ECE4513 Computer Vision Assignment 2

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Part I Written Exercises

Problem 1 Lighting (20 points)

A.

Answer the following regarding the above image (photo credit: ColinBrough from RGBStock.com). Short answers (several words) are sufficient (8 points):

a. In what direction is the dominant light source: left and above, directly above, or right and above?

right and above, observed from the shadow

b. Why is one of the temple tips (the part that rests on the ear) so bright, considering that the other tip which has the same material is very dark?

By the Lambertian model:

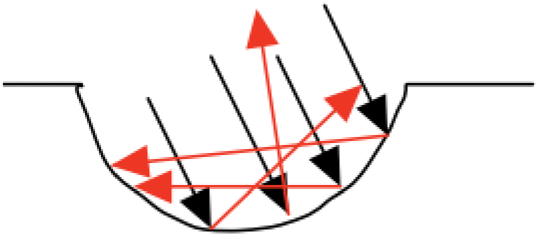
Intensity depends on illumination angle. Less light comes in at oblique angles.

$$I(x) = \rho(x)(\vec{s} \cdot \vec{n(x)})$$

The bright tip is facing to the light source, and reflects light, the angle between s and n is small.

The dark tip is reflect light at oblique angles in the Lambertian model.

c. What causes the dark streaks in the wood (in terms of shape, albedo, reflectance, etc.)?

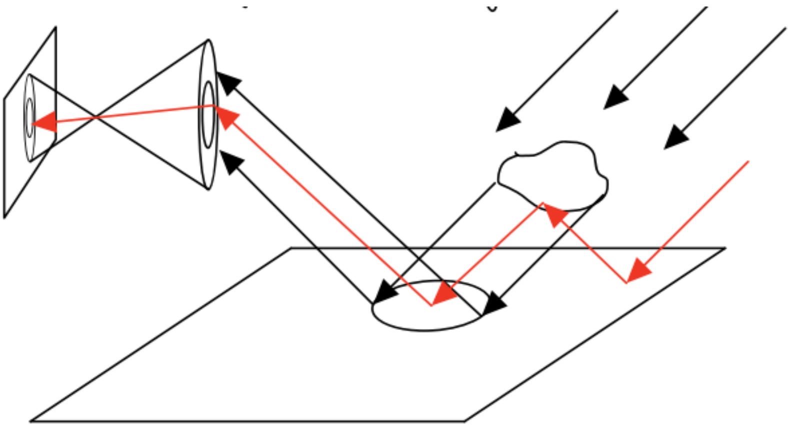
Shape: The wood gap has shape like this: 

Albedo: albedo is very small close to 0

Reflection: the light cast ito the gap and damped in each reflection, when the light reflect out, its intensity reduced/

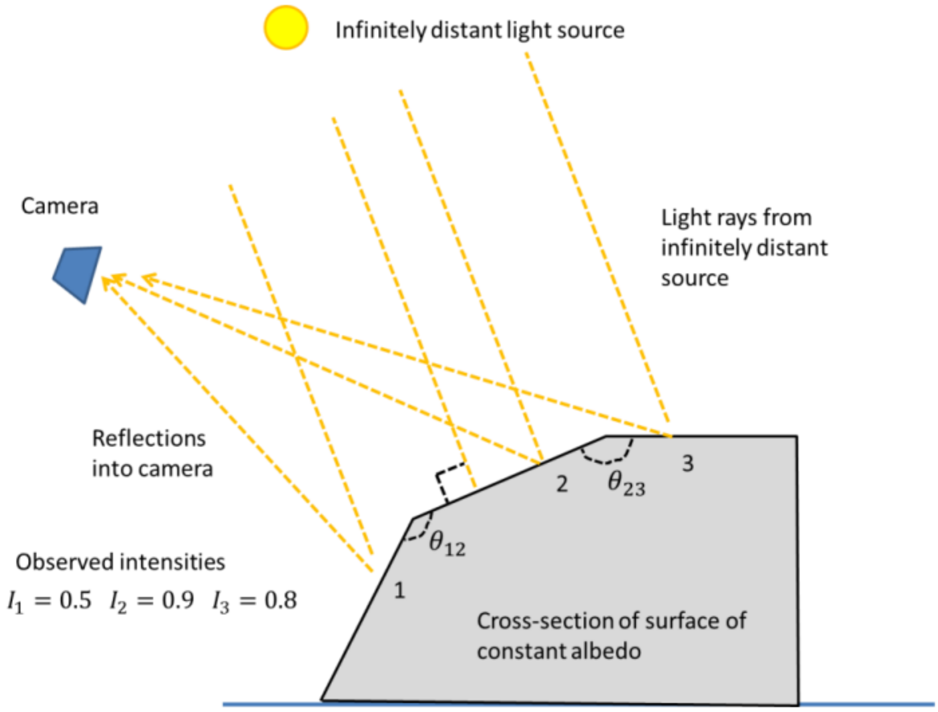
d. If the table were completely specular, would the glasses cast a shadow on it (explain why or why not)?

