Mathematical Modeling and Simulation

Obligatory assignment 1

Fall 2023

Solutions to be handed in 21.09

Western Norway University of Applied Sciences

This is the first obligatory assignment. Solutions are to be handed in by 21.09 in Canvas. Hand in one pdf with written solutions. You are allowed to discuss the exercises with others, but everyone has to hand in their own solutions.

Exercise 1: [**Optimization**]

Given the functions:

$$F(x,y) = 3x^2 + 2xy + 4y^2 - 6x - 8y + 5, \quad (1)$$

$$G(x,y) = (x^2 + 3y^2)^2 - 2xy \quad (2)$$

a) Find the partial derivatives and the gradients

b) Does the functions have one minima, several local minimum or no minimum at all? Make a theoretical or geometrical argument.

c) If possible, find the minima using calculation (analytical solution).

Exercise 2: [**Mixing alloys**]

Alloys A, B, C, D and E contain the following mass fractions (in %) of constituents m, n, o and p, as well as other constituents:

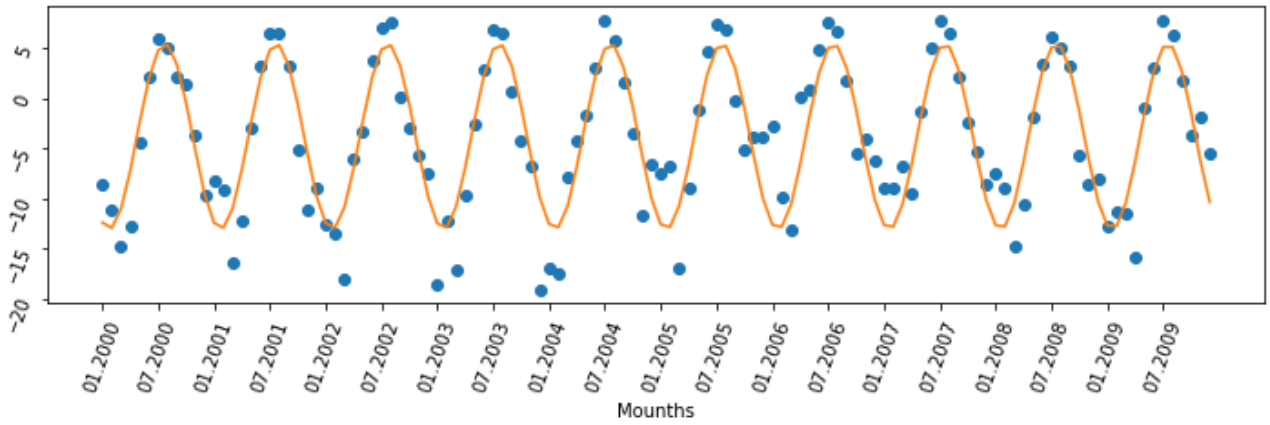
	A	B	C	D	E
m	11	7	5	13	0
n	21	23	10	11	35
o	0	5	15	20	25
p	12	0	23	17	13
other	56	65	47	39	27

If making a new mixture of 0.5 kg by combining alloys A, B, C, D and E such that the new mixture has 25.5 g of constituent m, 117.25 g of n, 87.5 g of o and 60.25 g of p. We will in the following try to find out how much is then needed of alloy E.

a) Formulate the mixing of the alloys and composition of constituents in new mix as system of equations. Write in matrix form.

b) Solve the above system of equations in python (or another programming language of your choice). How much is needed of alloy E? (And the other alloys?)

c) What is $\{S, Q, M\}$ here? What are the state variable(s) and system parameters? Which auxilliary variables did you need to formulate the mathematical model?



Exercise 3: [**Curve fitting**]

We consider the monthly average temperature at Svalbard Lufthavn from 2000- 2010. The observations can be fitted using a parameterized sin-function

$$S(x) = a_0 \sin(a_1 x + a_2) + a_3,$$

whera $\mathbf{a} = [a_0, a_1, a_2, a_3]$ is a parameter vector. The data and the fitted curve can be seen in the figure.

a) Extend the function (model) with one extra parameter. The new function should be able to account for the gradually increasing yearly middle-temperature in the data.

b) Propose a simpler model, that will be able to account for the long term change in yearly middle-temperature, but not necessarily the seasonal variations (from month to month).

c) The models in questions a) and b) are phenomenological models. If fitted to data from various 10-year periods, they may be used to show that there is an increased trend of higher annual average temperatures. Can this alone be used as an argument for human made climate changes? Make a short discussion.