

第二章 非线性器件的分析方法

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2.1 概述



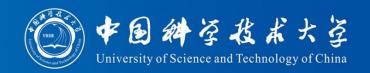
非线性器件物理特性复杂,线性电子线路的分析方法不适用于非线性电子线路,需要解非线性方程或时变系数的线性微分方程。

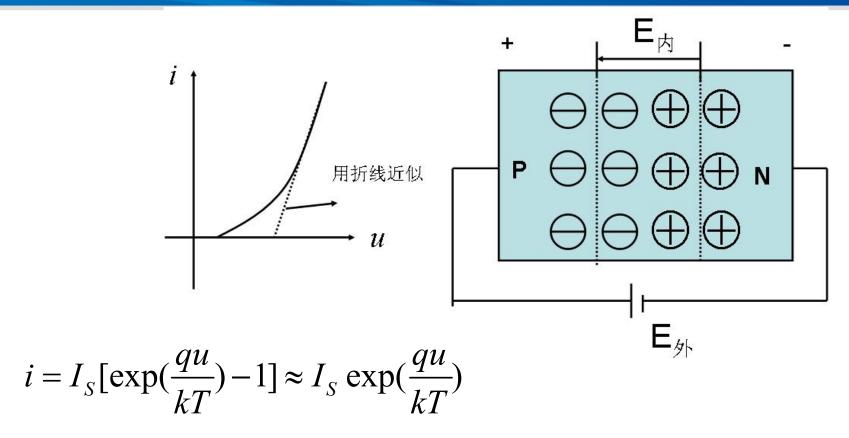
对策:对器件数学模型和电路工作条件进行合理近似,工程上用近似分析方法获得具有实用意义的结果。

近似分析法: 在一定的近似假设下,用于真实的非线性器件特性充分拟合的,在数学上比较易于处理的近似特性代替真实的特性,对此近似特性展开分析,得到初步结论,必要时再做修订和补充。

优点: 简单实用,基本满足"粗略描述非线性电子线路的特点和作用,为电路设计和调整提供理论指导"的要求。

PN结特性近似分析示例



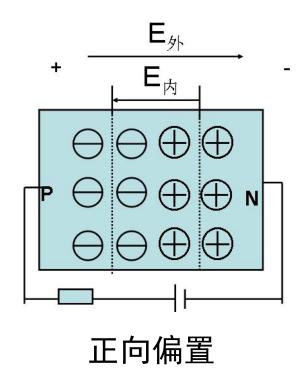


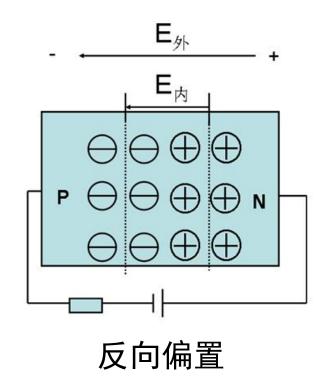
- 1) 将PN结近似看成一个非线性电阻,忽略非线性电容的影响;
- 2)特性曲线用折线近似;
- 3) 用几段折线近似逼近。



适用于以PN结为核心的非线性电阻。PN结特性决定了各种与 PN结有关的器件的特性,如二极管、晶体管等。

1. PN结的单向导电性







正向导通时, PN结伏安特性方程近似指数关系:

$$i = I_S[\exp(\frac{qu}{kT}) - 1] \approx I_S \exp(\frac{qu}{kT}) = I_S \exp(\frac{u}{U_r})$$

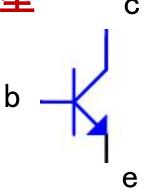
$$I_{S}$$
 - 反向饱和电流;常温下: $\frac{kT}{q} = 26mV = U_{r}$

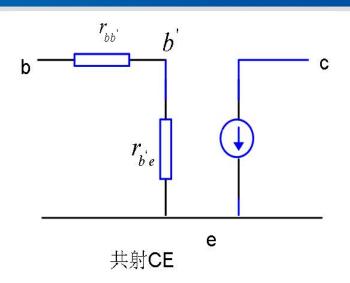
 I_{S} : 锘 $10^{-7} \sim 10^{-8} A$, 硅 $10^{-14} \sim 10^{-16} A$,

