第七章作业

7-1 在同一介质中同时传播两个均匀平面波，它们的电场强度分别为

$$\vec{E}_{1m}=E_1e^{-jw_1z/c}\hat{x},\vec{E}_{2m}=E_2e^{-jw_2z/c}\hat{x}$$

并且 $w_1\neq w_2$，证明总的平均能流密度等于波的平均能流密度之和

$$\vec{H}_{1m}=\frac{1}{\eta}\hat{z}\times\vec{E}_{1m}=\sqrt{\frac{\varepsilon}{\mu}}\hat{z}\times\vec{E}_{1m}=\sqrt{\frac{\varepsilon}{\mu}}E_1e^{-jw_1z/c}\hat{y}\qquad \vec{H}_{2m}=\sqrt{\frac{\varepsilon}{\mu}}\hat{z}\times\vec{E}_{2m}=\sqrt{\frac{\varepsilon}{\mu}}E_2e^{-jw_2z/c}\hat{y}$$

$$\vec{E}=\vec{E}_{1m}+\vec{E}_{2m}=\left(E_1e^{-jw_1z/c}+E_2e^{-jw_2z/c}\right)\hat{x}\qquad \vec{H}=\vec{H}_{1m}+\vec{H}_{2m}=\sqrt{\frac{\varepsilon}{\mu}}\left(E_1e^{-jw_1z/c}+E_2e^{-jw_2z/c}\right)\hat{y}$$

$$\bar{\vec{S}}=\mathrm{Re}\left(\frac{1}{2}\vec{E}\times\vec{H}^*\right)=\mathrm{Re}\left(\frac{1}{2}\left(\left(E_1e^{-jw_1z/c}+E_2e^{-jw_2z/c}\right)\hat{x}\times\sqrt{\frac{\varepsilon}{\mu}}\left(E_1e^{jw_1z/c}+E_2e^{jw_2z/c}\right)\hat{y}\right)\right)$$

$$=\frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}\left(E_1^2+2E_1E_2\cos((w_1-w_2)z/c)+E_2^2\right)\hat{z}$$ 总的平均能流密度

$$\bar{\vec{S}}_{11}=\mathrm{Re}\left(\frac{1}{2}\vec{E}_{1m}\times\vec{H}_{1m}^*\right)=\mathrm{Re}\left(\frac{1}{2}\left(E_1e^{-jw_1z/c}\hat{x}\times\sqrt{\frac{\varepsilon}{\mu}}E_1e^{jw_1z/c}\hat{y}\right)\right)=\frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}E_1^2\hat{z}\qquad \bar{\vec{S}}_{22}=\mathrm{Re}\left(\frac{1}{2}\vec{E}_{2m}\times\vec{H}_{2m}^*\right)=\frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}E_2^2\hat{z}$$

$$\bar{\vec{S}}_{12}=\mathrm{Re}\left(\frac{1}{2}\vec{E}_{1m}\times\vec{H}_{2m}^*\right)=\mathrm{Re}\left(\frac{1}{2}\left(E_1e^{-jw_1z/c}\hat{x}\times\sqrt{\frac{\varepsilon}{\mu}}E_2e^{jw_2z/c}\hat{y}\right)\right)=\frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}E_1E_2\cos((w_2-w_1)z/c)\hat{z}$$

$$\bar{\vec{S}}_{21}=\mathrm{Re}\left(\frac{1}{2}\vec{E}_{2m}\times\vec{H}_{1m}^*\right)=\frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}E_1E_2\cos((w_1-w_2)z/c)\hat{z}$$

$$\bar{\vec{S}}=\bar{\vec{S}}_{11}+\bar{\vec{S}}_{22}+\bar{\vec{S}}_{12}+\bar{\vec{S}}_{21}$$

7-2 自由空间中给定

$$\vec{E}(z,t)=30\pi\cos(10^8 t-kz)\hat{x}$$
$$\vec{H}(z,t)=H_m\cos(10^8 t-kz)\hat{y}$$

求磁场强度的幅度H_m和传播常数 k

$$H_m=\frac{E_m}{\eta}=\frac{30\pi}{120\pi}=0.25(A/m) \qquad k=w\sqrt{\mu\varepsilon}=\frac{w}{c}=\frac{10^8}{3\times10^8}=\frac{1}{3}(rad/m)$$

