数字信号处理B

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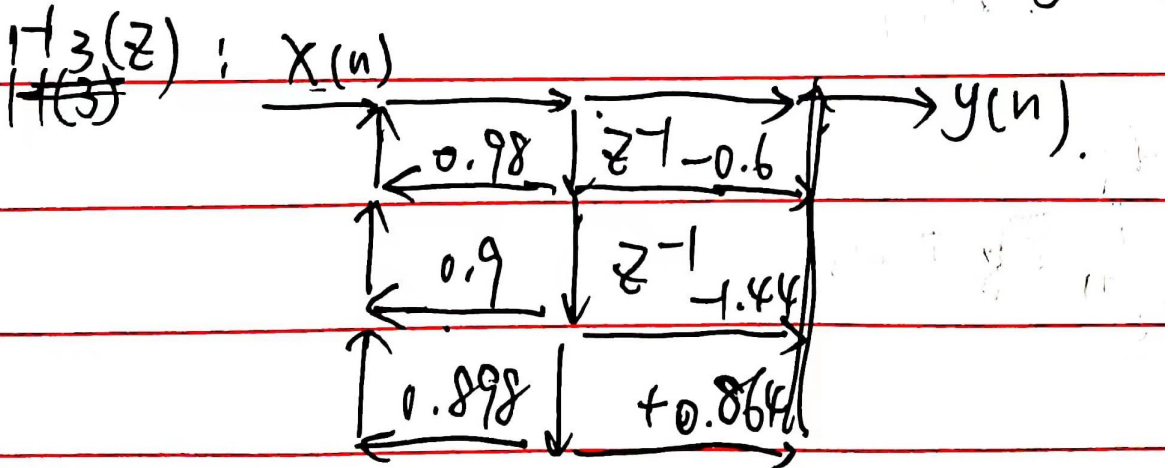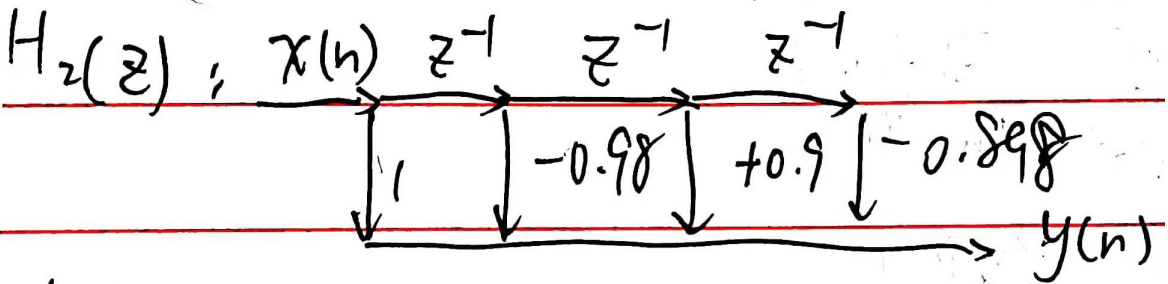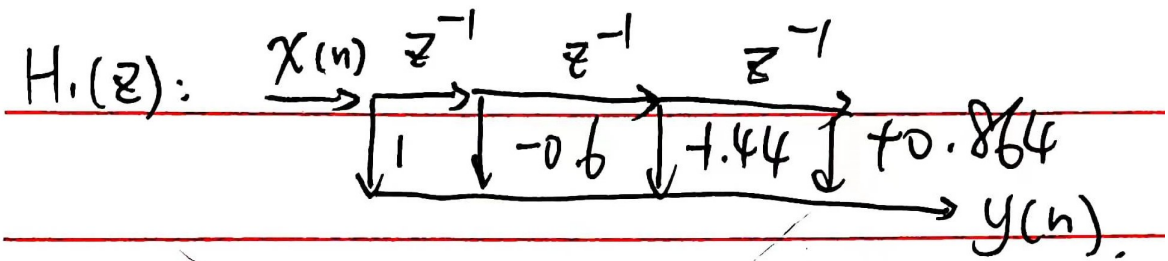## HW7

Exercise 1

(1)

$$
\begin{aligned}
|H_1(\omega)|^2 &= \frac{\frac{13}{9} - \frac{4}{3}\cos\omega}{\frac{10}{9} - \frac{2}{3}\cos\omega} \\
&= \frac{(1 - \frac{2}{3}e^{j\omega})(1 - \frac{2}{3}e^{-j\omega})}{(1 - \frac{1}{3}e^{j\omega})(1 - \frac{1}{3}e^{-j\omega})} \\
&= \frac{(1 - \frac{2}{3}e^{-j\omega})}{(1 - \frac{1}{3}e^{-j\omega})} \cdot \frac{(1 - \frac{2}{3}e^{j\omega})}{(1 - \frac{1}{3}e^{j\omega})} \\
&= H_1(\omega) \cdot H_1(-\omega)
\end{aligned}
$$

$$
H_1(\omega) = \frac{(1 - \frac{2}{3}e^{-j\omega})}{(1 - \frac{1}{3}e^{-j\omega})}
$$

