Raiiyf
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Chap 4.

正弦量相关的记号与书写规范

(什么是相量?)

小写字母 i_c、u_L：时域量　形式 ($A_m\cos(\omega t+\varphi)$)

其中幅值 $A_m>0$

大写字母 I_c、U_c：有效值

在正弦量中：$I=\frac{A_m}{\sqrt{2}}$　(A_m为 i 的幅值)　故有效值也大于0

☆相量：大写字母头上加一点，$\dot I_c$、$\dot U_L$

$\dot I_c = I\angle\varphi = a+jb$　(I为i_c在时域的有效值)

幅值相量，相量加上m脚标，$\dot I_{mc}$、$\dot U_{mL}$

$\dot I_{mc} = I_m\angle\varphi = \sqrt{2} I\angle\varphi = \sqrt{2}\dot I_c$　(幅值相量使用较少)。

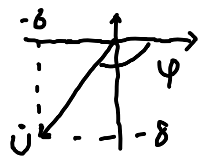
4.2

(a) $\dot U_m = 10\angle-10^\circ\ V \Rightarrow u=10\cos(\omega t-10^\circ)\ V$

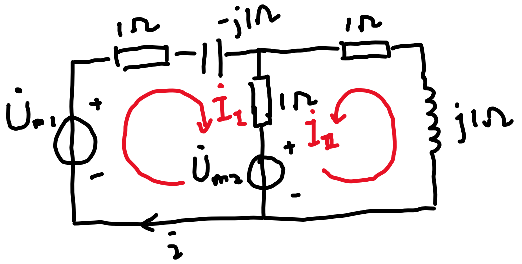
(b) $\dot U=(-6-j8)\ V \Rightarrow u=10\sqrt{2}\cos(\omega t-127^\circ)\ V$

(c) $\dot I_m=(0.2-j20.8)\ A \Rightarrow i=20.8\cos(\omega t-89.4^\circ)\ A$

(d) $\dot I=-30A \Rightarrow i=30\sqrt{2}\cos(\omega t+180^\circ)\ A$



4.10



$u_{s1}=u_{s2}=4\cos\omega t\ V \Rightarrow \dot U_{m1}=\dot U_{m2}=4$

回路电流法：

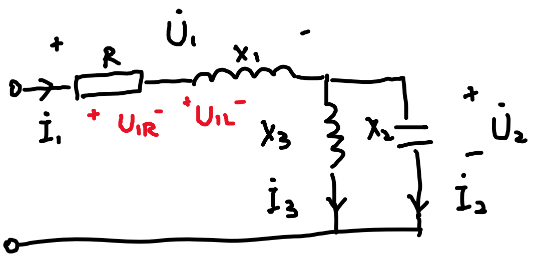
$$\begin{pmatrix}2-j & 1\\ 1 & 2+j\end{pmatrix}\begin{pmatrix}\dot I_I\\ \dot I_{II}\end{pmatrix}=\begin{pmatrix}0\\ -4\end{pmatrix}$$

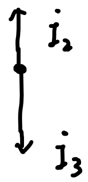
$\dot I_I = 1A$，$\dot I_{II}=-2+j\ A$

$\Rightarrow i=\cos\omega t=\cos 100t\ A$

相量图.

4.22

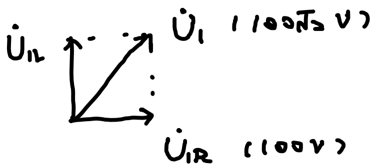




$\dot I_1 = \dot I_3 + \dot I_2$ 注意符号的区别

$I_1 = I_3 - I_2 = 10A$

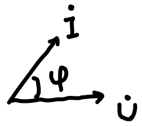
$P = I_1^2 \cdot R \Rightarrow R = 10\Omega$. $\dot U_{1R} = 100V$



$\Rightarrow \dot U_{1L} = j100V \Rightarrow X_1 = j10\Omega$

功率相关

复功率 $\tilde S = \dot U \cdot \dot I^* = U \cdot I \angle\varphi$ φ 为阻抗角.



