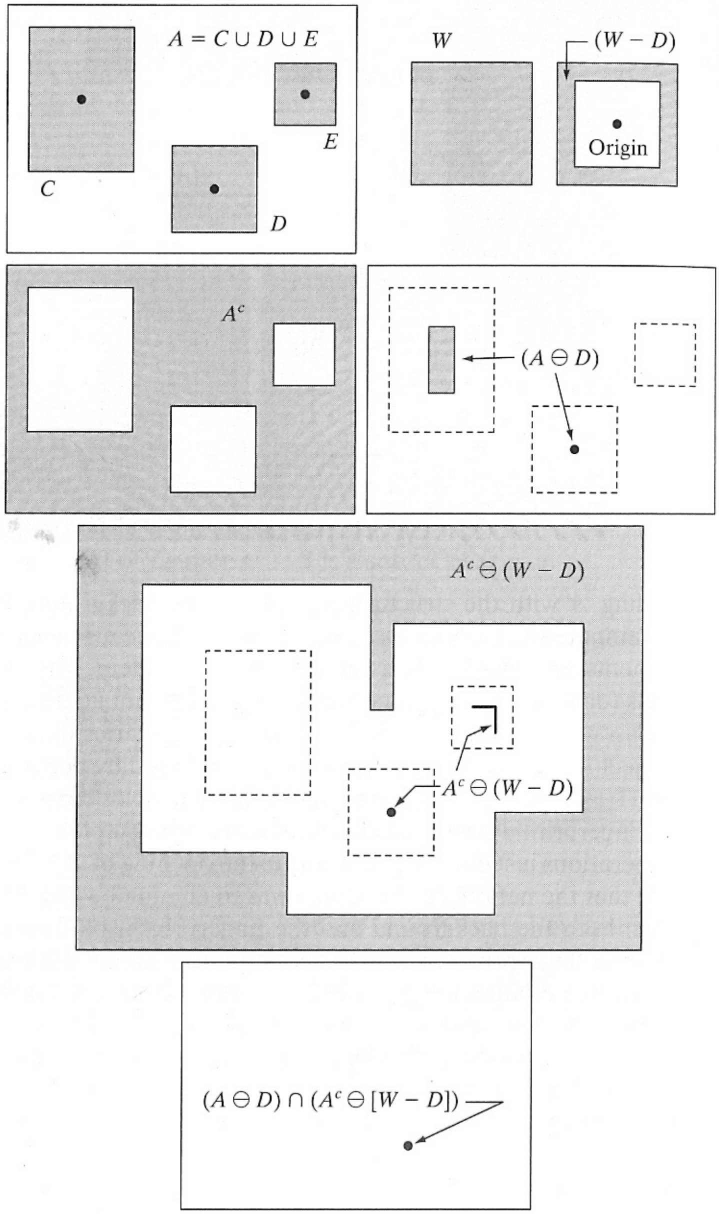
R. Gonzalez & R. Woods, Digital Image Processing, Third Edition, 2008. Chapter 9 excerpt.

9.4 The Hit-or-Miss Transformation

The morphological hit-or-miss transform is a basic tool for shape detection. We introduce this concept with the aid of Fig. 9.12, which shows a set A consisting of three shapes (subsets), denoted C, D, and E. The shading in Figs. 9.12(a) through (c) indicates the original sets, whereas the shading in Figs. 9.12(d) and (e) indicates the result of morphological operations. The objective is to find the location of one of the shapes, say, D.



a b
c d
e
f

FIGURE 9.12 (a) Set A. (b) A window, W, and the local background of D with respect to W, $(W - D)$. (c) Complement of A. (d) Erosion of A by D. (e) Erosion of A^c by $(W - D)$. (f) Intersection of (d) and (e), showing the location of the origin of D, as desired. The dots indicate the origins of C, D, and E.

Let the origin of each shape be located at its center of gravity. Let D be enclosed by a small window, W. The *local background* of D with respect to W is defined as the set difference $(W - D)$, as shown in Fig. 9.12(b). Figure 9.12(c) shows the complement of A, which is needed later. Figure 9.12(d) shows the erosion of A by D (the dashed lines are included for reference). Recall that the erosion of A by D is the set of locations of the *origin* of D, such that D is completely contained in A. Interpreted another way, $A \ominus D$ may be viewed geometrically as the set of all locations of the origin of D at which D found a match (hit) in A. Keep in mind that in Fig. 9.12 A consists only of the three *disjoint* sets C, D, and E.

Figure 9.12(e) shows the erosion of the complement of A by the local background set $(W - D)$. The outer shaded region in Fig. 9.12(e) is part of the erosion. We note from Figs. 9.12(d) and (e) that the set of locations for which D *exactly* fits inside A is the *intersection* of the erosion of A by D and the erosion of A^c by $(W - D)$ as shown in Fig. 9.12(f). This intersection is precisely the location sought. In other words, if B denotes the set composed of D and its background, the match (or set of matches) of B in A, denoted $A \circledast B$, is

$$A \circledast B = (A \ominus D) \cap \left[A^c \ominus (W - D)\right] \tag{9.4-1}$$

We can generalize the notation somewhat by letting $B = (B_1, B_2)$, where B_1 is the set formed from elements of B associated with an object and B_2 is the set of elements of B associated with the corresponding background. From the preceding discussion, $B_1 = D$ and $B_2 = (W - D)$. With this notation, Eq. (9.4-1) becomes

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2) \tag{9.4-2}$$

Thus, set $A \circledast B$ contains all the (origin) points at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c. By using the definition of set differences given in Eq. (2.6-19) and the dual relationship between erosion and dilation given in Eq. (9.2-5), we can write Eq. (9.4-2) as

$$A \circledast B = (A \ominus B_1) - (A \oplus \hat{B}_2) \tag{9.4-3}$$

However, Eq. (9.4-2) is considerably more intuitive. We refer to any of the preceding three equations as the *morphological hit-or-miss transform*.

