

Exercise Sheet 3
Fall 2023

Exercise 1: [Optimization]

Given the functions:

$$F(x, y) = x^2 + y^2 - 4x + 6y + 9, \quad G(x, y) = (x + 2y - 7)^2 + (2x + y - 5)^2$$

$$H(x, y) = (x^2 + y - 11)^2 + (x + y^2 - 7)^2$$

- a) Plot the functions. Vary the domain (x and y -values) to capture the essential part of the functions.
- b) Are the functions bounded? Give a mathematical argument.
- c) Are the functions convex?
- d) For functions F and G : Find the partial derivatives and the gradients
- e) Find the minima using calculations. (analytical solution)
- f) Find the minima using a numerical solution

Exercise 2: [parameter estimation]

Load temperature data from Svalbard lufthavn from the file `svalbard_met.csv` (open data from Meteorologisk institutt (MET)). This is the same file as for the last exercise.

- a) Plot the data for the last five years.
- b) We have the function

$$S(x) = a_0 \sin(a_1 x + a_3) + a_4.$$

Explain the role of the different parameters.

- c) Plot the function S for $x \in [0, 10\pi]$ and your own choice of \mathbf{a} -values.
 - d) Formulate a cost-function $C(\mathbf{a})$ that fits the function S to the observations.
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- e) It is natural to assume that the observations are from a observation site on the planet earth. If we do not make this assumption, the length of the year is unknown. Based on the data and the cost-function above. Estimate the length of the year on the observation site. (HINT-1: Try using data from a short period, i.e. some years, when fitting the model to the data.)
(HINT-2: The parameter estimation will be sensitive to the initialization of the \mathbf{a} -values. Use the answers a-c to find resonable initial values.)