Solutions Manual to Accompany

SEMICONDUCTOR DEVICES
Physics and Technology
2nd Edition

S. M. SZE
UMC Chair Professor
National Chiao Tung University
National Nano Device Laboratories
Hsinchu, Taiwan

John Wiley and Sons, Inc
New York. Chicester / Weinheim / Brisband / Singapore / Toronto

Contents

CHAPTER 2

1. (a) From Fig. 11a, the atom at the center of the cube is surround by four equidistant nearest neighbors that lie at the corners of a tetrahedron. Therefore the distance between nearest neighbors in silicon (a = 5.43 Å) is

$$1/2\ [(a/2)^2 + (\sqrt{2a}\ /2)^2]^{1/2} = \sqrt{3a}\ /4 = 2.35 \text{ Å}.$$

(b) For the (100) plane, there are two atoms (one central atom and 4 corner atoms each contributing 1/4 of an atom for a total of two atoms as shown in Fig. 4a) for an area of a^2, therefore we have

$$2/\ a^2 = 2/\ (5.43 \times 10^{-8})^2 = 6.78 \times 10^{14} \text{ atoms / cm}^2$$

Similarly we have for (110) plane (Fig. 4a and Fig. 6)

$$(2 + 2 \times 1/2 + 4 \times 1/4)\ /\ \sqrt{2a}^{\ 2} = 9.6 \times 10^{15} \text{ atoms / cm}^2,$$

and for (111) plane (Fig. 4a and Fig. 6)

$$(3 \times 1/2 + 3 \times 1/6)\ /\ 1/2(\sqrt{2a}\ )(\sqrt{\left(\frac{3}{2}\right)}a\ ) = \frac{2}{\left(\frac{\sqrt{3}}{2}\right)a^2} = 7.83 \times 10^{14} \text{ atoms / cm}^2.$$

2. The heights at X, Y, and Z point are $\frac{3}{4}$, $\frac{1}{4}$, and $\frac{3}{4}$.

3. (a) For the simple cubic, a unit cell contains 1/8 of a sphere at each of the eight corners for a total of one sphere.

Maximum fraction of cell filled

= no. of sphere × volume of each sphere / unit cell volume

$= 1 \times 4\square(a/2)^3\ /\ a^3 = 52\ \%$

(b) For a face-centered cubic, a unit cell contains 1/8 of a sphere at each of the eight corners for a total of one sphere. The fcc also contains half a sphere at each of the six faces for a total of three spheres. The nearest neighbor distance is $1/2(a\sqrt{2}\ )$. Therefore the radius of each sphere is $1/4\ (a\sqrt{2}\ )$.

Maximum fraction of cell filled

$= (1 + 3)\ \{4\square[(a/2)\ /\ 4\ ]^3\ /\ 3\}\ /\ a^3 = 74\ \%.$

(c) For a diamond lattice, a unit cell contains 1/8 of a sphere at each of the eight corners for a total of one sphere, 1/2 of a sphere at each of the six faces for a total of three spheres, and 4 spheres inside the cell. The diagonal distance between (1/2, 0, 0) and (1/4, 1/4, 1/4) shown in Fig. 9a is

$$D = \frac{1}{2}\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \frac{a}{4}\sqrt{3}$$

The radius of the sphere is D/2 = $\frac{a}{8}\sqrt{3}$

$\therefore$ Maximum fraction of cell filled

$$= (1 + 3 + 4)\left[\frac{4\pi}{3}\left(\frac{a}{8}\sqrt{3}\right)\right]^3 / a^3 = \pi\sqrt{3}/16 = 34\ \%.$$

This is a relatively low percentage compared to other lattice structures.

4. $|d_1| = |d_2| = |d_3| = |d_4| = \mathrm{d}$

$$\overline{d_1} + \overline{d_2} + \overline{d_3} + \overline{d_4} = 0$$

$$\overline{d_1} \bullet (\overline{d_1} + \overline{d_2} + \overline{d_3} + \overline{d_4}) = \overline{d_1} \bullet 0 = 0$$

$$|d_1|^2 + \overline{d_1} \bullet \overline{d_2} + \overline{d_1} \bullet \overline{d_3} + \overline{d_1} \bullet \overline{d_4} = 0$$

$$\therefore d^2 + d^2 \cos\theta_{12} + d^2\cos\theta_{13} + d^2\cos\theta_{14} = d^2 + 3\,d^2 \cos\theta = 0$$

$$\therefore \cos\theta = \frac{-1}{3}$$

$$\theta = \cos^{-1}\left(\frac{-1}{3}\right) = 109.47^\circ$$

5. Taking the reciprocals of these intercepts we get 1/2, 1/3 and 1/4. The smallest three integers having the same ratio are 6, 4, and 3. The plane is referred to as (643) plane.

6. (a) The lattice constant for GaAs is 5.65 Å, and the atomic weights of Ga and As are 69.72 and 74.92 g/mole, respectively. There are four gallium atoms and four arsenic atoms per unit cell, therefore

$$4/a^3 = 4/(5.65 \times 10^{-8})^3 = 2.22 \times 10^{22} \text{ Ga or As atoms/cm}^2,$$

$$\text{Density} = (\text{no. of atoms/cm}^3 \times \text{atomic weight}) / \text{Avogadro constant}$$

$$= 2.22 \times 10^{22}(69.72 + 74.92) / 6.02 \times 10^{23} = 5.33 \text{ g / cm}^3.$$

(b) If GaAs is doped with Sn and Sn atoms displace Ga atoms, donors are formed, because Sn has four valence electrons while Ga has only three. The resulting semiconductor is *n*-type.

7. (a) The melting temperature for Si is 1412 ºC, and for SiO_2 is 1600 ºC. Therefore, SiO_2 has higher melting temperature. It is more difficult to break the Si-O bond than the Si-Si bond.

(b) The seed crystal is used to initiated the growth of the ingot with the correct crystal orientation.

(c) The crystal orientation determines the semiconductor's chemical and electrical properties, such as the etch rate, trap density, breakage plane etc.

(d) The temperating of the crusible and the pull rate.

8. $E_g\,(T) = 1.17 - \dfrac{4.73\text{x}10^{-4}\,T^2}{(T+636)}$ for Si

$\therefore$ E_g (100 K) = 1.163 eV , and E_g (600 K) = 1.032 eV

$E_g(T) = 1.519 - \dfrac{5.405\text{x}10^{-4}\,T^2}{(T+204)}$ for GaAs

$\therefore E_g$(100 K) = 1.501 eV, and E_g (600 K) = 1.277 eV .

9. The density of holes in the valence band is given by integrating the product $N(E)[1\text{-}F(E)]\mathrm{d}E$ from top of the valence band (E_V taken to be E = 0) to the bottom of the valence band E_{bottom}:

$$p = \int_0^{E_{bottom}} N(E)[1 - F(E)]\mathrm{d}E \qquad (1)$$

where $1 - F(E) = 1 - \left\{1/\left[1 + e^{(E-E_F)/kT}\right]\right\} = \left[1 + e^{(E-E_F)/kT}\right]^{-1}$

If $E_F - E >> kT$ then

$$1 - F(E) \sim \exp\left[-\left(E_F - E\right)/kT\right] \qquad (2)$$

Then from Appendix H and , Eqs. 1 and 2 we obtain

$$p = 4\pi[2m_p / h^2]^{3/2} \int_0^{E_{bottom}} E^{1/2} \exp\,[-(E_F - E) / kT\,]\mathrm{d}E \qquad (3)$$

Let $x \mapsto E / kT$, and let $E_{bottom} = -\infty$, Eq. 3 becomes

$$p = 4\pi(2m_p / h^2)^{3/2}\,(k\mathrm{T})^{3/2} \exp\,[-(E_F / kT)] \int_0^{-\infty} x^{1/2} e^x \mathrm{d}x$$

where the integral on the right is of the standard form and equals $\sqrt{\pi}\ / 2$.

$$\therefore\ p = 2[2\pi m_p\, kT / h^2]^{3/2} \exp\,[-(E_F / kT)]$$

By referring to the top of the valence band as E_V instead of E = 0 we have,

$$p = 2(2\pi m_p\, kT / h^2)^{3/2} \exp\,[-(E_F - E_V) / kT\,]$$

or $$p = N_V \exp\,[-(E_F - E_V) / kT\,]$$

where $N_V = 2\,(2\pi m_p\, kT / h^2)^3$.

10. From Eq. 18

$$N_V = 2(2\pi m_p\, kT / h^2)^{3/2}$$

The effective mass of holes in Si is

$$m_p = (N_V / 2)^{2/3}\,(\,h^2 / 2\pi kT\,)$$

$$= \left(\frac{2.66\times10^{19}\times10^6\ \mathrm{m}^{-3}}{2}\right)^{2/3} \frac{\left(6.625\times10^{-34}\right)^2}{2\pi\left(1.38\times10^{-23}\right)(300)}$$

$= 9.4\times10^{-31}$ kg = 1.03 m_0.

Similarly, we have for GaAs

$m_p = 3.9\times10^{-31}$ kg = 0.43 m_0.

11. Using Eq. 19

$$E_i = (E_C + E_V)/2 + (kT/2)\ln(N_V/N_C)$$

$$= (E_C + E_V)/2 + (3kT/4)\ln\left[(m_p/m_n)(6)^{2/3}\right] \qquad (1)$$

At 77 K

$E_i = (1.16/2) + (3 \times 1.38 \times 10^{-23} T) / (4 \times 1.6 \times 10^{-19}) \ln(1.0/0.62)$

$= 0.58 + 3.29 \times 10^{-5}\ T = 0.58 + 2.54 \times 10^{-3} = 0.583$ eV.

At 300 K

$E_i = (1.12/2) + (3.29 \times 10^{-5})(300) = 0.56 + 0.009 = 0.569$ eV.

At 373 K

$E_i = (1.09/2) + (3.29 \times 10^{-5})(373) = 0.545 + 0.012 = 0.557$ eV.

Because the second term on the right-hand side of the Eq.1 is much smaller compared to the first term, over the above temperature range, it is reasonable to assume that E_i is in the center of the forbidden gap.

12. $$KE = \left.\frac{\int_{E_C}^{E_{top}} (E - E_C)\sqrt{E - E_C}\ e^{-(E-E_F)/kT}\,dE}{\int_{E_C}^{E_{top}} \sqrt{E - E_C}\, e^{-(E-E_F)/kT}\,dE}\right|_{x=(E-E_C)}$$

$$= kT\frac{\int_0^\infty x^{3/2} e^{-x}\,dx}{\int_0^\infty x^{1/2} e^{-x}\,dx} = kT\frac{\Gamma\left(\frac{5}{2}\right)}{\Gamma\left(\frac{3}{2}\right)} = kT\frac{1.5\times 0.5\times\sqrt{\pi}}{0.5\sqrt{\pi}}$$

$$= \frac{3}{2}kT.$$

13. (a) $p = mv = 9.109 \times 10^{-31} \times 10^5 = 9.109 \times 10^{-26}$ kg–m/s

$$\lambda = \frac{h}{p} = \frac{6.626\times10^{-34}}{9.109\times10^{-26}} = 7.27 \times 10^{-9} \text{ m} = 72.7 \text{ Å}$$

(b) $$\lambda_n = \frac{m_0}{m_p}\lambda = \frac{1}{0.063}\times 72.7 = 1154 \text{ Å}.$$

14. From Fig. 22 when $n_i = 10^{15}$ cm^{-3}, the corresponding temperature is $1000 / T = 1.8$.

So that $T = 1000/1.8 = 555$ K or 282

15. From $\quad E_c - E_F = kT \ln [N_C / (N_D - N_A)]$

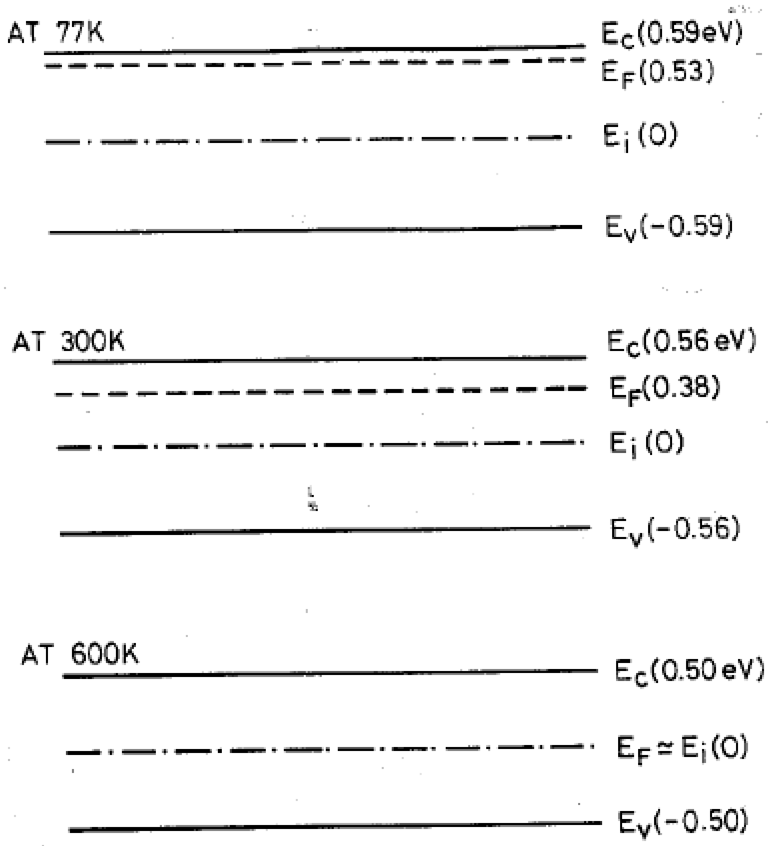
which can be rewritten as $\quad N_D - N_A = N_C \exp[-(E_C - E_F) / kT]$

Then $\quad N_D - N_A = 2.86 \times 10^{19} \exp(-0.20 / 0.0259) = 1.26 \times 10^{16}$ cm^{-3}

or $\quad N_D = 1.26 \times 10^{16} + N_A = 2.26 \times 10^{16}$ cm^{-3}

A compensated semiconductor can be fabricated to provide a specific Fermi energy level.