The reason for using a structuring element B_1 associated with objects and an element B_2 associated with the background is based on an assumed definition that two or more objects are distinct only if they form disjoint (disconnected) sets. This is guaranteed by requiring that each object have at least a one-pixel-thick background around it. In some applications, we may be interested in detecting certain patterns (combinations) of 1s and 0s within a set, in which case a background is not required. In such instances, the hit-or-miss transform reduces to simple erosion. As indicated previously, erosion is still a set of matches, but without the additional requirement of a background match for detecting individual objects. This simplified pattern detection scheme is used in some of the algorithms developed in the following section.

convexity. One simple approach to reduce this effect is to limit growth so that it does not extend past the vertical and horizontal dimensions of the original set of points. Imposing this limitation on the example in Fig. 9.19 resulted in the image shown in Fig. 9.20. Boundaries of greater complexity can be used to limit growth even further in images with more detail. For example, we could use the maximum dimensions of the original set of points along the vertical, horizontal, and diagonal directions. The price paid for refinements such as this is additional complexity and increased computational requirements of the algorithm.

9.5.5 Thinning

The thinning of a set A by a structuring element B, denoted $A \otimes B$, can be defined in terms of the hit-or-miss transform:

$$\begin{aligned} A \otimes B &= A - (A \circledast B) \\ &= A \cap (A \circledast B)^c \end{aligned} \tag{9.5-6}$$

As in the previous section, we are interested only in pattern matching with the structuring elements, so no background operation is required in the hit-or-miss transform. A more useful expression for thinning A symmetrically is based on a *sequence* of structuring elements:

$$\{B\} = \{B^1, B^2, B^3, \ldots, B^n\} \tag{9.5-7}$$

where B^i is a rotated version of B^{i-1}. Using this concept, we now define thinning by a sequence of structuring elements as

$$A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2) \ldots) \otimes B^n) \tag{9.5-8}$$

The process is to thin A by *one pass* with B^1, then thin the result with one pass of B^2, and so on, until A is thinned with one pass of B^n. The entire process is repeated until no further changes occur. Each individual thinning pass is performed using Eq. (9.5-6).

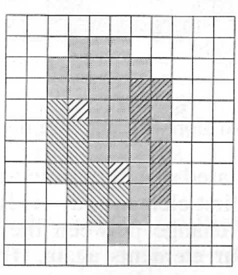


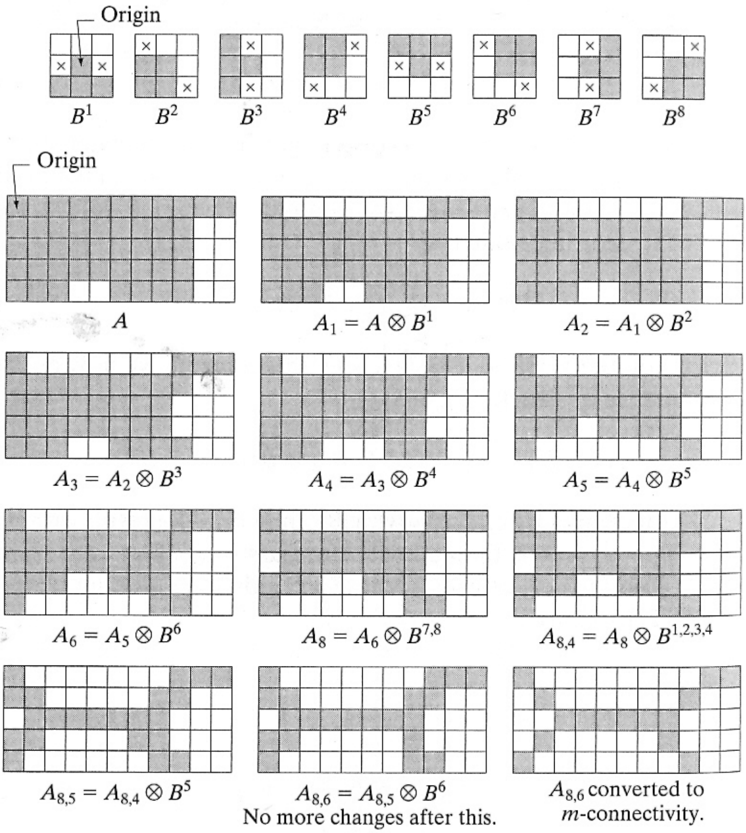
FIGURE 9.20 Result of limitin growth of the convex hull algorithm to the maximum dimensions of th original set of points along the vertical and horizontal directions.

Figure 9.21(a) shows a set of structuring elements commonly used for thinning, and Fig. 9.21(b) shows a set A to be thinned by using the procedure just discussed. Figure 9.21(c) shows the result of thinning after one pass of A with B^1, and Figs. 9.21(d) through (k) show the results of passes with the other structuring elements. Convergence was achieved after the second pass of B^6. Figure 9.21(l) shows the thinned result. Finally, Fig. 9.21(m) shows the thinned set converted to m-connectivity (see Section 2.5.2) to eliminate multiple paths.

9.5.6 Thickening

Thickening is the morphological dual of thinning and is defined by the expression

$$A \odot B = A \cup (A \circledast B) \tag{9.5-9}$$



a
b c d
e f g
h i j
k l m

FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first four elements again. (l) Result after convergence. (m) Conversion to m-connectivity.