(考虑到半导体材料的表面效应,硅PN结 $I_S = 10^{-9}$ A数量级)



2. 晶体管(三极管)模型





$$r_{bb}$$
 - 基区体电阻 $\begin{cases} 高频管 (PN结很薄): 20^{\sim}70\Omega \\ 低频管: 几百 $\Omega \end{cases}$$

 r_{be} - 发射结电阻 (PN结), r_{be} = 可达几千 Ω

$$i_E = I_{ES} \exp(\frac{u_{b'e}}{U_r}) \approx I_{ES} \exp(\frac{u_{be}}{U_r})$$
 高频下, $r_{bb'} < r_{b'e}$,可忽略 $r_{bb'}$



3. 指数律特性分析

设加在晶体管b, e间的电压为:

$$u_{BE} = U_{BE} + U_{be} \cos \omega t$$

则有:

$$i_{E} = I_{ES} \exp(\frac{U_{BE} + U_{be} \cos \omega t}{U_{r}}) = I_{ES} \exp(\frac{U_{BE}}{U_{r}}) \exp(x \cos \omega t)$$
$$=I_{ES} \exp(\frac{U_{BE}}{U_{r}}) \exp(x) \frac{\exp(x \cos \omega t)}{\exp(x)} = I_{EP} W_{x}(\omega t)$$

$$x = \frac{U_{be}}{U_r}$$
 - 归一化交流电压幅度
$$I_{EP} = I_{ES} \exp(\frac{U_{BE}}{U_r}) \exp(x) \qquad (\cos \omega t = 1) - 发射极电流峰值$$

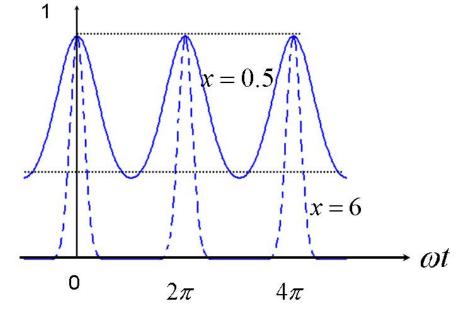


$$W_x(\omega t) = \frac{\exp(x\cos\omega t)}{\exp(x)} = \frac{i_E}{I_{EP}} -$$
归一化发射极电流

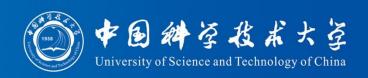
-最大值为1,与幅度和时间有关,波形如图所示。

周期性钟形脉冲

与输入电压周期性相同



x 越大钟形脉冲越窄(苗条),x越小,发射极电流越逼近正弦信号;即输入电压幅度越大,发射极电流失真越严重。



将周期性电流 i_E 展开成傅里叶级数:

$$i_{E}(\omega t) = I_{EP}W_{x}(\omega t) = \sum_{n=0}^{\infty} I_{En} \cos n\omega t$$

$$= I_{E0} + I_{E1} \cos \omega t + I_{E2} \cos 2\omega t + \dots + I_{En} \cos n\omega t + \dots$$

其中:

$$\begin{split} I_{E0} &= \frac{1}{2\pi} \int_0^{2\pi} i_E(\omega t) d\omega t = \frac{1}{2\pi} \int_0^{2\pi} I_{EP} W_x(\theta) d\theta \\ &= \frac{I_{EP}}{e^x} \left[\frac{1}{2\pi} \int_0^{2\pi} e^{x \cos \theta} d\theta \right] \\ &= \frac{I_{EP}}{e^x} I_0(x) - \mathbb{E}[\hat{M}] Bessel \mathbb{E}[\hat{M}] \end{split}$$

$$\begin{split} I_{En} &= 2 \times \frac{1}{2\pi} \int_{0}^{2\pi} I_{EP} W_{x}(\theta) \cos n\theta d\theta \\ &= \frac{2I_{EP}}{e^{x}} \left[\frac{1}{2\pi} \int_{0}^{2\pi} e^{x \cos \theta} \cos n\theta d\theta \right] \\ &= \frac{2I_{EP}}{e^{x}} I_{n}(x) - n$$
 第一类变态 $Bessel$ 函数



