# ECE4513 - Image Processing and Computer Vision

# Assignment #1

### Instructor: Zhen Li Due on 2022/10/15 23:59 pm

## Part I Written Exercises

**Problem 1 (7 points)**

Show that forming unweighted local averages, which yields an operation of the form

$$\mathcal{R}_{ij} = \frac{1}{(2k+1)^2} \sum_{u=i-k}^{u=i+k} \sum_{v=j-k}^{v=j+k} \mathcal{F}_{uv}$$

is a convolution. What is the kernel of this convolution?

**Problem 2 (7 points)**

Write $\mathcal{E}_0$ for an image that consists of all zeros with a single one at the center. Show that convolving this image with the kernel

$$H_{ij} = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{((i-k-1)^2 + (j-k-1)^2)}{2\sigma^2}\right)$$

(which is a discretised Gaussian) yields a circularly symmetric fuzzy blob.

**Problem 3 (7 points)**

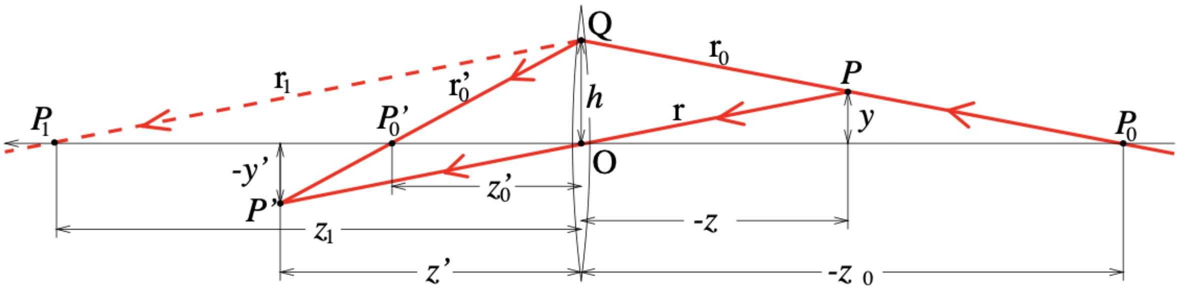
Show that convolving a function with a $\delta$ function simply reproduces the original function. Now show that convolving a function with a shifted $\delta$ function shifts the function.

**Problem 4 (7 points)**

Derive the perspective equation projections for a virtual image located at a distance $f'$ *in front* of the pinhole.

**Problem 5 (7 points)**

Derive the thin lens equation.
Hint: consider a ray $r_0$ passing through the point $P$ and construct the rays $r_1$ and $r_2$ obtained respectively by the refraction of $r_0$ by the right boundary of the lens and the refraction of $r_1$ by its left boundary.




**Problem 6 (7 points)**

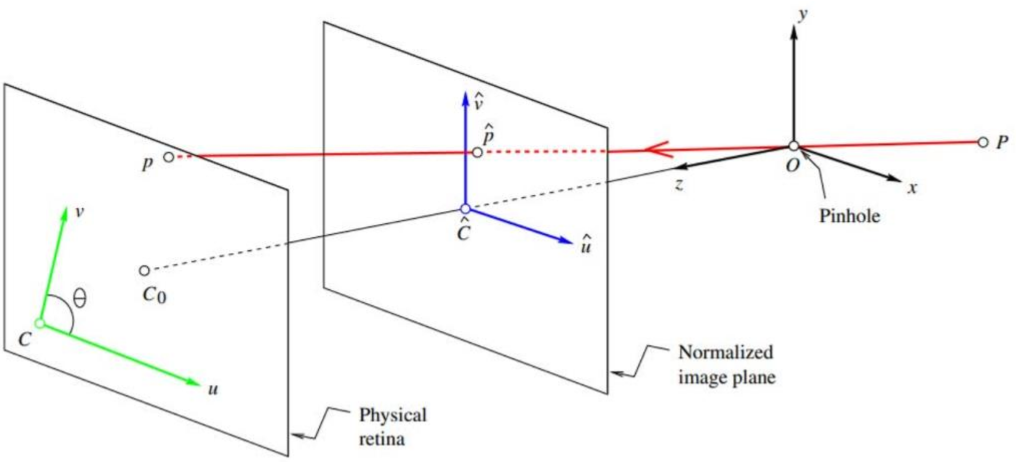
Give a geometric construction of the image $P'$ of a point $P$ given the two focal points $F$ and $F'$ of a thin lens.

**Problem 7 (7 points)**

Let $\boldsymbol{O}$ denote the *homogeneous* coordinate vector of the optical center of a camera in some reference frame, and let $\mathcal{M}$ denote the corresponding perspective projection matrix. Show that $\mathcal{M}\boldsymbol{O} = \boldsymbol{0}$.

**Problem 8 (7 points)**

Show that when the camera coordinate system is skewed and the angle $\theta$ between the two image axes is not equal to 90 degrees, then Eq. (2.11) transforms into Eq. (2.12).




$$\begin{cases} u = \alpha \dfrac{x}{z} + u_0 \\ v = \beta \dfrac{y}{z} + v_0 \end{cases} \qquad (2.11)$$

$$
\begin{cases} u = \alpha \frac{x}{z} - \alpha \cot\theta \frac{y}{z} + u_0 \\ v = \frac{\beta}{\sin\theta} \frac{y}{z} + v_0 \end{cases} \qquad (2.12)
$$

**Problem 9 (7 points)**

Write formulas for the matrices ${}^{A}_{B}\mathcal{R}$ when $(B)$ is deduced from $(A)$ via a rotation of angle $\theta$ about the axes $\boldsymbol{i_A}$, $\boldsymbol{j}_A$, and $\boldsymbol{k}_A$ respectively.

