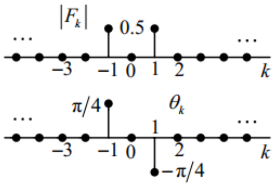
第五章作业答案

第六周

四：5.4（1，2，3，4）

5.4-1 $\tilde{x}(t)=0.5e^{-j(\pi/4)}e^{j(\pi/4)t}+0.5e^{j(\pi/4)}e^{-j(\pi/4)t}$ ，周期$T=8$ ，$\omega_0=\pi/4$

$F_1=0.5e^{-j(\pi/4)}$ ，$F_{-1}=0.5e^{j(\pi/4)}$ ，$F_k=0, k\neq\pm1$



5.4-2 $\omega_0=2\pi$

$$x(t)=[1+\cos(2\pi t)]\cos\left(6\pi t+\frac{\pi}{4}\right)$$
$$=\cos\left(6\pi t+\frac{\pi}{4}\right)+\frac{1}{2}\cos\left(8\pi t+\frac{\pi}{4}\right)+\frac{1}{2}\cos\left(4\pi t+\frac{\pi}{4}\right)$$
$$=\cos\left(3\omega_0 t+\frac{\pi}{4}\right)+\frac{1}{2}\cos\left(4\omega_0 t+\frac{\pi}{4}\right)+\frac{1}{2}\cos\left(2\omega_0 t+\frac{\pi}{4}\right)$$
$$=\frac{1}{2}e^{j\frac{\pi}{4}}e^{j3\omega_0 t}+\frac{1}{2}e^{-j\frac{\pi}{4}}e^{-j3\omega_0 t}+\frac{1}{4}e^{j\frac{\pi}{4}}e^{j4\omega_0 t}+\frac{1}{4}e^{-j\frac{\pi}{4}}e^{-j4\omega_0 t}+\frac{1}{4}e^{j\frac{\pi}{4}}e^{j2\omega_0 t}+\frac{1}{4}e^{-j\frac{\pi}{4}}e^{-j2\omega_0 t}$$

所以有$F_{-4}=\frac{1}{4}e^{-j\frac{\pi}{4}}, F_4=\frac{1}{4}e^{j\frac{\pi}{4}},$

$F_{-3}=\frac{1}{2}e^{-j\frac{\pi}{4}}, F_3=\frac{1}{2}e^{j\frac{\pi}{4}}$

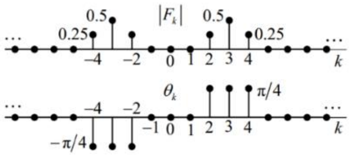
$F_{-2}=\frac{1}{4}e^{-j\frac{\pi}{4}}, F_2=\frac{1}{4}e^{j\frac{\pi}{4}},$

可以得到$|F_{-4}|=|F_4|=\frac{1}{4}$

$|F_{-3}|=|F_3|=\frac{1}{2}$

$|F_{-2}|=|F_2|=\frac{1}{4}$

$\theta_4=-\theta_{-4}=\frac{\pi}{4}, \theta_3=-\theta_{-3}=\frac{\pi}{4}, \theta_2=-\theta_{-2}=\frac{\pi}{4}$



5.4-3 $\tilde{x}(t)=\sin^2(2\pi t)=\frac{1}{2}(1-\cos 4\pi t)=\frac{1}{2}-\frac{1}{4}e^{-j4\pi t}-\frac{1}{4}e^{j4\pi t}$

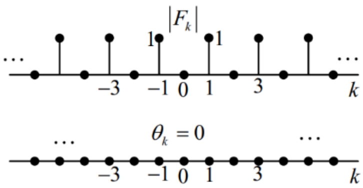
$T=\frac{1}{2}$，$F_0=\frac{1}{2}, F_{\pm1}=-\frac{1}{4}$ ，其余为 0

