Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
E		2205011

The MM Model: the Multivariant Forestry Based on Optimization and K-means Clustering

Summary

It is well known that global warming has posed a threat to life on earth. Under this backdrop, we should not only curb carbon dioxide emissions but also reduce the amount of carbon dioxide that already exists in the atmosphere by natural and artificial means. On this basis, carbon sequestration has become the focus of the global environment. This paper focuses on the amount of carbon sequestration that forests and their related products can provide and proposes better forest management and development strategies. In addition, we combine forest management strategies with the culture, climate, and lifestyle of different regions to provide diverse decision-making options for forest managers everywhere.

Our model consists of three parts. Before making the decision model, we visualize a forest and divide it into woods. Then we construct our carbon sequestration model by using optimization. On this basis, our forest management plan model then focuses on achieving a win-win for biodiversity, regional products, and carbon sequestration. We adopt a multi-objective programming model to realize the coordination of these three objectives.

Firstly, we searched the satellite forest photo and obtained the tree coverage rate of certain areas. The diffusion partial function could well simulate forest expansion. We calculated the future increased area. In this part of the model, we could only demonstrate the algorithm, but we couldn't really develop it because the environment resistance is unknown. We used a set of random vectors to implement it.

Secondly, to further refine the forest management plan, we separate the forest into regions. Then we use the simulated annealing algorithm to give how many trees that need to be cut to make them into products. We then create a very accurate management plan describing cutting which trees are in each region by sorting. After applying our model, the old trees will be cut down, while the young tree will stay to maximize the carbon sequestration amount.

Thirdly, we introduce the demand for biodiversity in our model. Using the K-means clustering algorithm, fundamental inequality, and basic log function, we get a function that describes biodiversity. We do the same optimization as the previous step. After applying our model, we know which trees should not be cut. The rarer trees should not be cut down.

After modeling, we apply our model to actual forests and conduct management plans at different stages. Then we discussed our results and summarized our strengths and weaknesses. We also wrote an untechnical newspaper which will be attached to this paper.

Keywords: Multivariant Forestry; optimization; K-means clustering; Diffusion partial differential equations;

Contents

1 Introduction **3**
1.1 Background 3
1.2 Our Work 3
1.3 Assumptions and Justifications 4

2 Notations **5**

3 The MM Model **6**
3.1 Data Description 6
3.2 The establishment of MM Model 6
3.3 Implementation 10

4 maturer MM Model for forestry **12**
4.1 Multi-optimize and Equilibrium Entropy Model **12**
Society Factors 12
Biodiversity 12
Micro Model 14
4.2 Result 14
Macro Model 14
Micro Model 15

5 Sensitivity Analysis **15**
Macro Model 15
Micro Model 16

6 Model Evaluation and Further Discussion **17**
6.1 Analysis of the Forest Management Plan 17
6.2 Strengths 17
6.3 Weaknesses 17
6.4 Further Discussion 18

7 Conclusion **18**

References **19**

Appendices **20**

1 Introduction

1.1 Background

Global warming is one of the biggest threats to life on earth. Carbon sequestration has become one of the environmental protection focuses as the greenhouse effect growth. Carbon sequestration refers to the conversion of carbon dioxide from the air into organic matter through the photosynthesis of plants and the retention of organic matter in trees and animals for as long as possible without decomposition.

Proper management of the forest ecosystem is an excellent way to realize carbon sequestration. By calculating the amount and duration of carbon sequestration in forest ecosystems and their products, we can maximize carbon sequestration. (Shown in Fig.1)

However, due to the impact of ecological and regional culture, the forest management plan with the largest carbon sequestration may not be the best one suitable for the local economic development level and ecology.

So how to develop a suitable forest management plan for each stage of forests in a different region is now meaningful to carbon sequestration.

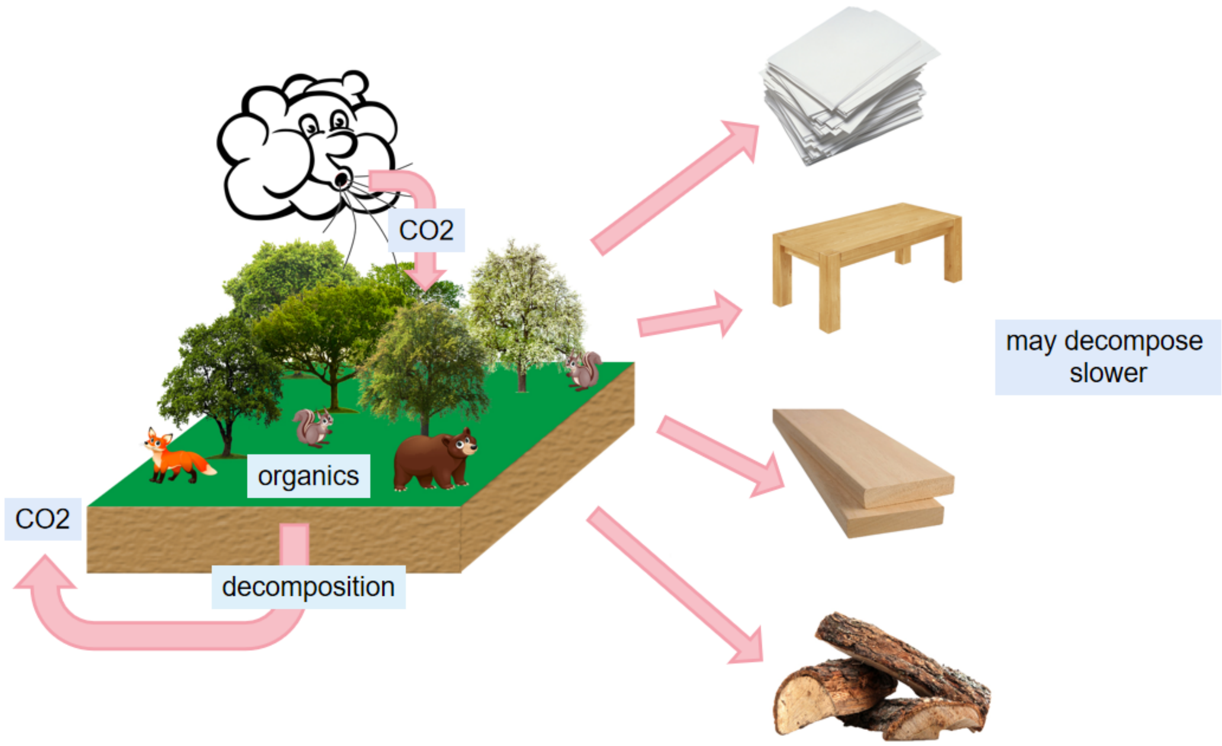


Fig 1: the forest ecosystem and carbon sequestration

1.2 Our Work

Task 1: Finding the boundary of the forest and separating it into different woods. Then calculate the density of trees for each wood. We use the diffusion partial differential equation to simulate the forest expansion. It's an analogy between heat diffusion and tree growth.

Task 2: Use the optimization to calculate how many trees should be cut in one wood. Then sorted the trees in each wood to decide which kind of trees we need to cut.

We use the logistic-like function to represent the relationship between the age of the wood and the carbon sequestration amount. Then we use it to construct a convex function when the products' lifetime is larger than the period of testing. Otherwise, we won't cut any trees. Then we sorted all the trees in each wood by their carbon-absorbing rate and cut the older trees with lower carbon-absorbing rates.

Task 3: Adding biodiversity and demand factors to develop a multi-objective model.

We use K-means clustering, basic inequality to quantify the dispersity of woods, and the diversity of the trees' species for ever woods. Then we ass this factor together with the tree's growth rate to decide which woods should we cut first. We use all the amount of trees from optimization to satisfy the demand until the demand for the product is satisfied. Otherwise, we will then separate the rest of the demand to each wood equally.

A clearer illustration of our work can be seen in the following picture.