16. From Fig. 28a we can draw the following energy-band diagrams:



17. (a) The ionization energy for boron in Si is 0.045 eV. At 300 K, all boron impurities are ionized. Thus $p_p = N_A = 10^{15}$ cm^{-3}

$$n_p = n_i^2 / n_A = (9.65 \times 10^9)^2 / 10^{15} = 9.3 \times 10^4 \text{ cm}^{-3}.$$

The Fermi level measured from the top of the valence band is given by:

$$E_F - E_V = kT \ln(N_V/N_D) = 0.0259 \ln (2.66 \times 10^{19} / 10^{15}) = 0.26 \text{ eV}$$

(b) The boron atoms compensate the arsenic atoms; we have

$$p_p = N_A - N_D = 3 \times 10^{16} - 2.9 \times 10^{16} = 10^{15} \text{ cm}^{-3}$$

Since p_p is the same as given in (a), the values for n_p and E_F are the same as in (a). However, the mobilities and resistivities for these two samples are different.

18. Since $N_D >> n_i$, we can approximate $n_0 = N_D$ and

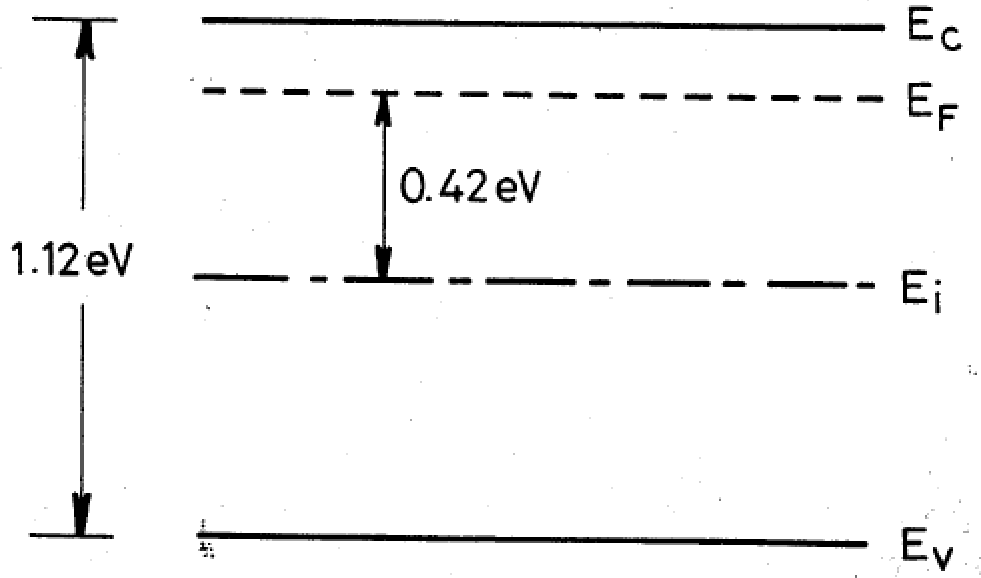
$$p_0 = n_i^2 / n_0 = 9.3 \times 10^{19} / 10^{17} = 9.3 \times 10^2 \text{ cm}^{-3}$$

From $n_0 = n_i \exp\left(\dfrac{E_F - E_i}{kT}\right)$,

we have

$$E_F - E_i = kT \ln (n_0 / n_i) = 0.0259 \ln (10^{17} / 9.65 \times 10^9) = 0.42 \text{ eV}$$

The resulting flat band diagram is :



19. Assuming complete ionization, the Fermi level measured from the intrinsic Fermi level is 0.35 eV for 10^{15} cm^{-3}, 0.45 eV for 10^{17} cm^{-3}, and 0.54 eV for 10^{19} cm^{-3}.

The number of electrons that are ionized is given by

$$n \cong N_D[1 - F(E_D)] = N_D / [1 + e^{-(E_D - E_F)/kT}]$$

Using the Fermi levels given above, we obtain the number of ionized donors as

$$n = 10^{15}\ \text{cm}^{-3} \qquad \text{for } N_D = 10^{15}\ \text{cm}^{-3}$$
$$n = 0.93 \times 10^{17}\ \text{cm}^{-3} \qquad \text{for } N_D = 10^{17}\ \text{cm}^{-3}$$
$$n = 0.27 \times 10^{19}\ \text{cm}^{-3} \qquad \text{for } N_D = 10^{19}\ \text{cm}^{-3}$$

Therefore, the assumption of complete ionization is valid only for the case of 10^{15} cm^{-3}.

20. $$N_D^+ = \frac{10^{16}}{1 + e^{-(E_D - E_F)/kT}} = \frac{10^{16}}{1 + e^{-0.135}}$$

$$= \frac{10^{16}}{1 + \frac{1}{1.145}} = 5.33 \times 10^{15}\ \text{cm}^{-3}$$

The neutral donor = $10^{16} - 5.33 \times 10^{15}$ cm^{-3} = 4.67×10^{15} cm^{-3}

__The ratio of $\frac{N_D^{O}}{N_D^{+}} = \frac{4.76}{5.33} = 0.876$.

CHAPTER 3

1. (a) For intrinsic Si, $\mu_n = 1450$, $\mu_p = 505$, and $n = p = n_i = 9.65\times10^9$

   We have $\rho = \dfrac{1}{qn\mu_n + qp\mu_p} = \dfrac{1}{qn_i(\mu_n + \mu_p)} = 3.31\times10^5\ \ \Omega\text{-cm}$

   (b) Similarly for GaAs, $\mu_n = 9200$, $\mu_p = 320$, and $n = p = n_i = 2.25\times10^6$

   We have $\rho = \dfrac{1}{qn\mu_n + qp\mu_p} = \dfrac{1}{qn_i(\mu_n + \mu_p)} = 2.92\times10^8\ \ \Omega\text{-cm.}$

2. For lattice scattering, $\mu_n \propto T^{-3/2}$

$$\text{T = 200 K, } \mu_n = 1300\times\frac{200^{-3/2}}{300^{-3/2}} = 2388\ \ \text{cm}^2/\text{V-s}$$

$$\text{T = 400 K, } \mu_n = 1300\times\frac{400^{-3/2}}{300^{-3/2}} = 844\ \ \text{cm}^2/\text{V-s.}$$

3. Since $\dfrac{1}{\mu} = \dfrac{1}{\mu_1} + \dfrac{1}{\mu_2}$

   $\therefore\ \ \dfrac{1}{\mu} = \dfrac{1}{250} + \dfrac{1}{500}$ $\qquad\qquad \mu = 167\ \ \text{cm}^2/\text{V-s.}$

4. (a) $p = 5\times10^{15}\ \ \text{cm}^{-3}$, $n = n_i^2/p = (9.65\times10^9)^2/5\times10^{15} = 1.86\times10^4\ \ \text{cm}^{-3}$

   $\mu_p = 410\ \ \text{cm}^2/\text{V-s}$, $\mu_n = 1300\ \ \text{cm}^2/\text{V-s}$

   $\rho = \dfrac{1}{q\mu_n n + q\mu_p p} \approx \dfrac{1}{q\mu_p p} = 3\ \ \Omega\text{-cm}$

   (b) $p = N_A - N_D = 2\times10^{16} - 1.5\times10^{16} = 5\times10^{15}\ \ \text{cm}^{-3}$, $n = 1.86\times10^4\ \ \text{cm}^{-3}$

   $\mu_p = \mu_p\,(N_A + N_D) = \mu_p\,(3.5\times10^{16}) = 290\ \ \text{cm}^2/\text{V-s}$,

   $\mu_n = \mu_n\,(N_A + N_D) = 1000\ \ \text{cm}^2/\text{V-s}$

   $\rho = \dfrac{1}{q\mu_n n + q\mu_p p} \approx \dfrac{1}{q\mu_p p} = 4.3\ \ \Omega\text{-cm}$

(c) $p = N_A$ (Boron) $- N_D + N_A$ (Gallium) $= 5\times10^{15}$ cm^{-3}, $n = 1.86\times10^{4}$ cm^{-3}

$\mu_p = \mu_p\ (N_A + N_D + N_A) = \mu_p\ (2.05\times10^{17}) = 150$ cm^2/V-s,

$\mu_n = \mu_n\ (N_A + N_D + N_A) = 520$ cm^2/V-s

$\rho = 8.3$ Ω-cm.

5. Assume $N_D - N_A >> n_i$, the conductivity is given by

$$\sigma \approx qn\mu_n = q\mu_n(N_D - N_A)$$

We have that

$$16 = (1.6\times10^{-19})\mu_n(N_D - 10^{17})$$

Since mobility is a function of the ionized impurity concentration, we can use Fig. 3 along with trial and error to determine μ_n and N_D . For example, if we choose $N_D = 2\times10^{17}$, then $N_I = N_D^+ + N_A^- = 3\times10^{17}$, so that $\mu_n \approx 510$ cm^2/V-s which gives $\sigma = 8.16$.

Further trial and error yields

$$N_D \approx 3.5\times10^{17}\ \text{cm}^{-3}$$

and

$$\mu_n \approx 400\ \text{cm}^2/\text{V-s}$$

which gives

$$\sigma \approx 16\ (\Omega\text{-cm})^{-1}.$$

6. $\sigma = q(\mu_n n + \mu_p p) = q\mu_p(bn + n_i^2/n)$

From the condition $d\sigma/dn = 0$, we obtain

$$n = n_i/\sqrt{b}$$

Therefore

$$\frac{\rho_m}{\rho_i} = \frac{\dfrac{1}{q\mu_p(bn_i/\sqrt{b}+\sqrt{b}n_i)}}{\dfrac{1}{q\mu_p n_i(b+1)}} = \frac{b+1}{2\sqrt{b}}.$$

7. At the limit when d >> s, $CF = \dfrac{\pi}{\ln 2} = 4.53$. Then from Eq. 16

$$\rho = \frac{V}{I} \times W \times CF = \frac{10\times10^{-3}}{1\times10^{-3}} \times 50\times10^{-4} \times 4.53 = 0.226 \quad \Omega\text{-cm}$$

From Fig. 6, $CF = 4.2$ ($d/s = 10$); using the $a/d = 1$ curve we obtain

$$V = \rho \cdot I/(W \cdot CF) = \frac{0.226\times10^{-3}}{50\times10^{-4}\times4.2} = 10.78 \quad \text{mV}.$$

8. Hall coefficient,

$$R_H = \frac{V_H A}{IB_z W} = \frac{10\times10^{-3}\times1.6\times10^{-3}}{2.5\times10^{-3}\times(30\times10^{-9}\times10^{4})\times0.05} = 426.7 \quad \text{cm}^3/\text{C}$$

Since the sign of R_H is positive, the carriers are holes. From Eq. 22

$$p = \frac{1}{qR_H} = \frac{1}{1.6\times10^{-19}\times426.7} = 1.46\times10^{16} \quad \text{cm}^{-3}$$

Assuming $N_A \approx p$, from Fig. 7 we obtain $\rho = 1.1$ Ω-cm

The mobility μ_p is given by Eq. 15b

$$\mu_p = \frac{1}{qp\rho} = \frac{1}{1.6\times10^{-19}\times1.46\times10^{16}\times1.1} = 380 \quad \text{cm}^2/\text{V-s}.$$

9. Since $R \propto \rho$ and $\rho = \dfrac{1}{qn\mu_n + qp\mu_p}$, hence $R \propto \dfrac{1}{n\mu_n + p\mu_p}$

From Einstein relation $D \propto \mu$

$$\mu_n/\mu_p = D_n/D_p = 50$$

$$\frac{R_1}{0.5R_1} = \frac{\dfrac{1}{N_D \mu_n}}{\dfrac{1}{N_D\mu_n + N_A\mu_p}}$$

We have $N_A = 50\ N_D$.

10. The electric potential ϕ is related to electron potential energy by the charge $(-q)$

$$\phi = +\frac{1}{q}(E_F - E_i)$$

The electric field for the one-dimensional situation is defined as

$$\mathcal{E}(x) = -\frac{d\phi}{dx} = \frac{1}{q}\frac{dE_i}{dx}$$

$$n = n_i \exp\left(\frac{E_F - E_i}{kT}\right) = N_D(x)$$

Hence

$$E_F - E_i = kT \ln\left(\frac{N_D(x)}{n_i}\right)$$

$$\mathcal{E}(x) = -\left(\frac{kT}{q}\right)\frac{1}{N_D(x)}\frac{dN_D(x)}{dx}\ .$$

11. (a) From Eq. 31, $J_n = 0$ and

$$\mathcal{E}(x) = -\frac{D_n}{\mu_n}\frac{dn/dx}{n} = -\frac{kT}{q}\frac{N_0(-a)e^{-ax}}{N_0 e^{-ax}} = +\frac{kT}{q}a$$

(b) $\mathcal{E}(x) = 0.0259\ (10^4) = 259$ V/cm.