Bessel函数性质

$$x \to 0$$

$$\begin{cases} I_0(x) \to 1 & x \to \infty \\ I_1(x) \to \frac{x}{2} + \frac{x^3}{16} \approx \frac{x}{2} & \begin{cases} W_x(\omega t) \end{pmatrix}$$
 非常窄的钟形脉冲
$$\begin{cases} I_0(x) \to \frac{x^2}{2} + \frac{x^4}{96} \\ I_n(x) \to 0, \\ I_n(x$$



i_E 可进一步表示为:

$$i_{E}(\omega t) = I_{E0} + \sum_{i=1}^{\infty} I_{En} \cos n\omega t$$

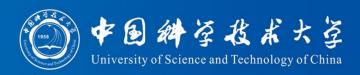
$$I_{E_n} = \frac{2I_{EP}}{e^x} I_n(x) = \frac{2I_{EP}I_0(x)}{e^x I_0(x)} I_n(x)$$

$$= I_{E_0} + \sum_{i=1}^{\infty} \frac{2I_{E_0}}{I_0(x)} I_n(x) \cos n\omega t$$

$$= I_{E_0} [1 + \sum_{n=1}^{\infty} \frac{2I_n(x)}{I_0(x)} \cos n\omega t]$$

1) 当输入为 ω 时,响应为无穷个频率 $n\omega$

(非线性器件的频率变换作用)



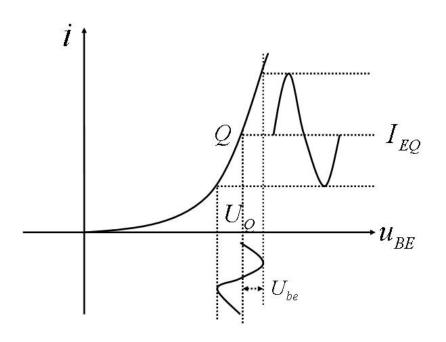
2) 直流分量 I_{E0} 与 U_{RE} 和 χ 有关

$$I_{E0} = \frac{I_{EP}}{e^{x}} I_{0}(x) = \frac{I_{ES} e^{\frac{U_{BE}}{U_{r}}} e^{x}}{e^{x}} I_{0}(x) = I_{ES} e^{\frac{U_{BE}}{U_{r}}} I_{0}(x)$$

3) 激励信号x 幅度很小时

$$I_{E0} = I_{EQ}$$

任何一个曲线在足够小的范围 观察均可看成直线,可用工作 点处的切线来代替,响应近似 为正弦波。





$$\begin{split} x &\to 0: I_{E0} = \frac{I_{EP}}{e^x} I_0(x) = I_{ES} e^{\frac{U_{BE}}{U_r}} = I_{EQ} \\ i_E(\omega t) &= I_{E0} [1 + \sum_{i=1}^{\infty} \frac{2I_n(x)}{I_0(x)} \cos n\omega t] \\ &= I_{EQ} [1 + \frac{2I_1(x)}{I_0(x)} \cos \omega t] \\ &= I_{EQ} [1 + \frac{2x}{2} \cos \omega t] \\ &= I_{EQ} [1 + \frac{U_{be}}{U_r} \cos \omega t] \\ &= I_{EQ} [1 + \frac{U_{be}}{U_r} \cos \omega t] \\ &= I_{EQ} [1 + \frac{U_{be}}{U_r} \cos \omega t] \\ &= I_{EQ} [1 + \frac{U_{be}}{U_r} \cos \omega t] \\ &= I_{EQ} [1 + \frac{U_{be}}{U_r} \cos \omega t] \end{split}$$

当 χ 很小时,指数律器件的作用如同跨导为 g_{mQ} 的线性放大器,即指数律器件退化成一个线性器件。

实际上,任何一个非线性器件,只要输入足够小,都退化成一个线性器件。



4) 激励信号x 幅度很大

$$I_{E0} = I_{ES}e^{\frac{U_{BE}}{U_r}}I_0(x) = I_{EQ}I_0(x) > I_{EQ}$$

均值电流 $I_{E0} >$ 静态电流 I_{EQ}

只考虑基波电流有:

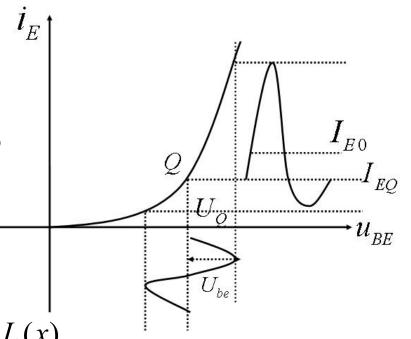
$$i_{C1} = \alpha i_{E1} = \alpha I_{E0} \frac{2I_1(x)}{I_0(x)} \cos \omega t = x\alpha I_{E0} \frac{2I_1(x)}{xI_0(x)} \cos \omega t$$

$$= \frac{1}{U_r} \alpha I_{E0} \frac{2I_1(x)}{xI_0(x)} U_{be} \cos \omega t$$

$$= \frac{1}{U_r} \alpha I_{E0} \frac{2I_1(x)}{xI_0(x)} u_{be} = G_{m1}(x) u_{be}$$

等效基波电导

$$G_{m1}(x) = \frac{\alpha I_{E0}}{U_r} \frac{2I_1(x)}{xI_0(x)}$$





大信号情况下,响应电流中基波分量的大小与输入信号的幅 度成非线性关系:

$$I_{C1} = G_{m1}(x)U_{be}$$

$$G_{m1}(x) = \frac{\alpha I_{E0}}{U_r} \frac{2I_1(x)}{xI_0(x)}$$
-等效基波跨导,与x有关

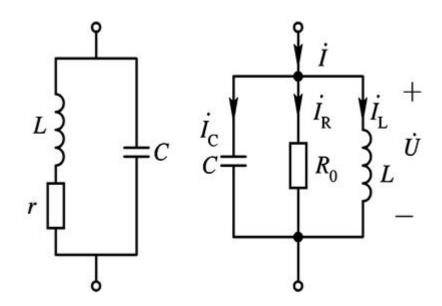
当输入信号幅度增大时,发射极或集电极电流失真随输入信号的增大而增大,必须用窄带滤波器(LBF)才能从失真电流中恢复出基波或某次谐波电流。



4. 补充知识点: 选频网络

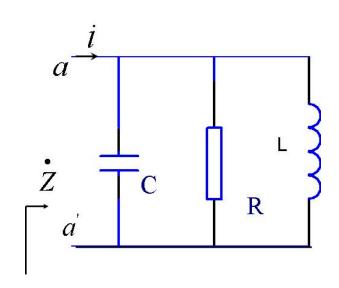
滤波器,又称选频网络,滤除无用信号,让有用信号通过。低频电路采用RC滤波器,高频电路采用LC滤波器。

带通滤波器(RLC并联谐振回路)





带通滤波器(RLC并联谐振回路)



特性阻抗:

谐振时回路的感抗或容抗

$$\rho = \omega_0 L = \frac{1}{\omega_0 C}$$

固有谐振频率:

$$\omega_0 = \frac{1}{\sqrt{LC}} rad / s$$

带宽: $BW = \frac{1}{RC}$

回路品质因数: 回路谐振时电感

(或电容)吸收的无功功率与电阻消耗

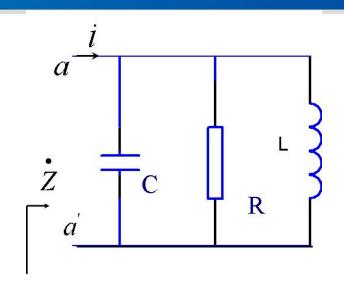
的有功功率之比

$$Q = \omega_0 RC = \frac{R}{\omega_0 L}$$



RLC并联谐振回路

$$\alpha = \frac{1}{2RC} = \frac{1}{2}BW$$
 半帶宽



$$\begin{split} \dot{Z} &= \frac{1}{\frac{1}{j\omega L} + \frac{1}{R} + j\omega C} = \frac{R}{1 + j\frac{\omega RC\omega_0}{\omega_0} + \frac{R\omega_0}{j\omega L\omega_0}} = \frac{R}{1 + jQ\frac{\omega}{\omega_0} - jQ\frac{\omega_0}{\omega}} \\ &= \frac{R}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)} = \frac{R}{1 + jQ\left(\frac{\omega^2 - \omega_0^2}{\omega\omega_0}\right)} \approx \frac{R}{1 + j\frac{\omega - \omega_0}{\omega}} \end{split}$$



