

1 Zahlenformate

1.1 Zweierkomplement

Umwandlung: Bsp. 8-Bit $(-4)_{10}$ (Funktioniert in beide Richtungen)

- Vorzeichen Ignorieren $(4)_{10} = (00000100)_2$
- Bits Invertieren $(0000\ 0100)_2 \rightarrow (1111\ 1011)_2$
- Eins Addieren $(1111\ 1011)_2 + (0000\ 0001)_2 = (1111\ 1100)_2$

1.2 Fixed Point (unsigned)

Qk.l mit k = Vorkomma und l = Nachkomma

$$x_{(10)} = \sum_{i=0}^{k-1} b_i \cdot 2^i + \sum_{j=-l}^{-1} b_j \cdot 2^j \quad (1)$$

Bsp Q4.5 $a = (01010110)_2$

$$0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 + 0 \cdot 2^{-1} + 1 \cdot 2^{-2} + 1 \cdot 2^{-3} + 0 \cdot 2^{-4} = 5.375$$

1.3 Fixed Point (signed)

Bsp. Q3.3 $(100.001)_2$

- Vorzeichen Merken $(100.001) \rightarrow -1$
- Bits Invertieren $(100\ 001)_2 \rightarrow (011\ 110)_2$
- $1 \cdot 2^{-k}$ Addieren $(011\ 110)_2 + (000\ 001)_2 = (011\ 111)_2 = -3.875$

Bits	Format	Numeric Range	Precision	Dynamic Range
8	Unsigned integer	$0 \rightarrow +255$	1	$\approx 48\text{ dB}$
8	Signed integer	$-128 \rightarrow +127$	1	$\approx 48\text{ dB}$
16	Unsigned integer	$0 \rightarrow +65,536$	1	$\approx 96\text{ dB}$
16	Signed integer	$-32,768 \rightarrow +32,767$	1	$\approx 96\text{ dB}$
16	Fixed-point (Q12)	$-8.0 \rightarrow \approx +7.999756$	≈ 0.000244	$\approx 96\text{ dB}$
16	Fixed-point (Q15)	$-1.0 \rightarrow \approx +0.9999695$	≈ 0.0000305	$\approx 96\text{ dB}$
32	Unsigned integer	$0 \rightarrow +4,294,967,296$	1	$\approx 193\text{ dB}$
32	Signed integer	$-2,147,483,648 \rightarrow +2,147,483,647$	1	$\approx 193\text{ dB}$
32	Single-precision	$\approx \pm 3.402823 \times 10^{38}$	$\approx 1.19 \times 10^{-7}$	$\approx 138\text{ dB}$
64	Double-precision	$\approx \pm 1.797693 \times 10^{308}$	$\approx 2.22 \times 10^{-16}$	$\approx 314\text{ dB}$

2 Filter in c

2.1 FIR

$$A(0)y(k) = \sum_{i=0}^N B(i)x(k-i) \quad (2)$$

```
void fir()
{
    CodecDataIn.UINT = ReadCodecData(); // get input data samples
    int i;
    xLeft[0] = CodecDataIn.Channel[LEFT]; // current input value
    yLeft = 0; // initialize the output
    for ( i = 0 ; i <= N ; i++) { // x is length N+1
        yLeft += xLeft[i] * B[i]; // perform the dot-product
    }
    for ( i = N ; i > 0 ; i--) { // shift for the next input
        xLeft[i] = xLeft[i-1];
    }
    CodecDataOut.Channel[LEFT] = yLeft; // output the value
}
```

2.2 IIR

$$A(0)y(k) = \sum_{i=0}^N B(i)x(k-i) - \sum_{i=0}^N A(i)y(k-i) \quad (3)$$

```
void iir()
{
    xLeft[0] = CodecDataIn.Channel[LEFT] + CodecDataIn.Channel[RIGHT];
    yLeft[0] = 0; // initialize the output value

    for (i = 0; i <= N; i++){
        yLeft[0] += B[i] * xLeft[i];
    }
    for (i = 1; i <= N; i++){
        yLeft[0] -= A[i] * yLeft[i];
    }
    for (i = N; i > 0; i--)
    {
        xLeft[i] = xLeft[i-1];
        yLeft[i] = yLeft[i-1];
    }
    CodecDataOut.Channel[LEFT] = yLeft[0]; // output the filtered value
}
```

2.3 Notch-IIR-Filter

$$H(z) = k \frac{(z - \beta_1)(z - \beta_2)}{(z - \alpha_1)(z - \alpha_2)} = k \frac{1 - 2 \cos(\omega_0)z^{-1} + z^{-2}}{1 - 2r \cos(\omega_0)z^{-1} + r^2 z^{-2}} \quad (4)$$

f_0 = Kerbfrequenz, f_s = Abtastfrequenz B_{3dB} = Kerbbreite

$$k = \frac{1 - 2r \cos(\omega_0) + r^2}{1 - 2 \cos(\omega_0) + 1} \quad (5)$$

$$\omega_0 = 2\pi \frac{f_0}{f_s} \quad (6)$$

$$r = 1 - \left(\frac{B_{3dB}}{f_s} \right) \pi \quad (7)$$

2.4 IIR-Oszillator

$$\sin(\omega_0 k) \leftrightarrow \frac{\sin(\omega_0)z^{-1}}{1 - 2 \cos(\omega_0)z^{-1} + z^{-2}} \quad (8)$$