7-4 证明电磁波在导电媒质内传播时场量的衰减约为每波长55dB。

证明：$k=\beta-j\alpha \qquad \alpha=\omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]} \quad \beta=\omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}$

在良导体中，有$\alpha\approx\beta=\sqrt{\frac{\mu\sigma\omega}{2}} \qquad \lambda=\frac{2\pi}{\beta} \quad \alpha\lambda=2\pi$

电磁波传播距离z后，场量的衰减为$e^{-\alpha z}$

故则每波长的衰减量为：$-20\log e^{-\alpha\lambda}=20\alpha\lambda\log e=20\beta\frac{2\pi}{\beta}\log e=40\pi\log e\approx 55dB$

7-5 电场为

$$\vec{E}(z,t)=\mathrm{Re}\left[\left(E_{x0}\hat{x}+E_{y0}e^{j\phi}\hat{y}\right)e^{j(wt-kz)}\right]$$

的椭圆极化均匀平面波，在波阻抗为Z_c的介质中传播，其中E_{x0},E_{y0}是实数

求（1）该波的磁场强度

（2）该波的波印亭矢量的瞬时值和平均值

$$\vec{H}(z,t)=\frac{1}{\eta}\hat{z}\times\vec{E}(z,t)=\mathrm{Re}\left[\frac{1}{\eta}\hat{z}\times\left(E_{x0}\hat{x}+E_{y0}e^{j\varphi}\hat{y}\right)e^{j(wt-kz)}\right]=\mathrm{Re}\left[\left(\frac{E_{x0}}{\eta}\hat{y}-\frac{E_{y0}}{\eta}e^{j\varphi}\hat{x}\right)e^{j(wt-kz)}\right]$$

$$\vec{S}=\vec{E}\times\vec{H}=\left(E_{x0}\cos(wt-kz)\hat{x}+E_{y0}\cos(wt-kz+\varphi)\hat{y}\right)\times\left(\frac{E_{x0}}{\eta}\cos(wt-kz)\hat{y}-\frac{E_{y0}}{\eta}\cos(wt-kz+\varphi)\hat{x}\right)$$

$$=\frac{E_{x0}^2}{\eta}\cos^2(wt-kz)\hat{z}+\frac{E_{y0}^2}{\eta}\cos^2(wt-kz+\varphi)\hat{z}$$

$$\bar{\vec{S}}=\mathrm{Re}\left(\frac{1}{2}\vec{E}(z)\times\vec{H}^*(z)\right)=\mathrm{Re}\left(\frac{1}{2}\left(\left(E_{x0}\hat{x}+E_{y0}e^{j\varphi}\hat{y}\right)e^{-jkz}\times\left(\frac{E_{x0}}{\eta}\hat{y}-\frac{E_{y0}}{\eta}e^{-j\varphi}\hat{x}\right)e^{jkz}\right)\right)$$

$$=\mathrm{Re}\left(\frac{1}{2}\left(\frac{E_{x0}^2+E_{y0}^2}{\eta}\hat{z}\right)\right)=\frac{1}{2}\left(\frac{E_{x0}^2+E_{y0}^2}{\eta}\hat{z}\right)$$

7-6 证明任何椭圆极化波均可分解为两个向相反方向旋转的圆极化波之和

任何椭圆极化波可表示成 $\vec{E}=(E_x\hat{x}+jE_y\hat{y})e^{-jkz}$ 其中 $E_x\neq E_y$

任何圆极化波可表示成 $\vec{E}=(A\hat{x}+jA\hat{y})e^{-jkz}$

设任何椭圆极化波均可分解为两个向相反方向旋转的圆极化波 $\vec{E}_1,\vec{E}_2$ 之和, 即

$$\vec{E}=\vec{E}_1+\vec{E}_2$$

$$(E_x\hat{x}+jE_y\hat{y})e^{-jkz}=(A_1\hat{x}+jA_1\hat{y})e^{-jkz}+(A_2\hat{x}-jA_2\hat{y})e^{-jkz}$$

$$E_xe^{-jkz}\hat{x}+jE_ye^{-jkz}\hat{y}=\left(A_1e^{-jkz}+A_2e^{-jkz}\right)\hat{x}+\left(jA_1e^{-jkz}-jA_2e^{-jkz}\right)\hat{y}$$

$$\begin{cases}E_xe^{-jkz}=A_1e^{-jkz}+A_2e^{-jkz}\\ jE_ye^{-jkz}=jA_1e^{-jkz}-jA_2e^{-jkz}\end{cases}$$