Yes, it will cast a shadow, because the object can reflect the light and cast the light on the region whose light was blocking by the object.



B.

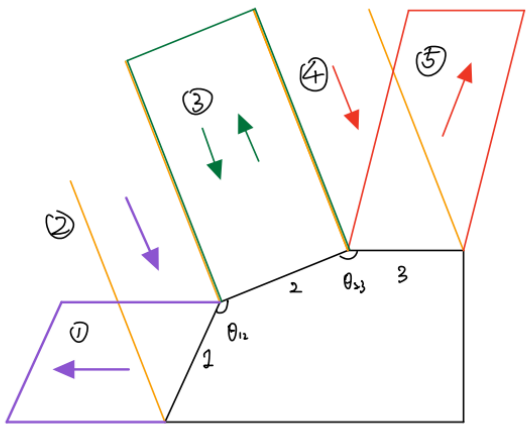
Answer the following using the above illustration. Suppose you have observed the intensities of three points on an object ($I1$,$I2$,$I3$), which are lit by an infinitely distant point source (the sun). The surface normal at point 2 is exactly perpendicular to the sun. The surface normals of points 1 and 3 differ in only one angle (θ), as shown in the cross-section.



a. Suppose the surface has a specular component. Will the observed intensities change as the camera moves (if so why/how)? (4 points)

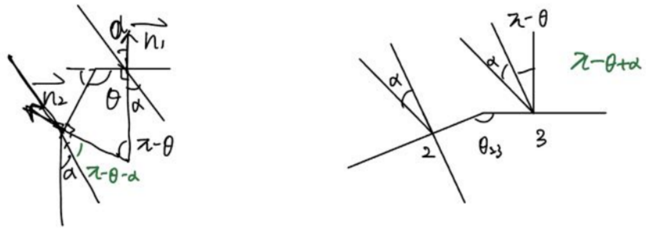
(a) the observed intensity will be different because the specular will only reflect in one direction

the ①, ③, ⑤ region will recieve the same intensity, while ②, ④ will observe darkness.



b. Suppose the surface material is Lambertian and has uniform (constant) albedo and that the camera response function is linear (and ignore effects due to interreflections in the scene). Express the intensities in terms of the between the surface normal and the lighting direction. Then, show (with equations for arbitrary observed intensities) how to compute the angles $\theta 12$, $\theta 23$ between surfaces containing points 1 and 2 and points 2 and 3. Finally, compute the values of $\theta 12$, $\theta 23$ for the observed intensities (0.5, 0.9, 0.8). (8 points)

(b)



$$I_2 = A \cdot \vec{n_2} \cdot \vec{s_1} = A|\vec{n_2}||\vec{s}| \cdot \cos\alpha$$

$$I_1 = A \cdot \vec{n_1} \cdot \vec{s_1} = A|\vec{n_1}||\vec{s}| \cdot \cos(\pi - \theta_{12} - \alpha)$$

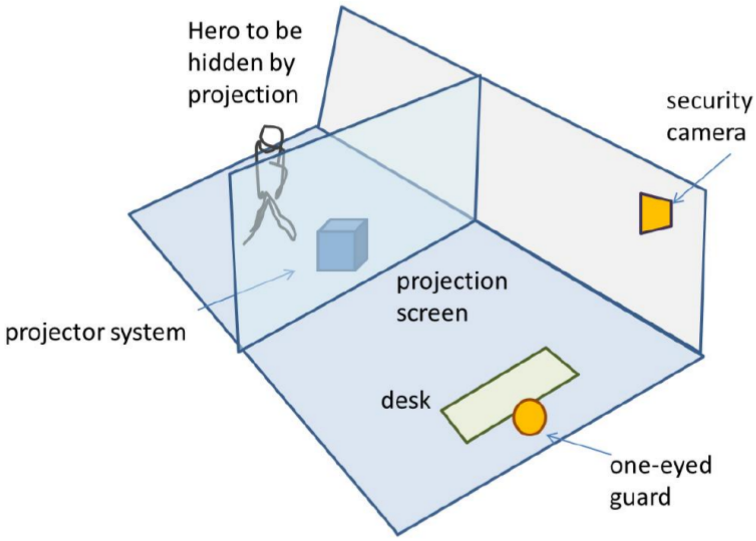
$$I_3 = A \cdot \vec{n_3} \cdot \vec{s_1} = A|\vec{n_3}||\vec{s}| \cdot \cos(\pi - \theta_{23} + \alpha)$$