12. At thermal and electric equilibria,

$$J_n = q\mu_n n(x)\mathcal{E} + qD_n\frac{dn(x)}{dx} = 0$$

$$\mathcal{E}(x) = -\frac{D_n}{\mu_n}\frac{1}{n(x)}\frac{dn(x)}{dx} = -\frac{D_n}{\mu_n}\frac{1}{N_0 + (N_L - N_0)(x/\ L)}\frac{N_L - N_0}{L}$$

$$= -\frac{D_n}{\mu_n}\frac{N_L - N_0}{LN_0 + (N_L - N_0)x}$$

$$V = \int_0^L -\frac{D_n}{\mu}\frac{N_L - N_0}{LN_0 + (N_L - N_0)x} = -\frac{D_n}{\mu_n}\ln\frac{N_L}{N_0}.$$

13. $\Delta n = \Delta p = \tau_p G_L = 10\times10^{-6}\times10^{16} = 10^{11}\ \ \text{cm}^{-3}$

$$n = n_{no} + \Delta n = N_D + \Delta n = 10^{15} + 10^{11} \approx 10^{15}\quad \text{cm}^{-3}$$

$$p = \frac{n_i^2}{N_D} + \Delta p = \frac{(9.65\times10^9)^2}{10^{15}} + 10^{11} \approx 10^{11}\quad \text{cm}^{-3}.$$

14. (a) $\tau_p \approx \dfrac{1}{\sigma_p v_{th} N_t} = \dfrac{1}{5\times10^{-15}\times10^7\times2\times10^{15}} = 10^{-8}\ \ \text{s}$

$$L_p = \sqrt{D_p \tau_p} = \sqrt{9\times10^{-8}} = 3\times10^{-4}\ \ \text{cm}$$

$$S_{lr} = v_{th}\sigma_s N_{sts} = 10^7\times2\times10^{-16}\times10^{10} = 20\ \ \text{cm/s}$$

(b) The hole concentration at the surface is given by Eq. 67

$$\begin{aligned} p_n(0) &= p_{no} + \tau_p G_L\left(1 - \frac{\tau_p S_{lr}}{L_p + \tau_p S_{lr}}\right) \\ &= \frac{(9.65\times10^9)^2}{2\times10^{16}} + 10^{-8}\times10^{17}\left(1 - \frac{10^{-8}\times20}{3\times10^{-4} + 10^{-8}\times20}\right) \\ &\approx 10^9\ \ \text{cm}^{-3}. \end{aligned}$$

15. $\sigma = qn\mu_n + qp\mu_p$

Before illumination

$$n_n = n_{no},\ \ p_n = p_{no}$$

After illumination

$$n_n = n_{no} + \Delta n = n_{no} + \tau_p G,$$

$$p_n = p_{no} + \Delta p = p_{no} + \tau_p G$$

$$\begin{aligned} \Delta\sigma &= [q\mu_n(n_{no} + \Delta n) + q\mu_p(p_{no} + \Delta p)] - (q\mu_n n_{no} + q\mu_p p_{no}) \\ &= q(\mu_n + \mu_p)\tau_p G. \end{aligned}$$

16. (a) $J_{p,\text{diff}} = -qD_p \dfrac{dp}{dx}$

$$= -1.6\times10^{-19}\times12\times\frac{1}{12\times10^{-4}}\times10^{15}\exp(-x/12)$$

$$= 1.6\exp(-x/12) \quad \text{A/cm}^2$$

(b) $J_{n,\text{drift}} = J_{total} - J_{p,\text{diff}}$

$$= 4.8 - 1.6\exp(-x/12) \quad \text{A/cm}^2$$

(c) $\because J_{n,\text{drift}} = qn\mu_n\mathcal{E}$

$$\therefore 4.8 - 1.6\exp(-x/12) = 1.6\times10^{-19}\times10^{16}\times1000\times\mathcal{E}$$

$$\mathcal{E} = 3 - \exp(-x/12) \quad \text{V/cm.}$$

17. For $\mathcal{E} = 0$ we have

$$\frac{\partial p}{\partial t} = -\frac{p_n - p_{no}}{\tau_p} + D_p\frac{\partial^2 p_n}{\partial x^2} = 0$$

at steady state, the boundary conditions are $p_n\ (x = 0) = p_n\ (0)$ and $p_n\ (x = W) = p_{no}$.

Therefore

$$p_n(x) = p_{no} + [p_n(0) - p_{no}]\left[\frac{\sinh\left(\dfrac{W-x}{L_p}\right)}{\sinh\left(\dfrac{W}{L_p}\right)}\right]$$

$$J_p(x=0) = -qD_p\left.\frac{\partial p_n}{\partial x}\right|_{x=0} = q[p_n(0) - p_{no}]\frac{D_p}{L_p}\coth\left(\frac{W}{L_p}\right)$$

$$J_p(x=W) = -qD_p\left.\frac{\partial p_n}{\partial x}\right|_{x=W} = q[p_n(0) - p_{no}]\frac{D_p}{L_p}\frac{1}{\sinh\left(\dfrac{W}{L_p}\right)}.$$

18. The portion of injection current that reaches the opposite surface by diffusion is

given by

$$\alpha_0 = \frac{J_p(W)}{J_p(0)} = \frac{1}{\cosh(W/L_p)}$$

$$L_p \equiv \sqrt{D_p \tau_p} = \sqrt{50\times 50\times 10^{-6}} = 5\times 10^{-2} \text{ cm}$$

$$\therefore \alpha_0 = \frac{1}{\cosh(10^{-2}/5\times 10^{-2})} = 0.98$$

Therefore, 98% of the injected current can reach the opposite surface.

19. In steady state, the recombination rate at the surface and in the bulk is equal

$$\frac{\Delta p_{n,\,\text{bulk}}}{\tau_{p\,,\,\text{bulk}}} = \frac{\Delta p_{n,\,\text{surface}}}{\tau_{p,\,\text{surface}}}$$

so that the excess minority carrier concentration at the surface

$$\Delta p_{n,\ \text{surface}} = 10^{14}\cdot\frac{10^{-7}}{10^{-6}} = 10^{13}\ \text{cm}^{-3}$$

The generation rate can be determined from the steady-state conditions in the bulk

$$G = \frac{10^{14}}{10^{-6}} = 10^{20}\ \text{cm}^{-3}\text{s}^{-1}$$

From Eq. 62, we can write

$$D_p\frac{\partial^2 \Delta p}{\partial x^2} + G - \frac{\Delta p}{\tau_p} = 0$$

The boundary conditions are $\Delta p(x = \infty) = 10^{14}$ cm^{-3} and $\Delta p(x = 0) = 10^{13}$ cm^{-3}

Hence $\Delta p(x) = 10^{14}(1 - 0.9e^{-x/L_p})$

where $L_p = \sqrt{10\cdot 10^{-6}} = 31.6\ \mu\text{m}$.

20. The potential barrier height

$$\phi_B = \phi_m - \chi = 4.2 - 4.0 = 0.2 \text{ volts.}$$

21. The number of electrons occupying the energy level between E and $E+dE$ is

$dn = N(E)F(E)dE$

where $N(E)$ is the density-of-state function, and $F(E)$ is Fermi-Dirac distribution function. Since only electrons with an energy greater than $E_F + q\phi_m$ and having a velocity component normal to the surface can escape the solid, the thermionic current density is

$$J = \int q v_x = \int_{E_F+q\phi_m}^{\infty} \frac{4\pi(2m)^{3/2}}{h^3} v_x E^{1/2} e^{-(E-E_F)/kT} dE$$

where v_x is the component of velocity normal to the surface of the metal. Since the energy-momentum relationship

$$E = \frac{P^2}{2m} = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2)$$

Differentiation leads to $dE = \frac{PdP}{m}$

By changing the momentum component to rectangular coordinates, $4\pi P^2 dP = dp_x dp_y dp_z$

Hence
$$J = \frac{2q}{mh^3}\int_{p_{x0}}^{\infty}\int_{p_y=-\infty}^{\infty}\int_{p_z=-\infty}^{\infty} p_x e^{-(p_x^2+p_y^2+p_z^2-2mE_f)/2mkT} dp_x dp_y dp_z$$
$$= \frac{2q}{mh^3}\int_{p_{x0}}^{\infty} e^{-(p_x^2-2mE_f)/2mkT} p_x dp_x \int_{-\infty}^{\infty} e^{-p_y^2/2mkT} dp_y \int_{-\infty}^{\infty} e^{-p_z^2/2mkT} dp_z$$

where $p_{x0}^2 = 2m(E_F + q\phi_m)$.

Since $\int_{-\infty}^{\infty} e^{-ax^2} dx = \left(\frac{\pi}{a}\right)^{1/2}$, the last two integrals yield $(2\pi mkT)^{1/2}$.

The first integral is evaluated by setting $\frac{p_x^2 - 2mE_F}{2mkT} = u$.

Therefore we have $du = \frac{p_x dp_x}{mkT}$

The lower limit of the first integral can be written as

$$\frac{2m(E_F + q\phi_m) - 2mE_F}{2mkT} = \frac{q\phi_m}{kT}$$

so that the first integral becomes $mkT\int_{q\phi_m/kt}^{\infty} e^{-u} du = mkT\, e^{-q\phi_m/kT}$

Hence $J = \frac{4\pi qmk^2}{h^3} T^2 e^{-q\phi_m/kT} = A^* T^2 \exp\left(\frac{-q\phi_m}{kT}\right).$

22. Equation 79 is the tunneling probability

$$\beta = \sqrt{\frac{2m_n(qV_0 - E)}{\hbar^2}} = \sqrt{\frac{2(9.11\times10^{-31})(20-2)(1.6\times10^{-19})}{(1.054\times10^{-34})^2}} = 2.17\times10^{10}\,\mathrm{m}^{-1}$$

$$T = \left\{1 + \frac{\left[20\times\sinh(2.17\times10^{0}\times3\times10^{-10}\right]^2}{4\times2\times(20-2)}\right\}^{-1} = 3.19\times10^{-6}.$$

23. Equation 79 is the tunneling probability

$$\beta = \sqrt{\frac{2m_n(qV_0 - E)}{\hbar^2}} = \sqrt{\frac{2(9.11\times10^{-31})(6-2.2)(1.6\times10^{-19})}{(1.054\times10^{-34})^2}} = 9.99\times10^{9}\,\mathrm{m}^{-1}$$

$$T(10^{-10}) = \left\{1 + \frac{\left[6\times\sinh(9.99\times10^{9}\times10^{-10})\right]^2}{4\times2.2\times(6-2.2)}\right\}^{-1} = 0.403$$

$$T(10^{-9}) = \left\{1 + \frac{\left[6\times\sinh\left(9.99\times10^{9}\times10^{-9}\right)\right]^2}{4\times2.2\times(6-2.2)}\right\}^{-1} = 7.8\times10^{-9}.$$

24. From Fig. 22

As $\mathcal{E} = 10^3$ V/s

$$v_d \approx 1.3\times10^6 \text{ cm/s (Si) and } v_d \approx 8.7\times10^6 \text{ cm/s (GaAs)}$$

$$t \approx 77 \text{ ps (Si) and } t \approx 11.5 \text{ ps (GaAs)}$$

As $\mathcal{E} = 5\times10^4$ V/s

$$v_d \approx 10^7 \text{ cm/s (Si) and } v_d \approx 8.2\times10^6 \text{ cm/s (GaAs)}$$

$$t \approx 10 \text{ ps (Si) and } t \approx 12.2 \text{ ps (GaAs).}$$

25. Thermal velocity $v_{th} = \sqrt{\dfrac{2E_{th}}{m_0}} = \sqrt{\dfrac{2kT}{m_0}}$

$$= \sqrt{\frac{2\times1.38\times10^{-23}\times300}{9.1\times10^{-31}}}$$

$$= 9.5\times10^4 \text{ m/s} = 9.5\times10^6 \text{ cm/s}$$

For electric field of 100 v/cm, drift velocity

$$v_d = \mu_n \mathcal{E} = 1350\times100 = 1.35\times10^5 \text{ cm/s} << v_{th}$$

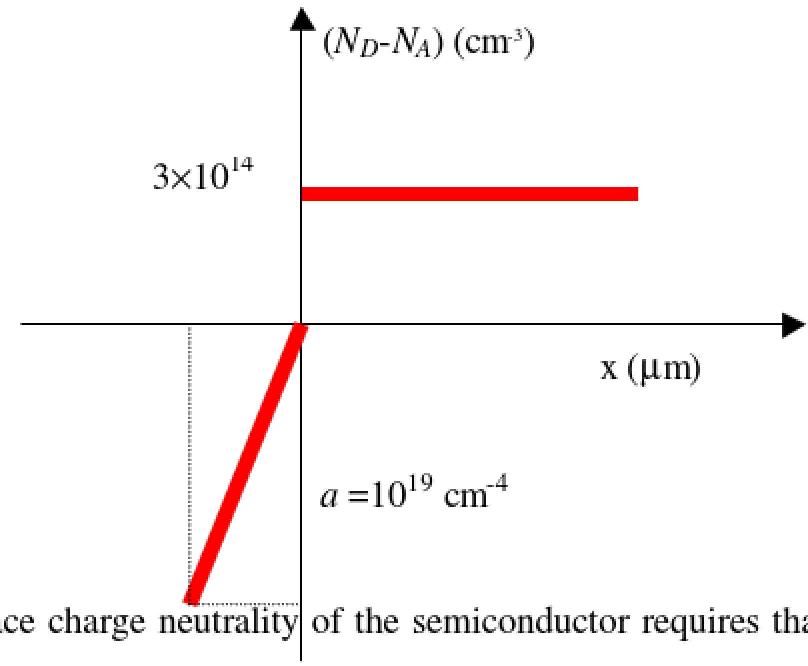
For electric field of 10^4 V/cm.

$$\mu_n \mathcal{E} = 1350\times10^4 = 1.35\times10^7 \text{ cm/s} \approx v_{th}.$$

The value is comparable to the thermal velocity, the linear relationship between drift velocity and the electric field is not valid.

CHAPTER 4

1. The impurity profile is,



The overall space charge neutrality of the semiconductor requires that the total negative space charge per unit area in the p-side must equal the total positive space charge per unit area in the n-side, thus we can obtain the depletion layer width in the n-side region:

$$\frac{0.8\times8\times10^{14}}{2} = W_n\times3\times10^{14}$$

Hence, the n-side depletion layer width is:

$$W_n = 1.067\,\mu\text{m}$$

The total depletion layer width is 1.867 μm.

We use the Poisson's equation for calculation of the electric field $\mathcal{E}(x)$.