$$y(n) = \sin(\omega_0)x(n-1) + 2\cos(\omega_0)y(n-1) - y(n-2) \quad (9)$$

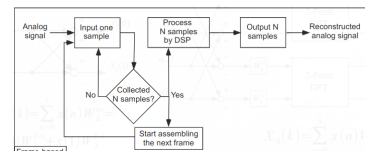
2.5 DDS-Oszillator

$$\varphi_{inc} = 2\pi \frac{f_0}{f_s} \quad (10)$$

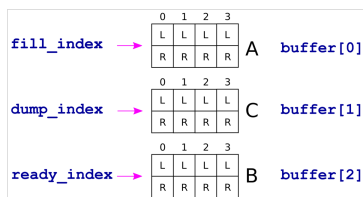
$$\varphi = \varphi + \varphi_{inc} \quad (11)$$

$$x(n) = A \sin(n\varphi) \quad (12)$$

3 Blocksignalverarbeitung



ISR schreibt N samples nach `buffer[fill_index]` und setzt `ready_index = fill_index` (`buffer[fill_index]` ist jetzt dran mit `ProcessBuffer`). Jeder Sample generiert einen Interrupt



- `fill_index` wird von ADC gefüllt
- `dump_index` wird an DAC geschrieben
- `ready_index` Buffer für Blocksignalverarbeitung

```
#define BUFFER_LENGTH      1024      // buffer length in samples
#define NUM_BUFFERS        3
volatile float buffer[NUM_BUFFERS][2][BUFFER_LENGTH];
```

```
void ProcessBuffer()
// Processes the data in buffer[ready_index]
{
    volatile float *pL = buffer[ready_index][LEFT];
    volatile float *pR = buffer[ready_index][RIGHT];
    // Do the Process
    // ...
    buffer_ready = 0; // means were done here
}
```

```
interrupt void Codec_ISR()
{
    static Uint8 fill_index = INITIAL_FILL_INDEX; // index of buffer to fill
    static Uint8 dump_index = INITIAL_DUMP_INDEX; // index of buffer to dump
    static Uint32 sample_count = 0; // current sample count in buffer

    // get input data samples
    CodecDataIn.UINT = ReadCodecData();
    // IN
    buffer[fill_index][LEFT][sample_count] = CodecDataIn.Channel[LEFT];
    buffer[fill_index][RIGHT][sample_count] = CodecDataIn.Channel[RIGHT];
    // OUT
    CodecDataOut.channel[LEFT] = buffer[dump_index][LEFT][sample_count];
    CodecDataOut.channel[RIGHT] = buffer[dump_index][RIGHT][sample_count];

    // update sample count and swap buffers when filled
    if(++sample_count >= BUFFER_LENGTH) {
        sample_count = 0;
        ready_index = fill_index;
        if(++fill_index >= NUM_BUFFERS)
            fill_index = 0;
        if(++dump_index >= NUM_BUFFERS)
            dump_index = 0;
        if(buffer_ready == 1) // set a flag if buffer isn't processed in time
            over_run = 1;
        buffer_ready = 1;
    }

    WriteCodecData(CodecDataOut.UINT); // send output data to
}
```

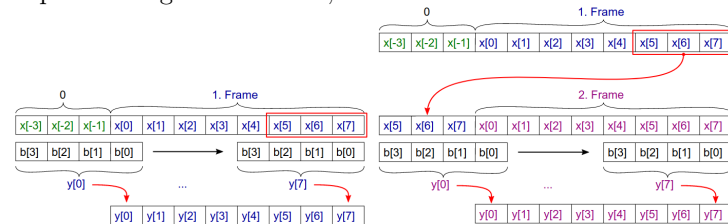
3.1 Blocksignalverarbeitung mit DMA

- DMA kopiert Sample von ADC nach Eingangsbuffer
- DMA kopiert Processed von Ausgangsbuffer nach DAC
- DMA generiert Interrupt, wenn N Samples transfert wurden → Buffer-Swap

```
interrupt void EDMA_ISR()
{
    if(++ready_index >= NUM_BUFFERS)
        ready_index = 0;
    if(buffer_ready == 1) //buffer isnt processed in time
        over_run = 1;
    buffer_ready = 1; //buffer is now ready for processing
}
```

3.2 FIR mit Blocksignalverarbeitung

Bsp. Ordnung Filter N = 4, Framesize = 8



Problem: Bei Frame-Übergängen müssen die N-1 letzten Werte des letzten Frames berücksichtigt werden.

