>0$，故 $K_{10}(x)=K_9(x)$.

至此，模式样本已全分类正确，故算法已收敛.

因此，判别函数为

$d(x)=K_{10}(x)=e^{-[x_1^2+(x_2+1)^2]}+e^{-[x_1^2+(x_2-1)^2]}-e^{-[(x_1-1)^2+x_2^2]}-e^{-[(x_1+1)^2-x_2^2]}$.