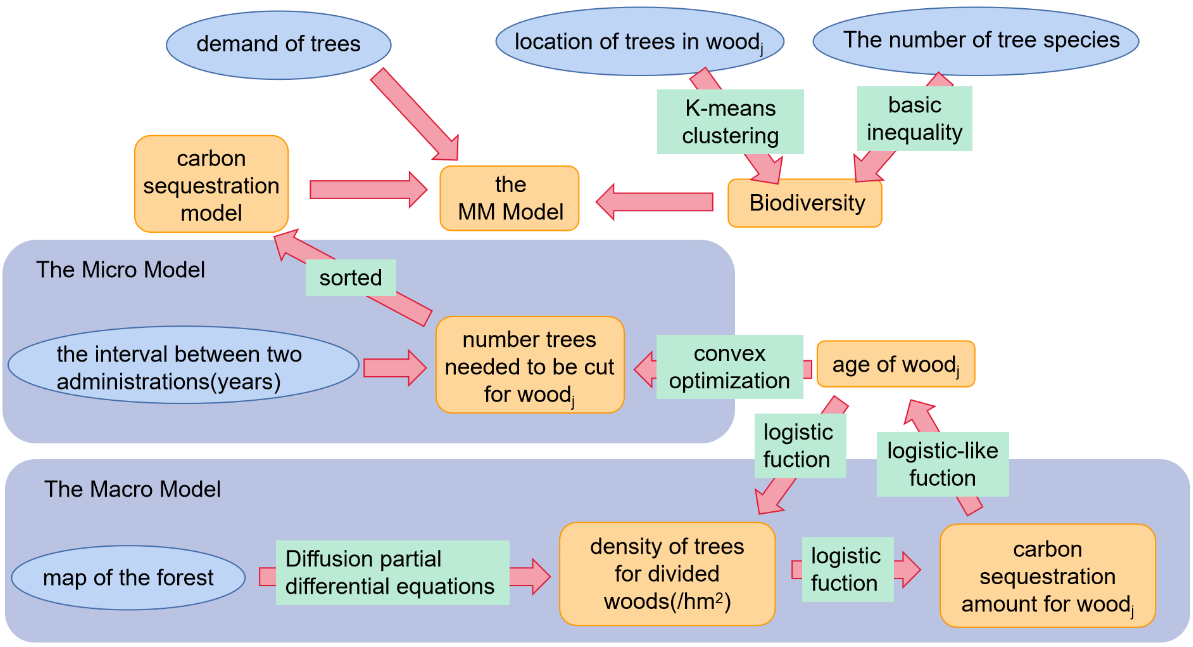


Fig 1: Model building chart

1.3 Assumptions and Justifications

To better describe the question, we made several assumptions with justifications:

Assumption 1: The carbon sequestration of one tree has a logistic relationship to the age of that particular tree.

Justification:

If one tree is young, it can not sequestrate much carbon. Since the number of its leaves is not large, it only transforms a small amount of sunlight into energy. Therefore the sequestration rate is relatively low. In the middle part of its growth, it carries on exuberant photosynthesis, which means It absorbs carbon at a fast rate. At the end part of the growth, a tree has its height limitation. Only lateral growth limits the rate at which he can convert carbon. It cannot grow as fast as earlier in the future. Therefore, we assume that one tree has a logistic relationship to the age of that particular tree.

To improve our assumption is accurate in some sense, we use the species group and total trees volume data from NFI.

Assumption 2: We have biodiversity consideration.

We have three assumptions:

1. The more the species are, the better the biodiversity is.
2. The number of each species is more balance, the better the biodiversity is.
3. The entropy of one species is higher, the better the biodiversity is.

Justification:

Biodiversity is a very important factor in forestry. It mainly contains the species of trees in one area, the number of trees in each species, and the entropy.

Assumption 3:

1. The cutdown of the forest is periodic because the worker is not always available to cut the trees. Assigning a long-term contract with the logging company is a suitable and reasonable choice for arrangement.
2. There is a limitation of the logging period. If the period is too short, the logging company will be overwhelmed. If the period is too long, the logging company may not consider that long-term contract.
3. It costs the same to reach any part of a forest. They don't take into account the distractions of the journey

Justification:

The testing of the forest is periodic since we need time for trees to grow. If the time is too short, the tree won't change too much. If the time is too long, we can't catch the change of the trees timely.

2 Notations

The key mathematical notations used in this paper are listed in Table 1.

Table 1: Notations used in this paper

Symbol	Description	Unit
Tj	The time when the jth tree was planted	year
t	The time now	year
Ai	A constant describes the max carbon accumulation rate of the ith tree. This constant is a fixed number for one type of tree	m3/ year
Bi	A constant describes the max size of the ith tree. This constant is a fixed number for one type of tree.	m3
Bio_j	The bio-term of j^{th} tree which will decide which trees should be cut in the micro model	/
Ei	The number of the type of ith trees in that area	/
N	The number of trees in that area	/
T	The length of time describes how long the next cut begins.	year
T0i	The initial age of woods i	year
v_i	the carbon sequestration rate of each tree	t/year
age_j	the age of the j th tree	year
x_{ik}	the number of trees for the kth type in the ith woods	/
m_i	the type of trees in the ith woods	/
N_t	The number of the types of trees in that wood	/
n_{ti}	The number of the i^{th} type of trees in that area	/

Bio_D	The bio-diversity term, describe the biodiversity in a particular area	/

3 The MM Model

3.1 Data Description

Carbon sequestration capacity of different tree species, measured by carbon density, is shown in figure 2.

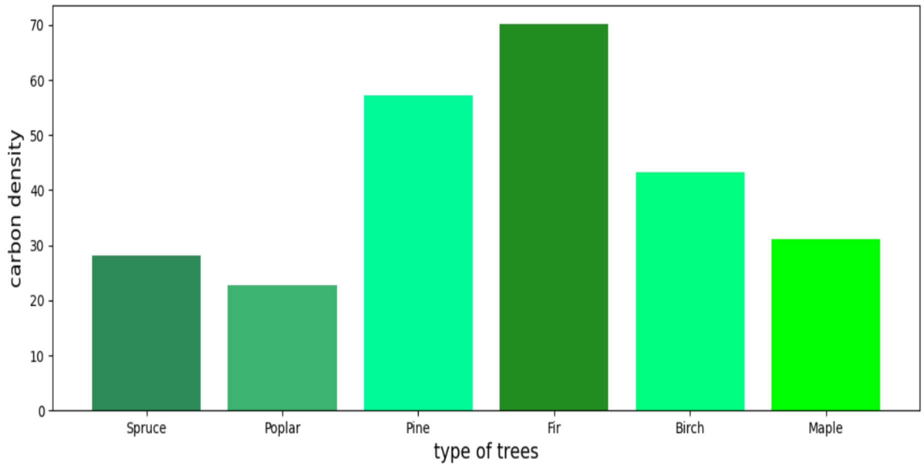


Fig 3: the carbon density of different types of trees

By using the data from NFI in CaUsing also roughly processes the number of trees varies from species to age.

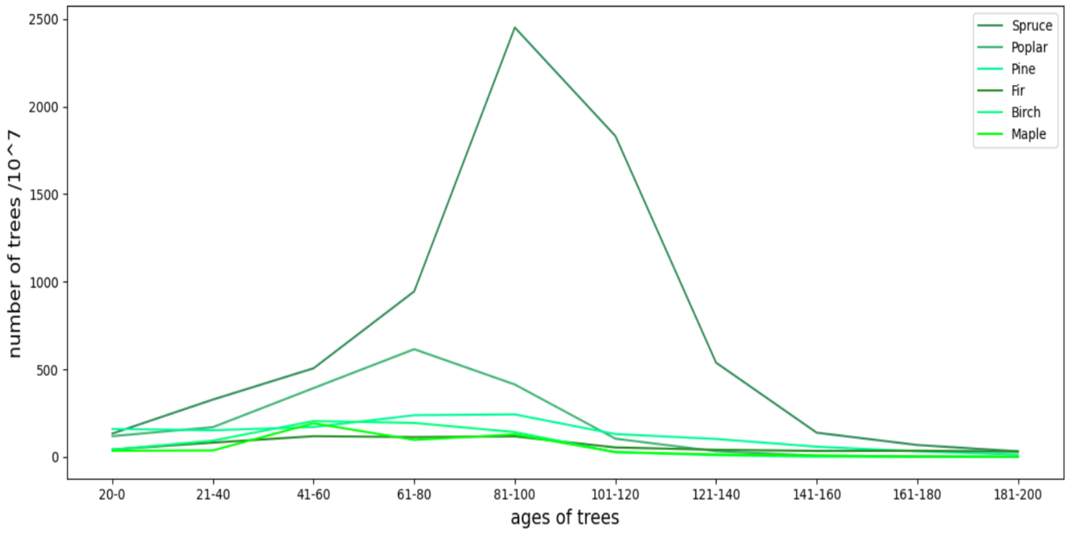


Fig 4: The number of different kinds of trees in Canada

3.2 The establishment of MM Model

We divide our model into two parts. The first part is macroscopic. The second part is microscopic.

In the macroscopic model, we divide the forest into several rectangle parts with equal areas. Each rectangle is recorded with its tree density. The histogram of color can measure this density. We first obtain a satellite picture of the whole forest. Then, divide the entire forest picture into several squares. We calculate the tree density in every square. After that, we gather the data of the green color density of one particular part of the forest. The green color density is the density of the woods. According to Loades(2010), Rational dense planting affects the growth of forests. We also use a logistic function to simulate the thickness of a forest and its carbon sequestration volume. Furthermore, we observe that the density increasing rate is proportional to density itself. As the density increases, it also limits its density increasing rate. In other words, the relationship between density and time is also a logistic model. After a series of calculations, we can also use the logistic model to fit the relationship between carbon accumulation volume and time. The detailed information is given in the following section.

The MM model is divided into macro and micro models. The macro model aims to divide a large forest into small regions based on the density of trees and decide how many trees need to be cut in each region. The micro model will then decide which type and how many trees we need to cut based on their Carbon sequestration ability and ages in each region.

Macro Model

In the forest area increasing model, we assume that each point on a piece of land has its own most suitable forest expansion direction, and we assume that this expansion direction can be represented by a continuous field. We call the F field the expansion field and F(x,y) the expansion vector

$$F(x, y) = M(x, y)\hat{i} + N(x, y)\hat{j} \tag{1}$$

The forest expansion model uses Green's theorem to derive the expansion rate per unit time.