$\alpha = 0$,

$$\therefore \quad \cos(\pi - \theta_{12}) = \frac{I_1}{I_2} \qquad \theta_{12} = \cos^{-1}\left(-\frac{I_1}{I_2}\right) = \cos^{-1}\left(-\frac{5}{9}\right) = 123.71^\circ$$

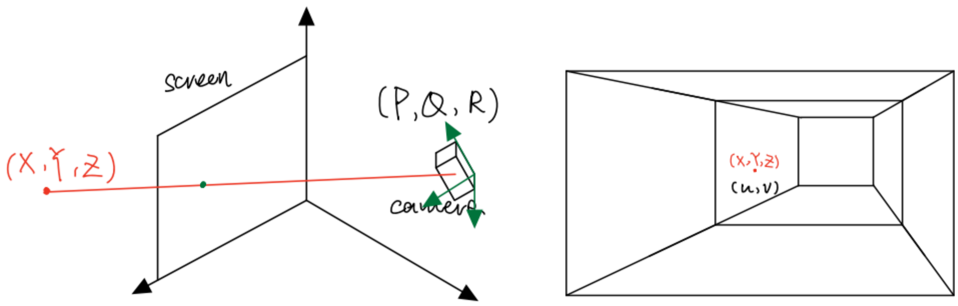
$$\cos(\pi - \theta_{23}) = \frac{I_3}{I_2} \qquad \theta_{23} = \cos^{-1}\left(-\frac{I_3}{I_2}\right) = \cos^{-1}\left(-\frac{8}{9}\right) = 152.73^\circ$$

Problem 2 Mission Impossible? (10 points)



a. A security camera is looking down the hallway at the screen. The camera can freely rotate but cannot otherwise move. Is it possible to put an image on the projection screen, such that the screen is undetectable for someone monitoring the camera? If not, why not? If so, what information is required? (4 points)

(a) It is possible to put image on the screen without being detected.



we need to know the pinhole position in the world coordinate and the position of all object that behind the screen. in the world coordinate, and the screen in the world corrodinate.

connect everything back the screen with the pinhole the points cross the screen is the fake img position

b. A security guard is sitting behind his desk looking down the hallway. Recently, while inspecting a pencil, the guard poked his eye, and now he has a patch covering that eye. But he can still move around and has one good eye. Is it possible to fool the security guard? If not, why not? If so, what information is required? (3 points)

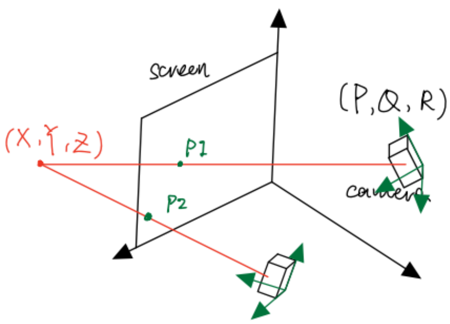
(b) It is possible to fool the guard with one eye.

We need to know the fine eye position all the object position behind. the Screen, and the screen in the world corrodinate.

the one eye situation is the same with the problem (a).

c. Is it possible to fool both the security camera and the one-eyed security man at the same time? If not, why not? If so, what information is required? (3 points)

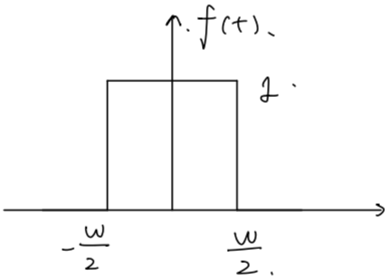
(c) Impossible. with 2 observer, it cause 2 cross point for one object project on the screen.



Problem 3 (10 points)

As the figure below shows, the Fourier transform of a "tent" function (on the left) is a squared sinc function (on the right). Advance an argument that shows that the Fourier transform of a tent function can be obtained from the Fourier transform of a box function. (Hint: The tent itself can be generated by convolving two equal boxes.)

Problem 3.

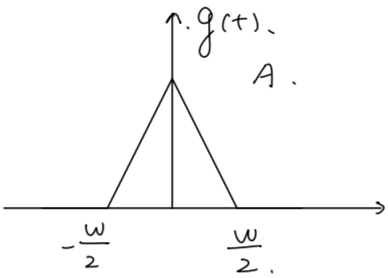


$$\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)\cdot e^{-j2\pi\mu t}\cdot dt$$

$$= \int_{-\frac{w}{2}}^{\frac{w}{2}} 1\, e^{-j2\pi\mu t}\cdot dt.$$

$$= \frac{1\left[e^{-j2\pi\mu t}\right]_{\frac{w}{2}}^{\frac{w}{2}}}{-j2\pi\mu t}.$$