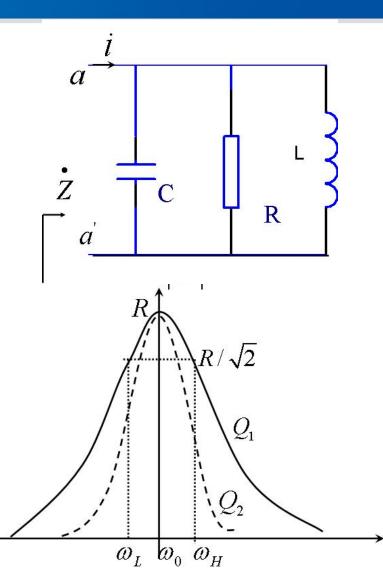
RLC并联谐振回路

幅频响应:

$$|Z| = \frac{R}{\sqrt{1 + Q^{2} \left[\frac{\omega^{2} - \omega_{0}^{2}}{\omega \omega_{0}}\right]^{2}}}$$

当 $\omega = \omega_0$ 时,Z = R-纯电阻

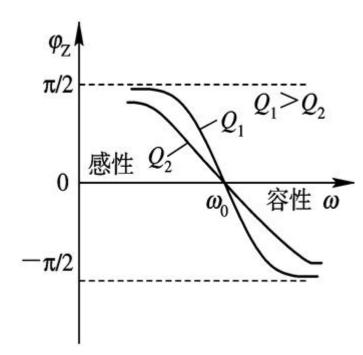
$$BW = \omega_H - \omega_L = \frac{1}{RC}, Q$$
越高,带宽越窄。





RLC并联谐振回路

相频响应:



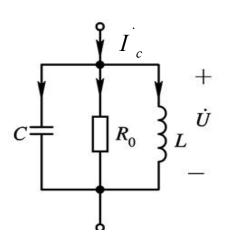
RLC并联谐振回路特点:

- (1) $\omega = \omega_0$ 谐振时,阻抗为最大值,呈纯电阻特性;
- (2) 谐振时,回路电流为(电感和电容中的电流)为输入电流(外部电路总电流)的Q倍。



RLC并联谐振回路提基波

$$i_C(\omega t) = I_{C0} + \sum_{n=1}^{\infty} I_{Cn} \cos n\omega t$$



回路电压=基波电压+二次谐波电压+···+ 高次谐波+···

$$\dot{U} = \dot{U_1} + \dot{U_2} + \dots + \dot{U_n} + \dots$$

当RLC网络起滤波作用提取基波时,调谐RLC并联网络,使 ω_0 =基波频率。

两种失真:

- ①基波幅度与激励信号幅度的高次方项有关,这是一种非线性失真;
- ②高次谐波的存在对基波分离产生影响,这将引起所谓的谐波失真。

总谐波失真 (THD)定义

提k次谐波:

$$THD_k = \sqrt{\sum_{\substack{n=1\\n \neq k}}^{\infty} \left(\frac{U_n}{U_k}\right)^2}$$

提基波:
$$THD_1 = \sqrt{\left(\frac{U_2}{U_1}\right)^2 + \left(\frac{U_3}{U_1}\right)^2 + \dots + \left(\frac{U_n}{U_1}\right)^2 + \dots} = \sqrt{\sum_{n=2}^{\infty} \left(\frac{U_n}{U_1}\right)^2}$$

指数律特性分析



RLC并联谐振回路提基波

$$I_{C1} = \alpha I_{E0} \frac{2I_1(x)}{I_0(x)}, I_{Cn} = \alpha I_{E0} \frac{2I_n(x)}{I_0(x)}$$

$$U_1 = I_{C1}R = I_{C1} |Z(\omega_0)|$$

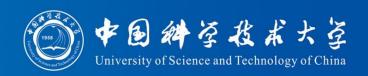
$$U_{n} = I_{Cn} | \dot{Z}(n\omega_{0}) | = I_{Cn} \frac{R}{\sqrt{1 + Q^{2} \left[\frac{n^{2}\omega_{0}^{2} - \omega_{0}^{2}}{n\omega_{0}\omega_{0}}\right]^{2}}} \approx I_{Cn} \frac{nR}{Q(n^{2} - 1)}$$