$= \underbrace{UI\cos\varphi}_{\text{平均(有功)}} + j\underbrace{UI\sin\varphi}_{\text{无功}}$

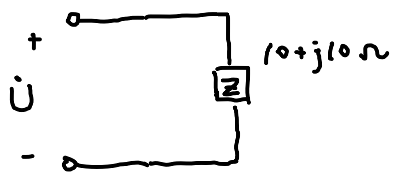
功率因数 $\lambda \triangleq \cos\varphi$

$\triangleq P + jQ$ (P: W, Q: var)

$|\tilde S| = S$ (视在功率) VA

4.25

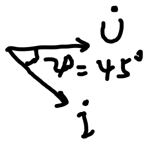
(1) 电路可以等效为:



$\omega = 2\pi f = 100\pi$ rad/s

$\dot I = \frac{\dot U}{Z} = 11 - j11$ A

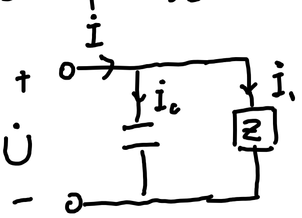
$\tilde S = \dot U \dot I^* = 2420 + j2420$ VA



$P = 2420$ W

$\lambda = \cos\varphi = \frac{\sqrt{2}}{2}$

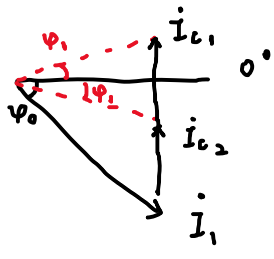
(2) 并上电容C后



$\dot I = \dot I_1 + \dot I_C$

$\varphi_0 = 45°$

$\cos\varphi_1 = 0.9$



$\left\{ \frac{11}{\sqrt{11^2 + (11 - I_C)^2}} = 0.9 \right.$

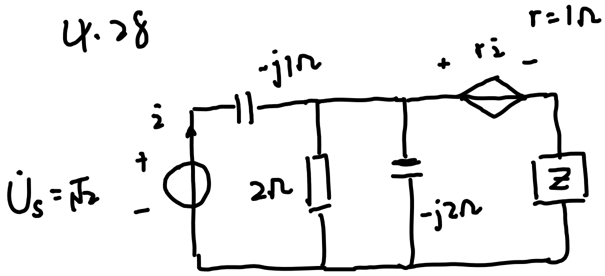
$\Rightarrow C = 82.1\mu F$ 或 $236.2\mu F$

$$\begin{cases} \sqrt{11^2+(11-I_c)^2} = 0.1 \\ I_c = \dot{U}/X_c \ , \ X_c = \dfrac{1}{\omega C} \end{cases} \Rightarrow C = 82.1\,\mu F \text{ 或 } 236.2\,\mu F$$

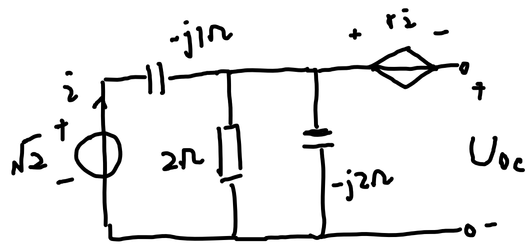
$\dot{I}_{c2}$ $\dot{I}_{c1}$

最大功率传输

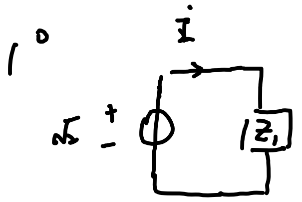
4.28



Thevenin 等效:



1°

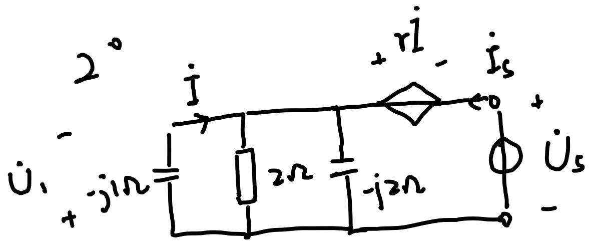


$$Z_1 = -j + \frac{2\times(-j2)}{2-j2} = 1-j2\,\Omega$$

$$\dot{I} = \frac{\dot{U}_s}{Z_1} = \frac{\sqrt{2}}{1-j2}\ A$$

$$\dot{U}_{oc} = \sqrt{2}\cdot\frac{1-j}{1-2j} - \frac{\sqrt{2}}{1-j2} = \frac{2}{5}\sqrt{2} - j\frac{\sqrt{2}}{5}\ V$$

2°



$$\dot{U}_s + r\dot{I} = -\dot{I}\cdot(-j1)$$

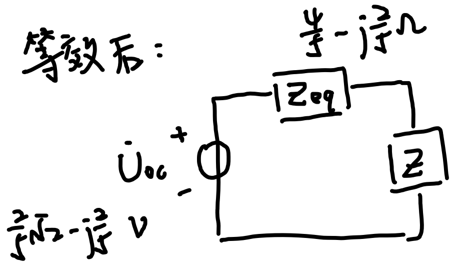
$$\dot{I} = \frac{\dot{U}_s}{j-1}\ ,\quad \dot{U}_1 = -j\dot{I}$$

$$\dot{I}_s + \dot{I} + \frac{\dot{U}_1}{2} + \frac{\dot{U}_1}{-j2} = 0$$

$$\dot{I}_s = \left(-\frac{3}{2} + j\frac{1}{2}\right)\dot{I}$$

$$Z_{eq} = \dot{U}_s/\dot{I}_s = \frac{j-1}{-\frac{3}{2}+j\frac{1}{2}} = \frac{4}{5} - j\frac{2}{5}\,\Omega$$

等效后:

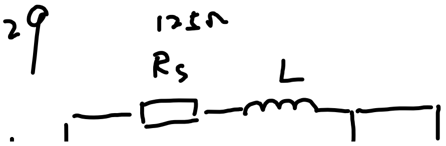


Z 可任意调节 ⇒ 共轭匹配

$$Z = Z_{eq}^* = \frac{4}{5} + j\frac{2}{5}\,\Omega$$

$$P_{max} = |U_{oc}|^2/4R = \frac{1}{8}\ W \qquad (R = Re\{Z\} = 4/5\,\Omega)$$

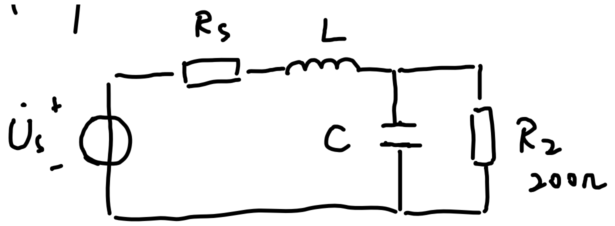
4.29



R_s . L . C 等效阻抗.

$-200\,jX_c$

1



R_2、L、C 等效阻抗

$$Z = jX_L + \frac{-200jX_C}{200-jX_C}$$

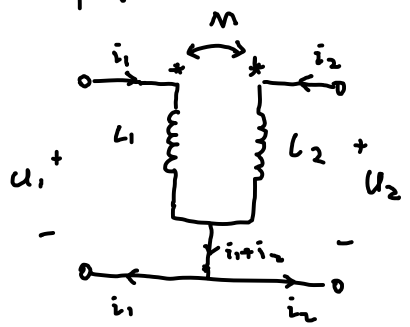
共轭匹配： $Z = R_s^* = R_s = 125\Omega$

$$jX_L + \frac{-200jX_C}{200-jX_C} = 125 \Rightarrow X_C X_L + j(200X_L - 75X_C) = 25000$$

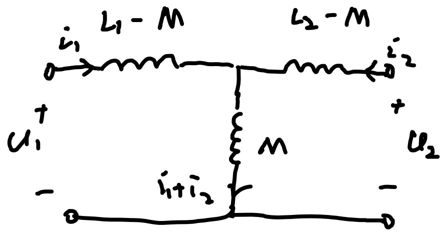
$$\Rightarrow L = 0.485\,mH,\quad C = 0.0194\,\mu F,\quad P_{max} = \frac{U_s^2}{4R_s} = \frac{1}{4\times125} = 2mW$$