$$= \frac{1}{j2\pi\mu t}\left[e^{j\pi\mu W} - e^{-j\pi\mu W}\right]$$



$$g(t) = Af(t) * f(t).$$

$$= A\int_{-\infty}^{\infty} f(\tau)\cdot f(t-\tau)\cdot d\tau$$

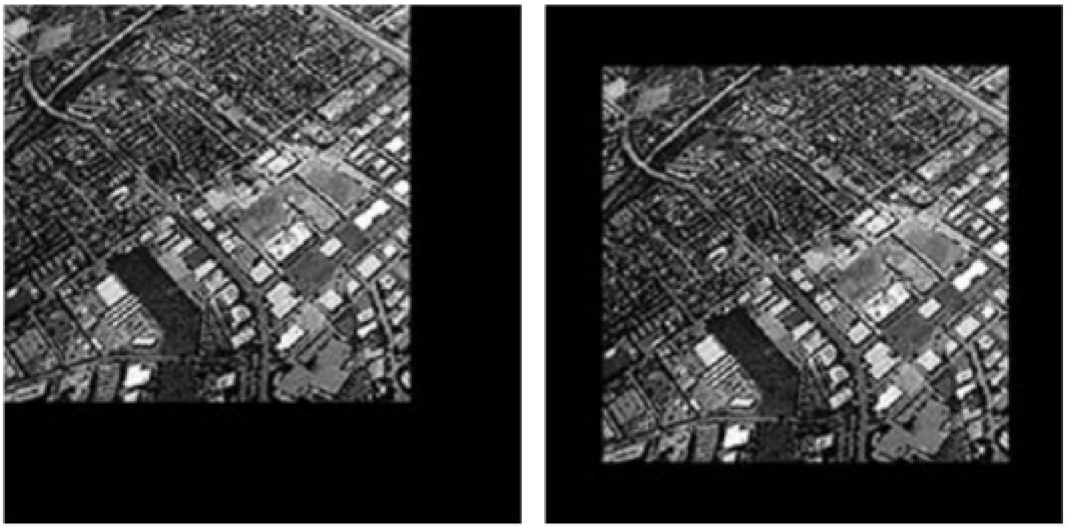
$$\mathcal{F}(g(t)) = A\mathcal{F}\{f(t) * f(t)\}$$

$$= A\,F(\mu)\cdot F(\mu)$$

$$= A\left(\frac{\sin(\pi\mu W)}{\pi\mu}\right)^2.$$

Problem 4 (10 points)

Images needed to be padded by appending zeros to the ends of rows and columns in the image (see the following image on the left). Do you think it would make a difference if we centered the image and surrounded it by a border of zeros instead (see image on the right), but without changing the total number of zeros used? Explain.

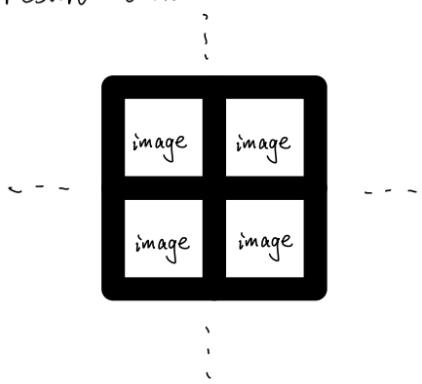


Problem 4. They are the same.

Zero padding is used to make zero fill the virtical & horizontal for the periods. Because.

$f(x, y) = f(x + k_1 M, y) = f(x, y + k_2 N) = f(x + k_1 M, y + k_2 N)$ in spatial domain.

Both 2 padding duplicate infinitely many times to cover the x-y plane, the result will be like



Problem 5 (10 points)

A continuous Gaussian lowpass filter in the continuous frequency domain has the transfer function

$$H(\mu, v) = Ae^{-(\mu^2+v^2)/2\sigma^2}$$

Show that the corresponding filter in the spatial domain is

$$h(t,z) = A2\pi\sigma^2 e^{-2\pi^2\sigma^2(t^2+z^2)}$$

Problem 5 ∴ is to Show $\mathbb{F}^{-1}\{Ae^{-(\mu^2+v^2)/2\sigma^2}\} = A2\pi\sigma^2 e^{-2\pi^2\sigma^2(t^2+z^2)}$

$$h(t,z) = \mathbb{F}^{-1}\{Ae^{-(\mu^2+v^2)/2\sigma^2}\}.$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(\mu^2+v^2)/2\sigma^2} \cdot e^{j2\pi(\mu t+vz)} d\mu \cdot dv.$$

$$= \int_{-\infty}^{\infty} e^{\frac{-\mu^2+4j\pi\mu t\sigma^2 - v^2 + 4j\pi v z\sigma^2}{2\sigma^2}} \cdot d\mu \cdot dv$$

$$= e^{-\frac{(2\pi)^2\sigma^2t^2}{2}} e^{\frac{-(2\pi)^2\sigma^2z^2}{2}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2}[\mu^2-j4\pi\sigma^2\mu t-(2\pi)^2\sigma^4t^2]+[v^2-j4\pi\sigma^2vz-(2\pi)^2\sigma^4z^2]} d\mu . dv.$$

$$= e^{-2\pi^2\sigma^2t^2} e^{-2\pi^2\sigma^2z^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2}[\mu^2-j4\pi\sigma^2\mu t-(2\pi)^2\sigma^4t^2]} \cdot d\mu \cdot e^{-\frac{1}{2\sigma^2}[v^2-j4\pi\sigma^2vz-(2\pi)^2\sigma^4z^2]} \cdot dv.$$

