# 1 Zahlenformate

## 1.1 Zweierkomplement

Umwandlung: Bsp. 8-Bit $(-4)_{10}$ (Funktioniert in beide Richtungen)

- Vorzeichen Ignorieren $(4)_{10} = (00000100)_2$
- Bits Invertieren $(0000\ 0100)_2 \rightarrow (1111\ 1011)_2$
- Eins Addieren $(1111\ 1011)_2 + (0000\ 0001)_2 = (1111\ 1100)_2$

## 1.2 Fixed Point (unsigned)

Qk.l mit k = Vorkomma und l = Nachkomma

$$x_{(10)} = \sum_{i=0}^{k-1} b_i \cdot 2^i + \sum_{j=-l}^{-1} b_j \cdot 2^j \tag{1}$$

Bsp Q4.5 $a = (01010110)_2$
$0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 + 0 \cdot 2^{-1} + 1 \cdot 2^{-2} + 1 \cdot 2^{-3} + 0 \cdot 2^{-4} = 5.375$

## 1.3 Fixed Point (signed)

Bsp. Q3.3 $(100.001)_2$

- Vorzeichen Merken $(100.001) \rightarrow -1$
- Bits Invertieren $(100\ 001)_2 \rightarrow (011\ 110)_2$
- $1 \cdot 2^{-k}$ Addieren $(011\ 110)_2 + (000\ 001)_2 = (011\ 111)_2 = -3.875$

| Bits | Format | Numeric Range | Precision | Dynamic Range |
|---|---|---|---|---|
| 8 | Unsigned integer | $0 \rightarrow +255$ | 1 | $\approx 48$ dB |
| 8 | Signed integer | $-128 \rightarrow +127$ | 1 | $\approx 48$ dB |
| 16 | Unsigned integer | $0 \rightarrow +65,536$ | 1 | $\approx 96$ dB |
| 16 | Signed integer | $-32,768 \rightarrow +32,767$ | 1 | $\approx 96$ dB |
| 16 | Fixed-point (Q12) | $-8.0 \rightarrow \approx +7.999756$ | $\approx 0.000244$ | $\approx 96$ dB |
| 16 | Fixed-point (Q15) | $-1.0 \rightarrow \approx +0.9999695$ | $\approx 0.0000305$ | $\approx 96$ dB |
| 32 | Unsigned integer | $0 \rightarrow +4,294,967,296$ | 1 | $\approx 193$ dB |
| 32 | Signed integer | $-2,147,483,648 \rightarrow +2,147,483,647$ | 1 | $\approx 193$ dB |
| 32 | Single-precision | $\approx \pm 3.402823 \times 10^{38}$ | $\approx 1.19 \times 10^{-7}$ | $\approx 138$ dB |
| 64 | Double-precision | $\approx \pm 1.797693 \times 10^{308}$ | $\approx 2.22 \times 10^{-16}$ | $\approx 314$ dB |

# 2 Filter in c

## 2.1 FIR

$$A(0)y(k) = \sum_{i=0}^{N} B(i)x(k-i) \tag{2}$$

```
void fir()
{
    CodecDataIn.UINT = ReadCodecData();      // get input data samples
    int i;
    xLeft[0] = CodecDataIn.Channel[LEFT];    // current input value
    yLeft = 0 ;                              // initialize the output
    for ( i = 0 ; i <= N ; i++) {            // x is length N+1
        yLeft += xLeft[i] * B[i];            // perform the dot-product
    }
    for ( i = N ; i > 0 ; i--) {
        xLeft[i] = xLeft[i-1];               // shift for the next input
    }
    CodecDataOut.Channel[LEFT] = yLeft;      // output the value
}
```

## 2.2 IIR

$$A(0)y(k) = \sum_{i=0}^{N} B(i)x(k-i) - \sum_{i=0}^{N} A(i)y(k-i) \tag{3}$$

```
void iir()
{
    xLeft[0]  = CodecDataIn.Channel[LEFT] + CodecDataIn.Channel[RIGHT];
    yLeft[0] = 0;                              // initialize the output value

    for (i = 0; i <= N; i++){
        yLeft[0] += B[i] * xLeft[i];
    }
    for (i = 1; i <= N; i++){
        yLeft[0] -=  A[i] * yLeft[i];
    }
    for (i = N; i > 0; i--)
    {
        xLeft[i] = xLeft[i-1];
        yLeft[i] = yLeft[i-1];
    }
    CodecDataOut.Channel[LEFT]  = yLeft[0]; // output the filtered value
}
```

