

数字信号处理B

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HW3

Exercise 1

(1)

$$\begin{aligned} Y_1(k) &= \sum_{n=0}^{2N-1} y_1(n) e^{-j\frac{2\pi}{2N}nk} \quad k = 0, 1, \dots, 2N-1 \\ &= \sum_{n \text{ 为偶数}} x\left(\frac{n}{2}\right) e^{-j\frac{2\pi}{2N}nk} \\ &\quad \text{令 } n = 2i \\ &= \sum_{i=0}^{N-1} x(i) e^{-j\frac{2\pi}{N}ik} \\ &= X_1(k) \quad k = 0, 1, \dots, 2N-1 \\ &= \begin{cases} X_1(k) & k = 0, 1, \dots, N-1 \\ X_1(k-N) & k = N, N+1, \dots, 2N-1 \end{cases} \end{aligned}$$

(2)

$$\begin{aligned} y_2(n) &= x(N-1-n) \\ Y_2(k) &= \sum_{n=0}^{N-1} y_2(n) e^{-j\frac{2\pi}{N}nk} \quad k = 0, 1, \dots, N-1 \\ &= \sum_{n=0}^{N-1} x(N-1-n) e^{-j\frac{2\pi}{N}nk} \\ &\quad \text{令 } i = N-1-n \\ &= \sum_{i=0}^{N-1} x(i) e^{-j\frac{2\pi}{N}(N-1-i)k} \\ &= e^{-j\frac{2\pi}{N}(N-1)k} \sum_{i=0}^{N-1} x(i) e^{-j\frac{2\pi}{N}i(-k)} \\ &= e^{-j\frac{2\pi}{N}(N-1)k} X(-k) \\ &= e^{-j\frac{2\pi}{N}(N-1)k} X^*(k) \end{aligned}$$

$$\begin{aligned}
y_3(n) &= (-1)^n x(n) \\
Y_3(k) &= \sum_{n=0}^{N-1} y_3(n) e^{-j\frac{2\pi}{N}nk} \quad k = 0, 1, \dots, N-1 \\
&= \sum_{n=0}^{N-1} (-1)^n x(n) e^{-j\frac{2\pi}{N}nk} \\
&= \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}nk - j\pi n} \\
&= \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}n(k+N/2)} \\
&= X(k + N/2)
\end{aligned}$$

Exercise 2

$$\begin{aligned}
\sum_{n=0}^{N-1} |x(n)|^2 &= \sum_{n=0}^{N-1} x(n) x^*(n) \\
&= \sum_{n=0}^{N-1} x(n) \left(\frac{1}{N} \sum_{k=0}^{N-1} X^*(k) W_N^{-kn} \right) \\
&= \frac{1}{N} \sum_{k=0}^{N-1} X^*(k) \left(\sum_{n=0}^{N-1} x(n) W_N^{-kn} \right) \\
&= \frac{1}{N} \sum_{k=0}^{N-1} X(k) X^*(k) \\
&= \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2
\end{aligned}$$

Exercise 3

$$\begin{aligned}
\sum_{n=0}^{N-1} x(n) y^*(n) &= \sum_{n=0}^{N-1} x(n) y^*(n) \\
&= \sum_{n=0}^{N-1} x(n) \left(\frac{1}{N} \sum_{k=0}^{N-1} Y^*(k) W_N^{-kn} \right) \\
&= \frac{1}{N} \sum_{k=0}^{N-1} Y^*(k) \left(\sum_{n=0}^{N-1} x(n) W_N^{-kn} \right) \\
&= \frac{1}{N} \sum_{k=0}^{N-1} X(k) Y^*(k)
\end{aligned}$$