(2)

$$
\begin{aligned}
|H_2(\omega)|^2 &= \frac{4(1 - \alpha^2)}{(1 + \alpha^2) - 2\alpha\cos\omega} \\
&= \frac{2(1+\alpha)2(1-\alpha)}{(1 - \alpha e^{j\omega/2})(1 - \alpha e^{-j\omega/2})} \\
&= \frac{2(1-\alpha)}{(1 - \alpha e^{-j\omega/2})} \cdot \frac{2(1+\alpha)}{(1 - \alpha e^{j\omega/2})}
\end{aligned}
$$

$$
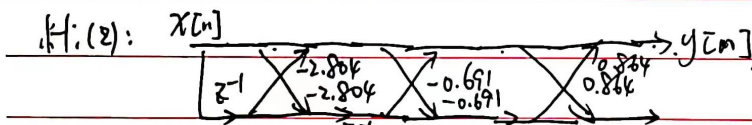H_2(\omega) = \frac{2(1-\alpha)}{(1 - \alpha e^{-j\omega/2})}
$$

Exercise 2
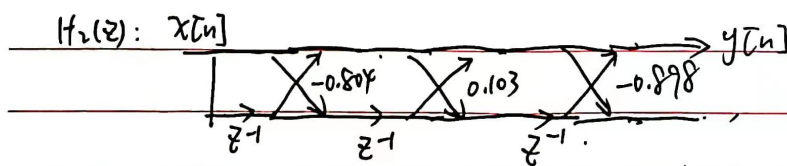
(1)

$H_1(z)$:

$$x(n) \xrightarrow{z^{-1}} \xrightarrow{z^{-1}} \xrightarrow{z^{-1}} +0.864$$

with taps 1, -0.6, $+1.44$ $\to y(n)$

$H_2(z)$:

$$x(n) \xrightarrow{z^{-1}} \xrightarrow{z^{-1}} \xrightarrow{z^{-1}} -0.898$$

with taps 1, -0.98, $+0.9$ $\to y(n)$

$H_3(z)$: ~~$H(3)$~~

$x(n) \to \to \to y(n)$

-0.98 $\quad z^{-1}-0.6$

0.9 $\quad z^{-1}$ $+1.44$

0.898 $\quad +0.864$

(2)

$H_1(z)$: ${}^A b_3^{(1)} = -0.6$ $\quad {}^D b_2^{(1)} = \frac{1}{1-k_3^2}\left[b_2^{(1)} + k_3 b_3^{(2)}\right] = -0.867$ $\quad b_1^{(1)} = \frac{1}{1-k_2^2}\left[b_2^{(1)} + k_2 b_2^{(1)}\right] = -2.804$

${}^A b_3^{(2)} = -1.44$ $\quad {}^E b_2^{(2)} = \frac{1}{1-k_3^2}\left[b_3^{(2)} + k_3 b_3^{(1)}\right] = -0.691$ $\quad {}^F k_2 = -b_2^{(2)} = 0.691$ $\quad k_1 = -b_1^{(1)}$

$b_3^{(3)} = 0.864$ $\quad {}^C k_3 = -b_3^{(3)} = -0.864$ $\quad = 2.804$

$H_1(z)$: $x[n] \to \to y[n]$

z^{-1} $\quad$ -2.804 / -2.804 $\quad$ -0.691 / -0.691 $\quad$ 0.864 / 0.864

$H_2(z)$: $b_3^{(1)} = -0.98$ $\quad b_2^{(1)} = \frac{1}{1-k_3^2}\left[b_3^{(1)} + k_3 b_3^{(2)}\right] = -0.887$ $\quad b_1^{(1)} = \frac{1}{1-k_2^2}\left[b_2^{(1)} + k_2 b_2^{(1)}\right] = -0.804$

$b_3^{(2)} = 0.9$ $\quad b_2^{(2)} = \frac{1}{1-k_3^2}\left[b_3^{(2)} + k_3 b_3^{(1)}\right] = 0.103$ $\quad k_2 = -b_2^{(2)} = -0.103$ $\quad k_1 = -b_1^{(1)} = 0.804$

$b_3^{(3)} = -0.898$ $\quad k_3 = -b_3^{(3)} = 0.898$

$H_2(z)$: $x[n] \to \to y[n]$

z^{-1} $\quad$ -0.804 $\quad z^{-1}$ 0.103 $\quad z^{-1}$ -0.898

$H_3(z) := \dfrac{H_1(z)}{H_2(z)}$. 由前面可知. $H_2(z)$ 的 $k_3 = 0.898$ $k_2 = -0.103$ $k_1 = 0.804$.

$\therefore \dfrac{1}{H_2(z)}$: $x[n]$

0.898   -0.103   0.804

-0.898 z^{-1} +0.103 z^{-1} -0.804 z^{-1} $y[n]$
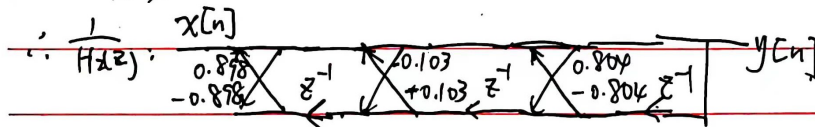
$c_3 = b_3 = 0.864$.

$c_2 = b_2 - c_3 a_3^{(1)} = -1.44 - 0.864 \times (-0.98) = -0.593$.

$c_1 = b_1 - c_2 a_2^{(1)} - c_3 a_3^{(2)} = -0.6 - (-0.593) \times (-0.887) - 0.864 \times 0.9 = -1.904$.

$\therefore H_3(z)$: $x[n]$ $c_0 = b_0 - c_1 a_1^{(1)} - c_2 a_2^{(2)} - c_3 a_3^{(3)} = 0.306$

0.898   -0.103   0.804

-0.898 z^{-1} +0.103 z^{-1} -0.804 z^{-1}

0.864   -0.593   -1.904   0.306

$y[n]$.