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120090694

CSC 3170 Assignment 2

This is an individual assignment and should be submitted by 5 pm, 25 March 2022 via Blackboard

Assignment Questions

1. Determine with explanations and examples (where appropriate) if each of the following is a trivial functional dependency, where Φ is the empty set, and A ≠ Φ,

   (a) A → Φ
   (b) Φ → A
   (c) Φ → Φ

(a) Trivial.

For an empty attribute set, any tuple

$\phi \subseteq A$, $f(A_i) = \phi$.

(b). Not Trivial

$\forall a \in A$, $a \notin \phi$, for there is no elements in an empty set

The $\phi \rightarrow A$ is only correct when
$\forall a_i \in A$, $a_j \in A$, s.t. $i \neq j$, $a_i = a_j$.

(c). Trivial $\quad \phi = \phi \quad \therefore \phi \subseteq \phi$

In the first set ϕ, every element is null, which is mapped to a null in the second set ϕ.

2. Consider the relation $R(A_1, A_2, \ldots, A_n)$, where each A_i, $i = 1, 2, \ldots, n$, is an atomic (i.e. simple) attribute. Let F be an arbitrary set of functional dependencies on R, show that

$$|F^+| \leq 2^{2n}.$$

α, β are two arbitany combination attributes in R.
Assume that $f(\alpha)=\beta \Leftrightarrow \alpha \to \beta$.
Because α is a subset of R, $\alpha \subseteq R$. there are 2^n possible different α combinations.

$|S_A| = |pow(\{A_i\})| = 2^{|\{A_i\}|} = 2$

Similarly. β is also a subset of R
there are 2^n different β combinations.
Therefore, there are $2^n \times 2^n$ possible functional dependent,

$|F^+| \leq |S_A \times S_A| = |S_A|^2 = 2^{2n}$

3. Consider a relation consisting of the attributes A, B, C, with the following set of functional dependencies F

$$A \to BC$$
$$B \to AC$$
$$C \to AB$$

Determine four different canonical covers for F.

$\begin{cases} A \to B \\ B \to C \\ C \to A \end{cases}$ $\begin{cases} A \to C \\ B \to A \\ C \to B \end{cases}$ $\begin{cases} A \to BC \\ C \to A \\ B \to A \end{cases}$ $\begin{cases} B \to AC \\ A \to B \\ C \to B \end{cases}$

4. Prove that functional dependency satisfies the formal definition of multivalued dependency.

For arbitary attribute α, β, γ in R, the Functional Dependency states that:

$\forall$ tuple t_1, t_2, t_3, t_4

s.t. if $t_1[\alpha] = t_2[\alpha] = t_3[\alpha] = t_4[\alpha]$

that $t_1[\beta] = t_2[\beta] = t_3[\beta] = t_4[\beta]$

$t[\gamma] = t_2[\gamma] = t_3[\gamma] = t_4[\gamma]$

that is $t_1[\alpha] = t_2[\alpha] = t_3[\alpha] = t_4[\alpha]$

$t_1[\beta] = t_3[\beta]$

$t[\gamma] = t_4[\gamma]$

$t_2[\beta] = t_4[\beta]$

$t_2[\gamma] = t_3[\gamma]$

which satisfied to the Multivalued Dependence

5. Consider the following relations for an order processing application database at company Global-UK.

Order (<u>O#</u>, Odate, Cust#, Total_amount)

Order-Item (<u>O#</u>, <u>I#</u>, Qty_ordered, Total_price, Discount%)

Here O#, I#, Cust# denote respectively the order number, item number, and customer number. Assume that each item has a different discount. The Total_price refers to the total price of one item, Odate is the date on which the order was placed, and the Total_amount is the amount of the order. Let us apply a natural join on the relations Order-Item and Order and call the result RelationX.

(i) Write down the schema of RelationX.

(ii) Determine the primary key for RelationX.

(iii) What are the functional dependencies of RelationX. You should state clearly any assumptions that you make. These assumptions should be reasonable assumptions.

(iv) Is RelationX in 2NF or 3NF? You should justify your answers.

(i) Relation X (<u>O#</u>, <u>I#</u>, Odate, Cust#, Total_amount, Qty_ordered, Total_price, Discount%)

(ii) P.K. = {<u>O#</u>, <u>I#</u>}

(iii) FD Assumptions,

O# → Odate — Each Order has only one date.

O# → Cust# — Each Order belongs to only one. customer.

O# → Total_amount — Total_amount for an order is unchangable.

O#, I# → Qty_ordered — The Quantity is determine by a speeific item in an order

O# I# → Total_price

I# → Dicount% . — An item has one discount rate.

(iv) Because Discount is not fully depend on P.K.
Relation X is not in 2NF
which also implies that
Relation X is not in 3NF.

6. Consider the relation concerning refrigerators

Ref (Model#, Year, Price, Manuf_Plant, Color)

and the following set of functional dependencies:

Model# → Manuf_Plant

Model#, Year → Price

Manuf_Plant → Color

(i) Evaluate each of the following as a candidate key for Ref, giving reasons why it can or cannot be a candidate key:

a. {Model#},
b. {Model#, Year},
c. {Model#, Color}.

(a) No {Model #} is not a candidate key because it can't identify a item.

for tuple t_1, t_2.

if t_1[Model #] = t_2[Model #]

then they should be same tuple.

However. because Model # → Price F.D. doesn't exist. then t_1[Price] ≠ t_2[Price]

Therefore Price is not in $\{\text{Model\#}\}^+$

(b) Yes . {Model #, Year} is a candidate key

Model # → Manuf - Plant

Model #, Year → Price

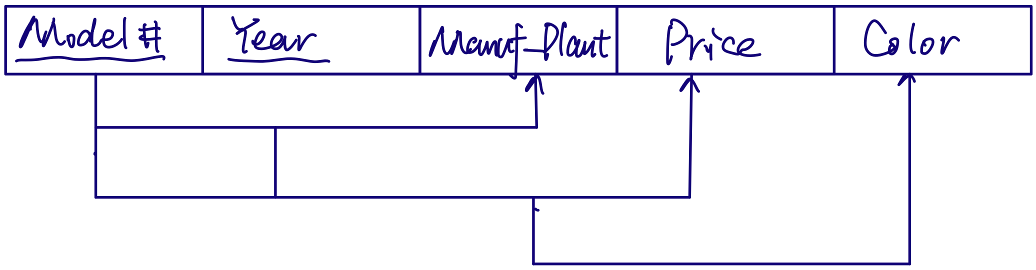
Model # → Color.

(c) No Model# is not a candidate key

{Model #, Color} can't be a candidate key

the two key can't determin price.

(ii) Based on the result of (i) above, determine whether the relation Ref is in 3NF and whether it is in BCNF. You should justify your answers.



The Schema isnot in BCNF. Manuf-Plant → Color is neither a trivial nor does Manuf-Plant serves as a supercode.

The Schema isnot a 3NF., because the Model# → Manuf_Plant. Manuf_Plant → Color is a transitive. F.D,

(iii) Consider the decomposition of Ref into

R_1 (Model#, Year, Price)
R_2 (Model#, Manuf_Plant, Color)

Determine whether this is a lossless decomposition. You should justify your answers.

$R_1 \cap R_2 = \{\text{Model\#}\}$, which is not a candidate key for Ref

The decomposition is not lossless.