In the n-side region,

$$\frac{d\mathcal{E}}{dx} = \frac{q}{\varepsilon_s}N_D \Rightarrow \mathcal{E}(x_n) = \frac{q}{\varepsilon_s}N_D x + K$$

$$\mathcal{E}(x_n = 1.067\,\mu\text{m}) = 0 \Rightarrow K = -\frac{q}{\varepsilon_s}N_D\times1.067\times10^{-4}$$

$$\therefore \mathcal{E}(x_n) = \frac{q}{\varepsilon_s}\times3\times10^{14}(x - 1.067\times10^{-4})$$

$$\mathcal{E}_{max} = \mathcal{E}(x_n = 0) = -4.86\times10^{3}\ \text{V/cm}$$

In the p-side region, the electrical field is:

$$\frac{d\mathcal{E}}{dx} = \frac{q}{\varepsilon_s} N_A \Rightarrow \mathcal{E}(x_p) = \frac{q}{2\varepsilon_s} \times ax^2 + K'$$

$$\mathcal{E}(x_p = -0.8\mu\text{m}) = 0 \Rightarrow K' = -\frac{q}{2\varepsilon_s} \times a \times \left(0.8 \times 10^{-4}\right)^2$$

$$\therefore \mathcal{E}(x_p) = \frac{q}{2\varepsilon_s} \times a \times \left[x^2 - \left(0.8 \times 10^{-4}\right)^2\right]$$

$$\mathcal{E}_{max} = \mathcal{E}(x_p = 0) = -4.86 \times 10^3 \text{ V/cm}$$

The built-in potential is:

$$V_{bi} = -\int_{-x_p}^{x_n} \mathcal{E}(x)dx = -\int_{-x_p}^{0} \mathcal{E}(x)dx\bigg|_{p-side} - \int_{0}^{x_n} \mathcal{E}(x)dx\bigg|_{n-side} = 0.52 \text{ V}.$$

2. From $V_{bi} = -\int \mathcal{E}(x)dx$, the potential distribution can be obtained

With zero potential in the neutral p-region as a reference, the potential in the p-side depletion region is

$$V_p(x) = -\int_0^x \mathcal{E}(x)dx = -\int_0^x \frac{q}{2\varepsilon_s} \times a \times \left[x^2 - \left(0.8 \times 10^{-4}\right)^2\right]dx = -\frac{qa}{2\varepsilon_s}\left[\frac{1}{3}x^3 - \left(0.8 \times 10^{-4}\right)^2 x - \frac{2}{3}\left(0.8 \times 10^{-4}\right.\right.$$

$$= -7.596 \times 10^{11} \times \left[\frac{1}{3}x^3 - \left(0.8 \times 10^{-4}\right)^2 x - \frac{2}{3}\left(0.8 \times 10^{-4}\right)^3\right]$$

With the condition $V_p(0)=V_n(0)$, the potential in the n-region is

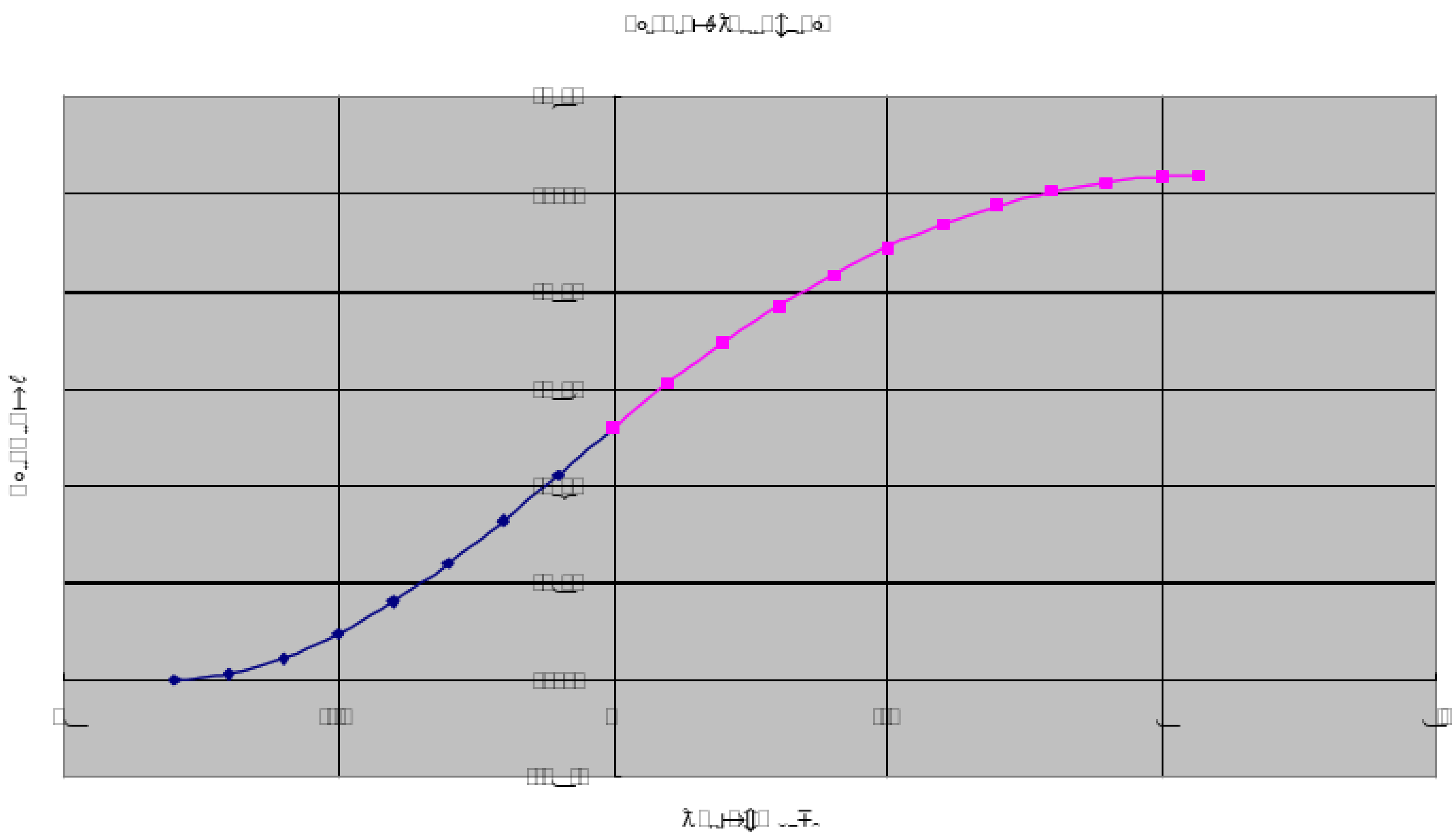
$$V_n(x) = -\frac{q}{\varepsilon_s} \times 3 \times 10^{14}\left(\frac{1}{2}x^2 - 1.067 \times 10^{-4} x + \frac{0.8^3}{9} \times 10^{-7}\right)$$

$$= -4.56 \times 10^7 \times \left(\frac{1}{2}x^2 - 1.067 \times 10^{-4} x - \frac{0.8^3}{9} \times 10^{-7}\right)$$

The potential distribution is

Distance	p-region	n-region
[illegible]	[illegible]	
[illegible]	[illegible]	
[illegible]	[illegible]	
[illegible]	[illegible]	
[illegible]	[illegible]	
[illegible]	[illegible]	
[illegible]	[illegible]	
[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]

[illegible]		[illegible]
[illegible]		[illegible]
[illegible]		[illegible]
[illegible]		[illegible]
[illegible]		[illegible]
[illegible]		[illegible]
[illegible]		[illegible]
[illegible]		[illegible]
[illegible]		[illegible]
[illegible]		[illegible]
[illegible]		[illegible]



3. The intrinsic carriers density in Si at different temperatures can be obtained by using Fig.22 in Chapter 2 :

Temperature (K)	Intrinsic carrier density (n_i)
250	1.50×10^{8}
300	9.65×10^{9}
350	2.00×10^{11}
400	8.50×10^{12}
450	9.00×10^{13}
500	2.20×10^{14}

The V_{bi} can be obtained by using Eq. 12, and the results are listed in the following table.

T	*ni*	*Vbi (V)*
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

Thus, the built-in potential is decreased as the temperature is increased.

The depletion layer width and the maximum field at 300 K are

$$W = \sqrt{\frac{2\varepsilon_s V_{bi}}{qN_D}} = \sqrt{\frac{2\times 11.9\times 8.85\times 10^{-14}\times 0.717}{1.6\times 10^{-19}\times 10^{15}}} = 0.9715\ \mu\text{m}$$

$$\mathcal{E}_{max} = \frac{qN_D W}{\varepsilon_s} = \frac{1.6\times 10^{-19}\times 10^{15}\times 9.715\times 10^{-5}}{11.9\times 8.85\times 10^{-14}} = 1.476\times 10^{4}\ \text{V/cm.}$$

4. $\mathcal{E}_{\max} \approx \left[\frac{2qV_R}{\varepsilon_s}\left(\frac{N_A N_D}{N_A + N_D}\right)\right]^{1/2} \Rightarrow 4\times 10^5 = \left[\frac{2\times 1.6\times 10^{-19}\times 30}{11.9\times 8.85\times 10^{-14}}\left(\frac{10^{18} N_D}{10^{18} + N_D}\right)\right]^{1/2}$

$$\Rightarrow 1.755\times 10^{16} = \frac{N_D}{1+\frac{N_D}{10^{18}}}$$

We can select n-type doping concentration of $N_D = 1.755\times 10^{16}$ cm^{-3} for the junction.

5. From Eq. 12 and Eq. 35, we can obtain the $1/C^2$ versus V relationship for doping concentration of 10^{15}, 10^{16}, or 10^{17} cm^{-3}, respectively.

For $N_D=10^{15}$ cm^{-3},

$$\frac{1}{C_j^{\,2}} = \frac{2(V_{bi} - V)}{q\varepsilon_s N_B} = \frac{2\times(0.837 - V)}{1.6\times 10^{-19}\times 11.9\times 8.85\times 10^{-14}\times 10^{15}} = 1.187\times 10^{16}(0.837 - V)$$

For $N_D=10^{16}$ cm^{-3},

$$\frac{1}{C_j^{\,2}} = \frac{2(V_{bi} - V)}{q\varepsilon_s N_B} = \frac{2\times(0.896 - V)}{1.6\times 10^{-19}\times 11.9\times 8.85\times 10^{-14}\times 10^{16}} = 1.187\times 10^{15}(0.896 - V)$$

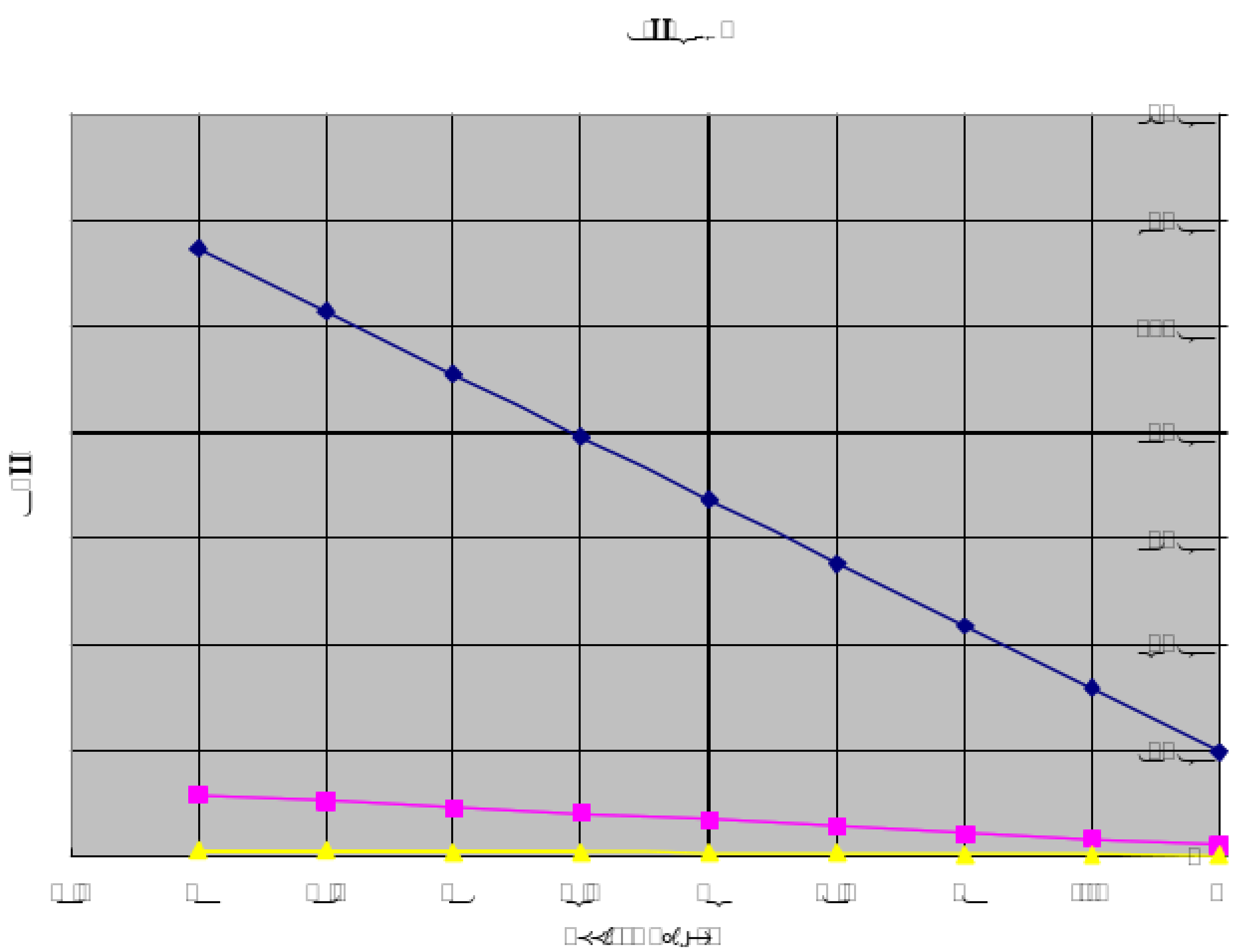
For $N_D=10^{17}$ cm^{-3},

$$\frac{1}{C_j^{\,2}} = \frac{2(V_{bi} - V)}{q\varepsilon_s N_B} = \frac{2\times(0.956 - V)}{1.6\times 10^{-19}\times 11.9\times 8.85\times 10^{-14}\times 10^{17}} = 1.187\times 10^{14}(0.956 - V)$$

When the reversed bias is applied, we summarize a table of $1/C_j^2$ vs V for various N_D values as following,

[illegible]	N_D=1E15	N_D=1E16	N_D=1E17
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

Hence, we obtain a series of curves of $1/C^2$ versus V as following,



The slopes of the curves is positive proportional to the values of the doping concentration.

The interceptions give the built-in potential of the *p*-*n* junctions.