The growth pattern of forest boundary density can be simulated by diffusion partial differential equation

$$\frac{\partial \rho(\hat{r},t)}{\partial t} = \nabla \cdot [D(\rho,\hat{r})\nabla\rho(\hat{r},t)] \tag{2}$$

We represent the forest coverage ratio at the time t as ϕ, where $\hat{r} = x\hat{i} + y\hat{j}$. $D(\phi,\hat{r})$is the diffusion coefficient at density $\phi(\hat{r}, t)$, which is related to the expansion vector $F(x, y)$of the environment.

In the two-dimensional case, we can write the diffusion control equation as

$$\frac{\partial u}{\partial t} = v\frac{\partial^2 u}{\partial x^2} + v\frac{\partial^2 u}{\partial y^2} \tag{3}$$

u is a coverage ratio matrix. By using forward difference and central difference for all spatial terms, we get the discrete equation:

$$\frac{u^{n+1}_{i,j} - u^{n}_{i,j}}{\Delta t} = v\frac{u^{n}_{i+1,j} - 2u^{n}_{i,j} + u^{n}_{i-1,j}}{\Delta x^2} + v\frac{u^{n}_{i,j+1} - 2u^{n}_{i,j} + u^{n}_{i,j-1}}{\Delta y^2} \tag{4}$$

n is the number of the time interval. From the above equation, we obtain the expression of the term to be solved

$$u_{i,j}^{n+1} = u_{i,j}^{n} + \frac{v\Delta t}{\Delta x^2}(u^{n}_{i+1,j} - 2u^{n}_{i,j} + u^{n}_{i-1,j}) + \frac{v\Delta t}{\Delta y^2}(u^{n}_{i,j+1} - 2u^{n}_{i,j} + u^{n}_{i,j-1}) \tag{5}$$

Referring to Global Forest Watch Data, on an area of 25 square kilometers, the red square in the forest area at the initial time. We assume that the coverage of an original forest is a superposition of a mixed binary Gaussian distribution.

$$D(\rho, \hat{r}) = ||F(x, y)|| \cdot u \tag{6}$$

The green part of the results is the one-year-stretch of the growth dynamic field simulation in a random environment. By iteration, we can calculate the total coverage change by

$$coverage\ change = \sum_{i}\sum_{j} u_{i,j}^{n+1} - u_{i,j}^{n} \tag{7}$$

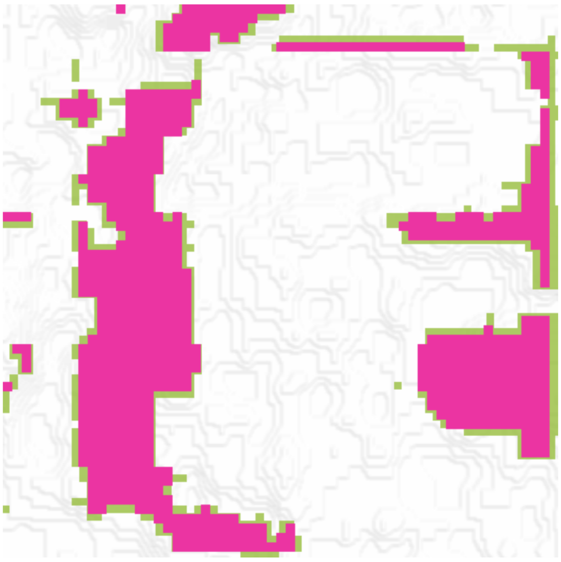


Fig. 5: The Forest boundary expansion

We then separate the forest into small pieces($1hm^2$). Our model then requires managers to offer the density of trees, the age composition, and the number of trees for different species of each region. They also need to offer the year Δt that they want so that after Δt years the carbon sequestration will be the maximum.

Based on our assumption, the relationship between forest density and forest age and the relationship between Carbon sequestration and forest density are both logistic functions. We proved that the relationship between Carbon sequestration and the age of the forest is a logistic function(The process is attached in Appendix 1.). So the relationship between Carbon sequestration and forest age can be represented as a logistic function.

$$C_i(T) = \frac{-wi}{(1+k_i \exp(a_i T - \rho_{0i})} a_i + w_i \tag{8}$$

, where Where ai, ki, wi, ρ0i are constants that represent two constants, the ability to absorb carbon and the original carbon sequestration density for the earth in woods i. C(T) represents the Carbon sequestration amount during t and t+Δt, where Δt is the next test time. Since we want to maximize the increased amount of Carbon sequestration, the equation would be

$$max\, C(t + \Delta t) - C(t) + I \cdot C(T_i) - I \cdot C(t) - C(T_i + \Delta t) + C(T_i) \tag{9}$$

If the value of the above function is smaller than 0, which means cutting trees will lead to the reduction of carbon sequestration amount after Δt years, then we choose not to cut the trees. Otherwise, based on our assumption, we translate the age into density since the time can be represented by density through a logistic function.

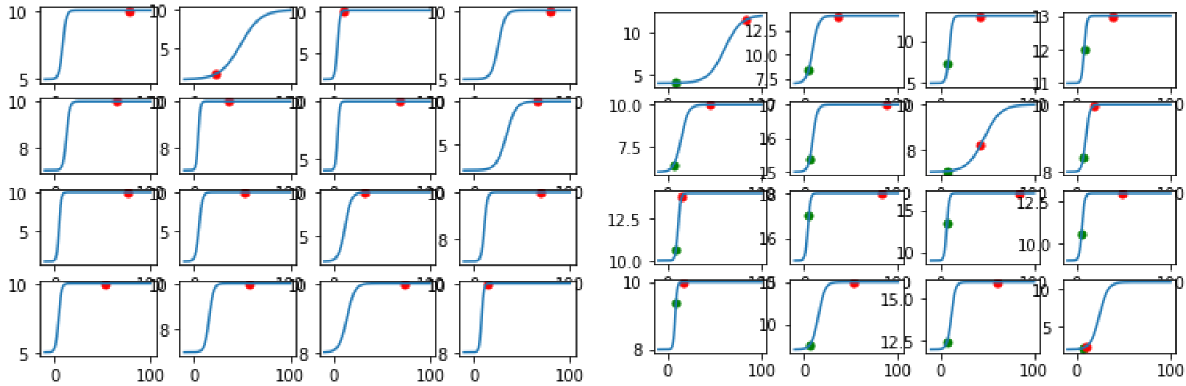


Fig. 6: woods carbon sequestration stage demonstration

In the first graph, A piece of forest is divided into 16 parts (much more woods in reality). The current time is signed by red points. By optimization procedure, the decision tree-cutting amount can be represented by a green point, which is equivalent to a time degeneration of the current carbon sequestration. The difference between red and green ordinates is the carbon exports from the woods during the harvest cycle. Our goal is to maximize the sum of them.

Micro Model

For the micro model, we first search all the trees available in the region and decide which trees we need to cut down. Some primary attributes like the type of the trees and the age of that trees are given. The constants configure initial carbon sequestration, carbon absorption rate, and final maximum w. The kind of the trees determines them. They are the maximum growing speed maximum size of this type of tree. In the previous paper, researchers used linear regression or logistic function to simulate the tree's mass growth over time.

Since cutting a tree down is a discrete problem, we cannot cut some tree parts and hope that tree could grow again. We can only cut down trees one by one. Therefore, we need to use an algorithm to describe how we choose the trees rather than using a function.

We use the logistic function in our assumptions to represent the relationship between a tree's age and its carbon sequestration amount. Then we take the derivative of this function to get the carbon sequestration rate of each tree, noted by v_j. For example, if the max-age of this tree is denoted

$$\frac{B_i \exp(-A_i(x-T_j))}{(i+\exp(-A_i(x-T_j)))^2} \tag{10}$$

Then we will sort v_j from small to large, then we get above to form a sequence. We potentially wanted to cut N trees with the lowest carbon sequestration rate, yet due to the symmetry of the carbon sequestration rate function, some young trees will be cut if we do so.

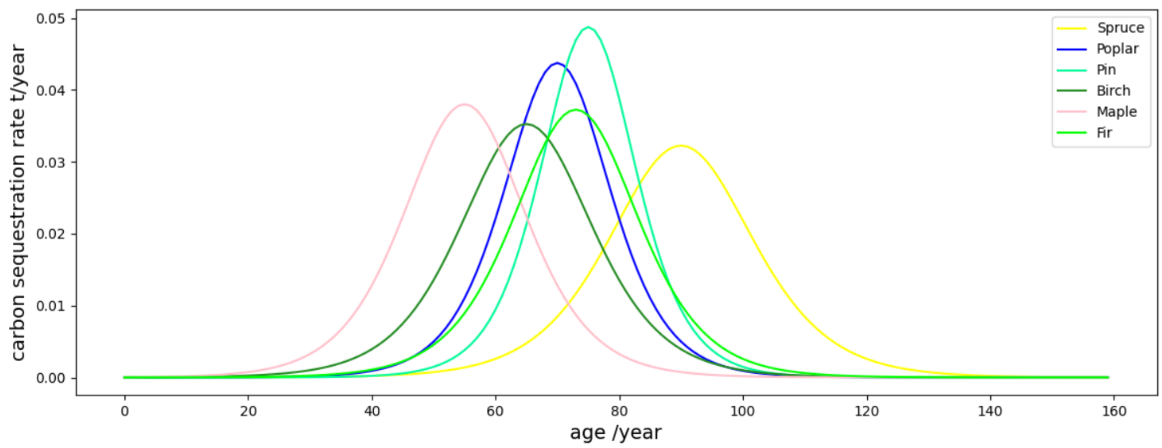


Fig4: The carbon sequestration rate of different trees over year

Assume that there are n young trees in N trees that need to be cut; we will then retrieve the ordered sequence up and replace these n trees with another n old trees.