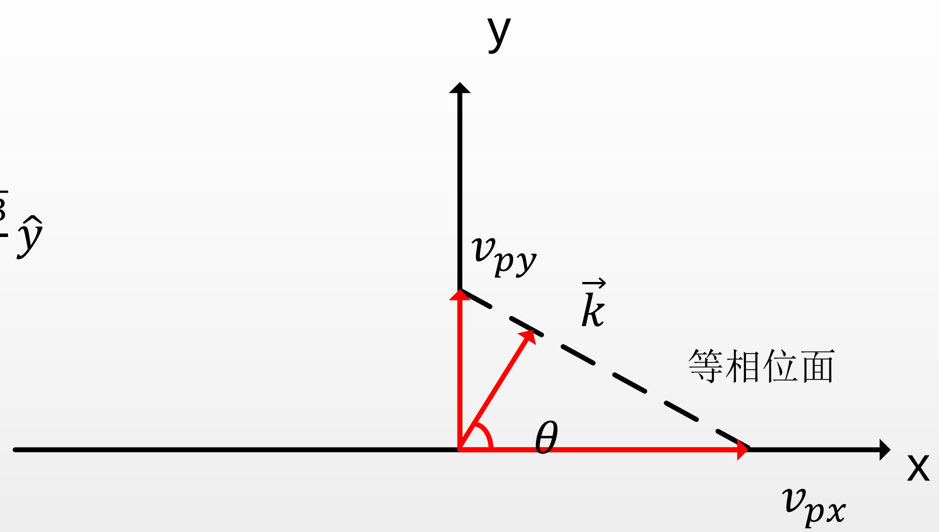
$$\begin{cases}A_1=\dfrac{\left(E_x+E_y\right)}{2}\\ A_2=\dfrac{\left(E_x-E_y\right)}{2}\end{cases}$$

$$\begin{cases}\vec{E}_1=\dfrac{1}{2}\left(\left(E_x+E_y\right)\hat{x}+j\left(E_x+E_y\right)\hat{y}\right)e^{-jkz}\\ \vec{E}_2=\dfrac{1}{2}\left(\left(E_x-E_y\right)\hat{x}-j\left(E_x-E_y\right)\hat{y}\right)e^{-jkz}\end{cases}$$

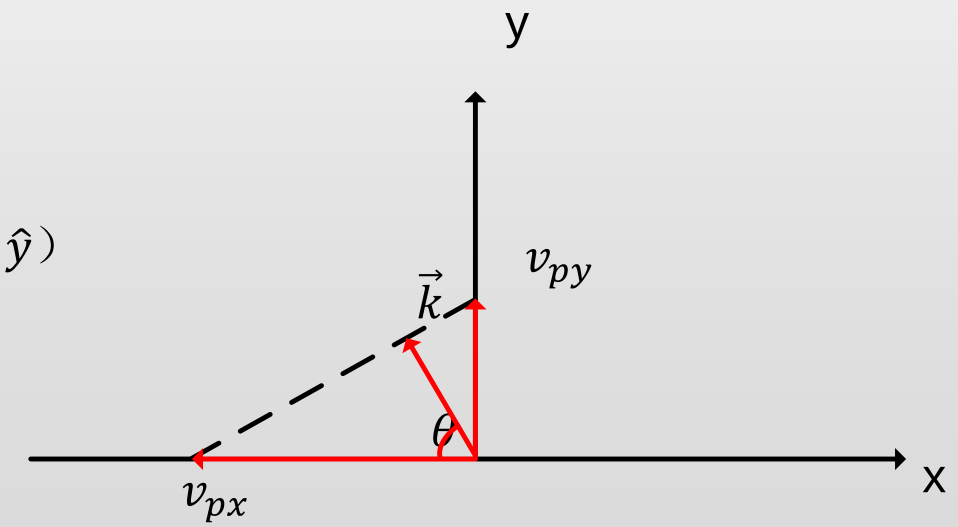
7-8 设有一均匀平面电磁波在自由空间传播，且$\vec{k}$位于xoy平面内，沿y轴的相速度为$2\sqrt{3}\times10^8 m/s$，求波的传播方向及其沿x轴的相速度。

