量子物理期中整理

中国科学技术大学

频率 ω, k 波长 $\lambda k = \omega T = 2\pi$ 相速 $\omega = vk$

E, p 频 ν, λ

粒子性与波动性
$$\begin{cases} E = h\nu = \hbar\omega \\ \vec{p} = \frac{h}{\lambda}\vec{e}_k = \hbar \vec{k} \\ p = \sqrt{2mE} \end{cases}$$

常数: m_e: 电子质量: 03.

e: 电子电荷: 23.

k: 玻尔兹曼常数: 25

h: 普朗克常数: 06.

Schrödinger equation:

$$i\hbar\frac{\partial\psi}{\partial t} = E\psi = \frac{\hat{p}^2}{2m}\psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + V\psi = \hat{H}\psi$$

$\psi|_{t=t} = \psi|_{t=0}\, e^{-\frac{iE}{\hbar}t}$. ψ为本征函数.

$\cancel{\hat{H}\psi = E\psi}$ 当$V=0$. $\psi_p(x) = \frac{1}{\sqrt{2\pi\hbar}} e^{\frac{i}{\hbar}px}$ 本征函数.

$E_p = \frac{p^2}{2m}$ 本征值.

波函数: 归一化: $\int_{-\infty}^{+\infty}\psi^*(x)\psi(x)dx = 1$.

概率: $P_{[a,b]} = \int_a^b \psi^*\psi\, dx$.

期望 $\langle f(x)\rangle = \frac{\int\psi^* f(x)\psi\, dx}{\int\psi^*\psi\, dx}$.

$\sigma_{f(x)}^2 = \langle f(x)^2\rangle - \langle f(x)\rangle^2$.

$\langle\sigma_x\sigma_p\rangle = \frac{\hbar}{2}$.

$\psi(x) = \int_{-\infty}^{+\infty} C(p)\psi_p(x)\,dp$. $\psi(x)$是 $\psi_p(x)$线性组合.

$C(p) = \int_{-\infty}^{+\infty}\psi_p^*(x)\psi(x)\,dx$. $C(p)$为系数.

期望组合: $\psi = c_1\psi_1 + c_2\psi_2$.

$\langle\psi\rangle = \frac{|c_1|^2\langle\psi_1\rangle + |c_2|^2\langle\psi_2\rangle}{|c_1|^2 + |c_2|^2}$

$\psi_p(x) = \frac{1}{\sqrt{2\pi\hbar}} e^{\frac{i}{\hbar}px}$ $\psi_p(x)$: 平面波,有确定动量.

$\psi_p^*(x) = \frac{1}{\sqrt{2\pi\hbar}} e^{-\frac{i}{\hbar}px}$

$\hat{f} \to \hat{f}\psi_n = \lambda_n\psi_n \to \det|\hat{f} - \lambda_n I| = 0 \to \lambda_n \to \psi_n$ (归一化) $\to U = (|\psi_1\rangle \cdots |\psi_n\rangle) \to \begin{cases}\hat{f}' = U^\dagger \hat{f} U = \mathrm{diag}\,\lambda_n \\ |u'\rangle = U^\dagger|u\rangle\end{cases}$

$\langle x|\psi\rangle = \psi(x)$. $\psi = \int\langle x|\psi\rangle|x\rangle\,dx$

投影.

$\hat{p} = -i\hbar\frac{\partial}{\partial x}$

$\hat{x}\hat{p} - \hat{p}\hat{x} = [\hat{x},\hat{p}] = i\hbar$.

$\hat{L}^2|l,m\rangle = l(l+1)\hbar^2|l,m\rangle$

$\hat{L}_z|l,m\rangle = m\hbar|l,m\rangle$.

m 取值$[-l, -l+1, \dots, l-1, l]$.

电子: $l = \frac{1}{2}$. $\Rightarrow m = \pm\frac{1}{2}$

$$\begin{cases}\hat{S}^2 = \frac{3}{4}\hbar^2 I \\ \hat{S}_z = \frac{\hbar}{2}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} = \frac{\hbar}{2}\sigma_z\end{cases}$$

$\sigma_x = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ $\sigma_y = \begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}$ $\sigma_z = \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$

eg. 测量 eg. $\lambda_1|a_1\rangle$、$\lambda_2|a_2\rangle|a_3\rangle$

$|\psi\rangle \xrightarrow{\lambda_1} |a_1\rangle$ $p = \langle a_1|\psi\rangle^2$?

$\xrightarrow{\lambda_2} \langle a_2|\psi\rangle|a_2\rangle + \langle a_3|\psi\rangle|a_3\rangle$ $p = \langle a_2|\psi\rangle^2 + \langle a_3|\psi\rangle^2$?

$$\psi(x,t=\tau)=\int\frac{1}{\sqrt{2\pi\hbar}}e^{-\frac{i}{\hbar}\frac{p^2}{2m}\tau}\psi_p(x)dp$$

$$=\int\frac{1}{\sqrt{2\pi\hbar}}e^{-\frac{i}{\hbar}\frac{p^2}{2m}\tau+\frac{i}{\hbar}px}dp$$

$$=\int\frac{1}{\sqrt{2\pi\hbar}}e^{-\frac{i\tau}{2m\hbar}\left(p^2-\frac{2m}{\tau}px+\frac{m^2x^2}{\tau^2}\right)+\frac{imx^2}{2\hbar\tau}}dp$$

$$=\frac{1}{\sqrt{2\pi\hbar}}e^{\frac{imx^2}{2\hbar\tau}}\int e^{-\frac{i\tau}{2m\hbar}\left(p-\frac{m}{\tau}x\right)^2}dp$$

$$=\frac{1}{\sqrt{2\pi\hbar}}e^{\frac{imx^2}{2\hbar\tau}}\sqrt{\frac{\pi}{\frac{i\tau}{2m\hbar}}}=\sqrt{\frac{m}{i\tau}}e^{\frac{imx^2}{2\hbar\tau}}$$