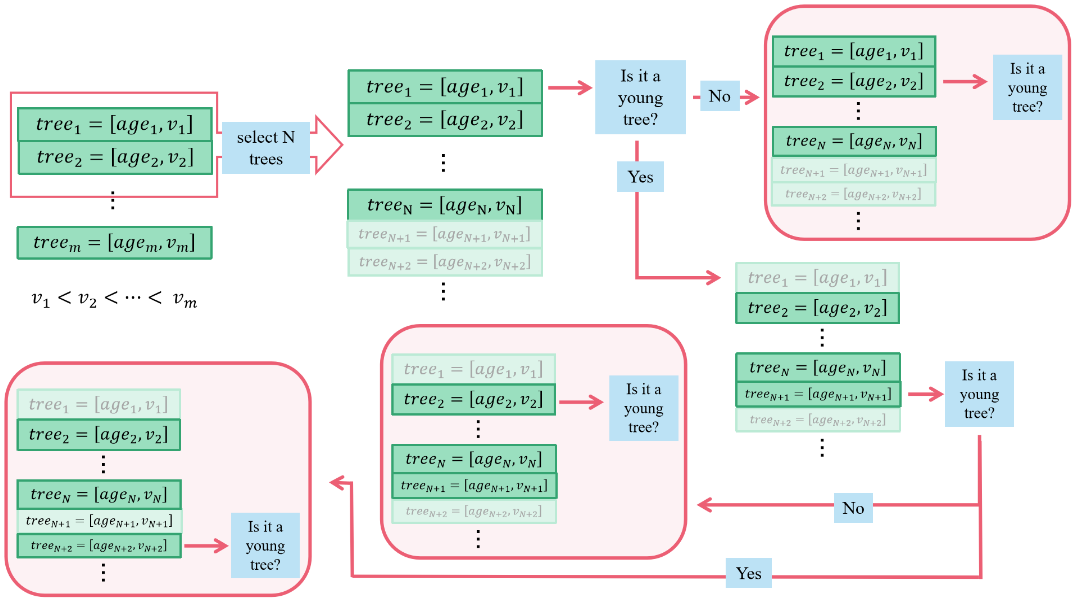


Fig 5: the main algrithm for micro model

3.3 Implementation

Macro model

After the forest boundary was retrieved, the forest was divided into small woods (/ hm^2). With the density of woods and the age of each wood as inputs, the decision to maximize the carbon sequestration was made through simulated annealing. We also require a manager to offer the year when the carbon sequestration amount is maximized and the maximum of products lifetime. Since we want to maximize the carbon sequestration amount, we only produce the product with a maximum lifetime.

We used python and a simulated annealing algorithm to solve this problem. The program is attached in Appendix. 2.

The below table shows the original density. The green block means we decided to cut the trees in that woods. Since the space is limited, we only list 100 blocks of wood.

593.7	1771	1755.5	428.2	1793.8	1626.9	1793.9	1793.7	1397.7	1136.6
332.7	1658.2	1634.4	1787	299.2	378.9	1792.3	1792.7	1787.6	874.6
1792.6	1787.2	1790.6	1667.3	993	1785	1773	1725.7	1791.1	1793.7
1787.1	1714.8	1792.5	1791.6	1769.1	347.6	1792.6	1777.6	1356.4	1790.9
1791.3	1780.2	971.1	1792.9	1720	1792.8	1469.3	1786.8	342.3	1488.4
1373.5	1667.3	1729.9	1784.5	1217.1	1790.4	1611.2	1792.1	1138.6	1750.2
1174.8	1790	1792.7	1652.5	1598.4	407.3	1714	1793.2	1651.5	837.8
1789.9	1084	319.4	1291.1	1790.6	1647.2	1786.1	1793.6	1189.2	860.3
1711.2	1087.4	1718.4	1008.1	266	1793.9	789.2	489.1	258.7	1793.3
1793.7	336.9	1490.6	453.3	582.6	886.7	1763.4	394.6	1784.8	1789.2

Table1: the result of the Macro Model

The number of trees needed to be cut is 1297, 58, 148, 7, 96, 1, 141, 82, 4, 145, 15, 166, 151, 1345, 2, 467, 54, 146 (corresponding to the green blocks from left to right, from up to down). The total increase of carbon sequestration of these 100 blocks of wood is 38.32t among 10 years after our cutting decision.

Micro Model

As the Macro model illustrates above, we need to cut down at least 1000 units of wood to meet the company's demand. Since there is relatively little information we can obtain from the internet, we can only make up a large dataset for us to run. An input form is a CSV form that is appended below after applying our algorithm to the data input. We know the id of trees we need to cut down. Since the dataset is too large to illustrate here, we only illustrate the first 100 data points to show in the graph. This graph shows the age of the trees we need to cut before logging and after logging. The age transforms to 0 means we need to cut it down.

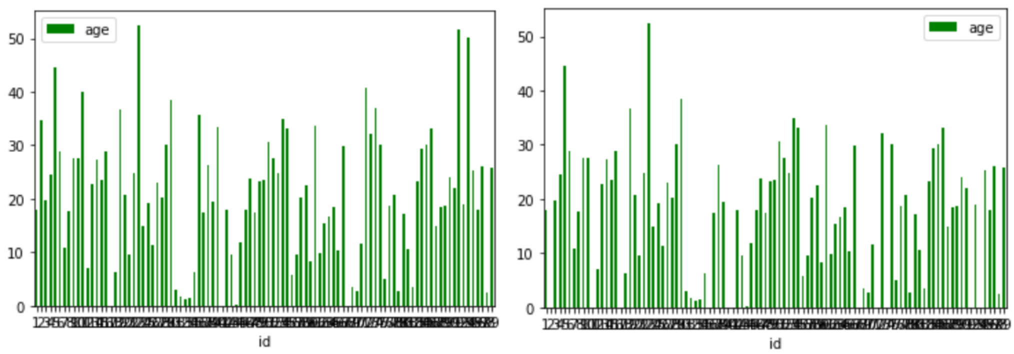


Fig 7: the age of trees before and after cutting

The first graph illustrates the trees' age before logging, and the second illustrates that of after the logging. We repeat this algorithm many times; we will get a final answer. By using the algorithm we illustrate above, we find an optimal solution to the following question.

1. Question 1: If the government or the company wants 20000 units (one can change this

number if one wants more wood, in this paper, we only present an example) of wood in the following 100 years, which trees should one cut down. And when to cut it down?
2. Question 2: If there is no particular demand of how much wood one needs. The logging team comes as frequently as one year(one can modify this variable, in this paper, we only present an example) to be more precise. How to manage the woods to maximize the volume of carbon we sequestrate?

4 maturer MM Model for forestry

4.1 Multi-optimize and Equilibrium Entropy Model

Society Factors

We assume that the forestry manager can receive a $y\ (tons)$ wood order from the market. Our goal is to balance the carbon sequestration amount biodiversity of trees according to the market order.

We assume that each block of woods' mass growth is a linear function of time $mass = kt + b$, By substituting time t, we get that z tons wood decrease lead to carbon sequestration phase go back to $\frac{z}{k}$ a year. Therefore, the problem of cutting how much of woods y_i becomes:

$$max_y \sum_{i=1}^{n} \rho_C(t - \frac{y_i}{k_i} + \Delta t) - \sum_{i=1}^{n} \rho_C(t - \frac{y_i}{k_i}) \tag{11}$$

Biodiversity

Based on the assumptions above, we now focus on balancing biodiversity and carbon sequestration. When we only plan to maximize carbon sequestration, we neglect the importance of biodiversity. We can only plan to grow the tree with the best carbon sequestration ability. Nowadays, European countries use mixed forests to enhance biodiversity.

We break the biodiversity goal into three parts to quantify it. First, a diverse eco-system should have relatively close trees' number of the different types. To do so, we bring in polynomial multiplication to measure how average the number they have. We added an adjustment term before the multiplication to eliminate the downtrend brought by multiplying too many terms.

$$(\frac{N_t}{N_t-1})^{N_t} \prod_{i=1}^{N_t} (1-\frac{n_{ti}}{N}) \tag{12}$$

Second, to have reasonably many kinds of trees, the $k\frac{ln(N_t-a)}{(N_t-a)} + b$ can simulate a first grow then slightly decrease model to represent how trees category can affect the biodiversity. Where the constant 'a' represent the suitable amount of trees.

Thirdly, a dispersive location of the same kind of trees to make the forest has homogenization, which means if the trees are cluster by their type, the biodiversity is lower. In Fig.7, the disparity in the left picture is lower than in the right image.

