

Oblig 1

$$F(x, y) = 3x^2 + 2xy + 4y^2 - 6x - 8y + 5$$

$$F_x = 6x + 2y - 6$$

$$F_y = 2x + 8y - 8$$

$$\bar{F}_{xx} = 6$$

$$F_{yy} = 8$$

$$F_{xy} = 2$$

Gradient:

$$\vec{\nabla} F = [F_x, F_y] \Rightarrow [6x + 2y - 6, 2x + 8y - 8]$$

$$G(x, y) = (x^2 + 3y^2)^2 - 2xy \Rightarrow (x^2 + 3y^2)(x^2 + 3y^2) - 2xy \\ 9y^4 + x^4 + 6x^2y^2 - 2xy$$

$$G_x = 4x^3 + 12xy^2 - 2y$$

$$G_y = 36y^3 + 12x^2y - 2x$$

$$G_{xx} = 12x^2 + 12y^2$$

$$G_{yy} = 108y^2 + 12x^2$$

$$G_{xy} = 24xy - 2$$

$$\vec{\nabla} G = [G_x, G_y] \Rightarrow [4x^3 + 12xy^2 - 2y, 36y^3 + 12x^2y - 2x]$$

b)

Githuly

$$c) \quad F_x = 0, \quad F_y = 0$$

$$(1) \quad 6x + 2y - 6 = 0$$

$$(2) \quad 2x + 8y - 8 = 0$$

$$(1) \quad 2y = 3(-x+1)$$

$$(2) \quad 2x + 8(-3x+3) - 8 = 0$$

$$2x - 24x + 24 - 8 = 0$$

$$-22x + 16 = 0$$

$$22x = 16$$

$$x = \frac{16}{22}$$

$$(1) \quad 6\left(\frac{16}{22}\right) + 2y - 6 = 0$$

$$(1) \quad y = 3\left(-\frac{16}{22} + \frac{22}{22}\right)$$

$$y = 3\left(\frac{6}{22}\right)$$

$$y = \frac{18}{22}$$

Kritisk punkt : $\left(\frac{-16}{22}, \frac{18}{22}\right)$

Kritisk punkt : $\left(\frac{8}{11}, \frac{9}{11}\right)$

Hessc

$$\begin{bmatrix} F_{xx} & F_{xy} \\ F_{yx} & F_{yy} \end{bmatrix} \Rightarrow \begin{bmatrix} 6 & 2 \\ 2 & 8 \end{bmatrix}$$

Hessc determinant

$$6 \cdot 8 - 2^2 = 44$$

$$F_{xx} = 6 \quad F_{yy} = 8$$

$$F_{xx} > 0$$

$$\text{Hesscdd} > 0$$

Minimum

Oppgave 3

a) $s(x) = a_0 \sin(a_1 x + a_2) + a_3 + a_4 x$

b) $ax + b$

c) Github

Oppgave 2 - Github

A

$$\begin{bmatrix} 11 & 7 & 5 & 13 & 0 \\ 21 & 23 & 10 & 11 & 35 \\ 0 & 5 & 15 & 20 & 25 \\ 12 & 0 & 23 & 17 & 13 \\ 56 & 65 & 47 & 39 & 27 \end{bmatrix} \begin{bmatrix} x_e \\ x_b \\ x_c \\ x_d \\ x_e \end{bmatrix} = \begin{bmatrix} 25.5 \\ 117.25 \\ 57.5 \\ 60.25 \\ 504 \\ 209.5 \end{bmatrix}$$