$|F_0|=\frac{1}{2}, |F_{\pm1}|=\frac{1}{4}, \theta_0=0, \theta_1=\pi, \theta_{-1}=-\pi$

5.4-4 $T=2$ ，令$\omega_0=\frac{2\pi}{T}=\pi$ ，则有

$$F_k=\frac{1}{T}\int_{<T>}x(t)e^{-jk\omega_0 t}dt$$
$$=\frac{1}{2}[1-\cos(k\omega)]$$
$$=\frac{1-(-1)^k}{2}.$$

$\theta_k=0$



第七周

二：5.7（2，4）5.9（1，3）5.11（1）5.14（1-b）5.15（1）

四：5.17（1）5.18（1，4）

日：5.16（1，2）

5.7-2 $\tilde{x}[n] = cos(2\pi n/3) + sin(2\pi n/7)$

$N = 21$，$\omega_0 = \frac{2\pi}{21}$

$$\tilde{x}[n] = \frac{1}{2}\left(e^{j\frac{2}{3}\pi n} + e^{-j\frac{2}{3}\pi n}\right) + \frac{1}{2j}\left(e^{j\frac{2}{7}\pi n} - e^{-j\frac{2}{7}\pi n}\right)$$

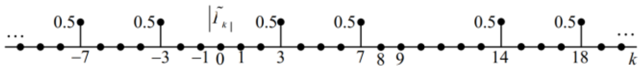
$$= \frac{1}{2}\left(e^{j7\frac{2\pi}{21}n} + e^{-j7\frac{2\pi}{21}n}\right) + \frac{1}{2j}\left(e^{j3\frac{2\pi}{21}n} - e^{-j3\frac{2\pi}{21}n}\right)$$

取一个周期$k = -10, -9, \ldots, 10$

$$F_7 = \frac{1}{2} \qquad F_{-7} = \frac{1}{2} \qquad F_3 = \frac{1}{2j} = -\frac{j}{2} \qquad F_{-3} = \frac{j}{2}$$

$$|F_7| = |F_{-7}| = |F_3| = |F_{-3}| = \frac{1}{2}$$

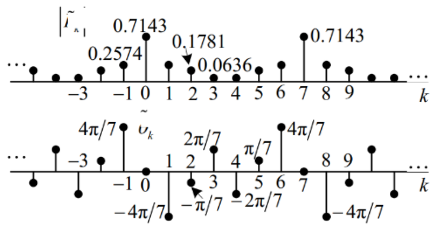
$$\theta_3 = -\frac{\pi}{2} \qquad \theta_{-3} = \frac{\pi}{2} \qquad \theta_k = 0(else)$$



5.7-4 基波频率 $\Omega_0 = \frac{2\pi}{7}$ ，DFS 系数为 $\tilde{F}_k = \frac{1}{7}\frac{\sin(5\pi k/7)}{\sin(\pi k/7)}e^{-j\frac{4\pi}{7}k}, k \neq 7m$ ，

而 $\tilde{F}_k = \frac{5}{7}, k = 7m, m \in$ 整数，

$\left|\tilde{F}_k\right| = \frac{1}{7}\left|\frac{\sin(5\pi k/7)}{\sin(\pi k/7)}\right|, k \neq 7m$ ，$\theta_k = -\frac{4\pi}{7}k, \tilde{F}_k > 0$



5.9-1 一周期内，$x[1] = x[7] = 4, x[3] = 4j, x[5] = -4j$ ，其余为 0

5.9-3 一周期内，$x[0] = 6, x[2] = 2, x[4] = -2, x[6] = 2$ ，其余为 0

5.11-1 $F_k = \frac{1}{2(a + jk\pi)} Sa\left(\frac{k\pi}{2}\right)$

5.14-1-b 1）b）$\tilde{x}[n] = j^n + (-1)^n$

$N = 4$，$\Omega_0 = \frac{\pi}{2}$

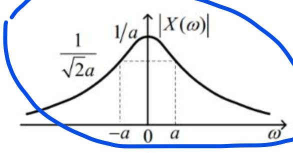
$$\tilde{x}[n] = j^n + (-1)^n = e^{j\Omega_0 n} + e^{j2\Omega_0 n}$$