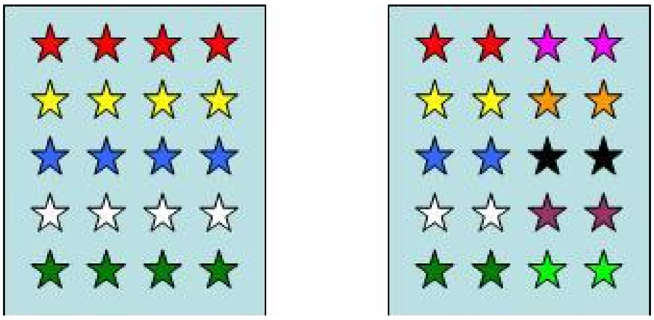


Fig 8: the illustration of dispersity

Here, we innovatively use an inverse K-means clustering model. Since the K-means clustering is used to determine the boundary of each type of point, the classification efficiency of K-means clustering represents the dispersity of the data. That is to say. The smaller efficiency means a better dispersity of the woods. So the third term is(1-K-efficient). Since the basic K-means algorithm only shows the difference of two families of points. It cannot point out the real type of the trees. The graph below illustrates types 1,2,3 may all come from one cluster. We cannot use K-cluster's number to represent the type number since K only represents the difference between two clusters. To quantify the outcome of the K-clustering, we use the following algorithm.

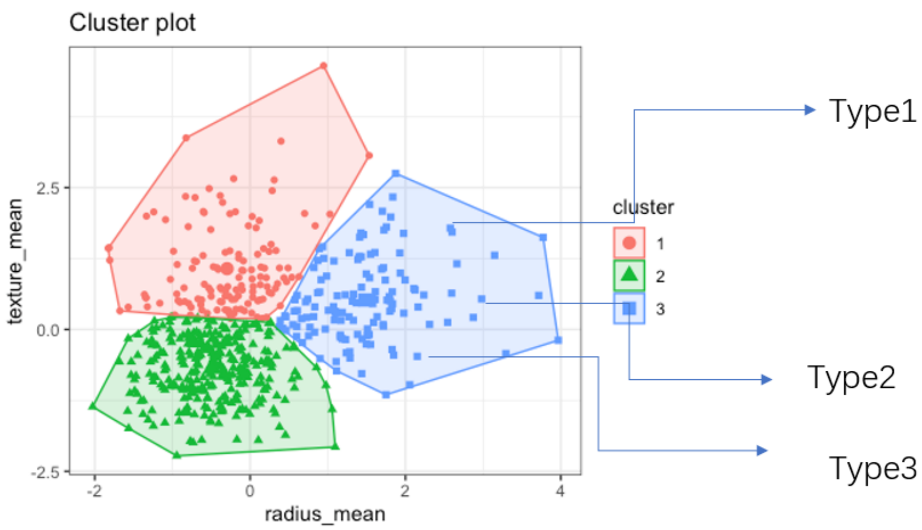


Fig.9 K-means Cluster Plot

In every K-cluster, we first transform the kind of trees into a sequence of id. For example, type A corresponds to $\frac{1}{N}$, and type B corresponds to $\frac{2}{N}$. If we have N types of trees, then the corresponding integer sequence is from $\frac{1}{N}$ to N. This operation neutralizes the difference brought by the number of types of the trees. Then we calculate the variance of the sequence corresponding to that K-cluster. We apply this method to all clusters. After that, we use the mean of the variance of all clusters to represent the outcome of the k-clustering. In this paper, we name this number(the mean of the variance of all clusters) K-efficient. Since this number cannot go over one and the bio-diversity decrease as K-efficient, we use (1-K-efficient) to describe the diversity on this term. Since we don't know how to balance these three factors, we set three weights for them. The final qualification of biodiversity could be

$$Bio_D = k\frac{ln(N_t - a)}{(N_t - a)} + (\frac{N_t}{N_t - 1})^{N_t} \prod_1^{N_t} (1 - \frac{n_{ti}}{N}) + k_2(1 - k_{efficient}) + b \tag{13}$$

for convenience, we call this term as biodiversity term. In the following Micro model, we will use this term and the carbon absorption rate to decide which trees should be cut in the micro model.

Macro Model

Due to the demand of yield, we need to prioritize each wood. We think it should be sorted by density and biodiversity and then sequestered by a carbon sequestration model for each area to see how many trees to cut down until the need is met. So the sorted value(score) for woods i is $\frac{c}{f(woods_i)} + speed_i$, where c is a constant based on the demand of biodiversity from managers. We think that the woods with larger biodiversity should be considered first to protect weaker ones. Using the above model, we can tell that the amount of trees that need to be cut is Dj under the consideration of carbon sequestration. Then we sorted all the woods by their score, the trees cut from woods with smaller scores will be first used to meet the need. If the trees are not enough, we will then cut the trees from all woods with the rest of the demand.

Micro Model

For the first model, we only use v_i, the carbon-absorbing rate, to sort all the trees in one wood. Now considering species diversity and preference for cutting dominant species, we decided to sort by a new value. We name this value by bio-term. The following function illustrates how to calculate the bio-term.

$$v_i + \frac{C}{Bio_D}(\frac{N}{n_{ti}}) \tag{14}$$

The algorithm after sorted is the same as the previous model.

4.2 result

Macro Model

We basically use the model raised in the above part, then we add biodiversity to sequence all the woods, in order to decide which woods we need to cut first. Considering the economic and social benefits, we assume that we need to cut 4000 trees per decade in one hectare. Here we only show the results of ten blocks of wood because of the limitation.

score	amount	id	score	amount	id
0.037	12	46	0.048	267	84
0.042	13	13	0.053	6	85
0.042	615	93	0.058	10	0
0.045	26	7	0.059	118	19
0.046	62	6	0.059	23	50

Table3: the result of the Macro Model

where ‘amount’ is the number of trees that need to be cut in $wood_j$.

Micro Model

For the micro model, suppose the $\frac{C}{Biod} = 1$. Then, we apply this algorithm we mentioned above to get a result that considers biodiversity. The input form is the left one, and the output is the right one. Because the dataset is large, we only show the first 100 trees' age here. Since we already considered the real bio-diversity, we use the real trees data to form a new input in this graph. If we apply the algorithm in model 1 will see the green one below as our output. Notice that the sixth green line disappears while that of the left blue graph is still there.

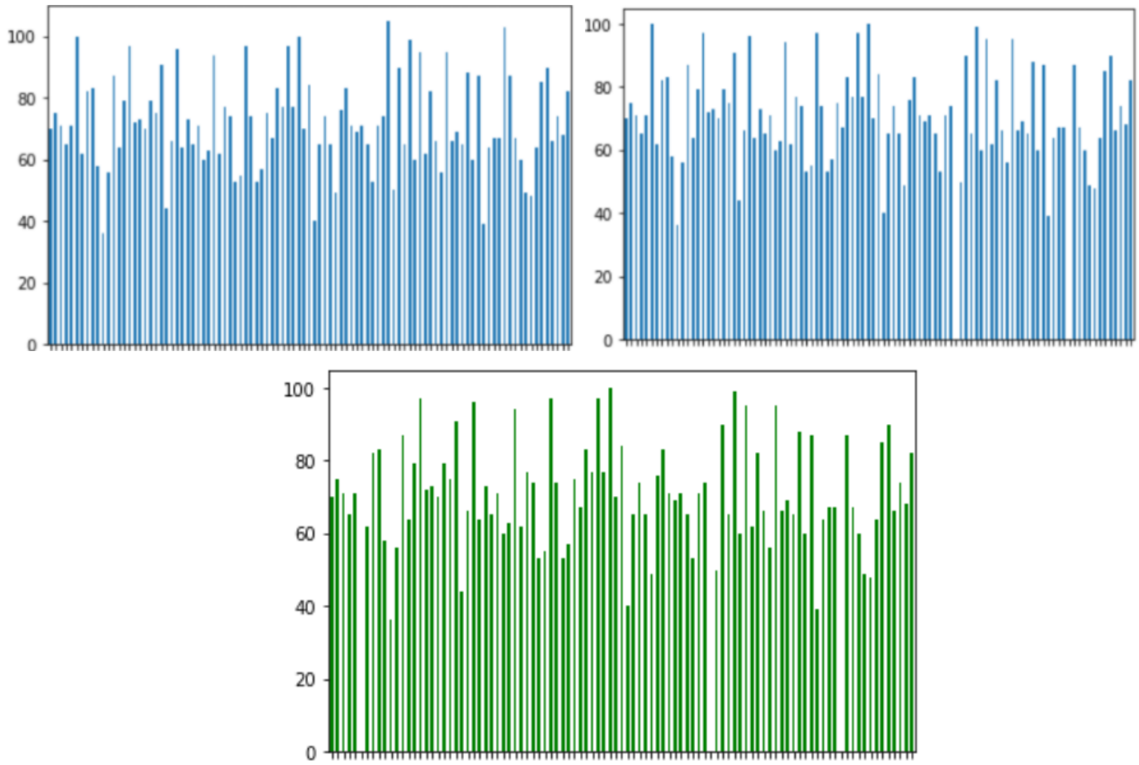


Fig.10: the age of trees before and after cutting

The reason why the sixth blue tree is not be cut is that it is a scarce plant. Other trees which are more flourishing are cut instead.