6. The built-in potential is

$$V_{bi} = \frac{2}{3}\frac{kT}{q}\ln\left(\frac{a^2\varepsilon_s kT}{8q^2 n_i^3}\right) = \frac{2}{3}\times 0.0259\times\ln\left(\frac{10^{20}\times 10^{20}\times 11.9\times 8.85\times 10^{-14}\times 0.0259}{8\times 1.6\times 10^{-19}\times\left(9.65\times 10^{9}\right)^3}\right)$$

$= 0.5686$ V

From Eq. 38, the junction capacitance can be obtained

$$C_j = \frac{\varepsilon_s}{W} = \left[\frac{qa\varepsilon_s^{\ 2}}{12(V_{bi} - V_R)}\right]^{1/3} = \left[\frac{1.6\times 10^{-19}\times 10^{20}\times\left(11.9\times 8.85\times 10^{-14}\right)^2}{12(0.5686 - V_R)}\right]^{1/3}$$

At reverse bias of 4V, the junction capacitance is 6.866×10^{-9} F/cm^2.

7. From Eq. 35, we can obtain

$$\frac{1}{C_j^{\ 2}} = \frac{2(V_{bi} - V)}{q\varepsilon_s N_B} \Rightarrow N_D = \frac{2(V_{bi} - V_R)}{q\varepsilon_s} C_j^{\ 2}$$

$$\because V_R >> V_{bi} \Rightarrow N_D \cong \frac{2(V_R)}{q\varepsilon_s} C_j^{\ 2} = \frac{2\times 4}{1.6\times 10^{-19}\times 11.9\times 8.85\times 10^{-14}}\times (0.85\times 10^{-8})^2$$

$$\Rightarrow N_d = 3.43\times 10^{15}\ \text{cm}^{-3}$$

We can select the n-type doping concentration of 3.43×10^{15}cm^{-3}.

8. From Eq. 56,

$$G = -U = \left[\frac{\sigma_p \sigma_n \upsilon_{th} N_t}{\sigma_n \exp\left(\frac{E_t - E_i}{kT}\right) + \sigma_p \exp\left(\frac{E_i - E_t}{kT}\right)}\right] n_i$$

$$= \left[\frac{10^{-15}\times 10^{-15}\times 10^{7}\times 10^{15}}{10^{-15}\exp\left(\frac{0.02}{0.0259}\right) + 10^{-15}\exp\left(\frac{-0.02}{0.0259}\right)}\right]\times 9.65\times 10^{9} = 3.89\times 10^{16}$$

and

$$W=\sqrt{\frac{2\varepsilon_s\left(V_{bi}+V\right)}{qN_A}}=\sqrt{\frac{2\times 11.9\times 8.85\times 10^{-14}\times(0.717+0.5)}{1.6\times 10^{-19}\times 10^{15}}}=12.66\times 10^{-5}\ \text{cm}=1.266\ \mu\text{m}$$

Thus

$$J_{gen}=qGW=1.6\times 10^{19}\times 3.89\times 10^{16}\times 12.66\times 10^{-5}=7.879\times 10^{-7}\ \text{A/cm}^2.$$

9. From Eq. 49, and $p_{no}=\dfrac{n_i^{\,2}}{N_D}$

We can obtain the hole concentration at the edge of the space charge region,

$$p_n=\frac{n_i^{\,2}}{N_D}e^{\left(\frac{0.8}{0.0259}\right)}=\frac{\left(9.65\times 10^{9}\right)^2}{10^{16}}e^{\left(\frac{0.8}{0.0259}\right)}=2.42\times 10^{17}\ \ \text{cm}^{-3}.$$

10. $J=J_p\left(x_n\right)+J_n\left(-x_p\right)=J_s\left(e^{qV/kT}-1\right)$

$$\Rightarrow \frac{J}{J_s}=e^{\frac{V}{0.0259}}-1$$

$$\Rightarrow 0.95=e^{\frac{V}{0.0259}}-1$$

$$\Rightarrow V=0.017\ \text{V}.$$

11. The parameters are

n_i= 9.65×10^9 cm^{-3} D_n= 21 cm^2/sec

D_p=10 cm^2/sec $\tau_{p0}=\tau_{n0}$=5×10^{-7} sec

From Eq. 52 and Eq. 54

$$J_p\left(x_n\right)=\frac{qD_p p_{no}}{L_p}\left(e^{qV/kT}-1\right)=q\sqrt{\frac{D_p}{\tau_{po}}}\times\frac{n_i^{\,2}}{N_D}\times\left[e^{\left(\frac{qV_a}{kT}\right)}-1\right]$$

$$\Rightarrow$$

$$7=1.6\times 10^{-19}\times\sqrt{\frac{10}{5\times 10^{-7}}}\times\frac{\left(9.65\times 10^{9}\right)^2}{N_D}\times\left[e^{\left(\frac{0.7}{0.0259}\right)}-1\right]$$

$$\Rightarrow$$

$$N_D=5.2\times 10^{15}\ \text{cm}^{-3}$$

$$J_n(-x_p) = \frac{qD_n n_{po}}{L_n}\left(e^{qV/kT} - 1\right) = q\sqrt{\frac{D_n}{\tau_{no}}} \times \frac{n_i^{\ 2}}{N_A} \times \left[e^{\left(\frac{qV_a}{kT}\right)} - 1\right]$$

$\Rightarrow$

$$25 = 1.6 \times 10^{-19} \times \sqrt{\frac{21}{5\times 10^{-7}}} \times \frac{\left(9.65\times 10^9\right)^2}{N_A} \times \left[e^{\left(\frac{0.7}{0.0259}\right)} - 1\right]$$

$\Rightarrow$

$$N_A = 5.278 \times 10^{16}\,\text{cm}^{-3}$$

We can select a *p-n* diode with the conditions of N_A= 5.278×10^{16}cm^{-3} and N_D = 5.4×10^{15}cm^{-3}.

12. Assume $\tau_g = \tau_p = \tau_n$ = 10^{-6} s, D_n= 21 cm^2/sec, and D_p = 10 cm^2/sec

(a) The saturation current calculation.
From Eq. 55a and $L_p = \sqrt{D_p \tau_p}$, we can obtain

$$J_s = \frac{qD_p p_{n0}}{L_p} + \frac{qD_n n_{p0}}{L_n} = qn_i^2\left(\frac{1}{N_D}\sqrt{\frac{D_p}{\tau_{p0}}} + \frac{1}{N_A}\sqrt{\frac{D_n}{\tau_{n0}}}\right)$$

$$= 1.6\times 10^{-19} \times \left(9.65\times 10^9\right)^2 \left(\frac{1}{10^{18}}\sqrt{\frac{10}{10^{-6}}} + \frac{1}{10^{16}}\sqrt{\frac{21}{10^{-6}}}\right)$$

$$= 6.87 \times 10^{-12}\,\text{A/cm}^2$$

And from the cross-sectional area A = 1.2×10^{-5} cm^2, we obtain

$$I_s = A \times J_s = 1.2\times 10^{-5} \times 6.87 \times 10^{-12} = 8.244 \times 10^{-17}\,\text{A}.$$

(b) The total current density is

$$J = J_s\left(e^{\frac{qV}{kt}} - 1\right)$$

Thus

$$I_{0.7V} = 8.244\times 10^{-17}\left(e^{\frac{0.7}{0.0259}} - 1\right) = 8.244\times 10^{-17} \times 5.47 \times 10^{11} = 4.51 \times 10^{-5}\,\text{A}$$

$$I_{-0.7V} = 8.244\times 10^{-17}\left(e^{\frac{-0.7}{0.0259}} - 1\right) = 8.244 \times 10^{-17}\,\text{A}.$$

13. From $J = J_s\left(e^{\frac{qV}{kt}} - 1\right)$

we can obtain

$$\frac{V}{0.0259} = \ln\left[\left(\frac{J}{J_s}\right)+1\right] \Rightarrow V = 0.0259\times\ln\left[\left(\frac{10^{-3}}{8.244\times10^{-17}}\right)+1\right] = 0.78\ \text{V}.$$

14. From Eq. 59, and assume D_p= 10 cm^2/sec, we can obtain

$$J_R \cong q\sqrt{\frac{D_p}{\tau_p}}\frac{n_i^{\ 2}}{N_D} + \frac{qn_iW}{\tau_g}$$

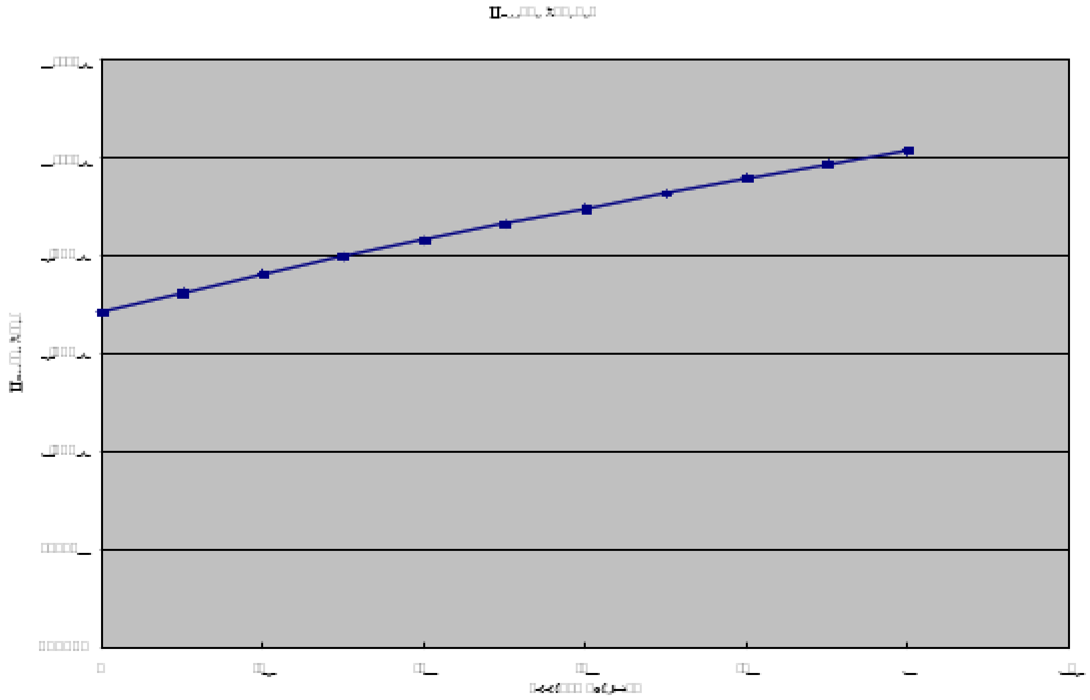
$$= 1.6\times10^{-19}\sqrt{\frac{10}{10^{-6}}}\frac{\left(9.65\times10^{9}\right)^2}{10^{15}} + \frac{1.6\times10^{-19}\times9.65\times10^{9}}{10^{-6}}\sqrt{\frac{2\times11.9\times8.85\times10^{-14}\times\left(V_{bi}+V_R\right)}{1.6\times10^{-19}\times10^{15}}}$$

$$V_{bi} = 0.0259\ln\frac{10^{19}\times10^{15}}{\left(9.65\times10^{9}\right)^2} = 0.834\ \text{V}$$

Thus

$$J_R = 5.26\times10^{-11} + 1.872\times10^{-7}\sqrt{0.834+V_R}$$

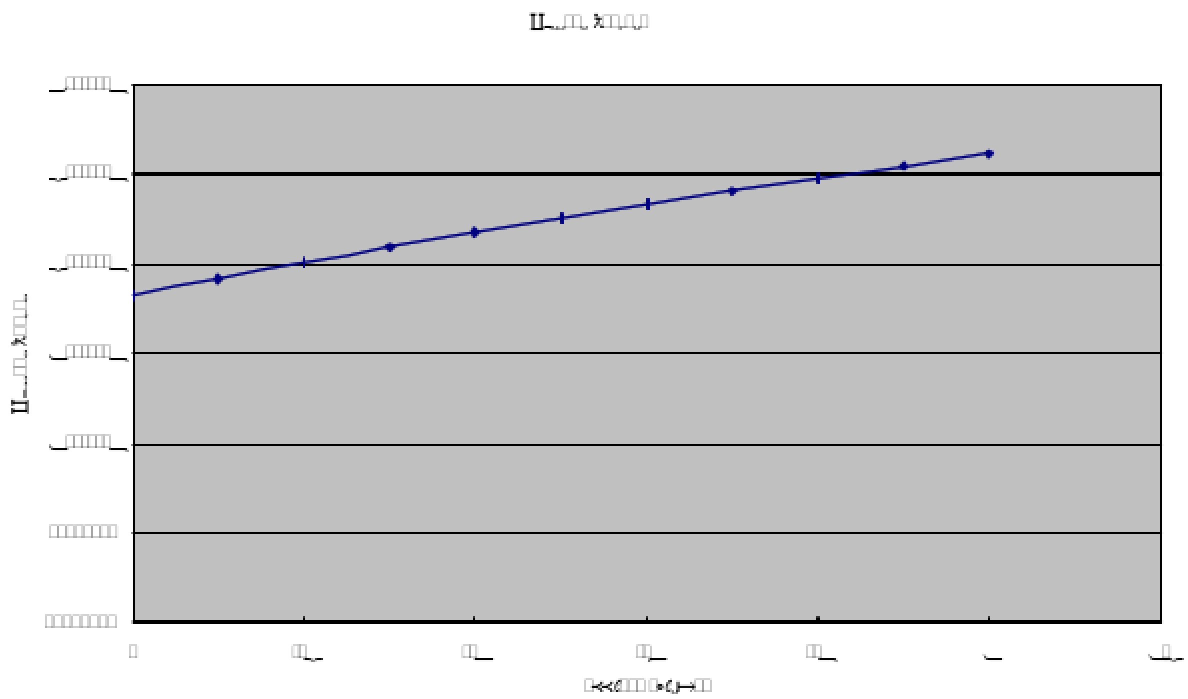
VR	Js
0	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]