耦合电感及等效

a. 三端等效

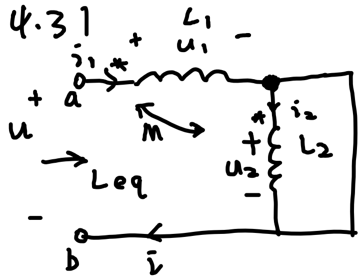


去耦合 ⇒



*若为异名端相连则将 M 改成 (−M) 即可

4.31



法一（从性质出发）

$$\begin{cases} u_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \\ u_2 = M\frac{di_1}{dt} + L_2\frac{di_2}{dt} \end{cases}$$

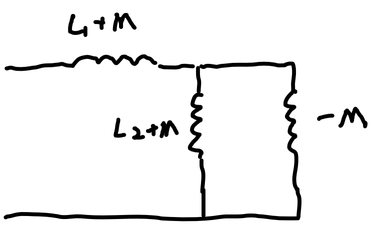
$$U_2 = 0 \Rightarrow \frac{di_2}{dt} = -\frac{M}{L_2}\frac{di_1}{dt}$$

$$U_1 = L_1\frac{di_1}{dt} - \frac{M^2}{L_2}\frac{di_1}{dt} = \left(L_1 - \frac{M^2}{L_2}\right)\frac{di_1}{dt}$$

对于端口 a.b，有 $u = L_{eq}\frac{di}{dt}$， $u = u_1 + u_2 = u_1$， $i = i_1$

$$\Rightarrow u_1 = L_{eq}\frac{di_1}{dt} \Rightarrow L_{eq} = L_1 - \frac{M^2}{L_2}$$

法二. 三端等效

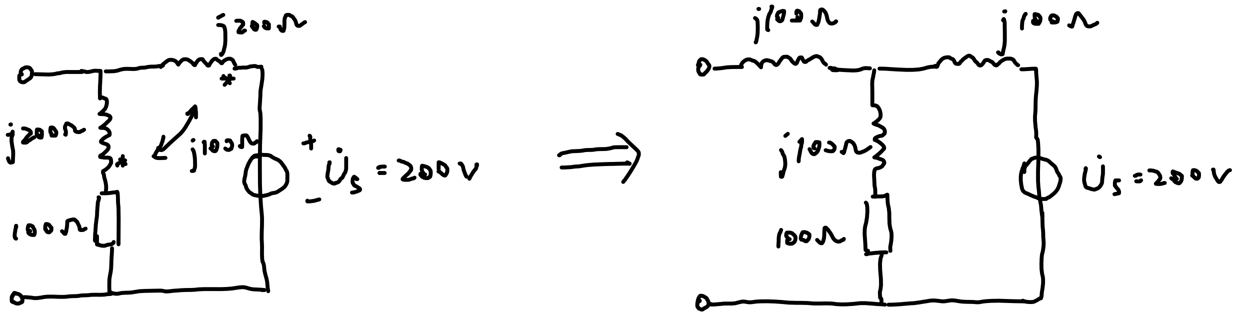


$$L_{eq} = L_1 + M + \frac{-M(L_2+M)}{L_2+M-M}$$

$$= L_1 + M - M - \frac{M^2}{L_2}$$

$$= L_1 - \frac{M^2}{L_2}$$

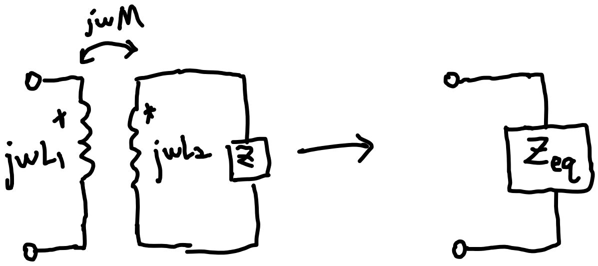
4.33 (b)



$$1^\circ \quad \dot{U}_{oc} = \frac{100+j100}{100+j200} \dot{U}_s = 120 - j40 \text{ V} = 126.49\angle -18.43^\circ \text{ V}$$