Let $x = \mu - j2\pi\sigma^2 t$

$dx = d\mu.$

$y = v - j2\pi\sigma^2 z$

$dz = dy$

$$h(t.z) = e^{-2\pi^2\sigma^2(t^2+z^2)} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{x^2}{2\sigma^2}} \cdot dx \; e^{-\frac{y^2}{2\sigma^2}} \cdot dy.$$

$$= e^{-2\pi^2\sigma^2(t^2+z^2)} \cdot 2\pi\sigma^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} \cdot dx \; \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} \cdot dy.$$

By the Gaussian distribution. $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} \cdot dx = 1.$

$$\therefore \; h(t,z) = 2\pi\sigma^2 e^{-2\pi^2\sigma^2(t^2+z^2)}$$

Problem 6 (10 points)

Show that the Fourier transform of the 2-D continuous sine function

$$f(x,y) = Asin(u_0x + v_0y)$$

is the pair of conjugate impluses

$$F(u,v) = -j\frac{A}{2}[\delta(u - \frac{u_0}{2\pi}, v - \frac{v_0}{2\pi}) - \delta(u + \frac{u_0}{2\pi}, v + \frac{v_0}{2\pi})]$$

(Hint: Use the continuous version of the Fourier transform and express the sine in terms of exponentials.)

Problem 6.

$$e^{\theta j} \quad \cos\theta + j\sin\theta .$$
$$e^{-j\theta} = \cos\theta - j\sin\theta .$$
$$\frac{e^{j\theta} - e^{-j\theta}}{2j} = \sin\theta .$$

$$\mathcal{F}\{\sin(u_0x+v_0y)\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \sin(u_0x+v_0y)\cdot e^{-j2\pi(\mu x+\nu y)} \cdot dx\,dy.$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{e^{j(u_0x+v_0y)} - e^{-j(u_0x+v_0y)}}{2j} \; e^{-j2\pi(\mu x+\nu y)} \cdot dx\cdot dy.$$

$$= \frac{j}{2}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{j(u_0x+v_0y)-j2\pi(\mu x+\nu y)}dx\cdot dy - \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-j(u_0x+v_0y)-j2\pi(\mu x+\nu y)}dx\cdot dy\right]$$

Because. $\mathcal{F}(e^{j2\pi u_0 t}) = \int_{-\infty}^{\infty} e^{j2\pi u_0 t}\, e^{-j2\pi\mu t}\cdot dt$

$$= \int_{-\infty}^{\infty} e^{-j2\pi(\mu-\mu_0)t}\cdot dt = \delta(\mu-\mu_0)$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{j(u_0x+v_0y)-j2\pi(\mu x+\nu y)}dx\cdot dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-j2\pi(\mu-\frac{u_0}{2\pi})x - j2\pi(\nu-\frac{v_0}{2\pi})y}\cdot dx\,dy.$$

$$= \delta\left(\mu-\frac{u_0}{2\pi},\ \nu-\frac{v_0}{2\pi}\right)$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-j(u_0x+v_0y)-j2\pi(\mu x+\nu y)}dx\cdot dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-j(\mu+\frac{u_0}{2\pi})x - j(\nu+\frac{v_0}{2\pi})y}\cdot dx\,dy.$$

$$= \delta\left(\mu+\frac{u_0}{2\pi},\ \nu+\frac{v_0}{2\pi}\right)$$

$$\mathcal{F}\{A\sin(u_0x+v_0y)\} = -j\frac{A}{2}\left[\delta\left(\mu-\frac{u_0}{2\pi},\ \nu-\frac{v_0}{2\pi}\right) - \delta\left(\mu+\frac{u_0}{2\pi},\ \nu+\frac{v_0}{2\pi}\right)\right]$$

Part II Programming Exercises

Problem 1

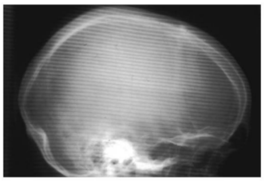
Run

Library import `matplotlib.pyplot`, `cv2`, `matplotlib.pyplot`

images in relative directory `./images`

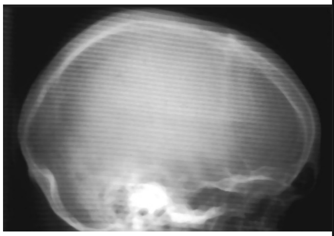
```
python main.py
```

Moire Pattern Suppression in Radiographs (30 points)