5 Sensitivity Analysis

Macro Model

In this model we require managers to offer the time they want the carbon sequestration to be maximized. Before we set the value to 10. Here we test 3 values.

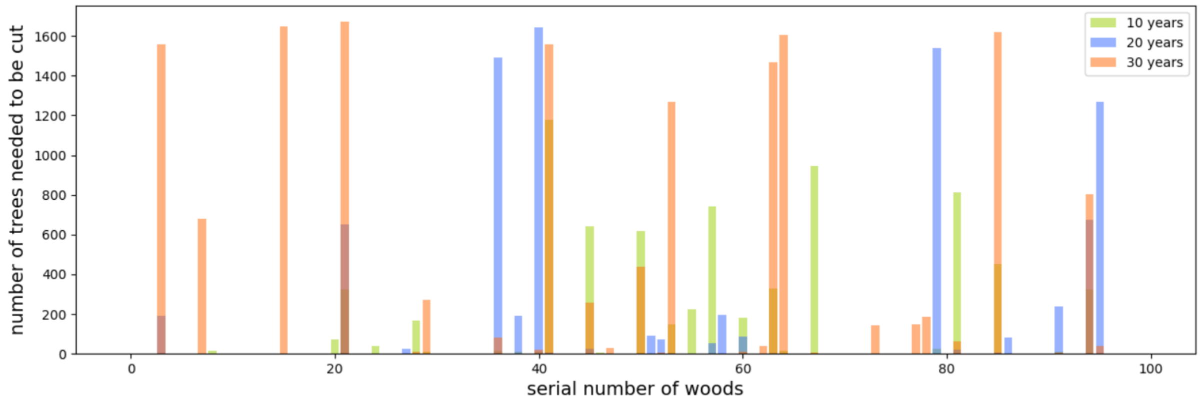


Fig.11 the decisions among three different demand

Noticing that the decision to each woods might change, yet from the graph, we can still observe that the trees we cut are increasing with the demanding year. A more intuitive graph about the relationship between demand year and the number of trees is shown below.

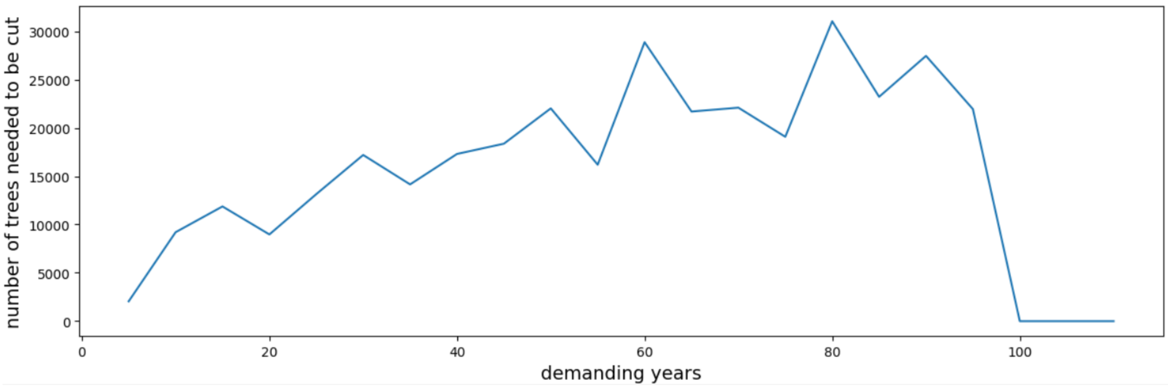


Fig.12 Tree Cutting v.s. Demanding years

The total number of trees for 100 blocks of wood goes up back and forth and then comes back to 0 when the demanding time is larger than the product's maximum lifetime.

Micro Model

Using the above model to solve the following question. During the solving progress to see the model's responses. We give the cut-down period p, cut-down volume v, considering the bio-diversity or not, and woods' detailed information. We want to test how sensitive our model responds to these parameters. Notice that most of the information above is not directly measured from the natural. We collect some of the data. We use those data to produce an extensive data set for the analysis.

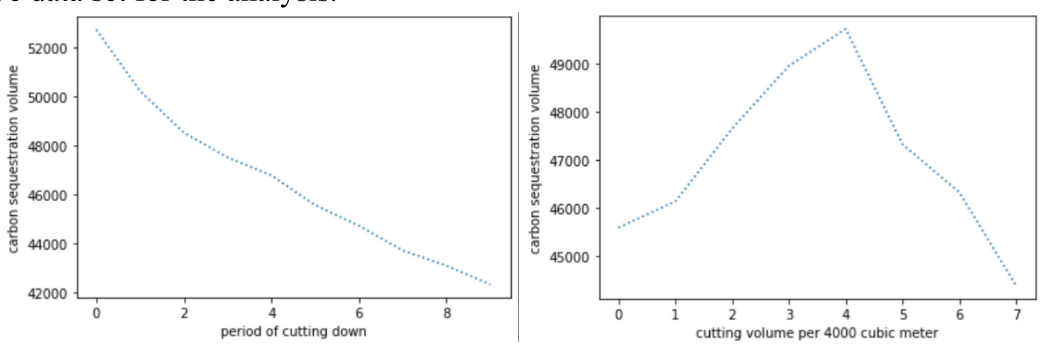


Fig.13 The carbon sequestration volume with different factors

Given that the cut-down volume is 20000 m^3 for the next 100 years want to know what is the relationship between carbon sequestration volume and period of logging as the figure above illustrates, as the period of logging increases, the carbon sequestration volume decrease steadily. (Notice that we provide a whole tree since the company only orders a fixed volume. Therefore, we assume that those parts of the trees are useless, for companies will waste companies. If that is not the reality, you only need to add about 10000 m^3 to get it back(see in the above-left picture).

Now fix the logging period as two years per log. Want to know how much volume should be cut in the following 100 years to maximize the carbon sequestration. The following figure shows that carbon sequestration volume peaks at 12000-14000 cubic meters. (see in the above right picture)

6 Model Evaluation and Further Discussion

6.1 Analysis of the Forest Management Plan

If we apply our model to the forest mentioned in the second model part. Every 1000 trees can grow about 49897 cubic meters for 100 years. That is about 9000 tons of pure carbon.

We should cut down the old trees to maintain the average age of the forest is young. One may look up the graph we draw above or the program we append to get a more detailed answer. In the forest we discuss above, we should control the average age around 80 years old.

If the time between harvests increases by ten years, we have to cut down more trees to younger the average age. For example, if the average now is 80, and the time between harvests increases from 10 to 20, in the harvest, we have to change the forest average age to around 70. Notice that by cutting one 110-year-old tree down, its age turns to 0 instantly.

6.2 Strengths

Our model is relatively accurate since we can identify which trees should be cut according to their age and type. We also provide a quantitative way to measure bio-diversity accurately. We link biodiversity to cutting management.

Our model is very flexible since there are many constants you can adjust to achieve users' requirements. For example, if one forest manager focuses more on bio-diversity, one can simply change the constant we mentioned above to change one's management plan. Furthermore, one can change the constant above to change how we calculate the biodiversity level.

1. We create a two layers model to ease the calculation pressure.
2. By using the macro model, we identify the density of the forest. This action will distribute different logging plans to different regions. Since we believe that the forest's ecosystem with lower density is more fragile than other parts of the forest, this action will allow us to notice the section of the forest that needs to be protected easier.

6.3 Weaknesses

In the assumption of forestland area expansion, the expansion force is designed as a continuous vector field, but in reality, cliffs and rivers inevitably make the continuous field disconnected. To use this model in practice, we need to measure many data. It may cost much time in reality. There is a flaw in the measurement of biodiversity. When one type of tree is much larger than others, the last term of the biodiversity is lower. However, the actual biodiversity should be higher. That is because the K-means clusters algorithm cannot handle the situation below. The left graph shows that too many trees are the orange type. The k-efficient of the left one is almost as same as that of the right one. Since ½ of the trees in one cluster are the same type.

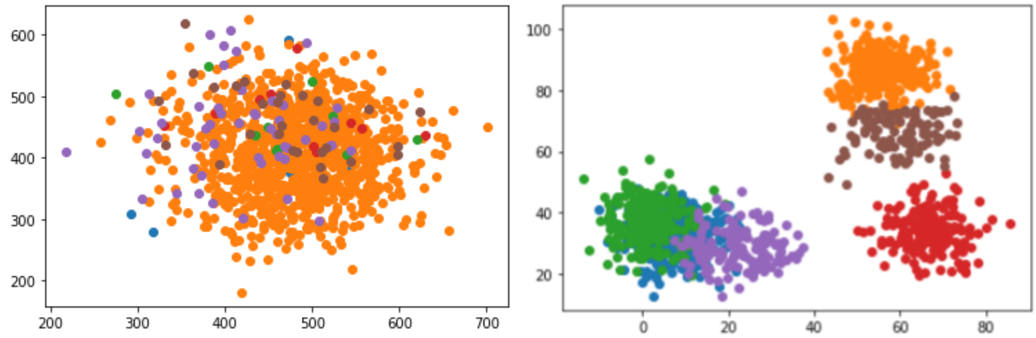


Fig.14 Bad Cluster Situation

6.4 Further Discussion

We have not considered our model's geography and the weather. We can use the AI model, which is applied by *Sid Meier's civilization vi,* to describe the growth of a city. We can also consider the competition of multiple species. In reality, the companies who need the wood may offer their contract to achieve the sustainable development goal according to this model.