$$500 - 25.5 - 117.25 - 57.5 - 60.25 = 209.5$$

1c

$$G_x = 4x^3 + 12xy^2 - 2y$$

$$G_y = 36y^3 + 12x^2y - 2x$$

$$(1) \quad 2x^3 + 6xy^2 - y = 0$$

$$(2) \quad 18y^3 + 12x^2y - x = 0$$

$$(1) \quad 2x^3 + y(6xy - 1) = 0$$

$$(2) \quad 18y^3 + x(6xy - 1) = 0$$

$$(1) \quad \boxed{6xy - 1 = \frac{-2x^3}{y}}$$

$$(2) \quad 18y^3 + x\left(\frac{-2x^3}{y}\right) = 0$$

$$18y^3 - \frac{2x^4}{y} = 0$$

$$18y^4 - 2x^4 = 0$$

$$9y^4 = x^4$$

$$x = \sqrt[4]{9}y$$

$$\boxed{x = \sqrt[4]{3}y}$$

$$(1) \quad 6(\sqrt[4]{3}y)y - 1 = \frac{-2(\sqrt[4]{3}y)^3}{y}$$

$$6\sqrt[4]{3}y^2 - 1 = \frac{-6\sqrt[4]{3}y^2}{y}$$

$$6\sqrt[4]{3}y^2 + 6\sqrt[4]{3}y^2 - 1 = 0$$

$$12\sqrt[4]{3}y^2 - 1 = 0$$

$$y^2 = \frac{1}{12\sqrt[4]{3}}$$

$$y = \sqrt{\frac{1}{12\sqrt[4]{3}}}$$

$$\boxed{y = \pm \frac{3^{3/4}}{6}}$$

wolfran

$$\boxed{y = \pm \frac{1}{\sqrt{12\sqrt[4]{3}}}}$$

$$X = \sqrt{3} \cdot \frac{1}{\sqrt{12} \cdot \sqrt[4]{3}}$$

$$\boxed{x = \frac{1}{2 \cdot \sqrt[4]{3}}}$$

Wolfram

~~$$\boxed{y = \frac{1}{\sqrt{12} \cdot \sqrt[4]{3}}}$$~~

wolfr

~~$$\boxed{y = \frac{\sqrt[4]{3}}{6}}$$~~

$$\boxed{y = \pm \frac{3^{3/4}}{6}}$$

Kritische Punkte:

$$(0,0)$$

$$\boxed{\left(\frac{1}{2 \sqrt[4]{3}}, \frac{3^{3/4}}{6} \right)}$$

$$\boxed{\left(\frac{1}{2 \sqrt[4]{3}}, -\frac{3^{3/4}}{6} \right)}$$

$$\boxed{\left(-\frac{1}{2 \sqrt[4]{3}}, \frac{3^{3/4}}{6} \right)}$$

$$\boxed{\left(-\frac{1}{2 \sqrt[4]{3}}, -\frac{3^{3/4}}{6} \right)}$$

Hessamatrix G:

$$\begin{bmatrix} 12x^2 & 24xy-2 \\ 24xy-2 & 12x^2 + 104y^2 \end{bmatrix}$$

$$\begin{bmatrix} x^2 & 2xy - \frac{1}{6} \\ 2xy - \frac{1}{6} & x^2 + 9y^2 \end{bmatrix}$$

$$\begin{bmatrix} 6x^2 & 12xy-1 \\ 12xy-1 & 6x^2 + 54y^2 \end{bmatrix}$$

$$\begin{pmatrix} x^2 & 2xy - \frac{1}{6} \\ 2xy - \frac{1}{6} & x^2 + 9y^2 \end{pmatrix}$$

$$\begin{cases} x = \pm \frac{1}{2\sqrt[4]{3}} \\ y = \pm \frac{3^{3/4}}{6} \end{cases}$$

$$\begin{cases} x^2 = \frac{1}{4\sqrt{3}} \\ y^2 = \frac{1}{4\sqrt{3}} \end{cases} \text{ wolf}$$

(0, 0)

$$f_{xx} \cdot f_{yy} - (f_{xy})^2 \quad \text{Funkt}$$

$$\begin{pmatrix} 0 & -\frac{1}{6} \\ -\frac{1}{6} & 0 \end{pmatrix}$$

\$

$$0 \cdot 0 - \left(-\frac{1}{6}\right)^2 \Rightarrow -\frac{1}{36} \quad \text{ff}$$

$$-\frac{1}{36} < 0 \Rightarrow \text{Selpunkt}$$

$$\frac{1}{2\sqrt[4]{3}} \left(\frac{1}{2\sqrt[4]{3}}, \frac{3^{3/4}}{6} \right)$$

$$\begin{pmatrix} \frac{1}{4\sqrt{3}} & 2 \cdot \frac{1}{2\sqrt[4]{3}} \cdot \frac{3^{3/4}}{6} & -\frac{1}{6} \\ 2 \cdot \frac{1}{2\sqrt[4]{3}} \cdot \frac{3^{3/4}}{6} & 10 \cdot \frac{1}{4\sqrt{3}} & \end{pmatrix}$$

$$\frac{1}{4\sqrt{3}} \cdot \frac{10}{4\sqrt{3}} - \left(\frac{8}{6\sqrt[4]{3}} - \frac{1}{6} \right)^2$$

$$\frac{5\sqrt{3}}{6} - \left(\frac{1}{2\sqrt{3}} - \frac{1}{6} \right)^2 \quad \underline{\text{Wolfram}}$$

$$\frac{5\sqrt{3}}{6} - \frac{1}{12} + \frac{1}{36}$$

$$\frac{6 \cdot 5\sqrt{3}}{36} + \frac{1}{36} - \frac{3}{36}$$

$$\frac{30\sqrt{3} - 2}{36} \approx 1.38 \quad \underline{\text{wolfram}}$$

$1.38 > 0 \rightarrow$ Max oder min

$f_{xx} > 0 \rightarrow$ Minimum

$$\left(-\frac{1}{2\sqrt{3}}, \frac{3^{3/4}}{6} \right)$$

$$\begin{aligned} & \left[\frac{1}{4\sqrt{3}} \quad 2 \cdot -\frac{1}{2\sqrt{3}} \cdot \frac{3^{3/4}}{6} - \frac{1}{6} \right] \\ & \left[2 \cdot -\frac{1}{2\sqrt{3}} \cdot \frac{3^{3/4}}{6} - \frac{1}{6} \quad \frac{10}{4\sqrt{3}} \right] \end{aligned}$$