$$\therefore THD_{1} = \sqrt{\sum_{n=2}^{\infty} \left[\frac{I_{cn}}{I_{c1}} \left| \frac{\dot{Z}(n\omega_{0})}{\dot{Z}(\omega_{0})} \right| \right]^{2}} = \sqrt{\sum_{n=2}^{\infty} \left[\frac{I_{n}(x)}{I_{1}(x)} \frac{1}{Q} \frac{n}{n^{2} - 1} \right]^{2}}$$

$$= \frac{1}{Q} \sqrt{\sum_{n=2}^{\infty} \left[\frac{I_n(x)}{I_1(x)} \frac{n}{n^2 - 1} \right]^2} = \frac{1}{Q} D(x)$$

$$D(x) \text{ bis it it is possible.}$$

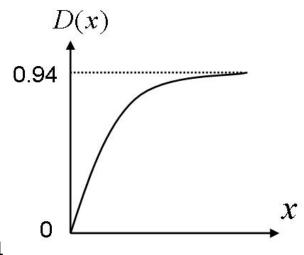
可查附录B(2)表得到



 $(1) \quad x \to 0$

$$D(x) = \sqrt{\sum_{n=2}^{\infty} \left[\frac{I_n(x)}{I_1(x)} \frac{n}{n^2 - 1}\right]^2} = \sqrt{\left[\frac{I_2(x)}{I_1(x)} \frac{2}{3}\right]^2 + \left[\frac{I_3(x)}{I_1(x)} \frac{3}{8}\right]^2 + \dots} \approx \sqrt{\left[\frac{I_2(x)}{I_1(x)} \frac{2}{3}\right]^2}$$

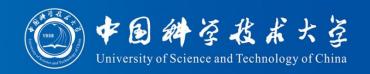
$$\approx \frac{I_2(x)}{I_1(x)} \frac{2}{3} = \frac{\frac{x^2}{8}}{\frac{x}{2}} \times \frac{2}{3} = \frac{1}{6}x$$
 失真很小



② $x \to \infty$, (极限状态)

$$\frac{I_n(x)}{I_1(x)} = 1 \implies D(x) \to \sqrt{\sum_{n=2}^{\infty} \left(\frac{n}{n^2 - 1}\right)^2} = 0.94$$
若 $Q = 30$, 则 $THD = \frac{1}{30} \times 0.94 = 3\%$

可见高次谐波所占比例很小,可忽略。



总结

以高 Q_T 调谐回路为负载提取基波(也可提谐波)时,对基波电压的作用如同一个线性放大器,此线性放大器的电压增益与输入电压幅度有关。

指数律器件 为例

$$U_{0} = I_{C1}R_{L} = G_{m1}(x)U_{be}R_{L} = G_{m1}(x)R_{L}U_{be} = A_{u}U_{be}$$

$$A_{u} = \frac{U_{0}}{U_{be}} = G_{m1}(x)R_{L}$$

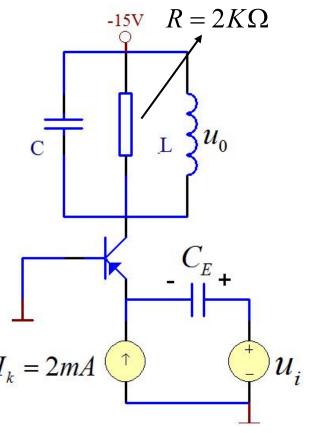
$$G_{m1}(x) = \frac{\alpha I_{E0}}{U_{r}} \frac{2I_{1}(x)}{xI_{0}(x)}$$

$$u_0(t) = G_{m1}(x)R_L U_{be} \cos \omega t$$



6. 指数律特性分析举例

例1: 恒流偏置晶体管放大电路



已知:
$$I_{ES} = e^{-30} mA$$
, $Q_T > 10$, $\alpha = 0.98$, $u_i = 0.1 \cos \omega t(V)$

输出回路调谐于输入信号上, C_E 容量足够大。

求:

- ① 集电极输出电压的表达式 u_o ;
- ② C_E上稳态直流压降。



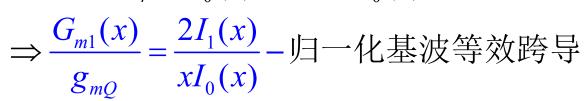
恒流偏置晶体管放大电路

解答:

$$I_{E0}$$
与 x 无关, $I_{E0} = I_{K}$

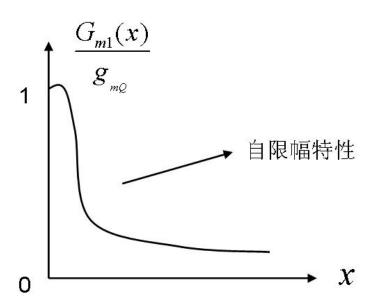
$$\therefore g_{mQ} = \frac{\alpha I_{E0}}{U_r} = \frac{\alpha I_K}{U_r}$$

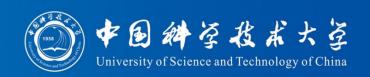
$$G_{m1}(x) = \frac{\alpha I_{E0}}{U_r} \frac{2I_1(x)}{xI_0(x)} = g_{mQ} \frac{2I_1(x)}{xI_0(x)}$$



(1) 静态时, I_K 向 C_E 充电,达到 $-U_{CE0} = U_{EB}$ 时充电结束。

$$i_{E} = I_{K} = I_{ES} e^{\frac{U_{EB}}{U_{r}}} = I_{ES} e^{\frac{-U_{CE0}}{U_{r}}} \Rightarrow U_{CE0} = -U_{r} \ln \frac{I_{K}}{I_{ES}} = -26 \ln \frac{2}{e^{-30}} = -798 mV$$



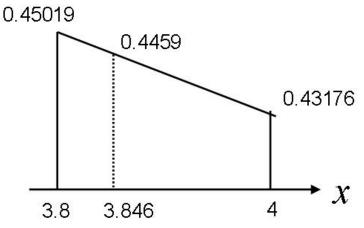


(2) 输入交流信号 $u_i = 0.1\cos\omega t(V)$

$$G_{m1}(x) = \frac{\alpha I_{E0}}{U_r} \frac{2I_1(x)}{xI_0(x)}$$

$$\begin{cases} x = \frac{U_i}{U_r} = \frac{0.1V}{26mV} = 3.846 \\ I_{E0} = I_{EQ} = I_K = 2mA \end{cases}$$

内插法求 $\frac{2I_1(x)}{xI_0(x)}$



$$y = \frac{2I_1(x)}{xI_0(x)}$$

$$= 0.45019 + \frac{0.43176 - 0.45019}{4 - 3.8}(x - 3.8)$$

$$\begin{array}{c} x = 0.3846 \\ x \end{array} \Rightarrow y = 0.4459$$



$$\therefore G_{m1}(x) = \frac{\alpha I_{E0}}{U_r} \frac{2I_1(x)}{xI_0(x)} = \frac{0.98 \times 2}{26} \times 0.4459 = 33.614 mS$$

$$u_0 = G_{m1}(x)Ru_i - 15 = 33.614 \times 2 \times 0.1\cos\omega t - 15$$
$$= -15 + 6.72\cos\omega t(V)$$

(3) C_F上稳态直流压降

$$I_{E0} = I_{ES}e^{\frac{-U_{CE}}{U_r}}I_0(x) = I_K$$

由附录B. 1查 $\frac{I_0(x)}{\rho^x}$ 得:

$$\Rightarrow U_{CE} = -U_r \left[\ln \frac{I_K}{I_{ES}} - \ln I_0(x) \right]$$

 $I_0(3.8)=0.2$ 由结果可见,当外加信号时,电 $I_0(4.0)=0.20$ 容两端的静态电压会发生变化

$$\therefore I_0(3.846) = 9.51663 + \frac{11.3018 - 9.51663}{4 - 3.8}(3.846 - 3.8) = 9.9272$$

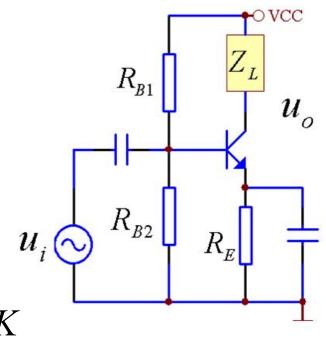
$$\therefore U_{CE} = -26[\ln \frac{2}{e^{-30}} - \ln 9.9272] = -798 + 26\ln 9.9272 = -738mV$$