7 Conclusion

Our mathematical model presents a tree felling model that takes into account the goals of low-carbon globalization, ecological diversity, and social market needs from both macro and micro perspectives. We used continuous functions to simulate forest area growth and carbon sequestration for trees. In computer simulation and optimization of objective problems, we use discrete methods. After solving the problem, we conclude that the carbon absorption capacity of the forest increases with the increase of felling frequency. This is because the frequent prediction of forest growth status and keeping forest carbon sink capacity in a strong range at all times can satisfy our objective function. But considering too frequent slashing will encounter the case of cutting one or two trees at a time. We set a minimum frequency of cutting every six months to balance out the cost of cutting that is not taken into account. Future research can continue in the direction of a dynamic game between market and forest. Our putative forest expansion fields can also be precisely constructed by geology and climate newspaper part.

References

Li Haikui. "Research progress on forest carbon sink assessment methods and carbon sequestration potential estimation under the carbon neutral vision." *Geological Survey of China* 8.4: 79-86.

Pingoud, K., Perälä, A. L., Soimakallio, S., & Pussinen, A. (2003). Greenhouse gas impacts of harvested wood products. *Evaluation and development of methods. VTT Research Notes*, *2189*(13), 8.

Hartigan, John A., and Manchek A. Wong. "Algorithm AS 136: A k-means clustering algorithm." *Journal of the royal statistical society. series c (applied statistics)* 28.1 (1979): 100-108.
https://www.pnas.org/content/116/29/14526#ref-list-1

Johnston, Craig MT, and Volker C. Radeloff. "Global mitigation potential of carbon stored in harvested wood products." *Proceedings of the National Academy of Sciences* 116.29 (2019): 14526-14531.
NFI_T14_LSAGE20_AREA_MEAS_T0 (nfis.org)

Tang Haofan, Xu Yanhong, and Ai Jianlin. "Carbon storage and carbon density of forest vegetation and their spatial distribution patterns in Yunnan Province." *Forestry Resources Management* 5 (2019): 37.
https://www.forestry.gov.cn/html/ghy/ghy_4828/20191129093421780132285/file/20191129094448089585887.pdf

Appendices

Appendix 1

Introduce: The proof about logistic function.

Want to show that given f(y) is a logistic function about y, and y(x) is a logistic function about x, f(y(x)) is a function looks like a logistic function about x.

1. h(y) = f(y)-½, then h(y) is an odd function. g(x) = y(x)-½, g(x) is an odd function
2. show that h(g(x)) is an odd function by showing h(g(-x)) = -h(g(x))
3. show that $\frac{d^2 h\cdot g}{dx^2} < 0$ when x>0

   Q.E.D

Appendix 2

Introduce: core codes

```
def cutdown_store(series, cutdownNumber, freedomValue = 120):
  edge = series.nsmallest(2*cutdownNumber,'bio_value')
  edge_speed = edge.iloc[2*cutdownNumber-1, 5]
  sum_vector = series.loc[(series['bio_value']<=edge_speed) &
(series['age']>series['Maxage']/2)]
  print(sum_vector)
  series['age'].loc[(series['bio_value']<=edge_speed) & (series['age']>series['Maxage']/2)]
= 0
  store = sum_vector.sum()
  return store
def cutdown_volume(series, volume, freedomValue = -1, cutdown = False):
  if freedomValue == -1:#it means we do not consider the biodiversity
    series['carbon_accumulation'] = logistic_col(series)
    #series['carbon_increasing_rate'] = logistic_derivative(series)
    series['carbon_increasing_rate'] = diversity_value(series)
    saliteta = series.sort_values('carbon_increasing_rate')
    sum = 0
    tree_id = []
    for i in range(len(series)):
      if saliteta.iloc[i]['age']/saliteta.iloc[i]['Maxage']<0.5:#if this tree is young
        continue
      sum += saliteta.iloc[i]['carbon_accumulation']
      tree_id.append(saliteta.iloc[i]['id'])
      if(sum>volume):
        break
    if cutdown:
      for i in range(len(tree_id)):
        series.iloc[tree_id[i]-1, series.columns.get_loc('age')] = 0
    return tree_id
id = [i for i in range(1,len(df)+1)]
df['id'] = id
#initialize with an id
cutdown_id = cutdown_volume(df,1000,cutdown = True)
```

```
df.to_csv('df.csv')
files.download('df.csv')
print(cutdown_id)
def cut(data, volume, period, times):
  cut_down_id = []
  data['carbon_accumulation'] = logistic_col(data)
  sum1 = data['carbon_accumulation'].sum()
  sum3 = 0
  for i in range(times):
    tempa = cutdown_volume(data,volume,cutdown = True)
    cut_down_id.append(tempa)
    data['age'] = data['age']+period
  sum2 = data['carbon_accumulation'].sum()
  print(sum2-sum1+volume*times)
    return cut_down_id
def optimize(data,volume_need, years,steps = 1):
  for i in range(years):
    period = years/(1+i)
    if(period!=100/50):
      continue
    volume = volume_need/(1+i)
    new_data = d
def logistic_col(series):
  A = series['age']
  X = df['age']/df['Maxage']-0.5
  returnValue = A/(1+np.exp(-X))
  return returnValue
def logistic_derivative(series):
  A = series['carbonA']
  X = df['age']/df['Maxage']-0.5
  C = np.exp(-X)
  returnvalue = A*C/(1+C)/(1+C)
  return returnvalue
def diversity_value(data,freedomValue=120):
  size = len(data)
  treesnumber = get_treesnum(data)
  bio = size/treesnumber[data['type']]/freedomValue
  bio.to_frame
  x = logistic_derivative(data)+bio[1]
  return x
from enum import unique
scaler = StandardScaler()
df[['x_stn', 'y_stn']] = scaler.fit_transform(df[['1','2']])
s = {'A':0,'B':1,'C':2,'D':3,'E':4,'F':5,'G':6}
import math
def varience(data, k):
  kmeans = KMeans(n_clusters = k)
  data[['x_stn', 'y_stn']] = scaler.fit_transform(df[['1','2']])
```

```
  kmeans.fit(data[['x_stn','y_stn']])
  data['kmeans'] = kmeans.labels_
  new_test = data[['0','kmeans']]
  varr = []
  for i in range(k):
    number = []
    temp = new_test[new_test['kmeans'] == i]
    for j in range(len(temp)):
      number.append(s[new_test.iloc[j]['0']])
    temp['new'] = number
    varr.append(temp.loc[:,'new'].var())
  return varr
def bio_diversity(data):
  unique_data = data['0'].value_counts()
  k = len(unique_data)
  rows_number = len(data)
  fix_multi = (k/(k-1))**k
  for i in range(k):
    fix_multi *= 1-unique_data.iloc[i]/rows_number
  discreate_level = sum(varience(data,k))/k
  unique_value = math.log(k)
  print(unique_value)
  print(fix_multi)
  print(discreate_level)
bio_diversity(df)
def C(x,w,a,k,rou):
    return -(a*w)/(1+k*np.exp(a*x-rou))+a*rou
def aimFunction(x,Tp,t_want,t_orign,w,a,k,rou):
    if Tp > t_want:
y=C(x+t_want,w,a,k,rou)-C(x,w,a,k,rou)-C(t_orign+t_want,w,a,k,rou)+C(t_orign,w,a,k,r
ou)
    else: y=C(x+t_want,w,a,k,rou)-C(t_orign+t_want,w,a,k,rou)
    return y
def convert(x):
    return 87.5*6.6*np.exp(0.1*x)/(6.6+0.875*(np.exp(0.1*x-1)-1))
for wood in range(0,200):
    T=1000 #initiate temperature
    Tmin=10 #minimum value of terperature
    x=np.random.uniform(low=0,high=100)#initiate x
    k=50 #times of internal circulation
    y=0#initiate result
    t=0#time
    while T>=Tmin:
        for i in range(k):
            y=aimFunction(x,Tp,t_want,E[wood],W[wood],A[wood],K[wood],ROU[wood])
            xNew=x+np.random.uniform(low=-0.055,high=0.055)*T
            if (0<=xNew and xNew<=100):
yNew=aimFunction(xNew,Tp,t_want,E[wood],W[wood],A[wood],K[wood],ROU[wood]
```

```
)
            if yNew-y<0:
               x=xNew
            else:
               p=math.exp(-(yNew-y)/T)
               r=np.random.uniform(low=0,high=1)
               if r<p:
                  x=xNew
      t+=1
      T=1000/(1+t)
   if x >= E[wood]:
      x = 0
 aimFunction(xNew,Tp,t_want,E[wood],W[wood],A[wood],K[wood],ROU[wood])<=0:
      x = 0
   if x != 0:
      x = convert(E[wood])-convert(x)
Carbon_increase.append(aimFunction(xNew,Tp,t_want,E[wood],W[wood],A[wood],K[
wood],ROU[wood]))
   else:  Carbon_increase.append(0)
   answer.append (x)
def tran(x):
      return
87.5*6.6*0.05*np.exp(0.1*x)*(6.6+0.875*(np.exp(0.1*x-1)-1)-0.875*np.exp(0.1*x-1))/(
6.6+0.875*((np.exp(0.1*x-1)-1)**2))
Bio = np.random.uniform(10,50,100)
demand = 4000
score = []
for woods in range(0,100):
   score.append((1/Bio[woods])+tran(E[woods]))
final = []
for i in range(0,100):
   final.append([score[i],answer[i],i])
def search():
   final.sort()
   for i in range(0,100):
      if demand > final[i][1]:
         demand = demand - final[i][1]
      elif demand <= final[i][1]:
         final[i][1] = final[i][1] - demand
         demand = 0
         for j in range(i+1,100):
            final[j][1] = 0
   if demand != 0:
      demand = demand/100
      for i in range(0,100):
         final[i][1] += round(demand)
```

