

第十周:

周二 6.4 (7, 8) 6.7 (5, 6, 10)

周四 6.1 (3, 11, 16) 6.3 (3) 6.7 (1, 3)

6.4-7

$$F(\omega) = \begin{cases} \cos^2(\frac{\pi\omega}{2W}), & |\omega| < W \\ 0, & |\omega| > W \end{cases} = \begin{cases} \frac{1}{2}(1 + \cos(\frac{\pi\omega}{W})), & |\omega| < W \\ 0, & |\omega| > W \end{cases}$$

$$\Rightarrow f(t) = \frac{W}{2\pi} \frac{\text{Sa}(Wt)}{1 - (Wt/\pi)^2}$$

6.4-8 $F(\Omega) = |F(\Omega)|e^{-j2\Omega}$

$$\begin{aligned} \text{令 } Y(\Omega) &= |F(\Omega)| = u\left(\Omega + \frac{\pi}{2}\right) + u\left(\Omega + \frac{\pi}{4}\right) - u\left(\Omega - \frac{\pi}{4}\right) - u\left(\Omega - \frac{\pi}{2}\right) \\ \therefore y[n] &= \frac{1}{2}\text{Sa}\left(\frac{\pi}{2}n\right) + \frac{1}{4}\text{Sa}\left(\frac{\pi}{4}n\right) \\ f[n] &= y[n-2] = \frac{1}{2}\text{Sa}\left(\frac{\pi}{2}(n-2)\right) + \frac{1}{4}\text{Sa}\left(\frac{\pi}{4}(n-2)\right) \end{aligned}$$

6.7-5 $f(t) = e^{-t}u(t) - e^{-(t-T)}u(t-T)$

6.7-6 $f[n] = a^n u[n] - a^{n-N} u[n-N]$

6.7-10 $f(t) = \cos \pi t u(t) + \cos \pi(t-1)u(t-1)$

6.1-3 $f(t) = [te^{-2t} \cos 4t]u(t)$

$$\begin{aligned} \because e^{-2t} \cos 4t u(t) &\Leftrightarrow \frac{2+j\omega}{(2+j\omega)^2 + 4^2} \\ \therefore [te^{-2t} \cos 4t]u(t) &\Leftrightarrow j \frac{d \frac{2+j\omega}{(2+j\omega)^2 + 4^2}}{d\omega} = -\frac{-(2+j\omega)^2 + 16}{[(2+j\omega)^2 + 16]^2} \\ \Rightarrow F(\omega) &= \frac{(2+j\omega)^2 - 16}{[(2+j\omega)^2 + 16]^2} \end{aligned}$$

6.1-11 $f[n] = (1-2n)0.5^n u[n+1] = 2 \cdot 0.5^{n+1} u[n+1] - 4n \cdot 0.5^{n+1} u[n+1]$

$$\begin{aligned} 0.5^n u[n] &\Leftrightarrow \frac{1}{1-0.5e^{-j\Omega}} \\ 0.5^{n+1} u[n+1] &\Leftrightarrow \frac{e^{j\Omega}}{1-0.5e^{-j\Omega}} \\ n \cdot 0.5^{n+1} u[n+1] &\Leftrightarrow j \frac{d \frac{e^{j\Omega}}{1-0.5e^{-j\Omega}}}{d\Omega} = -\frac{e^{j\Omega} - 1}{(1-0.5e^{-j\Omega})^2} \\ f[n] &\Leftrightarrow \frac{2e^{j\Omega}}{1-0.5e^{-j\Omega}} + 4 \frac{e^{j\Omega} - 1}{(1-0.5e^{-j\Omega})^2} \\ F(\Omega) &= \frac{6e^{j\Omega} - 5}{(1-0.5e^{-j\Omega})^2} \end{aligned}$$

$$\begin{aligned}
6.1-16 \quad f(t) &= -[u(t+2) - u(t+1)] + t[u(t+1) - u(t-1)] + [u(t-1) - u(t-2)] \\
&= (t+1)u(t+1) - (t-1)u(t-1) - u(t+2) - u(t-2) \\
\therefore u(t) &\Leftrightarrow \frac{1}{s}, tu(t) \Leftrightarrow \frac{1}{s^2} \\
\therefore (t+1)u(t+1) &\Leftrightarrow \frac{1}{s^2}e^s, (t-1)u(t-1) \Leftrightarrow \frac{1}{s^2}e^{-s}, \\
u(t+2) &\Leftrightarrow \frac{1}{s}e^{2s}, u(t-2) \Leftrightarrow \frac{1}{s}e^{-2s} \\
\therefore F(s) &= \frac{1}{s^2}e^s - \frac{1}{s^2}e^{-s} - \frac{1}{s}e^{2s} - \frac{1}{s}e^{-2s} = \frac{e^s - e^{-s}}{s^2} - \frac{e^{2s} + e^{-2s}}{s} \\
\Rightarrow F(\omega) &= \frac{e^{j\omega} - e^{-j\omega}}{(j\omega)^2} - \frac{e^{2j\omega} + e^{-2j\omega}}{j\omega} = -\frac{2j \sin \omega}{\omega^2} - \frac{2 \cos 2\omega}{j\omega}
\end{aligned}$$

$$\begin{aligned}
6.3-3 \quad x(t) &= te^{-a(t-1)}u(t+1) \\
&= te^{2a}e^{-a(t+1)}u(t+1) \\
e^{-at}u(t) &\Leftrightarrow \frac{1}{s+a} \\
e^{-a(t+1)}u(t+1) &\Leftrightarrow \frac{e^s}{s+a} \\
te^{2a}e^{-a(t+1)}u(t+1) &\Leftrightarrow X(s) = -e^{2a} \frac{d}{ds} \frac{e^s}{s+a} = -e^{2a} \frac{e^s(s+a-1)}{(s+a)^2}
\end{aligned}$$

$$\begin{aligned}
6.7-1 \quad X(s) &= \frac{s^2 e^{-2(s+1)}}{s^2 + 2s + 5} = \frac{s^2 e^{-2(s+1)}}{(s+1)^2 + 4} \\
\frac{e^{-2(s+1)}}{(s+1)^2 + 4} &\Leftrightarrow 0.5e^{-t} \sin(2t-4)u(t-2) \\
\frac{s^2 e^{-2(s+1)}}{(s+1)^2 + 4} &\Leftrightarrow e^{-2} \delta(t-2) - e^{-t} [2 \cos(2t-4) + 1.5 \sin(2t-4)]u(t-2)
\end{aligned}$$

$$\begin{aligned}
6.7-3 \quad X(z) &= \frac{2z}{(1-az^{-1})^3} \\
\frac{1}{(1-az^{-1})^3} &\leftrightarrow -\frac{(n+3-1)!}{n!(3-1)!} a^n u[-n-1] = -\frac{(n+2)(n+1)}{2} a^n u[-n-1] \\
\therefore x[n] &= -(n+3)(n+2)a^{n+1}u[-n-2]
\end{aligned}$$