$$\frac{1}{4\sqrt{3}} \cdot \frac{10}{4\sqrt{3}} - \left(2 \cdot -\frac{1}{2\sqrt{3}} \cdot \frac{3^{3/4}}{6} - \frac{1}{6} \right)^2$$

$$\frac{5\sqrt{3}}{6} - \left(\frac{3^{3/4}}{6\sqrt{3}} - \frac{1}{6} \right)^2 \quad \underline{\text{WolF}}$$

$$\frac{5\sqrt{3}}{6} - \left(\frac{3^{3/4}}{6\sqrt{3}} - \frac{1}{6} \right)^2 \quad \underline{\text{WolF}}$$

$$\frac{5\sqrt{3}}{6} - \frac{1}{12} - \frac{1}{36} = 1.38$$

$1.38 > 0$ Min / max

$f_{xx} > 0$ min

$$\left(\frac{1}{2\sqrt[4]{3}}, -\frac{3^{3/4}}{6} \right)$$

$$\left[\frac{1}{4\sqrt{3}}, 2 \cdot \frac{1}{2\sqrt{3}} \cdot \frac{3^{3/4}}{6} - \frac{1}{6} \right]$$

$$2 \cdot \frac{1}{2\sqrt{3}} \cdot \frac{3^{3/4}}{6} - \frac{1}{6}$$

$$10 \cdot \frac{1}{4\sqrt{3}}$$

$$\frac{1}{4\sqrt{3}} \cdot \frac{10}{4\sqrt{3}} - \left(\frac{1}{8\sqrt{3}} \cdot -\frac{3^{3/4}}{6} - \frac{1}{6} \right)^2$$

$$\frac{5\sqrt{3}}{6} - \left(-\frac{3^{3/4}}{8\sqrt{3} \cdot 6} - \frac{1}{6} \right)^2$$

~~$$\frac{5\sqrt{3}}{6} - \left(\frac{3^{3/4}}{4\sqrt{3} \cdot 6} + \frac{1}{6} \right)^2$$~~

$$\frac{5\sqrt{3}}{6} - \left(-\frac{1}{6} - \frac{1}{2\sqrt{3}} \right)^2 \quad \text{Wolfram}$$

$$\frac{5\sqrt{3}}{6} - \frac{1}{36} - \frac{1}{12} \dots$$

$$\frac{30\sqrt{3} - 1 - 1}{36} \Rightarrow \frac{30\sqrt{3} - 4}{36}$$

$$1 \cdot 37 > 0 \quad \text{min/max}$$

$$G_{xx} > 0 \quad \underline{\text{min}}$$

$$\left(-\frac{1}{2\sqrt[4]{3}}, -\frac{3^{3/4}}{6} \right)$$

$$2 \cdot -\frac{1}{2\sqrt[4]{3}} \cdot -\frac{3^{3/4}}{6} - \frac{1}{6}$$

$$2 \cdot -\frac{1}{2\sqrt[4]{3}} \cdot -\frac{3^{3/4}}{6} - \frac{1}{6}$$

Fra tidligere:

$$\frac{5\sqrt{3}}{6} - \left(2 \cdot -\frac{1}{2\sqrt[4]{3}} \cdot -\frac{3^{3/4}}{6} - \frac{1}{6} \right)^2$$

$$\frac{5\sqrt{3}}{6} - \left(\frac{3^{3/4}}{6\sqrt[4]{3}} - \frac{1}{6} \right)^2$$

$$\frac{5\sqrt{3}}{6} - \left(\sqrt{3} - \frac{1}{6} \right)^2 \quad \underline{\text{Wolf}}$$

~~$$\frac{5\sqrt{3}}{6} - \frac{6\sqrt{3}}{6} + \frac{1}{6} = \frac{5\sqrt{3} - 6\sqrt{3} + 1}{6}$$~~

~~$$\frac{5\sqrt{3}}{6} - \frac{6\sqrt{3}}{6} + \frac{1}{6}$$~~

$$\frac{5\sqrt{3}}{6} - \left(\frac{\sqrt{3}}{6} - \frac{1}{6} \right)^2 \quad \underline{\text{Wolf}}$$

$$\frac{30\sqrt{3}}{36} - \left(\frac{3}{36} - \frac{1}{36} \right)$$

$$\frac{30\sqrt{3}}{36} - \frac{2}{36}$$

$$1.38$$

~~$$\underline{\text{Wolf}}$$~~

$$1.38 > 0$$

$$\text{Min } / \text{McX}$$

$$t_{xx} > 0$$

~~$$\underline{\text{Min}}$$~~