Lösung: `Framesize += N`

- `Left[N|FRAMESIZE]`, `buffer[FRAMESIZE]`
- `Left[N:FRAMESIZE+N] = buffer[0:FRAMESIZE]`
- `buffer[0:FRAMESIZE] = Left * B` (B wird drüber `FRAMESIZE`-mal drüber geschoben s.o)
- `Left[0:N] = Left[FRAMESIZE:FRAMESIZE+N]`

```
void ProcessBuffer()
{
    short *pBuf = buffer[ready_index];
    // extra buffer room for convolution "edge effects"
    // N is filter order from coeff.h
    static float Left[BUFFER_COUNT+N]={0}, Right[BUFFER_COUNT+N]={0};
    float *pL = Left, *pR = Right;
    float yLeft, yRight;
    int i, j, k;
    // offset pointers to start filling after N elements

    pR += N;
    pL += N;

    // extract data to float buffers
    for(i = 0; i < BUFFER_COUNT; i++)
        *pL++ = *pBuf++;

    // reinitialize pointer before FOR loop
    pBuf = buffer[ready_index];

    // Implement FIR filter
    for(i=0; i < BUFFER_COUNT; i++)
    {
        yLeft = 0; // initialize the LEFT output value
        yRight = 0; // initialize the RIGHT output value

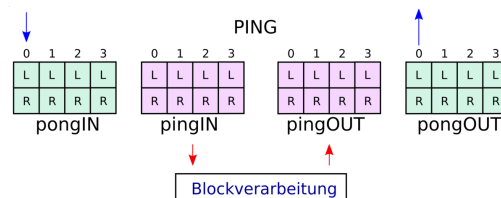
        for(j=0, k=i+N; j <= N; j++, k--)
        {
            yLeft += Left[k] * B[j]; // perform the LEFT dot-product
            yRight += Right[k] * B[j]; // perform the RIGHT dot-product
        }

        // put back into buffer after bounding (must be right then left)
        *pBuf++ = _sprint(yRight * 65536) >> 16;
        *pBuf++ = _sprint(yLeft * 65536) >> 16;
    }

    // save end values at end of buffer array for next pass
    // by placing at beginning of buffer array
    for(i=BUFFER_COUNT, j=0; i < BUFFER_COUNT+N; i++, j++)
    {
        Left[j] = Left[i];
        Right[j] = Right[i];
    }

    buffer_ready = 0; // signal we are done
}
```

3.3 Ping-Pong



- gleiche Latenz (=Durchlaufzeit 2 Buffer)

- Ping-Pong einfacher zu verwalten mit DMA

4 FFT

$$w_N^{nk} = e^{-j \frac{2\pi nk}{N}} \quad (13)$$

$$w_N^{2nk} = e^{-j \frac{4\pi nk}{N}} = w_{\frac{N}{2}}^{nk} \quad (14)$$

1. Aufteilung in $Y(n) = Y_{even}(n) + Y_{odd}(n)$

$$Y(n) = \sum_{k=0}^{N/2-1} y(2k)w_N^{2nk} + \sum_{k=0}^{N/2-1} y(2k+1)w_N^{(2k+1)n} \quad (15)$$

$$Y(n) = \sum_{k=0}^{N/2-1} y(2k)w_N^{2nk} + w_N^n \sum_{k=0}^{N/2-1} y(2k+1)w_N^{2kn} \quad (16)$$

$$Y(n) = \sum_{k=0}^{N/2-1} y(2k)w_{\frac{N}{2}}^{nk} + w_N^n \sum_{k=0}^{N/2-1} y(2k+1)w_{\frac{N}{2}}^{nk} \quad (17)$$

$$Y(n + \frac{N}{2}) = \sum_{k=0}^{N/2-1} y(2k)w_{\frac{N}{2}}^{(n+\frac{N}{2})k} + w_N^{n+\frac{N}{2}} \sum_{k=0}^{N/2-1} y(2k+1)w_{\frac{N}{2}}^{(n+\frac{N}{2})k} \quad (18)$$

2. Aufteilung in $Y(n) = Y_{even}(n) + Y_{odd}(n)$

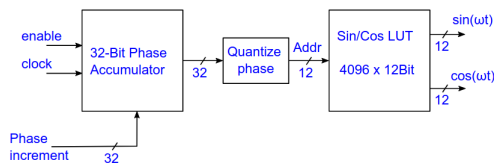
$$Y(n) = \sum_{k=0}^{N/2-1} y(2k)w_N^{2nk} + \sum_{k=0}^{N/2-1} y(2k+1)w_N^{(2k+1)n} \quad (19)$$

$$Y(n) = \sum_{k=0}^{N/2-1} y(2k)w_N^{2nk} + w_N^n \sum_{k=0}^{N/2-1} y(2k+1)w_N^{2kn} \quad (20)$$

$$Y(n) = \sum_{k=0}^{N/2-1} y(2k)w_{\frac{N}{2}}^{nk} + w_N^n \sum_{k=0}^{N/2-1} y(2k+1)w_{\frac{N}{2}}^{nk} \quad (21)$$

5 NCO

NCO = Counter, der Jeden Takt um ein Phaseninkrement μ (hier 32) erhöht wird. Der Ausgang des Counters wird mit LUT in Signalform (sin,cos,sägezahn) umgewandelt. Die LUT ist mit $N = 2^n$ 12-bit breiten Werten gefüllt



$$\mu = N \frac{f_d}{f_s} \quad (22)$$