例2: 电阻分压偏置晶体管放大电路

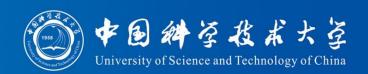
求图示电阻偏置晶体管放大器在x = 4时的基波跨导 $G_{m_1}(x)$ 和回路两端电压降之总谐波失真THD,设:

$$\begin{cases} Q_T = 20, \alpha = 0.99 \\ R_E = 1K, V_{CC} = 12V, R_L = 2K \\ R_{B1} = 5.1K, R_{B2} = 1.5K \end{cases}$$



归一化基波跨导

$$\frac{G_{m1}(x)}{g_{mQ}} = \left[1 + \frac{\ln I_0(x)}{x_{\lambda}}\right] \frac{2I_1(x)}{xI_0(x)}$$



直流分析

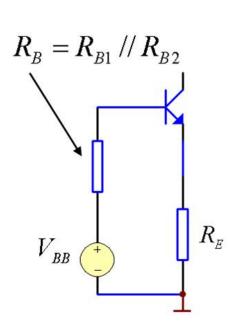
$$V_{BB} = \frac{R_{B2}}{R_{B1} + R_{B2}} V_{CC} = 0.7 + I_{EQ} R_E + I_{BQ} R_B$$

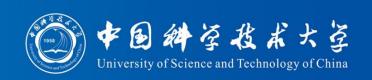
$$\Rightarrow I_{EQ} = \frac{V_{BB} - 0.7}{R_E + \frac{R_B}{1 + \beta}} = \frac{V_{BB} - 0.7}{R_E + (1 - \alpha)R_B}$$

$$I_{CQ} = \alpha I_{EQ} \Longrightarrow g_{mQ} = \frac{\alpha I_{EQ}}{U_r} = \frac{I_{CQ}}{U_r}$$

$$x_{\lambda} = \frac{U_{\lambda}}{U_{r}} = \frac{V_{BB} - 0.7}{U_{r}} = \frac{I_{EQ}R_{E} + (1 - \alpha)R_{B}I_{EQ}}{U_{r}} = \frac{I_{EQ}R_{E} + I_{BQ}R_{B}}{U_{r}}$$

$$U_{\lambda} = I_{EQ}[R_E + (1 - \alpha)R_B] = V_{BB} - 0.7$$





解答:
$$I_{EQ} = \frac{1.5}{S.1+1.5} \times 12-0.7$$

$$I_{EQ} = \frac{1.5}{R_E + (1-\alpha)(5.1/1.5)} \times 12-0.7$$

$$g_{mQ} = \frac{\alpha I_{EQ}}{U_r} = \frac{0.99 \times 1.55}{26} = 59.2 ms$$

$$x_{\lambda} = \frac{U_{\lambda}}{U_r} = \frac{2.27-0.7}{26} = 60.38$$

$$G_{m1}(x) = g_{mQ} \left[1 + \frac{\ln I_0(x)}{x_{\lambda}}\right] \frac{2I_1(x)}{xI_0(x)}$$

$$\Rightarrow G_{m1}(4) = 59.2 \times \left[1 + \frac{\ln 11.3}{60.38}\right] \times 0.43176 = 26.59 ms$$

$$D(x) = 0.4588$$

$$THD = \frac{1}{Q_T}D(x) = \frac{1}{20} \times 0.4588 = 2.3\%$$

$$u_o = V_{CC} - G_{m1}(4)R_L \cdot 4 \cdot 0.026\cos\omega t = 12 - 5.53\cos\omega t(V)$$



指数律电路分析程序:

电路模型 → 正弦信号激励下电流波形

→ Fourier展开得到各次谐波的表达式

$$\to G_{m,k} = \frac{I_k}{U_i} (尤其是 G_{m1}(x) = \frac{\alpha I_{E0}}{U_r} \cdot \frac{2I_1(x)}{xI_0(x)})$$

→ 查表得到D(x)

$$\to THD_1 = \frac{1}{Q_T}D(x)$$

$$\rightarrow U_1 = I_1 R_L = G_{m1}(x) U_i R_L$$



• 作业: 2.1, 2.2 (1), 2.3