High level:

input

a —

b —

c — $\boxed{\text{model}}$ $\rightarrow$ objective L

d —

⋮

assess change in L, ∇L

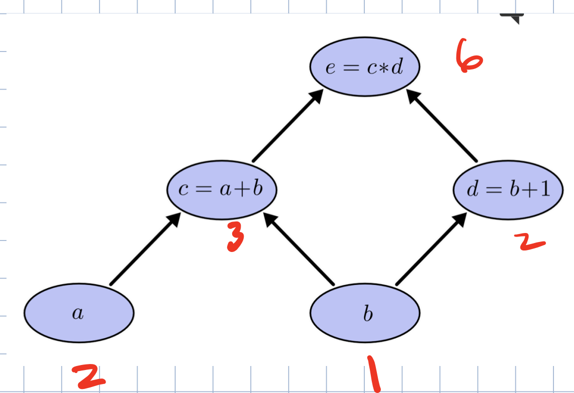
$$a, b \rightarrow (a+b)*(b+1) \rightarrow e$$

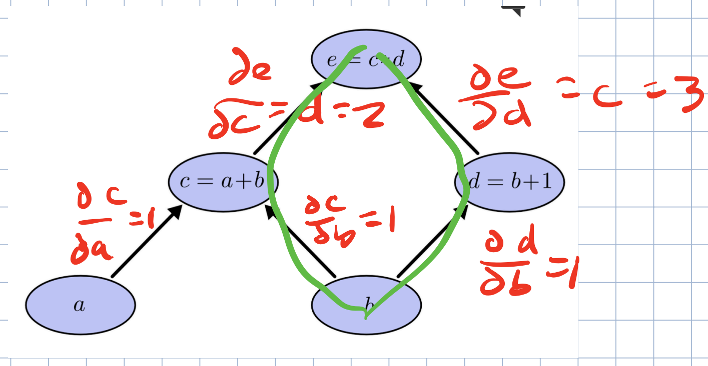
$$e = (a+b) * (b+1)$$

$$= c * d$$

$$d = b+1$$

$$c = a+b$$

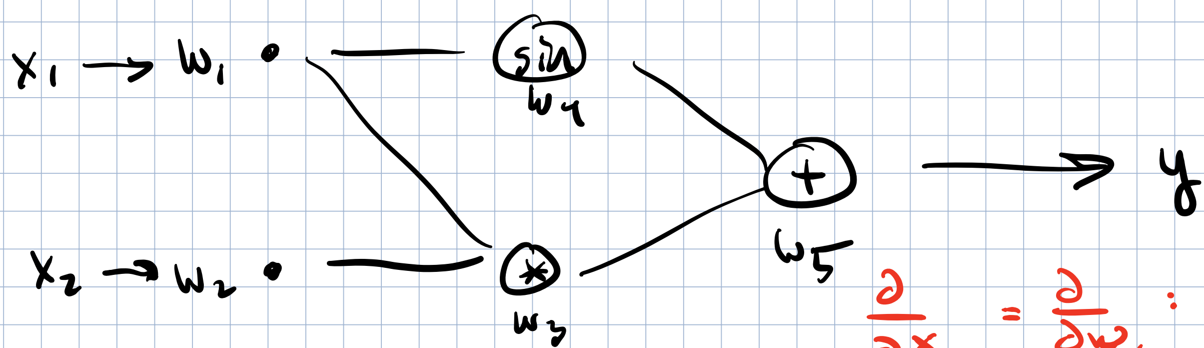




$$\frac{\partial e}{\partial b} = 1 \cdot 2 + 1 \cdot 3$$

$$y = x_1 * x_2 + \sin(x_1)$$

$x_1 = 2$
$x_2 = 5$



$$\frac{\partial}{\partial x_1} = \frac{\partial}{\partial w_1} :$$

2 $w_1 = x_1$

5 $w_2 = x_2$

$w_3 = w_1 * w_2$

$w_4 = \sin(w_1)$

$w_5 = w_3 + w_4$

rule $\sum_{j \text{ input}} \frac{\partial w_i}{\partial w_j} \dot{w}_j$

$\dot{w}_1 = 1$

$\dot{w}_2 = 0$

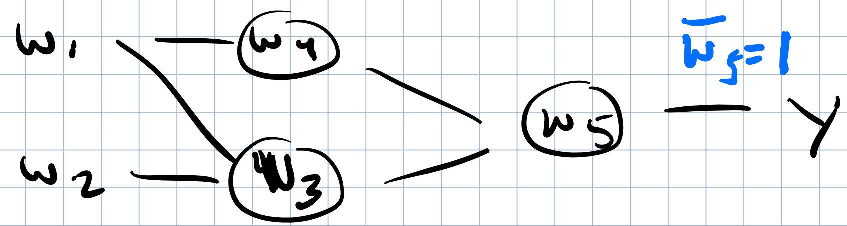
