一、

1. ×

$|\psi(x)|^2$ 的含义是几率密度.

$\psi(x)$ 代表几率幅

2. ×

$\psi(x)$ 与 $C\psi(x)$ 对应同一个量子状态

3. ×

由于量子态发生塌缩，这种方法不可行

需要制备大量处于 $|\psi\rangle$ 的体系，分别测量

再统计平均.

4. ×

多粒子体系遵循多粒子体系的薛定谔方程

5. ×

$[\hat{x},\hat{p}]=i\hbar$ 与表象无关.

二、

1. ABD

可观测力学量对应厄米矩阵：$A^{+}=A$

2. AB

A: $L_y = zp_x - xp_z$ 与 y 对易

B: 略.(作业)

C. $\hat{L}_x + i\hat{L}_y$ 不是厄米算符.

D. $\hat{x} + i\hat{p}$ 不是厄米算符.

3. BCD

代入验证 $-\hbar^2 \frac{\partial^2}{\partial \varphi^2}\psi = \lambda \psi$ 是否成立

4. C

代入验证 $\left(x - i\hbar\frac{\partial}{\partial x}\right)\psi(x) = \lambda\psi(x)$ 是否成立

5. BCD

代入方程 $i\hbar\frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t)$

验证是否成立

三 1 (1) $1=\int |\psi(x)|^2 dx = \int_0^a N^2 dx = N^2 a$

$\Rightarrow N = \sqrt{\frac{1}{a}}$

$\langle x \rangle = \int x |\psi(x)|^2 dx = \int_0^a \frac{x}{a} dx = \frac{1}{a}\frac{1}{2}a^2 = \frac{a}{2}$

$\langle x^2 \rangle = \int x^2 |\psi(x)|^2 dx = \int_0^a \frac{x^2}{a} dx = \frac{1}{a}\frac{1}{3}a^3 = \frac{a^2}{3}$

(2.) $C(p) = \frac{1}{\sqrt{2\pi\hbar}} \int e^{-\frac{i}{\hbar}px} \psi(x)\, dx$

$= \frac{1}{\sqrt{2\pi\hbar a}} \int_0^a e^{-\frac{i}{\hbar}px} dx$

$= \frac{i\hbar}{\sqrt{2\pi\hbar a}} \left(e^{-\frac{i}{\hbar}pa} - 1\right)/p$

$= \sqrt{\frac{2\hbar}{\pi a}}\, e^{-i\frac{pa}{2\hbar}} \frac{\sin\left(\frac{pa}{2\hbar}\right)}{p}$.

$\langle p \rangle = \int p |C(p)|^2 dp = \frac{2\hbar}{\pi a} \int_{-\infty}^{+\infty} \frac{\sin^2\left(\frac{pa}{2\hbar}\right)}{p} dp$ 奇函数

$= 0$

2. (1) $\hat{A}\hat{B} = \hat{B}\hat{A}$ ⇒ 可以同时测量.

$\hat{A}$ 的本征值与本征态:

$$\lambda_1^A = 1 \qquad |\psi_1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\lambda_2^A = 1 \qquad |\psi_2\rangle = \frac{1}{2}\begin{pmatrix} 0 \\ 1+i \\ \sqrt{2} \end{pmatrix}$$

$$\lambda_3^A = -1 \qquad |\psi_3\rangle = \frac{1}{2}\begin{pmatrix} 0 \\ 1+i \\ -\sqrt{2} \end{pmatrix}$$

这三个也是 $\hat{B}$ 的本征态, 对应的本征值分别是 $\lambda_1^B = -1$. $\lambda_2^B = 1$ $\lambda_3^B = 1$

(2)

$$\langle\psi_1|\psi\rangle = (1, 0, 0)\frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{3}}$$

$$\langle\psi_2|\psi\rangle = \frac{1}{2}(0, 1-i, \sqrt{2})\frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$= \frac{1}{2\sqrt{3}}(1+\sqrt{2}-i)$$

$$\langle\psi_3|\psi\rangle = \frac{1}{2}(0, 1-i, -\sqrt{2})\frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$= \frac{1}{2\sqrt{3}}(1-\sqrt{2}-i)$$

可能的结果为

① $A=1$. 几率为 $|\langle\psi_1|\psi\rangle|^2+|\langle\psi_2|\psi\rangle|^2$

$$=\frac{1}{3}+\frac{1}{4\times 3}\left[(1+\sqrt{2})^2+1\right]$$

$$=\frac{2}{3}+\frac{\sqrt{2}}{6}$$

② $A=-1$ 几率为 $|\langle\psi_3|\psi\rangle|^2$

$$=\frac{1}{4\times 3}\left[(1-\sqrt{2})^2+1\right]$$

$$=\frac{1}{3}-\frac{\sqrt{2}}{6}$$

(3) 第一步测得 $A=1$. 则 $|\psi\rangle$ 塌缩为

$$|\psi'\rangle=\langle\psi_1|\psi\rangle|\psi\rangle+\langle\psi_2|\psi\rangle|\psi_2\rangle$$

进行 B 测量. 有 $|\langle\psi_1|\psi\rangle|^2$ 的相对几率得 $B=-1$

$|\langle\psi_2|\psi\rangle|^2$ ...... $B=1$

绝对值:
$$P_{B=-1} = \frac{1}{3} \Big/ \left(\frac{2}{3}+\frac{\sqrt{2}}{6}\right) = \frac{4-\sqrt{2}}{7}$$

$$P_{B=1} = \left(\frac{1}{3}+\frac{\sqrt{2}}{6}\right) \Big/ \left(\frac{2}{3}+\frac{\sqrt{2}}{6}\right) = \frac{3+\sqrt{2}}{7}$$

(4) $|\psi\rangle = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\i\end{pmatrix}$

$$\langle\psi_1|\psi\rangle = (1,0,0)\frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\i\end{pmatrix} = \frac{1}{\sqrt{3}}$$

$$\langle\psi_2|\psi\rangle = \frac{1}{2}(0,1-i,\sqrt{2})\frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\i\end{pmatrix}$$

$$= \frac{1}{2\sqrt{3}}(1-i+i\sqrt{2})$$

$$= \frac{1}{2\sqrt{3}}\left[1+(\sqrt{2}-1)i\right]$$

$$\langle\psi_3|\psi\rangle = \frac{1}{2}(0,1-i,-\sqrt{2})\frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\i\end{pmatrix}$$

$$= \frac{1}{2\sqrt{3}}\left[1-(\sqrt{2}+1)i\right]$$

$$|\psi(t)\rangle = \langle\psi_1|\psi\rangle e^{-\frac{i}{\hbar}E_0\lambda_1^A t}|\psi_1\rangle$$
$$+\langle\psi_2|\psi\rangle e^{-\frac{i}{\hbar}E_0\lambda_2^A t}|\psi_2\rangle$$
$$+\langle\psi_3|\psi\rangle e^{-\frac{i}{\hbar}E_0\lambda_3^A t}|\psi_3\rangle$$
$$= \frac{1}{3}e^{-\frac{i}{\hbar}E_0 t}\begin{pmatrix}1\\0\\0\end{pmatrix}$$
$$+\frac{1}{4\sqrt{3}}\left[1+(\sqrt{2}-1)i\right]e^{-iE_0t}\begin{pmatrix}0\\1+i\\\sqrt{2}\end{pmatrix}$$
$$+\frac{1}{4\sqrt{3}}\left[1-(\sqrt{2}+1)i\right]e^{iE_0t}\begin{pmatrix}0\\1+i\\-\sqrt{2}\end{pmatrix}$$