When $N_D=10^{17}$ cm^{-3}, we obtain

$$V_{bi} = 0.0259 \ln \frac{10^{19} \times 10^{17}}{(9.65 \times 10^9)^2} = 0.953 \text{ V}$$

$$J_R = 5.26 \times 10^{-13} + 1.872 \times 10^{-8} \sqrt{0.956 + V_R}$$



15. From Eq. 39,

$$Q_p = q \int_{x_n}^{\infty} (p_n - p_{no})\ dx$$

$$= q\int_{x_n}^{\infty} p_{no}\left(e^{qV/kT} - 1\right)\ e^{-(x-x_n)/L_p}\,dx$$

The hole diffusion length is larger than the length of neutral region.

$$Q_p = q\int_{x_n}^{x_n'} (p_n - p_{no})\ dx$$

$$= q\int_{x_n}^{x_n'} p_{no}\left(e^{qV/kT} - 1\right)\ e^{-(x-x_n)/L_p}\,dx$$

$$= qp_{no}(-L_p)\left(e^{\frac{qV}{kT}} - 1\right)\left(e^{-\frac{x_n' - x_n}{L_p}} - e^{-\frac{x_n - x_n}{L_p}}\right)$$

$$= 1.6\times10^{-19}\times\frac{(9.65\times10^{9})^2}{10^{16}}(-5\times10^{-4})\left(e^{\frac{1}{0.0259}} - 1\right)\left(e^{-\frac{1}{5}} - e^{-\frac{0}{5}}\right)$$

$$= 8.784\times10^{-3}\ \text{C/cm}^2.$$

16. From Fig. 26, the critical field at breakdown for a Si one-sided abrupt junction is about 2.8×10^5 V/cm. Then from Eq. 85, we obtain

$$V_B(\text{breakdown voltage}) = \frac{\mathcal{E}_c W}{2} = \frac{\varepsilon_s \mathcal{E}_c^{\,2}}{2q}(N_B)^{-1}$$

$$= \frac{11.9\times8.85\times10^{-14}\times\left(2.8\times10^5\right)^2}{2\times1.6\times10^{-19}}\left(10^{15}\right)^{-1}$$

$$= 258\ \text{V}$$

$$W = \sqrt{\frac{2\varepsilon_s(V_{bi} - V)}{qN_B}} \cong \sqrt{\frac{2\times11.9\times8.85\times10^{-14}\times258}{1.6\times10^{-19}\times10^{15}}} = 1.843\times10^{-3}\ \text{cm} = 18.43\ \mu\text{m}$$

When the n-region is reduced to 5μm, the punch-through will take place first. From Eq. 87, we can obtain

$$\frac{V_B'}{V_B} = \frac{\text{shaded area in Fig. 29 insert}}{(\mathcal{E}_c W_m)/2} = \left(\frac{W}{W_m}\right)\left(2 - \frac{W}{W_m}\right)$$

$$V_B' = V_B\left(\frac{W}{W_m}\right)\left(2 - \frac{W}{W_m}\right) = 258\times\left(\frac{5}{18.43}\right)\left(2 - \frac{5}{18.43}\right) = 121\ \text{V}$$

Compared to Fig. 29, the calculated result is the same as the value under the conditions of $W = 5\ \mu$m and $N_B = 10^{15}$ cm^{-3}.

17. We can use following equations to determine the parameters of the diode.

$$J_F = q\sqrt{\frac{D_p}{\tau_p}}\frac{n_i^2}{N_D}e^{qV/kT} + \frac{qWn_i}{2\tau_r}e^{qV/2kT} \cong q\sqrt{\frac{D_p}{\tau_p}}\frac{n_i^2}{N_D}e^{qV/kT}$$

$$V_B = \frac{\mathcal{E}_c W}{2} = \frac{\varepsilon_s \mathcal{E}_c^2}{2q}(N_D)^{-1}$$

$$\Rightarrow$$

$$AJ_F = Aq\sqrt{\frac{D_p}{\tau_p}}\frac{n_i^2}{N_D}e^{qV/kT} \Rightarrow A\times 1.6\times 10^{-19}\times\sqrt{\frac{D_p}{10^{-7}}}\frac{(9.65\times 10^9)^2}{N_D}e^{\frac{0.7}{0.0259}} = 2.2\times 10^{-3}$$

$$V_B = \frac{\mathcal{E}_c W}{2} = \frac{\varepsilon_s \mathcal{E}_c^2}{2q}(N_D)^{-1} \Rightarrow 130 = \frac{11.9\times 8.85\times 10^{-14}\mathcal{E}_c^2}{2\times 1.6\times 10^{-19}}(N_D)^{-1}$$

Let $\mathcal{E}_c = 4\times 10^5$ V/cm, we can obtain $N_D = 4.05\times 10^{15}$ cm^{-3}.

The mobility of minority carrier hole is about 500 at $N_D = 4.05\times 10^{15}$

$\therefore D_p = 0.0259\times 500 = 12.95$ cm^2/s

Thus, the cross-sectional area A is 8.6×10^{-5} cm^2.

18. As the temperature increases, the total reverse current also increases. That is, the total electron current increases. The impact ionization takes place when the electron gains enough energy from the electrical field to create an electron-hole pair. When the temperature increases, total number of electron increases resulting in easy to lose their energy by collision with other electron before breaking the lattice bonds. This need higher breakdown voltage.

19. (a) The i-layer is easy to deplete, and assume the field in the depletion region is

constant. From Eq. 84, we can obtain.

$$\int_0^W 10^4\left(\frac{\mathcal{E}}{4\times10^5}\right)^6 dx = 1 \Rightarrow 10^4\left(\frac{\mathcal{E}}{4\times105}\right)^6 \times10^{-3} = 1 \Rightarrow \mathcal{E}_{critical} = 4\times10^5\times(10)^{1/6} = 5.87\times10^5 \text{ V/cm}$$

$$4\ V_B = 5.87\times10^5\times10^{-3} = 587\ \text{V}$$

(b) From Fig. 26, the critical field is $5\times\ 10^5$ V/cm.

$$V_B(\text{breakdown voltage}) = \frac{\mathcal{E}_c W}{2} = \frac{\varepsilon_s \mathcal{E}_c^{\ 2}}{2q}(N_B)^{-1}$$

$$= \frac{12.4\times8.85\times10^{-14}\times(5\times10^5)^2}{2\times1.6\times10^{-19}}(2\times10^{16})^{-1}$$

$$= 42.8\ \text{V}.$$

20. $a = \dfrac{2\times10^{18}}{2\times10^{-4}} = 10^{22}\,\text{cm}^{-4}$

$$V_B = \frac{2\mathcal{E}W}{3} = \frac{4\mathcal{E}_c^{\ 3/2}}{3}\left[\frac{2\varepsilon_s}{q}\right]^{1/2}(a)^{-1/2}$$

$$= \frac{4\mathcal{E}_c^{\ 3/2}}{3}\left[\frac{2\times11.9\times8.85\times10^{-14}}{1.6\times10^{-19}}\right]^{1/2}\times(10^{22})^{-1/2}$$

$$= 4.84\times10^{-8}\mathcal{E}_c^{\ 3/2}$$

The breakdown voltage can be determined by a selected $\mathcal{E}_c$.

21. To calculate the results with applied voltage of $V = 0.5\,\text{V}$, we can use a similar calculation in Example 10 with (1.6-0.5) replacing 1.6 for the voltage. The obtained electrostatic potentials are $1.1\,\text{V}$ and $3.4\times10^{-4}\,\text{V}$, respectively. The depletion widths are $3.821\times10^{-5}\,\text{cm}$ and $1.274\times10^{-8}\,\text{cm}$, respectively.

Also, by substituting $V = -5\,\text{V}$ to Eqs. 90 and 91, the electrostatic potentials are $6.6\,\text{V}$ and $20.3\times10^{-4}\,\text{V}$, and the depletion widths are $9.359\times10^{-5}\,\text{cm}$ and $3.12\times10^{-8}\,\text{cm}$, respectively.

The total depletion width will be reduced when the heterojunction is forward-biased from the thermal equilibrium condition. On the other hand, when the heterojunction is reverse-biased, the total depletion width will be increased.

22. $E_g(0.3) = 1.424 + 1.247 \times 0.3 = 1.789$ eV

$$V_{bi} = \frac{E_{g2}}{q} - \frac{\Delta E_C}{q} - (E_{F2} - E_{V2})/q - (E_{C1} - E_{F1})/q$$

$$= 1.789 - 0.21 - \frac{kT}{q}\ln\frac{4.7\times10^{17}}{5\times10^{15}} - \frac{kT}{q}\ln\frac{7\times10^{18}}{5\times10^{15}} = 1.273 \text{ V}$$

$$x_1 = \left[\frac{2N_A\varepsilon_1\varepsilon_2 v_{bi}}{qN_D(\varepsilon_1 N_D + \varepsilon_2 N_A)}\right]^{1/2} = \left[\frac{2\times12.4\times11.46\times8.85\times10^{-14}\times1.273}{1.6\times10^{-19}\times5\times10^{15}(12.4+11.46)}\right]^{1/2}$$

$$= 4.1\times10^{-5}\text{ cm.}$$

Since $N_D x_1 = N_A x_2$ $\quad\therefore x_1 = x_2$

$\therefore W = 2x_1 = 8.2\times10^{-5}\text{ cm} = 0.82\ \mu\text{m}$.

CHAPTER 5

1. (*a*) The common-base and common-emitter current gains is given by

$$\alpha_0 = \gamma\alpha_T = 0.997 \times 0.998 = 0.995$$
$$\beta_0 = \frac{\alpha_0}{1-\alpha_0} = \frac{0.995}{1-0.995}$$
$$= 199 \ .$$

(*b*) Since $I_B = 0$ and $I_{Cp} = 10 \times 10^{-9}$ A, then I_{CBO} is 10×10^{-9} A. The emitter current is

$$I_{CEO} = (1+\beta_0)I_{CBO}$$
$$= (1+199)\cdot 10 \times 10^{-9}$$
$$= 2 \times 10^{-6} \text{ A} \ .$$

2. For an ideal transistor,

$$\alpha_0 = \gamma = 0.999$$
$$\beta_0 = \frac{\alpha_0}{1-\alpha_0} = 999 \ .$$

I_{CBO} is known and equals to 10×10^{-6} A. Therefore,

$$I_{CEO} = (1+\beta_0)I_{CBO}$$
$$= (1+999)\cdot 10 \times 10^{-6}$$
$$= 10 \text{ mA} \ .$$

3. (a) The emitter-base junction is forward biased. From Chapter 3 we obtain

$$V_{bi} = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right) = 0.0259\ln\left[\frac{5\times10^{18}\cdot 2\times10^{17}}{(9.65\times10^9)^2}\right] = 0.956 \text{ V} \ .$$

The depletion-layer width in the base is

$$W_1 = \left(\frac{N_A}{N_A + N_D}\right) \text{(Total depletion - layer width of the emitter - base junction)}$$

$$= \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{N_A}{N_D}\right)\left(\frac{1}{N_A + N_D}\right)(V_{bi} - V)}$$

$$= \sqrt{\frac{2 \cdot 1.05\times10^{-12}}{1.6\times10^{-19}}\left(\frac{5\times10^{18}}{2\times10^{17}}\right)\left(\frac{1}{5\times10^{18} + 2\times10^{17}}\right)(0.956 - 0.5)}$$

$$= 5.364\times10^{-6}\ \text{cm} = 5.364\times10^{-2}\ \mu\text{m}\ .$$

Similarly we obtain for the base-collector function

$$V_{bi} = 0.0259 \ln\left[\frac{2\times10^{17}\cdot10^{16}}{(9.65\times10^9)^2}\right] = 0.795\ \text{V}\ .$$

and

$$W_2 = \sqrt{\frac{2 \cdot 1.05\times10^{-12}}{1.6\times10^{-19}}\left(\frac{10^{16}}{2\times10^{17}}\right)\left(\frac{1}{10^{16} + 2\times10^{17}}\right)(0.795 + 5)}$$

$$= 4.254\times10^{-6}\ \text{cm} = 4.254\times10^{-2}\ \mu\text{m}\ .$$

Therefore the neutral base width is

$$W = W_\text{B} - W_1 - W_2 = 1 - 5.364\times10^{-2} - 4.254\times10^{-2} = 0.904\ \mu\text{m}\ .$$

(b) Using Eq. 13*a*

$$p_n(0) = p_{no}e^{qV_{EB}/kT} = \frac{n_i^{\ 2}}{N_D}e^{qV_{EB}/kT} = \frac{(9.65\times10^9)^2}{2\times10^{17}}e^{0.5/0.0259} = 2.543\times10^{11}\ \text{cm}^{-3}\ .$$

4. In the emitter region

$$D_E = 52\ \text{cm/s} \quad L_E = \sqrt{52\cdot10^{-8}} = 0.721\times10^{-3}\ \text{cm}$$

$$n_{EO} = \frac{(9.65\times10^9)^2}{5\times10^{18}} = 18.625\ .$$

In the base region

$$D_p = 40\,\text{cm/s} \quad L_p = \sqrt{D_p \tau_p} = \sqrt{40 \cdot 10^{-7}} = 2\times10^{-3}\ \text{cm}$$
$$p_{no} = \frac{n_i^{\ 2}}{N_D} = \frac{\left(9.65\times10^{9}\right)^2}{2\times10^{17}} = 465.613\ .$$

In the collector region

$$D_C = 115\ \text{cm/s} \quad L_C = \sqrt{115\cdot10^{-6}} = 10.724\times10^{-3}\ \text{cm}$$
$$n_{CO} = \frac{\left(9.65\times10^{9}\right)^2}{10^{16}} = 9.312\times10^{3}\ .$$

The current components are given by Eqs. 20, 21, 22, and 23:

$$I_{Ep} = \frac{1.6\times10^{-19}\cdot0.2\times10^{-2}\cdot40\cdot465.613}{0.904\times10^{-4}} e^{0.5/0.0259} = 1.596\times10^{-5}\ \text{A}$$
$$I_{Cp} \cong I_{Ep} = 1.596\times10^{-5}\ \text{A}$$
$$I_{En} = \frac{1.6\times10^{-19}\cdot0.2\times10^{-2}\cdot52\cdot18.625}{0.721\times10^{-3}}\left(e^{0.5/0.0259}-1\right) = 1.041\times10^{-7}\ \text{A}$$
$$I_{Cn} = \frac{1.6\times10^{-19}\cdot0.2\times10^{-2}\cdot115\cdot9.312\times10^{3}}{10.724\times10^{-3}} = 3.196\times10^{-14}\ \text{A}$$
$$I_{BB} = I_{Ep} - I_{Cp} = 0\ .$$