Deforestation

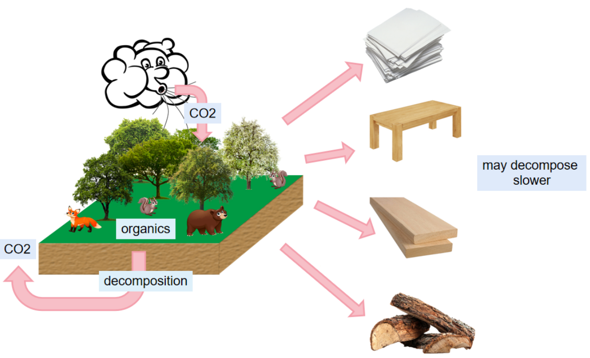
Maybe more Eco-friendly than Unattending

Introduction

Intuitively, leaving forests to grow and cutting them down as little as possible seems the best way to deal with the problem of excessive global carbon emissions. But our mathematical models tell us that reasonable deforestation by humans is not only not the destruction of forests but also the maximization of their carbon sequestration capacity.

Now a new plan is proposed.

Cutting down mature forest areas where carbon sequestration is slowing and plant new trees may increase the carbon sequeatration amount!



The carbon in the old trees is turned into products for human use. And the space freed up would provide the forest with young trees, increasing the rate of carbon sequestration. One might wonder, since all trees are planted, one might wonder why not expand the forest border instead of planting new trees in mature areas of the forest?

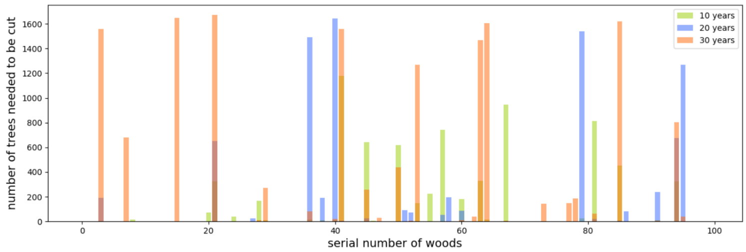
Here comes the MM Model! Read more about the models if you're interested!

The MM Model

The model simulated the growth of forest boundary. The reason is that And the products can actually save the carbon more than the trees did if their lifetime is long enough.

Think about a product has long enough life time, if we cut the trees, the carbon inside the trees will not go away. Instead, they stay in the product. After cutting the trees, they can abosorb more carbon during the secondary growth.

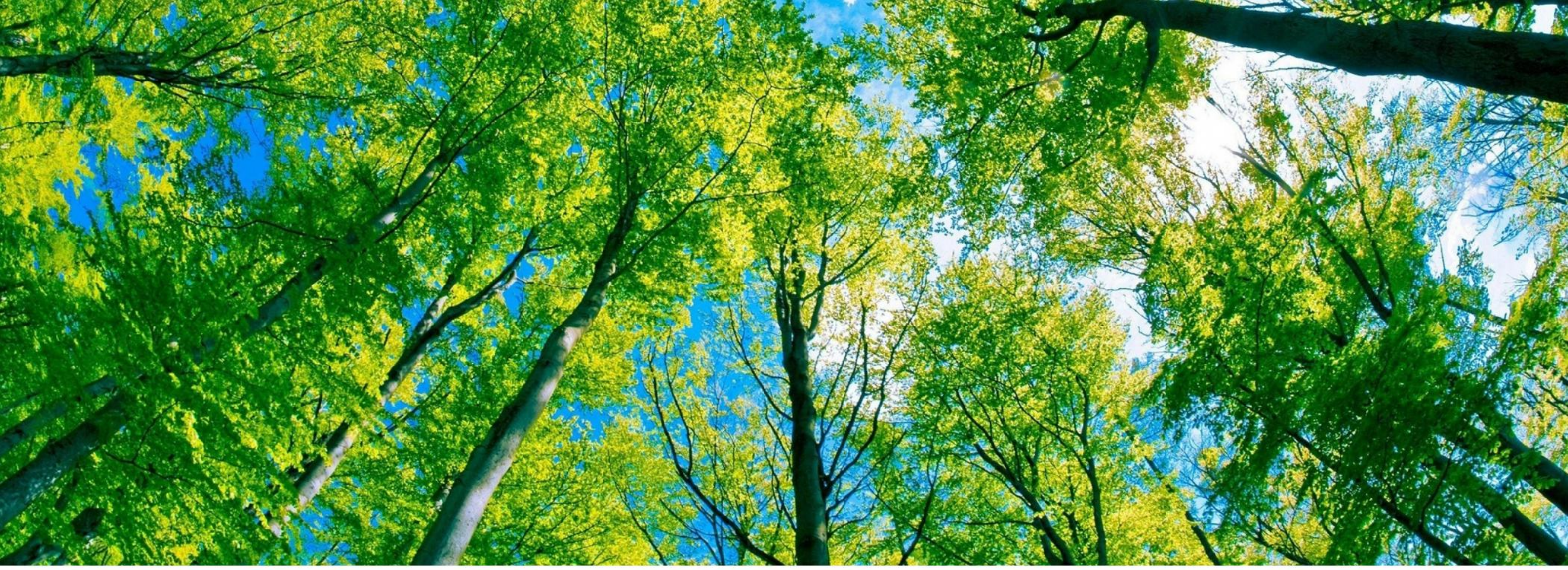
Base on this idea, our model offer a mangement plan that can maximize the carbon sequestration amount in a period with cutting down some old trees.



The key to our reasonable feeling plan is the period. In the sensitive analyses, tests show that we slash a few trees frequently can keep the forest in the high-carbon-absorption mode. That's because we're monitoring the absorption of the forest in real-time, eating small amounts and eating multiple meals. We recommend that foresters monitor every hectare of forest land containing about 2000 trees.

Monitoring methods are available through satellite forest maps.

However, we cannot only focus on carbon sequestration, the biodiversity is also very important for a forest ecosystem. The type of the trees in a wood, the dispersity of trees, the number of each type of trees are all important factors for the forest. The economic needs are also essential

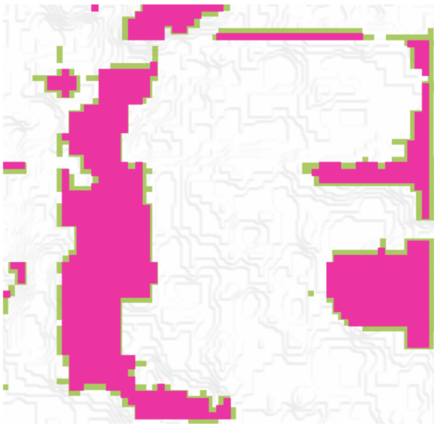


too. In order to provide a more realistic forestry plan for the managers, our model then consider the economic and biodiversity factors.

The meaning of MM Model

The MM Model is called so because it's made up of both macro model and micro model.

The macro moel is used to simulated the growth of forest boundary.



The reason is that the forest has its current shape is that the areas on the map where there is no forest are actually areas that are not suitable for forest growth, that is, areas where the scalar of our growth vector is small or even negative. It doesn't make sense to plant new trees in those places blindly. In contrast, previously densely forested areas, where land is fertile due to organic accumulation can be planted to maximize atmospheric carbon sequestration over the logging cycle.

Based on the assumptions of biodiversity in the micro model, we can selectively plant trees after reasonable felling, rather than blindly planting trees with the greatest carbon sequestration. Although planting a single, fast-growing tree can make the biggest short-term contribution to mitigating the planet's carbon problems, the long-term integrity of forest systems requires that we compensate for the loss of scarce trees in time.

Ovevrall, our model may offer a new plan for worldwide forestry!

Final words

Reasonable tree cutting can bring great guiding significance to the manufacture of wooden products in human society. In the past, the idea was to minimize the exploitation of forest resources and even cause more carbon emissions. For example, because wood is restricted to development, more concrete and steel are used in the design of buildings, but their production process has a lot of carbon emissions. However, when wood is used as a building material, this carbon is stored for a long time in places where forests do not exist in human societies.

Today, such planned logging models are increasingly possible thanks to satellite remote sensing technology and forest databases. We used satellite imagery to predict forest area expansion, density changes, and future boundary changes. They form the basis for data on deforestation cycles and volumes. In model 2, external market requirements are realized on the premise of forest data disclosure. In a more carbon-neutral future, forest carbon management and optimal forest development scenarios require more robust technical support.