

3.20

b. 解: 氦原子的质量 $m = 6.6969 \times 10^{-27} \text{ kg}$

普朗克常数 $h \approx 6.626 \times 10^{-34} \text{ J}\cdot\text{s}$

$$\lambda = \frac{h}{p}, \quad p = \sqrt{2mE_k} = \sqrt{2m \times \frac{3}{2} k_B T} = \sqrt{3mk_B T}$$

代入得

$$\lambda = \frac{6.626 \times 10^{-34}}{\sqrt{3 \times 6.6969 \times 10^{-27} \times 1.38 \times 10^{-23} \times 2}}$$

$$\approx 0.8898 \times 10^{-9} \text{ m}$$

$$= 0.8898 \text{ nm}$$

C. 解: $m_e = 9.1 \times 10^{-31} \text{ kg}$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

$$\text{则 } \lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e E_k}} = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 2 \times 1.6 \times 10^{-19}}} \approx 0.868 \text{ nm}$$

$$1. \int_{-\infty}^{+\infty} x^2 e^{-ax^2} dx = \frac{1}{2a} \sqrt{\frac{\pi}{a}} = \frac{\sqrt{\pi}}{2} a^{-\frac{3}{2}}$$

$$\begin{aligned} \text{则} \int_{-\infty}^{+\infty} x^4 e^{-ax^2} dx \\ &= -\frac{d}{da} \int_{-\infty}^{+\infty} x^2 e^{-ax^2} dx \\ &= \frac{3}{4} \sqrt{\pi} a^{-\frac{5}{2}} \end{aligned}$$

$$\begin{aligned} 2. \int_{-\infty}^{+\infty} f(x) \delta'(x) dx \\ &= \int_{-\infty}^{+\infty} \frac{d}{dx} [f(x) \delta(x)] dx - \int_{-\infty}^{+\infty} f'(x) \delta(x) dx \\ &= [f(x) \delta(x)] \Big|_{-\infty}^{+\infty} - f'(0) \\ &= -f'(0) \end{aligned}$$

$$\begin{aligned} 3. \int |\psi(x)|^2 dx &= \int_{-\infty}^{+\infty} x^2 e^{-m\omega x^2/\hbar} dx \\ &= \frac{\hbar}{2m\omega} \sqrt{\frac{\pi\hbar}{m\omega}} = \sqrt{\frac{\pi}{4}} \left(\frac{\hbar}{m\omega}\right)^{\frac{3}{2}} \end{aligned}$$

\Rightarrow 归一化后的波函数为:

$$\psi(x) = \frac{x e^{-m\omega x^2/\hbar}}{\left(\frac{\pi}{4}\right)^{\frac{1}{4}} \left(\frac{\hbar}{m\omega}\right)^{\frac{3}{4}}}$$