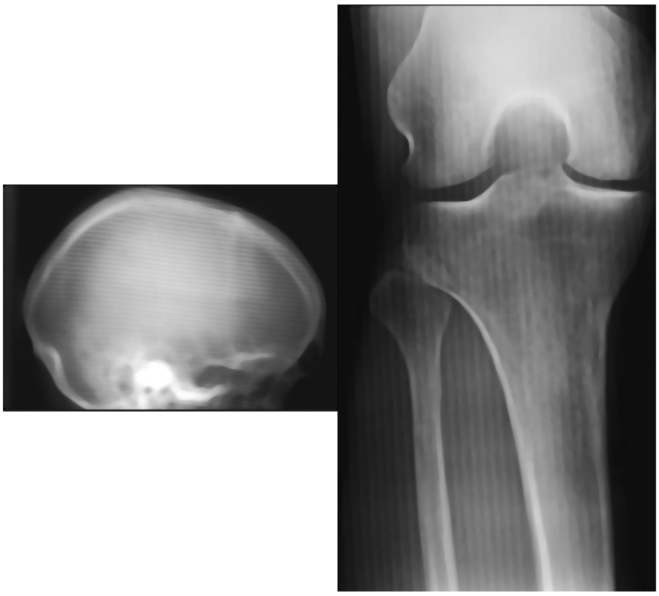
 

(a) For each image, apply an N ×N median filter in Problem 1 part (a). Adjust the window size N so that the Moire pattern is removed as much as possible while salient features are properly preserved. Report your choice of N. Display the filtered image, and comment on the quality of the filtered image.

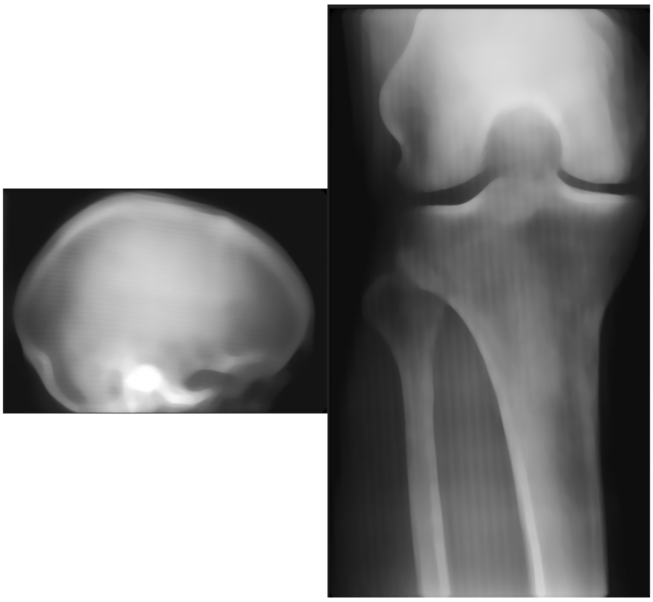
N = 5

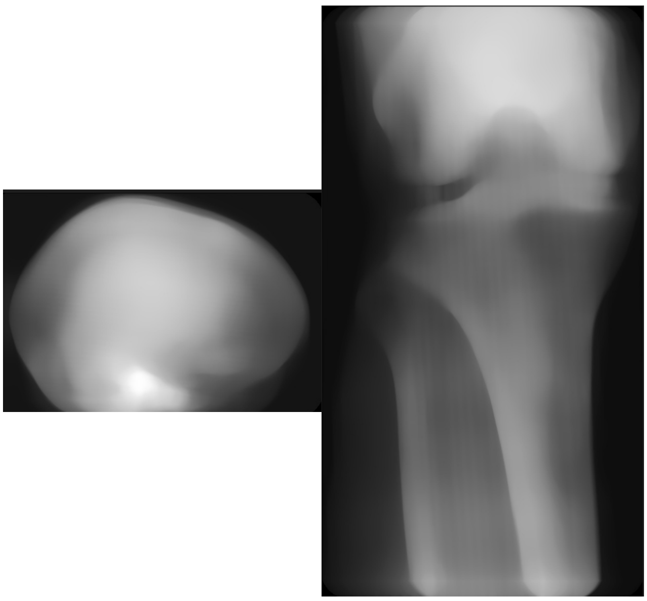
N = 10:



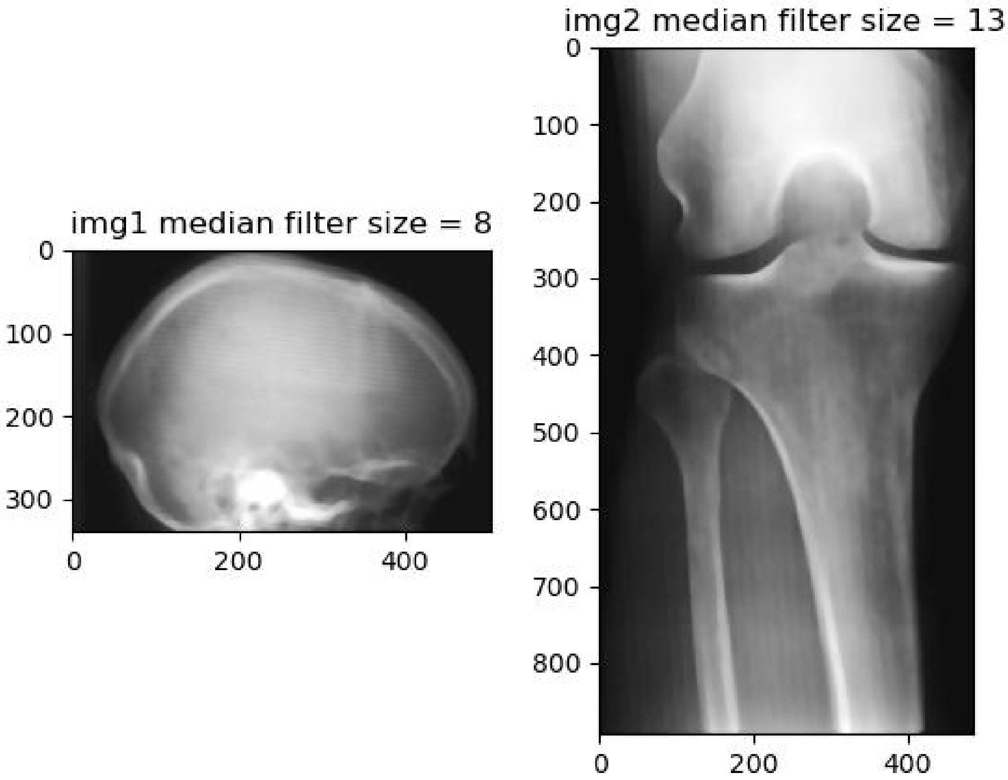
N = 20:



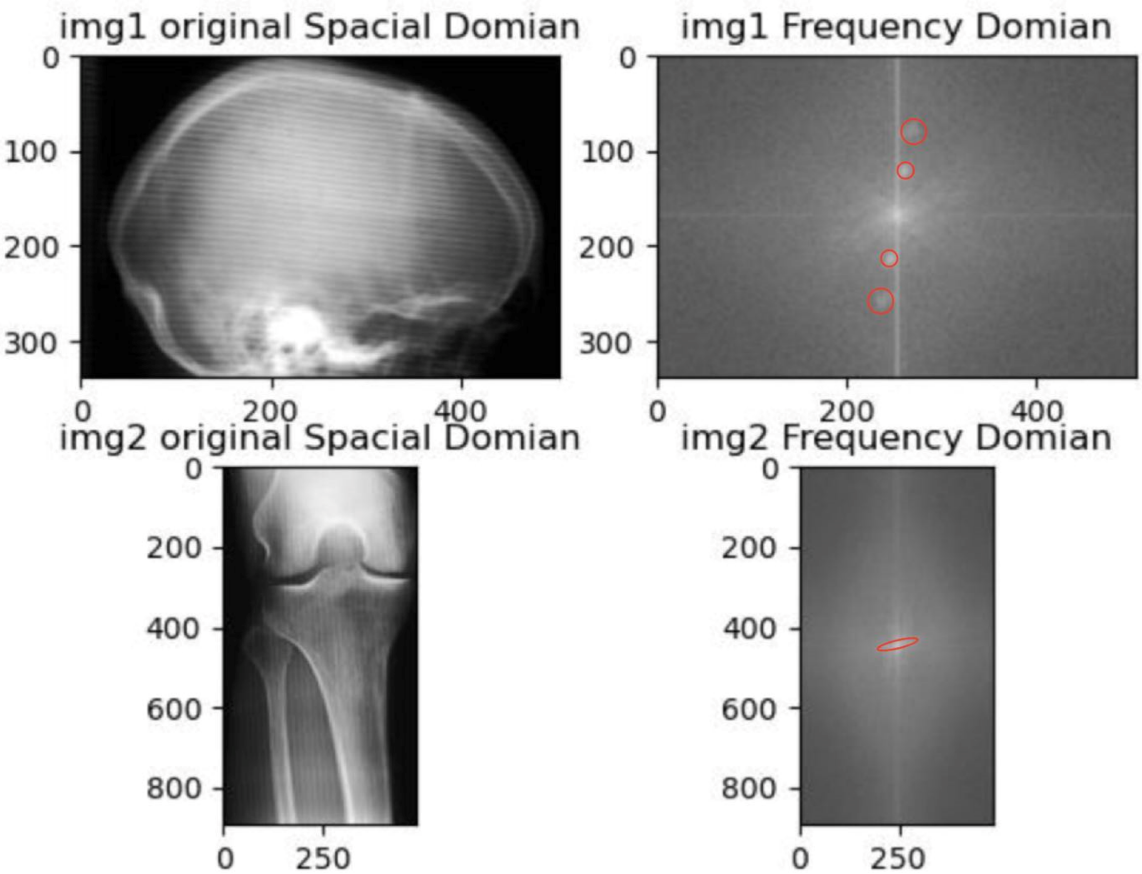
N = 50:



Picture between 10 to 20 window size is fine. Choose 8 ad 13 for each

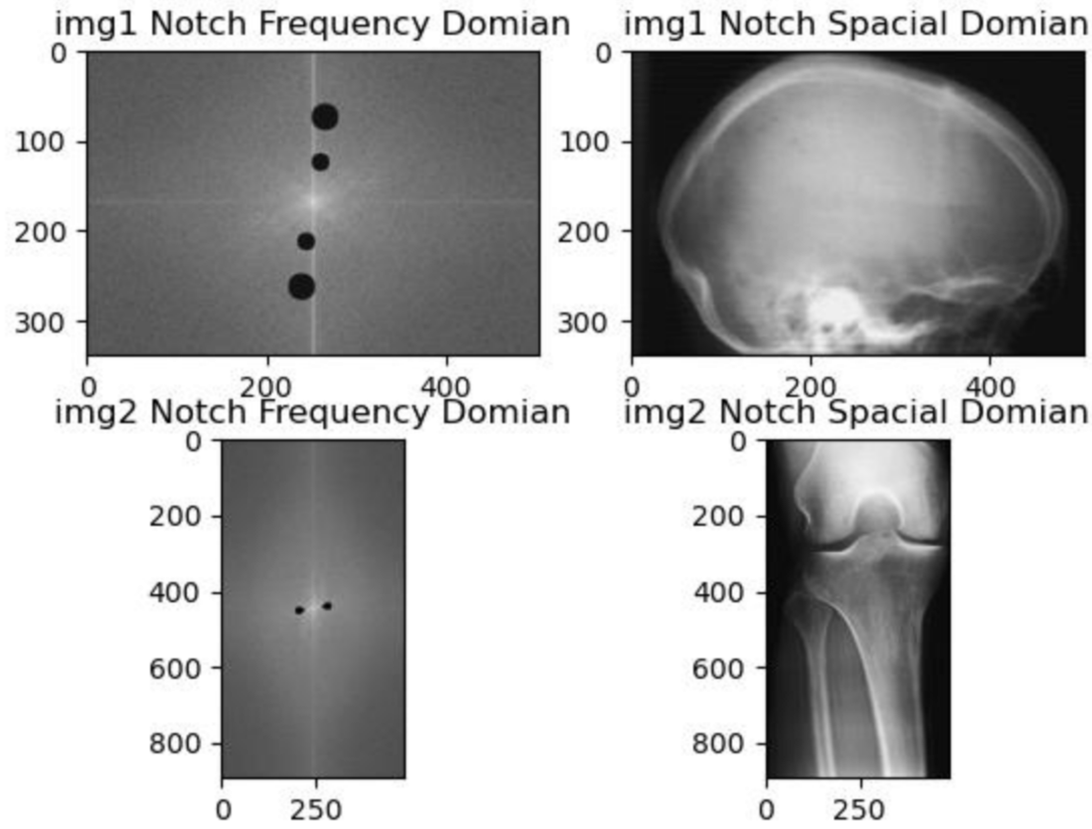


(b) For each image, compute its Discrete Fourier Transform (DFT) (numpy.fft) and display an image showing the DFT magnitude (numpy.abs). Clearly identify and label the frequency components that correspond to the Moire pattern.



The Morie pattern is circled in read

(c) For each image, design a notch filter notchFilter(img, parameter1, ...) so that the frequency components for the Moire pattern are suppressed as much as possible while other frequency components are preserved. Note that the parameters of the function are defined by yourself. Apply your notch filter to the image's DFT and display an image showing the filtered DFT magnitude. Display the filtered image in the spatial domain (numpy.fft). Compare the quality of the result to that of the filtered image from part (a). (Hint: numpy.meshgrid is useful for creating an (ωx, ωy) array.)



By taking away the frequency domain Morie pattern and do IFFT, the fine picture appeared.