## 2.3 Notch-IIR-Filter

$$H(z) = k\frac{(z-\beta_1)(z-\beta_2)}{(z-\alpha_1)(z-\alpha_2)} = k\frac{1 - 2\cos(\omega_0)z^{-1} + z^{-2}}{1 - 2r\cos(\omega_0)z^{-1} + r^2 z^{-2}} \tag{4}$$

$f_0$ = Kerbfrequenz, $f_s$ = Abtastfrequenz $B_{3dB}$ = Kerbbreite

$$k = \frac{1 - 2r\cos(\omega_0) + r^2}{1 - 2\cos(\omega_0) + 1} \tag{5}$$

$$\omega_0 = 2\pi\frac{f_0}{f_s} \tag{6}$$

$$r = 1 - (\frac{B_{3dB}}{f_s})\pi \tag{7}$$

## 2.4 IIR-Oszillator

$$sin(\omega_0 k) \leftrightarrow \frac{sin(\omega_0)z^{-1}}{1 - 2\cos(\omega_0)z^{-1} + z^{-2}} \tag{8}$$

$$y(n) = sin(\omega_0)x(n-1) + 2cos(\omega_0)y(n-1) - y(n-2) \tag{9}$$

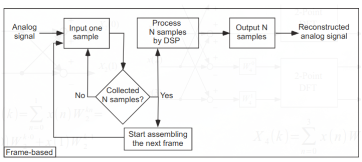
## 2.5 DDS-Oszillator

$$\varphi_{inc} = 2\pi\frac{f_0}{f_s} \tag{10}$$

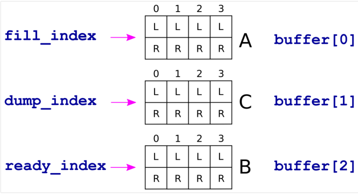
$$\varphi = \varphi + \varphi_{inc} \tag{11}$$

$$x(n) = Asin(n\varphi) \tag{12}$$

# 3 Blocksignalverarbeitung



ISR schreibt N samples nach buffer[fill_index] und setzt ready_index = fill_index (buffer[fill_index] ist jetzt dran mit ProcessBuffer). Jeder Sample generiert einen Interrupt




- fill_index wird von ADC gefüllt
- dump_index wird an DAC geschrieben
- ready_index Buffer für Blocksignalverarbeitung

```c
#define BUFFER_LENGTH          1024        // buffer length in samples
#define NUM_BUFFERS         3
volatile float buffer[NUM_BUFFERS][2][BUFFER_LENGTH];

void ProcessBuffer()
// Processes the data in buffer[ready_index]
{
    volatile float *pL = buffer[ready_index][LEFT];
    volatile float *pR = buffer[ready_index][RIGHT];
    // Do the Process
    // ...
    buffer_ready = 0;    // means were done here
}

interrupt void Codec_ISR()
{
    static Uint8 fill_index = INITIAL_FILL_INDEX; // index of buffer to fill
    static Uint8 dump_index = INITIAL_DUMP_INDEX; // index of buffer to dump
    static Uint32 sample_count = 0; // current sample count in buffer

    // get input data samples
          CodecDataIn.UINT = ReadCodecData();
          // IN
    buffer[fill_index][ LEFT][sample_count] = LEFT + RIGHT; // cropped
    buffer[fill_index][RIGHT][sample_count] = RIGHT + LEFT; // cropped
    // OUT
    CodecDataOut.channel[ LEFT] = buffer[dump_index][LEFT][sample_count];
    CodecDataOut.channel[RIGHT] = buffer[dump_index][RIGHT][sample_count];

    // update sample count and swap buffers when filled
    if(++sample_count >= BUFFER_LENGTH) {
        sample_count = 0;
        ready_index = fill_index;
        if(++fill_index >= NUM_BUFFERS)
            fill_index = 0;
        if(++dump_index >= NUM_BUFFERS)
            dump_index = 0;
        if(buffer_ready == 1) // set a flag if buffer isn't processed in time
            over_run = 1;
        buffer_ready = 1;
    }
        WriteCodecData(CodecDataOut.UINT);// send output data to  port
}
```