由于该波为均匀平面电磁波且在自由空间传播，则有$v_p = c$（光速）

① $v_{py} = \dfrac{c}{\sin\theta}$ $\sin\theta = \dfrac{\sqrt{3}}{2}$ $\theta = \dfrac{\pi}{3}$ $v_{px} = \dfrac{c}{\cos\theta} = 6\times10^8 m/s$ $\vec{k} = \dfrac{1}{2}\hat{x} + \dfrac{\sqrt{3}}{2}\hat{y}$



②$v_{px} = \dfrac{c}{\cos\theta} = -6\times10^8 m/s$（“-”表示x负方向） $\vec{k} =$ （$-\dfrac{1}{2}\hat{x} + \dfrac{\sqrt{3}}{2}\hat{y}$）



7-9一圆极化均匀平面波垂直投射于一介质板上，入射电场为

$$\vec{E}_m = E_m(\hat{x} + j\hat{y})e^{j\beta z}$$

求反射波与折射波的电场强度，并分析它们的极化

由任意圆极化波可分解为两个方向正交的线极化波

圆极化波可分解为x方向和y方向线极化波，设x方向为水平极化，y方向为垂直极化

$$\vec{E}_{ixm} = E_m e^{j\beta z}\hat{x} \qquad \vec{E}_{iym} = jE_m e^{j\beta z}\hat{y}$$

由平行极化波与垂直极化波的反射与折射公式知

平行极化波（x方向线极化波）：

$$\vec{E}_{rxm} = \frac{\eta_1\cos\theta_i - \eta_2\cos\theta_t}{\eta_1\cos\theta_i + \eta_2\cos\theta_t}E_{ixm} = \frac{\eta_1-\eta_2}{\eta_1+\eta_2}E_m e^{-j\beta z}(-\hat{x}) \qquad \vec{E}_{txm} = \frac{2\eta_2\cos\theta_i}{\eta_1\cos\theta_i + \eta_2\cos\theta_t}E_{ixm} = \frac{2\eta_2}{\eta_1+\eta_2}E_m e^{j\beta z}\hat{x}$$

垂直极化波（y方向线极化波）：

$$\vec{E}_{rym} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}E_{iym} = \frac{\eta_2-\eta_1}{\eta_2+\eta_1}jE_m e^{-j\beta z}\hat{y} \qquad \vec{E}_{tym} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}E_{iym} = \frac{2\eta_2}{\eta_2+\eta_1}jE_m e^{j\beta z}\hat{y}$$

$$\vec{E}_{rm} = \vec{E}_{rxm} + \vec{E}_{rym} = \frac{\eta_2-\eta_1}{\eta_1+\eta_2}E_m e^{-j\beta z}\hat{x} + \frac{\eta_2-\eta_1}{\eta_2+\eta_1}jE_m e^{-j\beta z}\hat{y} = \frac{\eta_2-\eta_1}{\eta_1+\eta_2}E_m(\hat{x}+j\hat{y})e^{-j\beta z}$$ 为左旋圆极化波

$$\vec{E}_{tm} = \vec{E}_{txm} + \vec{E}_{tym} = \frac{2\eta_2}{\eta_1+\eta_2}E_m e^{j\beta z}\hat{x} + \frac{2\eta_2}{\eta_2+\eta_1}jE_m e^{j\beta z}\hat{y} = \frac{2\eta_2}{\eta_2+\eta_1}E_m(\hat{x}+j\hat{y})e^{j\beta z}$$ 为右旋圆极化波

7-12试证一个圆极化波的瞬时坡印廷矢量是一个与时间无关的常数。

设圆极化波的电场为 $\vec{E}(z,t)=E_0\cos(wt-kz)\hat{x}+E_0\sin(wt-kz)\hat{y}$

则磁场为 $\vec{H}(z,t)=\frac{1}{\eta}\hat{z}\times\vec{E}(z,t)=\frac{1}{\eta}\hat{z}\times\left(E_0\cos(wt-kz)\hat{x}+E_0\sin(wt-kz)\hat{y}\right)=\frac{E_0}{\eta}\cos(wt-kz)\hat{y}-\frac{E_0}{\eta}\sin(wt-kz)\hat{x}$

$$\vec{S}(z,t)=\vec{E}(z,t)\times\vec{H}(z,t)=\left(E_0\cos(wt-kz)\hat{x}+E_0\sin(wt-kz)\hat{y}\right)\times\left(\frac{E_0}{\eta}\cos(wt-kz)\hat{y}-\frac{E_0}{\eta}\sin(wt-kz)\hat{x}\right)=\frac{E_0^2}{\eta}\hat{z}$$

