解：

势阱内的本征波函数（未归一化）为：

$$\psi_n(x) = \sin[k_n(x+a)]$$

其中 $k_n \cdot 2a = n \cdot \pi$

即 $k_n = \frac{n\pi}{2a}$ $\quad n = 1, 2, \cdots$

求归一化因子：

$$\int_{-a}^{a} |\psi_n(x)|^2 dx = \int_{-a}^{a} \sin^2[k_n(x+a)]\, dx$$

$$= \int_{-a}^{a} \left\{\frac{1}{2} - \frac{1}{2}\cos[2k_n(x+a)]\right\} dx$$

$$= a$$

$\Rightarrow$ 归一化后的波函数为：

$$\psi_n(x) = \frac{1}{\sqrt{a}} \sin[k_n(x+a)]$$

继续找跃迁概率量为：

$$\hat{H}' = \begin{cases} V_0 & -a < x < 0, \quad 0 < t < T \\ 0 & \text{otherwise} \end{cases}$$

故 $H'_{21} = \langle \psi_2 | \hat{H}' | \psi_1 \rangle$

$$= \int_{-\infty}^{+\infty} \psi_2^*(x) \hat{H}' \psi_1(x) \, dx$$

$$= \frac{V_0}{a} \int_{-a}^{0} \sin\left[\frac{\pi}{a}(x+a)\right] \sin\left[\frac{\pi}{2a}(x+a)\right] dx$$

$$= \frac{2V_0}{a} \int_{-a}^{0} \sin^2\left[\frac{\pi}{2a}(x+a)\right] \cos\left[\frac{\pi}{2a}(x+a)\right] dx$$

$$= \frac{2a}{\pi} \frac{2V_0}{a} \int_{-a}^{0} \sin^2\left[\frac{\pi}{2a}(x+a)\right] d \sin\left[\frac{\pi}{2a}(x+a)\right]$$

$$= \frac{4V_0}{\pi} \int_0^1 x^2 \, dx = \frac{4V_0}{3\pi}$$

故当 $t \le T$ 时，从 $|\psi_1\rangle$ 到 $|\psi_2\rangle$ 的跃迁几率为

$$W_{1\to2}(t) = \frac{1}{\hbar^2} |H'_{21}|^2 \frac{\sin^2(\omega_{12} t/2)}{(\omega_{12}/2)^2}$$

其中 $\omega_{12} = \frac{E_1 - E_2}{\hbar} = \frac{\hbar^2(k_1^2 - k_2^2)}{2m\hbar}$

$= -\frac{\hbar}{2m}\frac{3\pi^2}{4a^2}$

故T时刻，

$$W_{1\to 2}(T) = \frac{1}{\hbar^2}\left(\frac{4V_0}{3\pi}\right)^2 \frac{\sin^2\left(\frac{\hbar}{4m}\frac{3\pi^2}{4a^2}T\right)}{\left(\frac{\hbar}{4m}\frac{3\pi^2}{4a^2}\right)^2}$$

$$= \left[\frac{4V_0\, 4m\, 4a^2}{\hbar\, 3\pi\, \hbar\, 3\pi^2} \sin\left(\frac{\hbar}{4m}\frac{3\pi^2}{4a^2}T\right)\right]^2$$

$$= \left[\frac{64 V_0 m a^2}{9\hbar^2\pi^3} \sin\left(\frac{3\hbar\pi^2}{16 m a^2}T\right)\right]^2$$

T时刻以后，微扰为0，$|\psi_1\rangle$、$|\psi_2\rangle$又成为系统的本征态（定态波函数），故处于$|\psi_2\rangle$的几率不再变化，为 $W_{1\to 2}(T)$