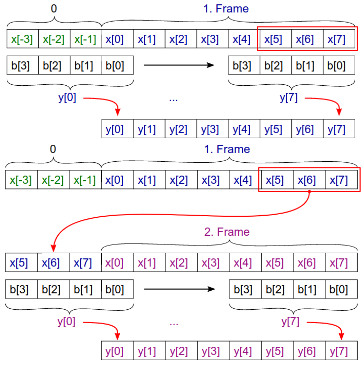
## 3.1 Blocksignalverarbeitung mit DMA

- DMA kopiert Sample von ADC nach Eingangsbuffer
- DMA kopiert Processed von Ausgangsbuffer nach DAC
- DMA generiert Interrupt, wenn N Samples transfert wurden → Buffer-Swap

```c
interrupt void EDMA_ISR()
{
    if(++ready_index >= NUM_BUFFERS)
        ready_index = 0;
    if(buffer_ready ==1) //buffer isnt processed in time
        over_run = 1;
    buffer_ready = 1; //buffer is now ready for processing
}
```

## 3.2 FIR mit Blocksignalverarbeitung

Bsp. Ordnung Filter N = 4, Framesize = 8

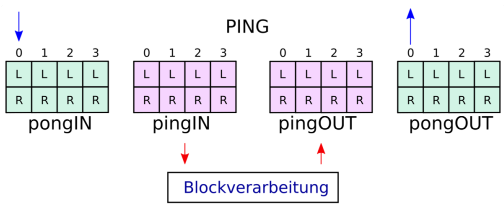



**Problem**: Bei Frame-Übergängen müssen die N-1 letzten Werte des letzten Frames berücksichtigt werden.
**Lösung**: Framesize += N

- Left[N|FRAMESIZE], buffer[FRAMESIZE]
- Left[N:FRAMESIZE+N] = buffer[0:FRAMESIZE]
- buffer[0:FRAMESIZE] = Left * B (B wird drüber FRAMESIZE-mal drüber geschoben s.o)
- Left[0:N] = Left[FRAMESIZE:FRAMESIZE+N]

```c
void ProcessBuffer()
{
    short *pBuf = buffer[ready_index];
    // extra buffer room for convolution "edge effects"
    // N is filter order from coeff.h
    static float Left[BUFFER_COUNT+N]={0}, Right[BUFFER_COUNT+N]={0};
    float *pL = Left, *pR = Right;
    float yLeft, yRight;
    int i, j, k;
    // offset pointers to start filling after N elements

    pR += N;
    pL += N;

    // extract data to float buffers
    for(i = 0; i < BUFFER_COUNT; i++)
    {
        *pR++ = *pBuf++;
        *pL++ = *pBuf++;
    }
    // reinitialize pointer before FOR loop
    pBuf = buffer[ready_index];

    // Implement FIR filter
    for(i=0; i < BUFFER_COUNT; i++)
    {
        yLeft = 0; // initialize the LEFT output value
        yRight = 0; // initialize the RIGHT output value

        for(j=0,k=i+N; j <= N; j++,k--)
        {
            yLeft += Left[k] * B[j]; // perform the LEFT dot-product
            yRight += Right[k] * B[j]; // perform the RIGHT dot-product
        }
        // pack into buffer after bounding (must be right then left)
        *pBuf++ = _spint(yRight * 65536) >> 16;
        *pBuf++ = _spint(yLeft * 65536) >> 16;
    }

    // save end values at end of buffer array for next pass
    // by placing at beginning of buffer array
    for(i=BUFFER_COUNT,j=0; i < BUFFER_COUNT+N; i++,j++)
    {
        Left[j] = Left[i];
        Right[j] = Right[i];
    }
    buffer_ready = 0; // signal we are done
}
```

## 3.3 Ping-Pong



- gleiche Latenz (=Durchlaufzeit 2 Buffer)
- Ping-Pong einfacher zu verwalten mit DMA

# 4 FFT

Die FFT basiert auf dem **Devide-and-Conquer** Prinzip, sodass schon berechnete Zwischenergebnisse wiederverwendet werden. Mögliche Realisierungsformen:

