DSP_HW9

msh

May 2024

Exercise 1

现希望设计一个巴特沃夫低通数字滤波器，其 3dB 带宽为 0.2π，阻带边缘频率为 0.5π，阻带衰减大于 30dB。给定抽样间隔为 $T_s = 10\mu s$

w9.1 $\quad w_p = 0.2\pi,\ w_s = 0.5\pi,\ \alpha_p = 3dB,\ \alpha_s = 30dB$ $\begin{cases} \text{双线性变换法} \\ \text{冲激响应不变法} \end{cases}$

$\Omega_p = \tan\frac{w_p}{2} = 0.3249,\ \Omega_s = \tan\frac{w_s}{2} = 1.\quad \alpha_p = 3dB,\ \alpha_s = 30dB.$

$\lambda = \Omega/\Omega_p,\ \Rightarrow \lambda_p = 1,\ \lambda_s = 3.078,\quad C = 1,$

$a = \sqrt{\frac{10^{3}-1}{10^{0.3}-1}} = 31.68,\ n = \operatorname{arccosh} a / \operatorname{arccosh} \lambda_s = 3.$

$p_k = \exp\left(j\frac{k+1}{3}\pi\right).\ k = 1, 2, 3.$

$$G(p) = \frac{1}{\prod_{k=1}^{3}(p-p_k)} = \frac{1}{(p^2+0.7654p+1)(p^2+1.8478p+1)}.$$

$$H(z) = G(p)\Big|_{p=\frac{1}{\Omega_p}\cdot\frac{z-1}{z+1}} = \frac{\Omega_p^3(z+1)^3}{[(z-1)^2+\Omega_p(z^2-1)+\Omega_p^2(z+1)^2]\cdot[z-1+\Omega_p(z+1)]}.$$

Exercise 2

给定待设计的数字高通滤波器的技术指标如下：

HP: $f_p = 400Hz, f_s = 300Hz, F_s = 1000Hz, \alpha_p = 3dB, \alpha_s = 35dB$

试用双线性 Z 变换法设计上述要求的切比雪夫滤波器，给出其转移函数、对数幅频以及相频曲线。(用 MATLAB 做图)

hw9.2 $\quad w_p = 0.8\pi,\ w_s = 0.6\pi.$

$$\Omega_p = \tan\frac{w_p}{2} = 3.078, \qquad \Omega_s = \tan\frac{w_s}{2} = 1.376.$$

$$\begin{cases} \lambda_p = 1, & \lambda_s = \dfrac{1}{\eta_s} = 2.2361 \\ \eta_p = 1, & \eta_s = \dfrac{\Omega_s}{\Omega_p} = 0.4472 \end{cases}$$

$$\varepsilon^2 = 10^{\alpha p/10} - 1 = 1.$$

$$n = \operatorname{arcosh}\left(\sqrt{\frac{10^{\alpha s/10}-1}{10^{\alpha p/10}-1}}\right) / \operatorname{arccosh}(\lambda_s) = 3.273, \quad N取4.$$

$$p_k = -\sin\left(\frac{2k-1}{2n}\pi\right)\cdot\sinh(\varphi_2) + j\cos\left(\frac{2k-1}{2n}\pi\right)\cosh(\varphi_2).$$

$$\varphi_2 = \frac{1}{n}\operatorname{arcsinh}\frac{1}{\varepsilon} = 0.2203$$

$$\Rightarrow \begin{cases} p_1 = -0.085 + j0.946 & p_3 = -0.205 + j0.392 \\ p_2 = -0.205 + j0.392 & p_4 = -0.085 + j0.946. \end{cases}$$

$$\therefore G(p) = \frac{1}{2^3 \prod_{k=1}^{4}(p - p_k)}$$

$$H(z) = G(p)\Big|_{p=\Omega_p\frac{z+1}{z-1}} = \frac{0.00105\,(1 - 4z^{-1} + 6z^{-2} - 4z^{-3} + z^{-4})}{1 + 3.27z^{-1} + 4.342z^{-2} + 2.743z^{-3} + 0.695z^{-4}}.$$

Exercise 3

一个数字系统的抽样频率 $F_s = 1000Hz$，试设计一个 50Hz 陷波器。要求下通带是 $0 - 44Hz$, 阻带在 47Hz，上通带与之对称；又要求通带衰减为 3dB，阻带衰减为 50dB。试用双线性 Z 变换法设计一个 50Hz 的切比雪夫数字陷波器来满足上述技术要求。

hw9.3 $\quad f_1 = 44\,Hz, \quad f_{sl} = 47\,Hz, \quad f_{sh} = 53\,Hz, \quad f_3 = 56\,Hz.$

$$\Rightarrow \begin{cases} w_1 = 0.088\pi, & w_{sl} = 0.094\pi \\ w_3 = 0.112\pi, & w_{sh} = 0.106\pi. \end{cases} \Rightarrow \begin{cases} \Omega_1 = \tan\frac{w_1}{2} = 0.1391 & \Omega_{sl} = 0.1487 \\ \Omega_3 = 0.1778 & \Omega_{sh} = 0.1681 \end{cases}$$

$\Omega_{BW} = \Omega_3 - \Omega_1 = 0.03865.$

$\Omega_2 = \sqrt{\Omega_1\Omega_3} = 0.15726$

$$\Rightarrow \begin{cases} \eta_1 = \Omega_1/\Omega_{BW} = 3.5994 & \eta_{sl} = 3.8483, \; \eta_3 = 4.5994 \\ \eta_2 = \Omega_2/\Omega_{BW} = 4.0688 & \eta_{sh} = 4.3483, \end{cases}$$

$\lambda_p = 1, \quad \lambda_s = \dfrac{\eta_{sh}}{\eta_{sh}^2 - \eta_2^2} = 1.84$[redacted]$49, \quad -\lambda_s = \dfrac{\eta_{sl}}{\eta_{sl}^2 - \eta_2^2} = -2.1966$

取 $\lambda_s = 1.8449 \quad . \; \varepsilon^2 = 10^{\alpha_P/10} - 1 = 1.$

$n = \operatorname{arccosh}\left(\sqrt{\dfrac{10^5-1}{10^{0.3}-1}}\right) / \operatorname{arccosh}(1.8449) = 5.27$, n[redacted]取 6.

$\varphi_2 = \dfrac{1}{n} \operatorname{arcsinh}\dfrac{1}{\varepsilon} = 0.1469$

$P_k = -\sin\left(\dfrac{2k-1}{12}\pi\right)\sinh(\varphi_2) + j\cos\left(\dfrac{2k-1}{12}\pi\right)\cosh(\varphi_2) \quad k = 1, 2, \cdots 6.$

$$\begin{cases} P_1 = -0.03816 + j0.97637 & P_4 = -0.1424 - j0.2612 \\ P_2 = -0.1042 + j0.7148 & P_5 = -0.1042 - j0.7148 \\ P_3 = -0.1424 + j0.2612 & P_6 = -0.03816 - j0.97637 \end{cases}$$

$$G(p) = \frac{1}{32(p^2 + 0.07632p + 0.95475)(p^2 + 0.20848p + 0.52177)(p^2 + 0.2848p + 0.08851)}$$

$H(z) = G(p)|_{p = \Omega_{BW}(z^2-1)}$