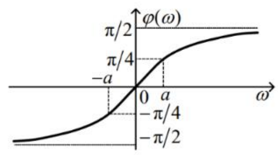
$$\tilde{y}[n] = H(\Omega_0)e^{j\Omega_0 n} + H(2\Omega_0)e^{j2\Omega_0 n} = \frac{3}{5}j^n + \frac{1}{3}(-1)^n$$

5.15-1 $x(t) = e^{at}u(-t) \qquad a > 0$

$$X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}dt = \int_{-\infty}^{0} e^{at}e^{-j\omega t}dt = \int_{0}^{\infty} e^{-at}e^{j\omega t}dt = \frac{a + j\omega}{a^2 + \omega^2}$$

$$|X(\omega)| = \frac{1}{\sqrt{a^2 + \omega^2}} \qquad \varphi(\omega) = arctan\frac{\omega}{a}$$





5.17-1

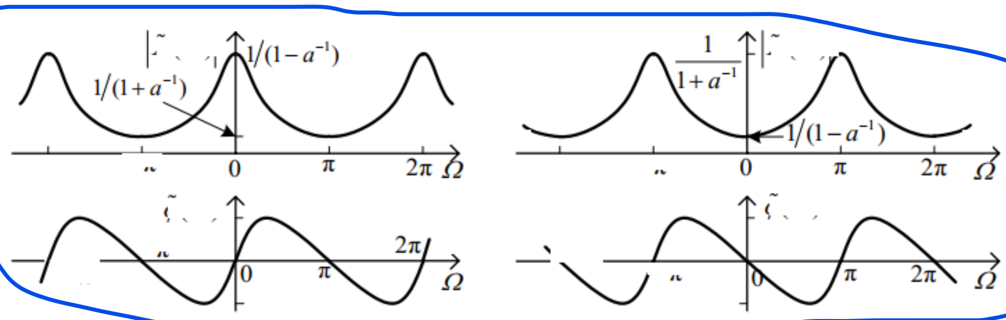
$$x[n] = a^n u[-n] \qquad |a| > 1$$

$$\tilde{X}(\Omega) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\Omega n} = \sum_{n=-\infty}^{0} a^n e^{-j\Omega n} = \sum_{n=0}^{+\infty} a^{-n} e^{j\Omega n}$$

$$= \frac{a}{a - e^{j\Omega}} = \frac{a}{a - \cos\Omega - j\sin\Omega}$$

$$|\tilde{X}(\Omega)| = \frac{|a|}{\sqrt{(a - \cos\Omega)^2 + (\sin\Omega)^2}} = \frac{|a|}{\sqrt{a^2 + 1 - 2a\cos\Omega}}$$

$$\varphi(\Omega) = -\arctan\frac{-\sin\Omega}{a - \cos\Omega} = \arctan\frac{\sin\Omega}{a - \cos\Omega}$$



對!

$0 < a^{-1} < 1$ $\qquad\qquad$ $-1 < a^{-1} < 0$

5.18-1 $x[-2] = 4, x[0] = 1, x[3] = -1, x[6] = 3$

$$x[n] = 4\delta[n+2] + \delta[n] - \delta[n-3] + 3\delta[n-6]$$

5.18-4 $x[n] = \delta[n] - \dfrac{W}{\pi}\mathrm{Sa}(Wn) = \dfrac{\pi - W}{\pi}\mathrm{Sa}[(\pi - W)n]$

5.16-1 $x(t) = 0.5e^{j\pi/3}\delta(t+4) + 0.5e^{-j\pi/3}\delta(t-4)$

5.16-2 $x(t) = \dfrac{2j}{\pi}\sin t + \dfrac{3j}{\pi}\sin(2\pi t)$