$$\dot{w}_3 = \frac{\partial w_3}{\partial w_1} \cdot \dot{w}_1 + \frac{\partial w_3}{\partial w_2} \dot{w}_2$$
$$= w_2 \cdot \dot{w}_1 + w_1 \dot{w}_2$$
$$= 5 \cdot 1 + 2 \cdot 0$$
$$= 5$$

$$\dot{w}_4 = \frac{\partial w_4}{\partial w_1} \cdot \dot{w}_1$$
$$= \cos(w_1) \cdot 1$$

$$\dot{w}_5 = 1 \cdot \dot{w}_3 + 1 \cdot \dot{w}_4$$
$$= 5 + \cos(2)$$

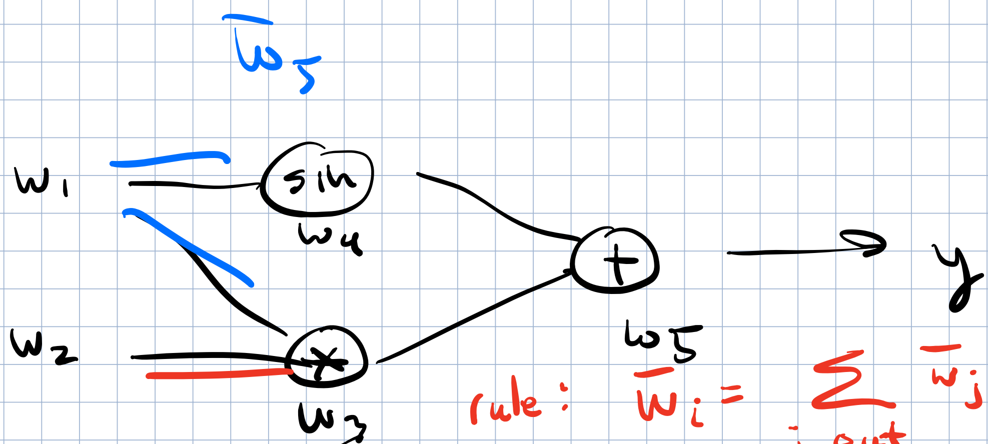
local gradient

$$\bar{w}_i = \frac{\partial y}{\partial w_i}$$



$$\frac{\partial y}{\partial w_1} = \frac{\partial y}{\partial w_4}\frac{\partial w_4}{\partial w_1} + \frac{\partial y}{\partial w_3}\frac{\partial w_3}{\partial w_1}$$

$$= \bar{w}_4 \frac{\partial w_4}{\partial w_1} + \bar{w}_3 \frac{\partial w_3}{\partial w_1}$$



rule: $\bar{w}_i = \sum_{j \in \text{out}} \bar{w}_j \frac{\partial w_j}{\partial w_i}$

