DSP_HW8

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Exercise 1

推导切比雪夫 I 型模拟低通滤波器设计的极点公式。根据理论推导，教材上给出了一套计算极点的公式：

$$p_k = -\sin(\frac{(2k-1)\pi}{2n})\sinh(\varphi_2) + j\cos(\frac{(2k-1)\pi}{2n})\cosh(\varphi_2) \tag{1}$$

其中 $k = 1, 2, \cdots, n; \varphi_2 > 0$ 现对上面公式做合理化变化以得到其它有意义的表达式，并做合理化解释。

hw 8.1 $|G(j\lambda)|^2 = \frac{1}{1+\varepsilon^2 C_n^2(\lambda)}$

$G(p)\cdot G(-p)\big|_{p=j\lambda} = |G(j\lambda)|^2$

极点 P_k 满足 $\left|\begin{array}{l} 1+\varepsilon^2 C_n^2(p/j)=0. \\ \Downarrow \\ \cos[n\arccos(-jp)] = \pm j\frac{1}{\varepsilon} \end{array}\right.$ $\Leftarrow C_n(\Omega) = \cos(n\arccos\Omega)$.

令 $\varphi = \arccos(-jp)$, $\varphi = \varphi_1 + \varphi_2$.

$p = j\cos(\varphi_1 + j\varphi_2) = \sin\varphi_1 \sinh\varphi_2 + j\cos\varphi_1 \cosh\varphi_2$.

又有 $\cos(j\varphi_2) = \frac{e^{jj\varphi_2} + e^{-jj\varphi_2}}{2} = \cosh(\varphi_2)$

$\sin(j\varphi_2) = j\sinh(\varphi_2)$

$\therefore \cos(n\varphi) = \cos(n(\varphi_1 + j\varphi_2)) = \pm j\frac{1}{\varepsilon}$. 展开实虚部相等得.

$\therefore \varphi_1 = \frac{(2k-1)\pi}{2n}$, $\varphi_2 = \frac{1}{n}\operatorname{arcsinh}\frac{1}{\varepsilon}$

$\therefore p = j\cos(\varphi_1 + j\varphi_2)$

$= \sin(\varphi_1)\sinh(\varphi_2) + j\cos(\varphi_1)\cosh(\varphi_2)$.

如果选取半平面, 需加负号, $k=1,\cdots n$. $\varphi_2>0$.

$\therefore P_k = \sin\left[-\frac{(2k-1)\pi}{2n}\right]\sinh(\varphi_2) + j\cos\left[-\frac{(2k-1)\pi}{2n}\right]\cosh(\varphi_2)$.

$k = 1, 2, \cdots, 2n$.

合理化: $P_k = j\cos\left(\frac{2k-1}{}\pi - j\varphi_2\right)$.

Exercise 2

完成教材例题 6.4.2 中的计算，

例 6.4.2 试设计一个低通数字滤波器，要求在通带 $0\sim 0.2\pi$ 内衰减不大于 3dB，在阻带 $0.6\pi\sim\pi$ 内衰减不小于 20dB，给定 $T_s=0.001\mathrm{s}$。

hw 6.2 [illegible]

由 $\omega = \Omega T_s$，得 $\Omega_p = \omega_p / T_s = 200\pi$，$\Omega_s = \omega_s / T_s = 600\pi$. $\alpha_p = 3\,dB$，$\alpha_s = 20\,dB$.

令 $\lambda = \Omega / \Omega_p$，得 $\lambda_p = 1$，$\lambda_s = 3$，得 $N = 2$ 阶 $\Longrightarrow N = \lg \sqrt{\dfrac{10^{\alpha_s/10} - 1}{10^{\alpha_p/10} - 1}} \Big/ \lg \lambda_s$

$$G(p) = \frac{1}{p^2 + \sqrt{2}p + 1}$$

$$G(s) = G(p)\Big|_{p = \frac{s}{\Omega_p}} = \frac{\Omega_p^2}{s^2 + \sqrt{2}\Omega_p s + \Omega_p^2}$$

归一化注：$\lambda_s = \Omega_s / \Omega_p$，$\lambda_p = \Omega_p / \Omega_p = 1$，$\lambda = \Omega / \Omega_p$.

$$= \frac{\sqrt{2}\,\Omega_p \cdot \Omega_p / \sqrt{2}}{\left[s - \left(-\frac{\sqrt{2}}{2}\Omega_p\right)\right]^2 + \left(-\frac{\Omega_p}{\sqrt{2}}\right)^2}$$