**Problem 10 (7 points)**

Show that rotation matrices are characterized by the following properties: (a) the inverse of a rotation matrix is its transpose and (b) its determinant is 1.

# Part II Programming Exercises

**Problem 1 Single-View Metrology (30 points)**

Please use “CIMG6476.JPG” and “kyoto_street.JPG” as the inputs. You can only use the following Python libraries: OpenCV, NumPy, math, matplotlib, and SciPy.

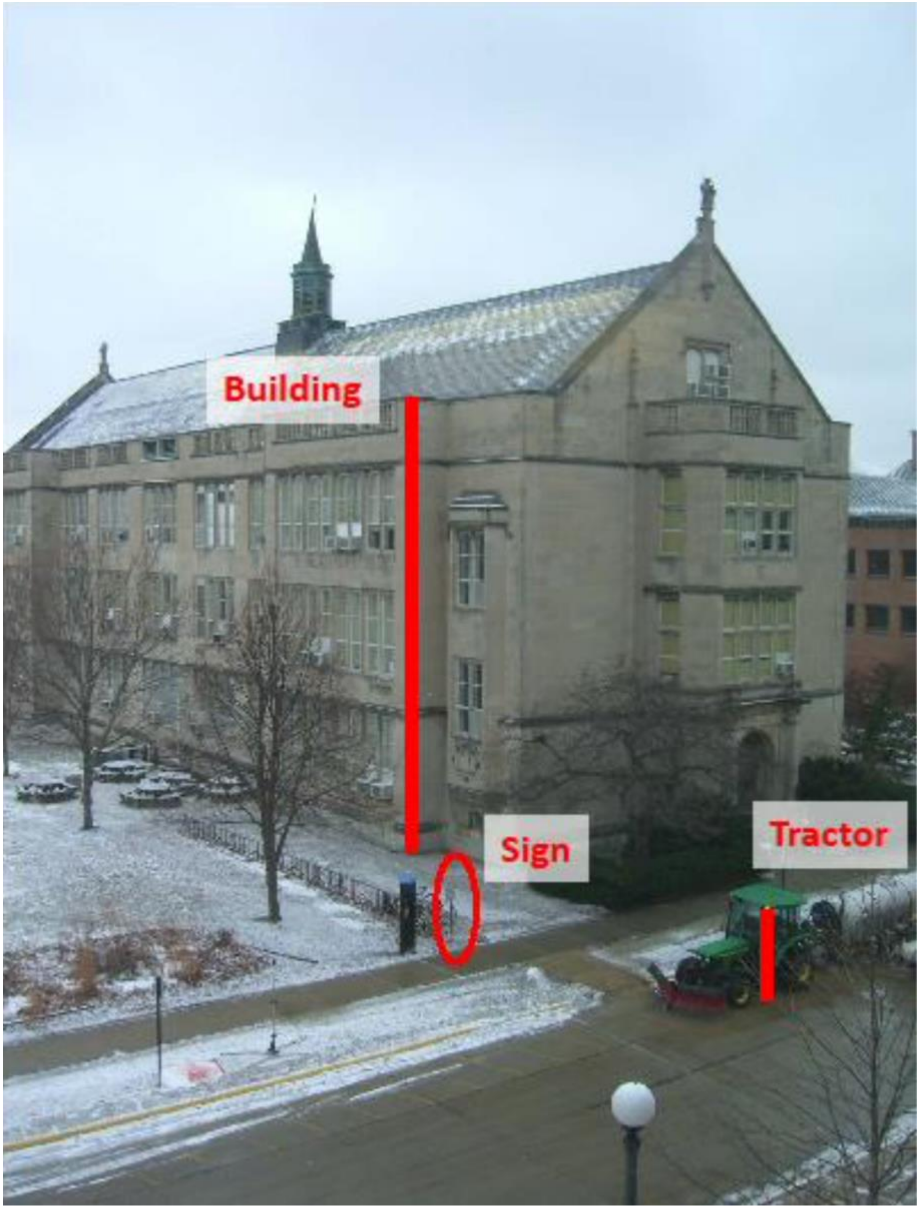
(a) For the Kyoto Street image, shown above, estimate the positions (in the image plane) of the three major orthogonal vanishing points (VPs), corresponding to the building orientations. Use at least three manually selected lines to solve for each vanishing point. The included code *getVanishingPoint.py* provides an interface for selecting and drawing the lines, but the code for computing the vanishing point needs to be inserted.

- Plot the VPs and the lines used to estimate them on the image plane. (1 pts)
- Specify the VPs ($u$, $v$). (1 pts)
- Plot the ground horizon line and specify its parameters in the form a$u$ + b$v$ + c = 0. Normalize the parameters so that: $a^2 + b^2 = 1$. (3 pts)

(b) Use the fact that the vanishing points are in orthogonal directions to estimate the camera focal length ($f$) and optical center ($u_0$, $v_0$). Show all work. (5 pts)

(c) Show how to compute the camera's rotation matrix when provided with vanishing points in the X, Y, and Z directions. (5 pts)

Now, compute the rotation matrix for this image, setting the vertical vanishing point as the Y-direction, the right-most vanishing point as the X-direction, and the left-most vanishing point as the Z-direction. (5 pts)



(d) The above photo is of the University High building, taken from the third floor in Siebel Center facing south. Estimate the horizon and draw/plot it on the image. Assume that the sign is 1.65m. Estimate the heights of the tractor, the building, and the camera (in meters). This can be done with PowerPoint, paper and a ruler, or Python.

- Turn in an illustration that shows the horizon line, and the lines and measurements used to estimate the heights of the building, tractor, and camera. (5 pts)

- Report the estimated heights of the building, tractor, and camera. (5 pts)