FWD

$w_1 = x_1 = 2$
$w_2 = x_2 = 5$
$w_3 = w_1 \times w_2$
$w_4 = \sin(w_1)$
$w_5 = w_3 + w_4$

$$\bar{w}_1 = \bar{w}_4 \frac{\partial w_4}{\partial w_1} + \bar{w}_3 \frac{\partial w_3}{\partial w_1}$$
$$= 1 \cos(w_1) + 1 \cdot 5$$

$$\bar{w}_2 = \bar{w}_3 \frac{\partial w_3}{\partial w_2} = \bar{w}_3 \cdot w_1 = 2$$

$$\bar{w}_3 = \bar{w}_5 \frac{\partial w_5}{\partial w_3} = 1 \cdot 1 = 1$$

$$\bar{w}_4 = \bar{w}_5 \frac{\partial w_5}{\partial w_4} = 1 \cdot 1 = 1$$

$$\bar{w}_5 = 1$$

Simpler

$a = 4$

$b = 3$

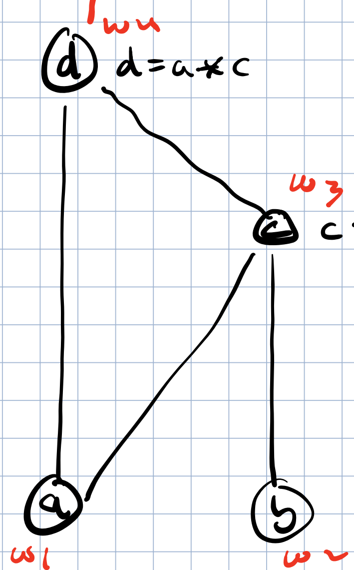
$y = a * (a + b)$

$a = 4$ $= w_1$

$b = 3$ $= w_2$

$c = a + b$ $= w_3$

$d = a * c$ $\bar{v}_4 = 1$ $= w_4$



Step 1: Fwd

$d = a * (a + b)$

$c = a + b = 4 + 3 = 7$

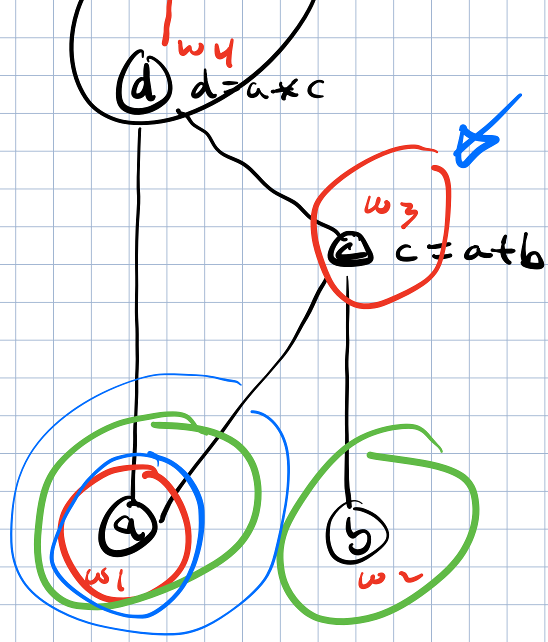
$"\frac{\partial}{\partial a}" = 1$, $"\frac{\partial}{\partial b}" = 1$

local partials

$d = a * c$

$d = a * c = 4 * 7 = 28$

$"\frac{\partial}{\partial a}" = c$, $"\frac{\partial}{\partial c}" = a$



Step 2:

① $\bar{w}_4 = 1$

② for the subnodes of w_4: w_1, w_3

$$\bar{w}_1 \Leftarrow \bar{w}_4 \cdot \frac{\partial w_4}{\partial w_1}$$

$$\bar{w}_3 \Leftarrow \bar{w}_4 \cdot \frac{\partial w_4}{\partial w_3}$$

③ subnodes of w_3

$$\bar{w}_1 \Leftarrow \bar{w}_1 + \bar{w}_3 \frac{\partial w_3}{\partial w_1}$$

$$\bar{w}_2 \Leftarrow \bar{w}_3 \frac{\partial w_3}{\partial w_2} + 0$$