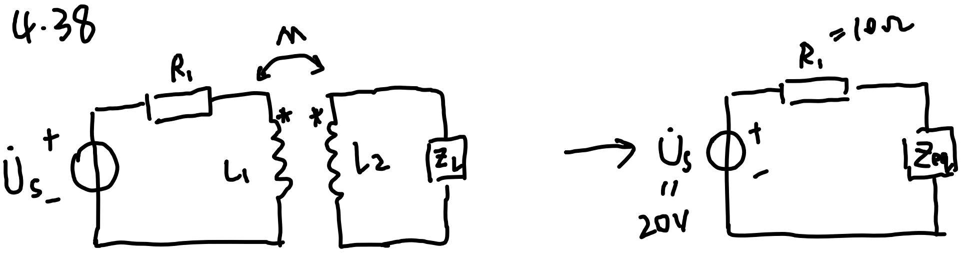
$$2^\circ \quad Z_{eq} = j100 + \frac{j100(100+j100)}{100+j200} = 20 + j60\ \Omega = 161.24\angle 82.87^\circ\ \Omega$$

6. 耦合电感阻抗变换



$$Z_{eq} = j\omega L_1 + \frac{(\omega M)^2}{Z + j\omega L_2}$$

4.38



$$Z_{eq} = \frac{(\omega M)^2}{Z_L + j\omega L_2} + j\omega L_1 = 10\ \Omega \quad , \quad M = K\sqrt{L_1 L_2} = 0.2\text{ H}$$

draft:

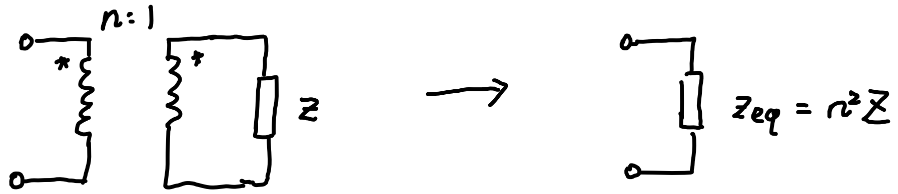
$$j100 + \frac{400}{Z_L + j100} = 10 \Rightarrow Z_L = \frac{400}{10 - j100} - j100\ \Omega$$

计算器

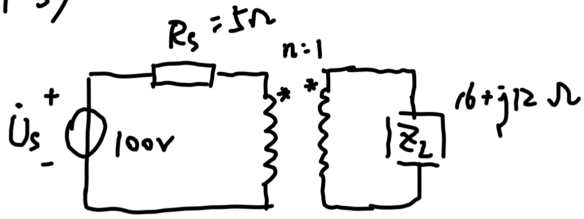
$\xrightarrow{计算器}$ $Z_L = \frac{40}{101} - j\frac{9700}{101} = 96.04\angle -89.76^\circ\ \Omega$

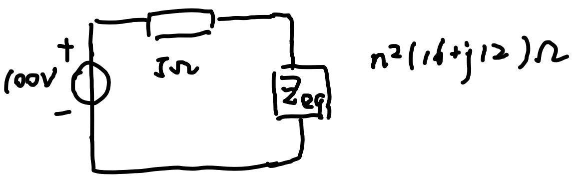
$P_{max} = \frac{U_s^2}{4R} = 10W.$

C. 理想变压器电阻变换.



4.37





由共模匹配:

$20 \cdot n^2 = 5$

$n = \frac{1}{2}$

$\dot{I} = \frac{\dot{U}_s}{Z_{eq}+R_s} = \frac{100}{9+j3} = 10 - j\frac{10}{3}\ A$

$\tilde{S} = \dot{I}\dot{I}^* \cdot Z_{eq} = \frac{4000}{9} + j\frac{1000}{3}\ VA$

$P_{max} = Re\{\tilde{S}\} = \frac{4000}{9}\ W$