## 数字信号处理B

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## HW8

## **Exercise 1**

切比雪夫I型模拟滤波器公式:

$$|G(j\Omega)|^2 = rac{1}{1+arepsilon^2 C_n^2(\Omega)} = rac{1}{1+arepsilon^2 \cos^2(n\cos^{-1}\Omega)}$$

令分母等于0,  $\cos^{-1}\Omega=\varphi$ , 可以求得极点

$$egin{aligned} 1+arepsilon^2\cos^2(narphi) &= 0 \ \cos^2(narphi) &= -rac{1}{arepsilon^2} \ rac{\cos(2narphi)+1}{2} &= -rac{1}{arepsilon^2} \ \cos(2narphi) &= rac{-2-arepsilon^2}{arepsilon^2} \ 2inarphi &= \cosh^{-1}rac{-2-arepsilon^2}{arepsilon^2} \ arphi &= rac{\cosh^{-1}rac{-2-arepsilon^2}{arepsilon^2}}{2ni} \ \Omega_p &= \cos(rac{\cosh^{-1}rac{-2-arepsilon^2}{arepsilon^2}}{2ni}) \end{aligned}$$

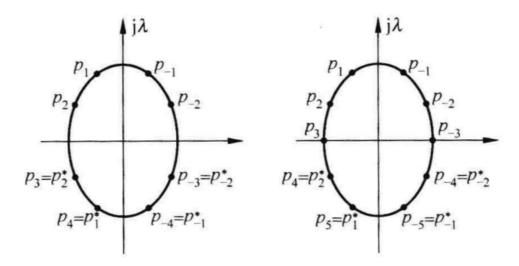
可以将(1)公式变化如下:

$$p_k = -\sin(\frac{(2k-1)\pi}{2n})\sinh(\varphi_2) + j\cos(\frac{(2k-1)\pi}{2n})\cosh(\varphi_2)$$

$$\Leftrightarrow \varphi_3 = \frac{(2k-1)\pi}{2n}$$

$$p_k = -\sin(\varphi_3)\sinh(\varphi_2) + j\cos(\varphi_3)\cosh(\varphi_2)$$

$$\Leftrightarrow \begin{cases} \sigma_k = -\sin(\varphi_3)\sinh(\varphi_2) \\ \lambda_k = \cos(\varphi_3)\cosh(\varphi_2) \end{cases}$$
则满足: 
$$\left(\frac{\sigma_k}{\sinh(\varphi_2)}\right)^2 + \left(\frac{\lambda_k}{\cosh(\varphi_2)}\right)^2 = 1$$
这表示极点在椭圆上均匀分布,且跟实轴、虚轴对称



## **Exercise 2**

$$\begin{split} \omega_p &= 0.2\pi, \omega_s = 0.6\pi \\ \Omega_p &= \frac{\omega_p}{T_s} = 200\pi, \Omega_s = \frac{\omega_s}{T_s} = 600\pi \\ \lambda_p &= \frac{\Omega_p}{\Omega_p} = 1, \lambda_s = \frac{\Omega_s}{\Omega_p} = 3 \\ \alpha_p &= 3, \alpha_s = 20 \\ \varepsilon &= \sqrt{10^{\alpha_p/10} - 1} = 1 \\ a &= \sqrt{\frac{10^{\alpha_s/10} - 1}{10^{\alpha_p/10} - 1}} = 9.97 \\ n &= \frac{\cosh^{-1}a}{\cosh^{-1}\lambda_s} = 1.70, N = 2 \\ G(p) &= \frac{1}{p^2 + \sqrt{2}p + 1} \\ G(s) &= G(p = \frac{s}{\Omega_p}) = \sqrt{2}\Omega_p \cdot \frac{\left(\frac{\Omega_p}{\sqrt{2}}\right)^2}{\left[s - \left(-\frac{\Omega_p}{\sqrt{2}}\right)\right]^2 + \left(\frac{\Omega_p}{\sqrt{2}}\right)^2} = \sqrt{2}\Omega_p \cdot \frac{\beta^2}{(s - \alpha)^2 + \beta^2} \\ \alpha &= -\frac{\Omega_p}{\sqrt{2}}, \beta = \frac{\Omega_p}{\sqrt{2}} \\ H(z) &= \sqrt{2}\Omega_p \cdot \frac{zT_s e^{\alpha T_s} \sin(\beta T_s)}{z^2 - z2e^{\alpha T_s} \cos(\beta T_s) + e^{2\alpha T_s}} \\ H(z) &= \frac{0.2449z^{-1}}{1 - 1.1580z^{-1} + 0.4112z^{-2}} \end{split}$$