数字信号处理B

PB21511897 李霄奕

HW8

Exercise 1

切比雪夫I型模拟滤波器公式:

$$|G(j\Omega)|^2 = \frac{1}{1+\varepsilon^2 C_n^2(\Omega)} = \frac{1}{1+\varepsilon^2\cos^2(n\cos^{-1}\Omega)}$$

令分母等于0，$\cos^{-1}\Omega = \varphi$，可以求得极点

$$\begin{aligned}
1+\varepsilon^2\cos^2(n\varphi) &= 0 \\
\cos^2(n\varphi) &= -\frac{1}{\varepsilon^2} \\
\frac{\cos(2n\varphi)+1}{2} &= -\frac{1}{\varepsilon^2} \\
\cos(2n\varphi) &= \frac{-2-\varepsilon^2}{\varepsilon^2} \\
2in\varphi &= \cosh^{-1}\frac{-2-\varepsilon^2}{\varepsilon^2} \\
\varphi &= \frac{\cosh^{-1}\frac{-2-\varepsilon^2}{\varepsilon^2}}{2ni} \\
\Omega_p &= \cos(\frac{\cosh^{-1}\frac{-2-\varepsilon^2}{\varepsilon^2}}{2ni})
\end{aligned}$$

可以将（1）公式变化如下:

$$p_k = -\sin(\frac{(2k-1)\pi}{2n})\sinh(\varphi_2) + j\cos(\frac{(2k-1)\pi}{2n})\cosh(\varphi_2)$$

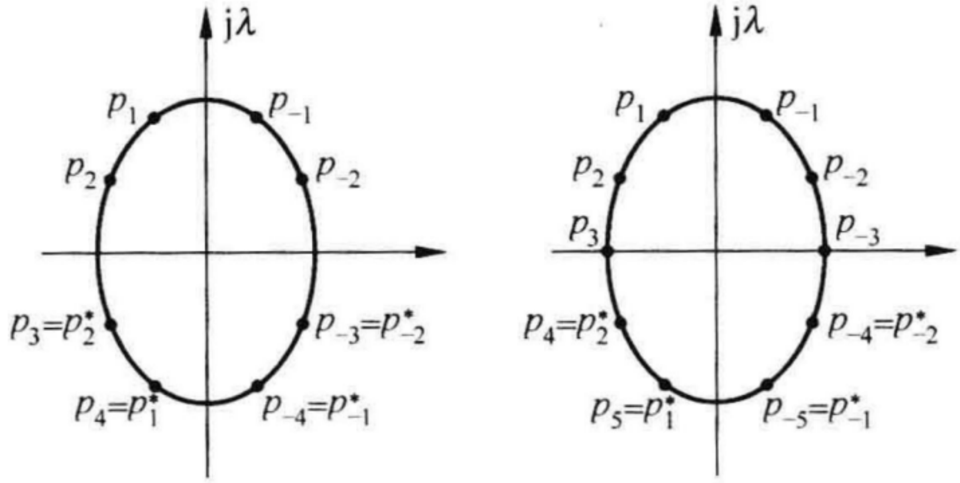
$$\text{令 } \varphi_3 = \frac{(2k-1)\pi}{2n}$$

$$p_k = -\sin(\varphi_3)\sinh(\varphi_2) + j\cos(\varphi_3)\cosh(\varphi_2)$$

$$\text{令 } \begin{cases} \sigma_k = -\sin(\varphi_3)\sinh(\varphi_2) \\ \lambda_k = \cos(\varphi_3)\cosh(\varphi_2) \end{cases}$$

$$\text{则满足：} \left(\frac{\sigma_k}{\sinh(\varphi_2)}\right)^2 + \left(\frac{\lambda_k}{\cosh(\varphi_2)}\right)^2 = 1$$

$$\text{这表示极点在椭圆上均匀分布，且跟实轴、虚轴对称}$$



Exercise 2

$$\omega_p = 0.2\pi, \omega_s = 0.6\pi$$

$$\Omega_p = \frac{\omega_p}{T_s} = 200\pi, \Omega_s = \frac{\omega_s}{T_s} = 600\pi$$

$$\lambda_p = \frac{\Omega_p}{\Omega_p} = 1, \lambda_s = \frac{\Omega_s}{\Omega_p} = 3$$

$$\alpha_p = 3, \alpha_s = 20$$

$$\varepsilon = \sqrt{10^{\alpha_p/10} - 1} = 1$$

$$a = \sqrt{\frac{10^{\alpha_s/10} - 1}{10^{\alpha_p/10} - 1}} = 9.97$$

$$n = \frac{\cosh^{-1} a}{\cosh^{-1} \lambda_s} = 1.70, N = 2$$

$$G(p) = \frac{1}{p^2 + \sqrt{2}p + 1}$$

$$G(s) = G(p = \frac{s}{\Omega_p}) = \sqrt{2}\Omega_p \cdot \frac{\left(\frac{\Omega_p}{\sqrt{2}}\right)^2}{\left[s - (-\frac{\Omega_p}{\sqrt{2}})\right]^2 + \left(\frac{\Omega_p}{\sqrt{2}}\right)^2} = \sqrt{2}\Omega_p \cdot \frac{\beta^2}{(s-\alpha)^2 + \beta^2}$$

$$\alpha = -\frac{\Omega_p}{\sqrt{2}}, \beta = \frac{\Omega_p}{\sqrt{2}}$$

$$H(z) = \sqrt{2}\Omega_p \cdot \frac{zT_s e^{\alpha T_s} \sin(\beta T_s)}{z^2 - z2e^{\alpha T_s} \cos(\beta T_s) + e^{2\alpha T_s}}$$

$$H(z) = \frac{0.2449z^{-1}}{1 - 1.1580z^{-1} + 0.4112z^{-2}}$$