由 (6.4.6) 式，令 $\alpha = -\frac{\sqrt{2}}{2}\Omega_p$；$\beta = \Omega_p / \sqrt{2}$，$\alpha T_s = -0.444$，$\beta T_s = 0.444$，则

$$H(z) = \frac{z T_s e^{\alpha T_s} \sin(\beta T_s)(\sqrt{2}\Omega_p)}{z^2 - z\,2e^{\alpha T_s}\cos(\beta T_s) + e^{2\alpha T_s}}$$

$$= \frac{0.2449\,z^{-1}}{1 - 1.1580\,z^{-1} + 0.4112\,z^{-2}}.$$

上式相比多了个 T_s，是由于 $h(nT_s)$ 是按 $h(nT_s) = T_s\, g(t)\big|_{t = nT_s}$ 抽样得的。

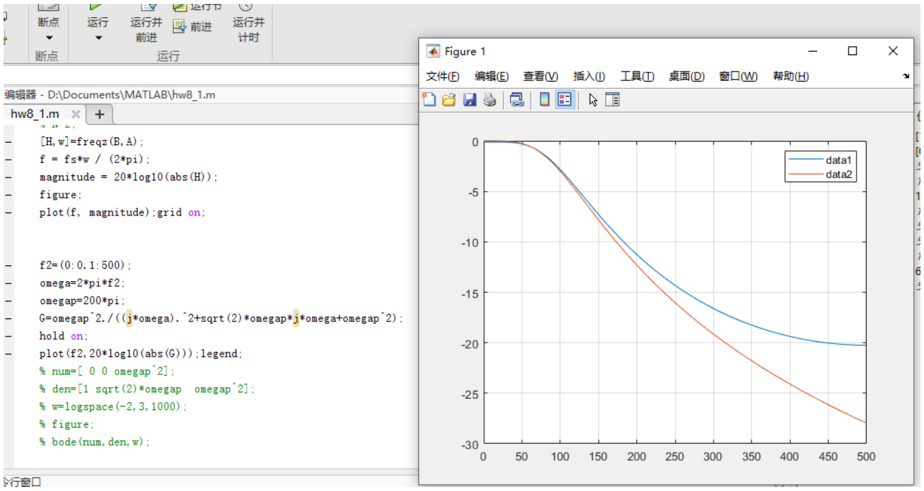
$G(j\Omega)$ 和 $H(e^{j\omega})$ 对数幅频曲线如图。

模拟满足要求，数字在阻带不符合要求，

$f_s = 300\,Hz$ 处，衰减为 $-16.8\,dB$，由于混叠造成的。

若 $G(s) = \beta / [(s-\alpha)^2 + \beta^2]$，这时 $G(s)$ 为一个二阶系统，$g(t) = e^{\alpha t}\sin(\beta t)u(t)$，则

$$H(z) = \frac{z e^{\alpha T_s}\sin(\beta T_s)}{z^2 - z[2e^{\alpha T_s}\cos(\beta T_s)] + e^{2\alpha T_s}} \qquad (6.4.6)$$



Exercise 3 (Optional)

巴特沃思、切比雪夫 I 型模拟低通归一化滤波器设计步骤、相关公式。

8.3 巴特沃思: 1. 实频率归一化,

$$|G(j\lambda)|^2=\frac{1}{1+C^2\lambda^{2N}},\quad \lambda_s=\frac{\Omega_s}{\Omega_p},\quad \lambda_p=1.$$

2. 求 C、N.

$$\begin{cases} C^2=10^{a_p/10}-1. \\ N=\lg\sqrt{\dfrac{10^{a_s/10}-1}{10^{a_p/10}-1}}\Big/\lg\lambda_s. \end{cases}$$

3. 求 $G(s)$.

$$p_k=\exp\left(j\frac{2k+N-1}{2N}\pi\right),\quad k=1,2,\cdots N$$

$$\Rightarrow G(p)=\frac{1}{\prod_{k=1}^{N}(p-p_k)}$$

去归一化得 $G(s)=G_p\big|_{p=\frac{s}{\Omega_p}}.$

切比雪夫. ①归一化指标. $|G(j\lambda)|^2=\dfrac{1}{1+\varepsilon^2C_n^2(\lambda)}$ $\quad C_n(\lambda)=\cos(n\arccos$

②求 ε、N.

$$\begin{cases} \varepsilon^2=10^{a_p/10}-1 \\ n=\dfrac{\operatorname{arcosh} a}{\operatorname{arcosh}\lambda_s} \end{cases}\qquad a=\sqrt{\frac{10^{a_s/10}-1}{10^{a_p/10}-1}}$$

$$\lambda_s=\frac{\Omega_s}{\Omega_p}$$

$$\lambda_p=1$$

③ 求 $G(s)$,

$$p_k=-\sin\left(\frac{2k-1}{2n}\pi\right)\sinh\varphi_2+j\cos\left(\frac{2k-1}{2n}\pi\right)\cosh\varphi_2,$$

$$k=1,2,\cdots n.$$

$$\Rightarrow G(p)=\frac{1}{\varepsilon\times 2^{n-1}\prod_{k=1}^{n}(p-p_k)}.$$

去归一化: $G(s)=G(p)\big|_{p=\frac{s}{\Omega_p}}$