- Decimation in Frequency
- Decimation in Time

$$w_N^{nk} = e^{-j\frac{2\pi nk}{N}} \tag{13}$$
$$w_N^{2nk} = e^{-j\frac{4\pi nk}{N}} = w_{\frac{N}{2}}^{nk} \tag{14}$$

## 4.1 Decimation in Time (DIT)

1. Aufteilung in $Y(n) = Y_{even}(n) + Y_{odd}(n)$

$$Y(n) = \sum_{k=0}^{N/2-1} y(2k)w_N^{2nk} + \sum_{k=0}^{N/2-1} y(2k+1)w_N^{(2k+1)n} \tag{15}$$
$$Y(n) = \sum_{k=0}^{N/2-1} y(2k)w_N^{2nk} + w_N^n \sum_{k=0}^{N/2-1} y(2k+1)w_N^{2kn} \tag{16}$$
$$Y(n) = \sum_{k=0}^{N/2-1} y(2k)w_{\frac{N}{2}}^{nk} + w_N^n \sum_{k=0}^{N/2-1} y(2k+1)w_{\frac{N}{2}}^{nk} \tag{17}$$
$$\tag{18}$$

2. Aufteilung in $Y(n) = Y_{left}(n) + Y_{right}(n)$

$$Y(n) = \sum_{k=0}^{N/2-1} y(2k)w_{\frac{N}{2}}^{nk} + w_N^n \sum_{k=0}^{N/2-1} y(2k+1)w_{\frac{N}{2}}^{nk} \tag{19}$$
$$Y(n+\frac{N}{2}) = \sum_{k=0}^{N/2-1} y(2k)w_{\frac{N}{2}}^{(n+\frac{N}{2})k} + w_N^{n+\frac{N}{2}} \sum_{k=0}^{N/2-1} y(2k+1)w_{\frac{N}{2}}^{(n+\frac{N}{2})k} \tag{20}$$

mit

$$w_{\frac{N}{2}}^{(n+\frac{N}{2})k} = w_{\frac{N}{2}}^{nk} \cdot \underbrace{w_{\frac{N}{2}}^{k\frac{N}{2}}}_{=1} = w_{\frac{N}{2}}^{nk} \tag{21}$$
$$w_N^{n+\frac{N}{2}} = w_N^n \cdot \underbrace{w_N^{\frac{N}{2}}}_{=-1} = -w_N^n \tag{22}$$

folgt

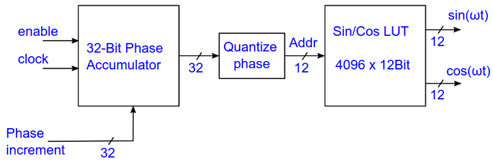
$$Y(n) = \sum_{k=0}^{N/2-1} y(2k)w_{\frac{N}{2}}^{nk} + w_{\frac{N}{2}}^n \sum_{k=0}^{N/2-1} y(2k+1)w_{\frac{N}{2}}^{kn} \tag{23}$$
$$Y(n+\frac{N}{2}) = \sum_{k=0}^{N/2-1} y(2k)w_{\frac{N}{2}}^{nk} - w_{\frac{N}{2}}^n \sum_{k=0}^{N/2-1} y(2k+1)w_{\frac{N}{2}}^{kn} \tag{24}$$

$$Y_{left}(n) = Y_{even}(n) + w_{\frac{N}{2}}^n Y_{odd}(n) \tag{25}$$
$$Y_{right}(n) = Y_{even}(n) - w_{\frac{N}{2}}^n Y_{odd}(n) \tag{26}$$

Die Komplexität ist $O(N\log_2(N))$:
Es gibt 2 $log_2(N)$ Splitting-Steps mit je $O(n)$

# 5 NCO

NCO = Counter, der Jeden Takt um ein Phaseninkrement $\mu$ (hier 32) erhöht wird. Der Ausgang des Counters wird mit LUT in Signalform (sin,cos,sägezahn) umgewandelt. Die LUT ist mit $N = 2^n$ 12-bit breiten Werten gefüllt




$$\mu = N\frac{f_d}{f_s} \tag{27}$$