5. (a) The emitter, collector, and base currents are given by

$$I_E = I_{Ep} + I_{En} = 1.606\times10^{-5}\ \text{A}$$
$$I_C = I_{Cp} + I_{Cn} = 1.596\times10^{-5}\ \text{A}$$
$$I_B = I_{En} + I_{BB} - I_{Cn} = 1.041\times10^{-7}\ \text{A}\ .$$

(b) We can obtain the emitter efficiency and the base transport factor:

$$\gamma = \frac{I_{Ep}}{I_E} = \frac{1.596\times10^{-5}}{1.606\times10^{-5}} = 0.9938$$

$$\alpha_T = \frac{I_{Cp}}{I_{Ep}} = \frac{1.596\times10^{-5}}{1.596\times10^{-5}} = 1\ .$$

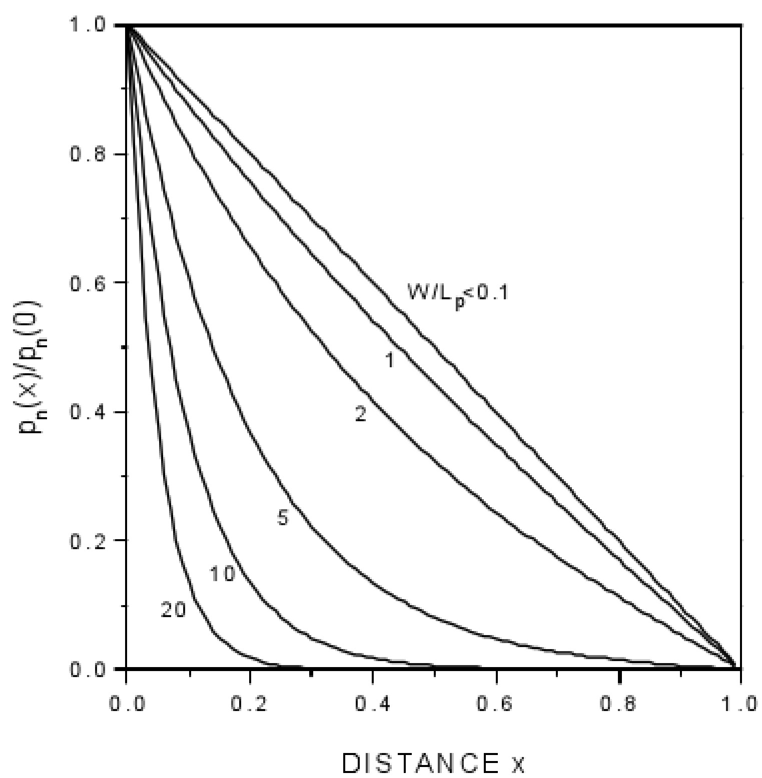
Hence, the common-base and common-emitter current gains are

$$\alpha_0 = \gamma\alpha_T = 0.9938$$
$$\beta_0 = \frac{\alpha_0}{1-\alpha_0} = 160.3\ .$$

(c) To improve γ, the emitter has to be doped much heavier than the base.

To improve α_T, we can make the base width narrower.

6. We can sketch $p_n(x)/p_n(0)$ curves by using a computer program:



In the figure, we can see when $W/L_p < 0.1$ ($W/L_p = 0.05$ in this case), the minority carrier distribution approaches a straight line and can be simplified to Eq. 15.

7. Using Eq.14, I_{Ep} is given by

$$I_{Ep} = A\left(-qD_p \frac{dp_n}{dx}\bigg|_{x=0}\right)$$

$$= A(-qD_p)\left\{ p_{no}\,(e^{qV_{EB}/kT}-1)\left[\frac{-\frac{1}{L_p}\cosh\left(\frac{W-x}{L_p}\right)}{\sinh\left(\frac{W}{L_p}\right)}\right] + p_{no}\left[\frac{-\frac{1}{L_p}\cosh\left(\frac{x}{L_p}\right)}{\sinh\left(\frac{W}{L_p}\right)}\right]\right\}_{x=0}$$

$$= qA\frac{D_p\,p_{no}}{L_p}\left\{(e^{qV_{EB}/kT}-1)\left[\frac{\cosh\left(\frac{W}{L_p}\right)}{\sinh\left(\frac{W}{L_p}\right)}\right] + \left[\frac{1}{\sinh\left(\frac{W}{L_p}\right)}\right]\right\}$$

$$= qA\frac{D_p\,p_{no}}{L_p}\coth\left(\frac{W}{L_p}\right)\left[\left(e^{qV_{EB}/kT}-1\right) + \frac{1}{\cosh\left(\frac{W}{L_p}\right)}\right].$$

Similarly, we can obtain I_{Cp} :

$$I_{Cp} = A\left(-qD_p \frac{dp_n}{dx}\bigg|_{x=W}\right)$$

$$= A(-qD_p)\left\{ p_{no}\,(e^{qV_{EB}/kT}-1)\left[\frac{-\frac{1}{L_p}\cosh\left(\frac{W-x}{L_p}\right)}{\sinh\left(\frac{W}{L_p}\right)}\right] + p_{no}\left[\frac{-\frac{1}{L_p}\cosh\left(\frac{x}{L_p}\right)}{\sinh\left(\frac{W}{L_p}\right)}\right]\right\}_{x=W}$$

$$= qA\frac{D_p\,p_{no}}{L_p}\left\{(e^{qV_{EB}/kT}-1)\left[\frac{1}{\sinh\left(\frac{W}{L_p}\right)}\right] + \left[\frac{\cosh\left(\frac{W}{L_p}\right)}{\sinh\left(\frac{W}{L_p}\right)}\right]\right\}$$

$$= qA\frac{D_p\,p_{no}}{L_p}\frac{1}{\sinh\left(\frac{W}{L_p}\right)}\left[\left(e^{qV_{EB}/kT}-1\right) + \cosh\left(\frac{W}{L_p}\right)\right].$$

8. The total excess minority carrier charge can be expressed by

$$
\begin{aligned}
Q_B &= qA\int_0^W [p_n(x) - p_{no}]dx \\
&= qA\int_0^W \left[p_{no} e^{qV_{EB}/kT} (1 - \frac{x}{W}) \right] dx \\
&= qAp_{no} e^{qV_{EB}/kT} (x - \frac{x^2}{2W}) \Big|_0^W \\
&= \frac{qAWp_{no} e^{qV_{EB}/kT}}{2} \\
&= \frac{qAWp_n(0)}{2} \ .
\end{aligned}
$$

From Fig. 6, the triangular area in the base region is $\frac{Wp_n(0)}{2}$. By multiplying this value by q and the cross-sectional area A, we can obtain the same expression as Q_B.

In Problem 3,

$$
\begin{aligned}
Q_B &= \frac{1.6\times 10^{-19} \cdot 0.2\times 10^{-2} \cdot 0.904\times 10^{-4} \cdot 2.543\times 10^{11}}{2} \\
&= 3.678\times 10^{-15} \text{ C} \ .
\end{aligned}
$$

9. In Eq. 27,

$$
\begin{aligned}
I_C &= a_{21}\left(e^{qV_{EB}/kT} - 1\right) + a_{22} \\
&\cong \frac{qAD_p\, p_n(0)}{W} \\
&= \frac{2D_p}{W^2} \frac{qAQp_n(0)}{2} \\
&= \frac{2D_p}{W^2} Q_B \ .
\end{aligned}
$$

Therefore, the collector current is directly proportional to the minority carrier charge stored in the base.

10. The base transport factor is

$$\alpha_T \cong \frac{I_{Cp}}{I_{Ep}} = \frac{\dfrac{1}{\sinh\left(\dfrac{W}{L_p}\right)}\left[\left(e^{qV_{EB}/kT}-1\right)+\cosh\left(\dfrac{W}{L_p}\right)\right]}{\coth\left(\dfrac{W}{L_p}\right)\left[\left(e^{qV_{EB}/kT}-1\right)+\dfrac{1}{\cosh\left(\dfrac{W}{L_p}\right)}\right]} .$$

For $W/L_p << 1$, $\cosh(W/L_p) \cong 1$. Thus,

$$\begin{aligned}\alpha_T &= \frac{1}{\sinh\left(\dfrac{W}{L_p}\right)\cdot\coth\left(\dfrac{W}{L_p}\right)} \\ &= \operatorname{sech}\left(\frac{W}{L_p}\right) \\ &= 1-\frac{1}{2}\left(\frac{W}{L_p}\right)^2 \\ &= 1-\left(W^2/2L_p^{\,2}\right) .\end{aligned}$$

11. The common-emitter current gain is given by

$$\beta_0 \equiv \frac{\alpha_0}{1-\alpha_0} = \frac{\gamma\alpha_T}{1-\gamma\alpha_T} .$$

Since $\gamma \cong 1$,

$$\begin{aligned}\beta_0 &\cong \frac{\alpha_T}{1-\alpha_T} \\ &= \frac{1-\left(W^2/2L_p^{\,2}\right)}{1-\left[1-\left(W^2/2L_p^{\,2}\right)\right]} \\ &= \left(2L_p^{\,2}/W^2\right)-1 .\end{aligned}$$

If $W/L_p << 1$, then $\beta_0 \cong 2L_p^{\,2}/W^2$.

12. $L_p = \sqrt{D_p\tau_p} = \sqrt{100\cdot 3\times10^{-7}} = 5.477\times10^{-3}\ \text{cm} = 54.77\ \mu\text{m}$

Therefore, the common-emitter current gain is

$$\beta_0 \cong 2L_p^{\ 2}/W^2 = \frac{2\left(54.77\times10^{-4}\right)^2}{\left(2\times10^{-4}\right)^2}$$
$$=1500.$$

13. In the emitter region,

$$\mu_{pE} = 54.3 + \frac{407}{1+0.374\times10^{-17}\cdot3\times10^{18}} = 87.6$$

$$D_E = 0.0259\cdot87.6 = 2.26 \text{ cm/s} .$$

In the base region,

$$\mu_n = 88 + \frac{1252}{1+0.698\times10^{-17}\cdot2\times10^{16}} = 1186.63$$

$$D_p = 0.0259\cdot1186.63 = 30.73 \text{ cm/s} .$$

In the collector region,

$$\mu_{pC} = 54.3 + \frac{407}{1+0.374\times10^{-17}\cdot5\times10^{15}} = 453.82$$

$$D_C = 0.0259\cdot453.82 = 11.75 \text{ cm/s} .$$

14. In the emitter region,

$$L_E = \sqrt{D_E\tau_E} = \sqrt{2.269\cdot10^{-6}} = 1.506\times10^{-3} \text{ cm}$$

$$p_{EO} = \frac{n_i^{\ 2}}{N_E} = \frac{\left(9.65\times10^9\right)^2}{3\times10^{18}} = 31.04 \text{ cm}^{-3} .$$

In the base region,

$$L_n = \sqrt{30.734\cdot10^{-6}} = 5.544\times10^{-3} \text{ cm}$$

$$n_{po} = \frac{\left(9.65\times10^9\right)^2}{2\times10^{16}} = 4656.13 \text{ cm}^{-3} .$$

In the collector region,

$$L_C = \sqrt{11.754\cdot10^{-6}} = 3.428\times10^{-3} \text{ cm}$$

$$p_{CO} = \frac{\left(9.65\times10^{9}\right)^2}{5\times10^{15}} = 18624.5\ \text{cm}^{-3}\ .$$

The emitter current components are given by

$$I_{En} = \frac{1.6\times10^{-19}\cdot10^{-4}\cdot30.734\cdot4656.13}{0.5\times10^{-4}} e^{0.6/0.0259} = 526.83\times10^{-6}\ \text{A}$$

$$I_{Ep} = \frac{1.6\times10^{-19}\cdot10^{-4}\cdot2.269\cdot31.04}{1.506\times10^{-3}}\left(e^{0.6/0.0259}-1\right) = 8.609\times10^{-9}\ \text{A}\ .$$

Hence, the emitter current is

$$I_E = I_{En} + I_{Ep} = 526.839\times10^{-6}\ \text{A}\ .$$

And the collector current components are given by

$$I_{Cn} = \frac{1.6\times10^{-19}\cdot10^{-4}\cdot30.734\cdot4656.13}{0.5\times10^{-4}} e^{0.6/0.0259} = 526.83\times10^{-6}\ \text{A}$$

$$I_{Cp} = \frac{1.6\times10^{-19}\cdot10^{-4}\cdot11.754\cdot18624.5}{3.428\times10^{-3}} = 1.022\times10^{-14}\ \text{A}\ .$$

Therefore, the collector current is obtained by

$$I_C = I_{Cn} + I_{Cp} = 5.268\times10^{-4}\ \text{A}\ .$$

15. The emitter efficiency can be obtained by

$$\gamma = \frac{I_{En}}{I_E} = \frac{526.83\times10^{-6}}{526.839\times10^{-6}} = 0.99998\ .$$

The base transport factor is

$$\alpha_T = \frac{I_{Cn}}{I_{En}} = \frac{526.83\times10^{-6}}{526.83\times10^{-6}} = 1\ .$$

Therefore, the common-base current gain is obtained by

$$\alpha_0 = \gamma\alpha_T = 1\times0.99998 = 0.99998\ .$$

The value is very close to unity.

The common-emitter current gain is

$$\beta_0 = \frac{\alpha_0}{1-\alpha_0} = \frac{0.99998}{1-0.99998} \cong 50000 \ .$$

16. (a) The total number of impurities in the neutral base region is

$$Q_G = \int_W^0 N_{AO} e^{-x/l} dx = N_{AO} l \left(1 - e^{-W/l}\right)$$
$$= 2\times10^{18} \cdot 3\times10^{-5} \left(1 - e^{-8\times10^{-5}/3\times10^{-5}}\right) = 5.583\times10^{13} \text{ cm}^{-2} \ .$$

(b) Average impurity concentration is

$$\frac{Q_G}{W} = \frac{5.583\times10^{13}}{8\times10^{-5}}$$
$$= 6.979\times10^{17} \text{ cm}^{-3} \ .$$

17. For $N_A = 6.979\times10^{17}$ cm^{-3}, $D_n = 7.77$ cm^3/s , and

$$L_n = \sqrt{D_n \tau_n} = \sqrt{7.77 \cdot 10^{-6}} = 2.787\times10^{-3} \text{ cm}$$

$$\alpha_T \cong 1 - \frac{W^2}{2L_n^{\ 2}} = 1 - \frac{\left(8\times10^{-5}\right)^2}{2\left(2.787\times10^{-3}\right)^2} = 0.999588$$

$$\gamma = \frac{1}{1 + \frac{D_E}{D_n}\frac{Q_G}{N_E L_E}} = \frac{1}{1 + \frac{1}{7.77}\cdot\frac{5.583\times10^{13}}{10^{19}\cdot10^{-4}}} = 0.99287 \ .$$

Therefore,

$$\alpha_0 = \gamma\alpha_T = 0.99246$$

$$\beta_0 = \frac{\alpha_0}{1-\alpha_0} = 131.6 \ .$$

18. The mobility of an average impurity concentration of 6.979×10^{17} cm^{-3} is about

300 cm^2/Vs . The average base resistivity $\overline{\rho}_B$ is given by

$$\overline{\rho}_B = \frac{1}{q\mu_n \left(Q_G / W\right)} = 0.0299 \ \Omega\text{-cm} \ .$$

Therefore,

$$R_B = 5\times10^{-3}\left(\overline{\rho}_B/W\right) = 5\times10^{-3}\cdot\left(0.0299/8\times10^{-5}\right) = 1.869\ \Omega\ .$$

For a voltage drop of kT/q,

$$I_B = \frac{kT}{qR_B} = 0.0139\ \text{A}\ .$$

Therefore,

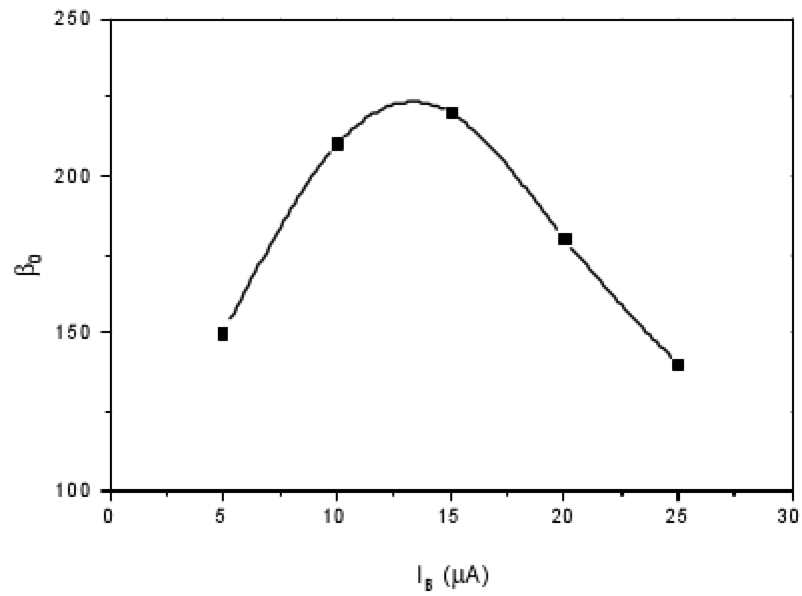
$$I_C = \beta_0 I_B = 131.6\cdot0.0139 = 1.83\ \text{A}\ .$$

19. From Fig. 10*b* and Eq. 35, we obtain

$I_B\,(\mu\text{A})$	$I_C\,(\text{mA})$	$\beta_0 = \frac{\Delta I_C}{\Delta I_B}$
0	0.20	---
5	0.95	150
10	2.00	210
15	3.10	220
20	4.00	180
25	4.70	140

β_0 is not a constant. At low I_B, because of generation-recombination current, β_0 increases with increasing I_B. At high I_B, V_{EB} increases with I_B, this in turn causes a reduction of V_{BC} since $V_{EB} + V_{BC} = V_{EC} = 5\ \text{V}$. The reduction of V_{BC} causes a widening of the neutral base region, therefore β_0 decreases.

The following chart shows β_0 as function of I_B. It is obvious that β_0 is not a constant.



20. Comparing the equations with Eq. 32 gives

$$I_{FO} = a_{11},\ \alpha_R I_{RO} = a_{12}$$

$$\alpha_F I_{FO} = a_{21}, \text{ and } I_{RO} = a_{22}\ .$$

Hence,

$$\alpha_F = \frac{a_{21}}{a_{11}} = \frac{1}{1 + \frac{W}{L_E} \cdot \frac{D_E}{D_p} \cdot \frac{n_{EO}}{p_{no}}}$$

$$\alpha_R = \frac{a_{12}}{a_{22}} = \frac{1}{1 + \frac{W}{L_C} \cdot \frac{D_C}{D_p} \cdot \frac{n_{CO}}{p_{no}}}\ .$$

21. In the collector region,

$$L_C = \sqrt{D_C \tau_C} = \sqrt{2 \cdot 10^{-6}} = 1.414 \times 10^{-3} \text{ cm}$$

$$n_{CO} = n_i^2 / N_C = \left(9.65 \times 10^9\right)^2 / 5 \times 10^{15} = 1.863 \times 10^4 \text{ cm}^{-3}\ .$$

From Problem 20, we have

$$\alpha_F = \frac{1}{1+\frac{W}{L_E}\cdot\frac{D_E}{D_p}\cdot\frac{n_{EO}}{p_{no}}} = \frac{1}{1+\frac{0.5\times10^{-4}}{10^{-3}}\cdot\frac{1}{10}\cdot\frac{9.31}{9.31\times10^{-2}}}$$
$$= 0.99995$$

$$\alpha_R = \frac{1}{1+\frac{W}{L_C}\cdot\frac{D_C}{D_p}\cdot\frac{n_{CO}}{p_{no}}} = \frac{1}{1+\frac{0.5\times10^{-4}}{1.414\times10^{-3}}\cdot\frac{2}{10}\cdot\frac{1.863\times10^{4}}{9.31\times10^{-2}}}$$
$$= 0.876$$

$$I_{FO} = a_{11} = qA\left(\frac{D_p p_{no}}{W} + \frac{D_E n_{EO}}{L_E}\right)$$
$$= 1.6\times10^{-19}\cdot 5\times10^{-4}\cdot\left(\frac{10\cdot 9.31\times10^{2}}{0.5\times10^{-4}} + \frac{1\cdot 9.31}{10^{-4}}\right)$$
$$= 1.49\times10^{-14}\ \text{A}$$

$$I_{RO} = a_{22} = qA\left(\frac{D_p p_{no}}{W} + \frac{D_C n_{CO}}{L_C}\right)$$
$$= 1.6\times10^{-19}\cdot 5\times10^{-4}\cdot\left(\frac{10\cdot 9.31\times10^{2}}{0.5\times10^{-4}} + \frac{2\cdot 1.863\times10^{4}}{1.414\times10^{-3}}\right)$$
$$= 1.7\times10^{-14}\ \text{A}\ .$$

The emitter and collector currents are

$$I_E = I_{FO}\left(e^{qV_{EB}/kT} - 1\right) + \alpha_R I_{RO}$$
$$= 1.715\times10^{-4}\ \text{A}$$

$$I_C = \alpha_F I_{FO}\left(e^{qV_{EB}/kT} - 1\right) + I_{RO}$$
$$= 1.715\times10^{-4}\ \text{A}\ .$$

Note that these currents are almost the same (no base current) for $W/L_p << 1$.

22. Referring Eq. 11, the field-free steady-state continuity equation in the collector region is

$$D_C\left[\frac{d^2 n_C(x')}{dx'^2}\right] - \frac{n_C(x') - n_{po}}{\tau_C} = 0\ .$$

The solution is given by ($L_C = \sqrt{D_C \tau_C}$)

$$n_C(x') = C_1 e^{x'/L_C} + C_2 e^{-x'/L_C}\ .$$

Applying the boundary condition at $x' = \infty$ yields

$$C_1 e^{\infty/L_C} + C_2 e^{-\infty/L_C} = 0 \ .$$

Hence $C_1 = 0$. In addition, for the boundary condition at $x' = 0$,

$$C_2 e^{-0/L_C} = C_2 = n_C(0)$$

$$n_C(0) = n_{CO}\left(e^{qV_{CB}/kT} - 1\right) .$$

The solution is

$$n_C(x) = n_{CO}\left(e^{qV_{CB}/kT} - 1\right)e^{-x'/L_C} \ .$$

The collector current can be expressed as

$$\begin{aligned} I_C &= A\left(-qD_p \left.\frac{dp_n}{dx}\right|_{x=W}\right) + A\left(-qD_C \left.\frac{dn_C}{dx'}\right|_{x'=0}\right) \\ &= qA\frac{D_p\, p_{no}}{W}\left(e^{qV_{EB}/kT} - 1\right) - qA\left(\frac{D_p\, p_{no}}{W} + \frac{D_C\, n_{CO}}{L_C}\right)\left(e^{qV_{CB}/kT} - 1\right) \\ &= a_{21}\left(e^{qV_{EB}/kT} - 1\right) - a_{22}\left(e^{qV_{CB}/kT} - 1\right). \end{aligned}$$

23. Using Eq. 44, the base transit time is given by

$$\tau_B = W^2/2D_p = \frac{\left(0.5\times10^{-4}\right)^2}{2\times10} = 1.25\times10^{-10}\ \text{s} \ .$$

We can obtain the cutoff frequency :

$$f_T \cong 1/2\pi\tau_B = 1.27\ \text{GHz} \ .$$

From Eq. 41, the common-base cutoff frequency is given by:

$$f_\alpha \cong f_T/\alpha_0 = \frac{1.27\times10^9}{0.998} = 1.275\ \text{GHz} \ .$$

The common-emitter cutoff frequency is

$$f_\beta = (1-\alpha_0) f_\alpha = (1-0.998)\times1.275\times10^9 = 2.55\ \text{MHz} \ .$$

Note that f_β can be expressed by

$$f_\beta = (1-\alpha_0) f_\alpha = (1-\alpha_0)/\alpha_0 \times f_T = \frac{1}{\beta_0} f_T \ .$$

24. Neglect the time delays of emitter and collector, the base transit time is given by

$$\tau_B = \frac{1}{2\pi f_T} = \frac{1}{2\pi \times 5 \times 10^9} = 31.83 \times 10^{-12}\,\text{s}\,.$$

From Eq. 44, W can be expressed by

$$W = \sqrt{2D_p \tau_B}\,.$$

Therefore,

$$\begin{aligned} W &= \sqrt{2 \times 10 \times 31.83 \times 10^{-12}} \\ &= 2.52 \times 10^{-5}\,\text{cm} \\ &= 0.252\,\mu\text{m}\,. \end{aligned}$$

The neutral base width should be 0.252 μm.

25. ΔE_g =9.8% ×1.12≅ 110 meV.

$$\beta_o \sim \exp\left(\frac{\Delta E_g}{kT}\right)$$

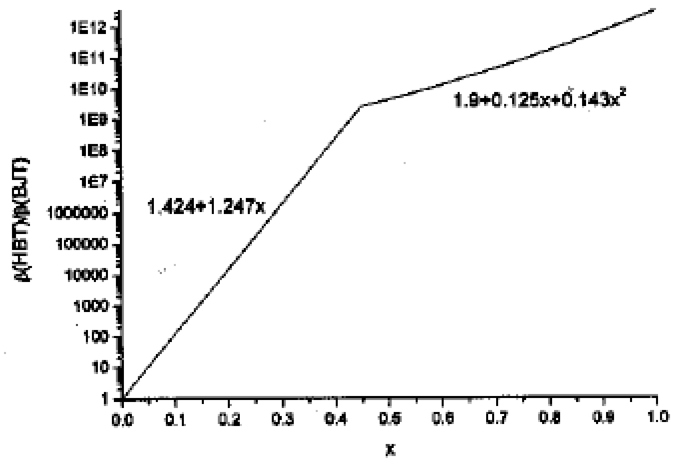
$$\therefore \frac{\beta_o(100\ °\text{C})}{\beta_o(0\ °\text{C})} = \exp\left(\frac{110\ \text{meV}}{373\ k} - \frac{110\ \text{meV}}{273\ k}\right) = 0.29.$$

26. $$\frac{\beta_0(\text{HBT})}{\beta_0(\text{BJT})} = \exp\left(\frac{E_{gE} - E_{gB}}{kT}\right) = \exp\left[\frac{E_{gE}(x) - 1.424}{0.0259}\right]$$

where

$$\begin{aligned} E_{gE}(x) &= 1.424 + 1.247x, \quad x \le 0.45 \\ &= 1.9 + 0.125x + 0.143x,\ \ 0.45 < x \le 1\,. \end{aligned}$$

The plot of $\beta_0(\text{HBT})/\beta_0\ (\text{BJT})$ is shown in the following graph.



Note that $\beta_0(\mathrm{HBT})$ increases exponentially when x increases.

27. The impurity concentration of the $n1$ region is $10^{14}\ \mathrm{cm^{-3}}$. The avalanche breakdown voltage (for $W > W_m$) is larger than 1500 V ($W_m > 100\,\mu\mathrm{m}$). For a reverse block voltage of 120 V, we can choose a width such that punch-through occurs, i.e.,

$$V_{PT} = \frac{qN_D W^2}{2\varepsilon_s} \ .$$

Thus,

$$W = \left(\frac{2\varepsilon_s V_{PT}}{qN_D}\right) = 3.96\times10^{-3}\ \mathrm{cm} \ .$$

When switching occurs,

$$\alpha_1 + \alpha_2 \cong 1 \ .$$

That is,

$$\alpha_1 = 0.5\sqrt{\frac{L_p}{W}}\ln\left(\frac{J}{J_0}\right)$$
$$= 1 - 0.4 = 0.6$$

$$\ln\left(\frac{J}{J_0}\right) = \frac{0.6}{0.5}\sqrt{\frac{25\times10^{-4}}{39.6\times10^{-4}}}$$
$$= 1.51 \ .$$