7-13一个右旋圆极化波垂直入射到位于z=0的理想导体板上，其电场为 $\vec{E}_m(z)=E_0(\hat{x}-j\hat{y})e^{-j\beta z}$

分析反射波的极化方式是什么，并求出z<0的半空间中电场和磁场的分布

由题可知，入射波由z<0区域，沿+z方向入射，故z<0的半空间内包含入射波与反射波

由电磁波对理想导体平面正入射的边界条件 $\vec{E}_{rm}(0)+\vec{E}_{im}(0)=0$ 知： $\vec{E}_{rm}(0)=-\vec{E}_m(0)=E_0(-\hat{x}+j\hat{y})$

$\vec{E}_{rm}(z)=E_0(-\hat{x}+j\hat{y})e^{j\beta z}$ $\quad\quad$ $\vec{H}_{rm}(z)=-\frac{1}{\eta}\hat{z}\times\vec{E}_{rm}(z)=\frac{1}{\eta}\hat{z}\times\left(E_0(-\hat{x}+j\hat{y})e^{j\beta z}\right)=\frac{E_0}{\eta}e^{j\beta z}(\hat{y}+j\hat{x})$ 左旋圆极化波

$$\vec{H}_{im}(z)=\frac{1}{\eta}\hat{k}\times\vec{E}_{im}(z)=\frac{1}{\eta}\hat{z}\times\left(E_0(\hat{x}-j\hat{y})e^{-j\beta z}\right)=\frac{E_0}{\eta}e^{-j\beta z}(\hat{y}+j\hat{x})$$

$$\vec{H}_m(z)=\vec{H}_{im}(z)+\vec{H}_{rm}(z)=\frac{E_0}{\eta}e^{-j\beta z}(\hat{y}+j\hat{x})+\frac{E_0}{\eta}e^{j\beta z}(\hat{y}+j\hat{x})=\frac{E_0}{\eta}\left(e^{-j\beta z}+e^{j\beta z}\right)(\hat{y}+j\hat{x})=\frac{2E_0}{\eta}\cos\beta z(\hat{y}+j\hat{x})$$

$$\vec{E}_m(z)=\vec{E}_{im}(z)+\vec{E}_{rm}(z)=E_0(\hat{x}-j\hat{y})e^{-j\beta z}+E_0(-\hat{x}+j\hat{y})e^{j\beta z}=E_0\left(e^{-j\beta z}-e^{j\beta z}\right)(\hat{x}-j\hat{y})=-2jE_0\sin\beta z(\hat{x}-j\hat{y})$$

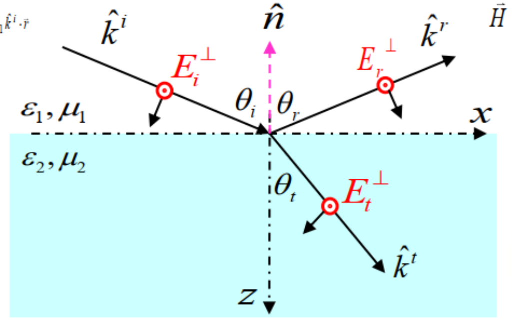
7-18 一垂直极化的均匀平面波投射到一介质分界面上，证明当发生全反射时，$(\theta_i > \theta_c)$，在两种介质分界面上坡印廷矢量 S 的平均值为零。（PPT CH7 P92）

$$\vec{E}^i = \hat{y}E_0^i e^{-jk_1\hat{k}^i\cdot\vec{r}}$$

$$\vec{H}^i = \frac{E_0^i}{\eta_1}(-\hat{x}\cos\theta_i + \hat{z}\sin\theta_i)e^{-jk_1\hat{k}^i\cdot\vec{r}}$$

$$\vec{E}^r = \hat{y}E_0^r e^{-jk_1\hat{k}^r\cdot\vec{r}}$$

$$\vec{H}^r = \frac{E_0^r}{\eta_1}(\hat{x}\cos\theta_i + \hat{z}\sin\theta_i)e^{-jk_1\hat{k}^r\cdot\vec{r}}$$



$$\vec{E}^t = \hat{y}E_0^t e^{-jk_2\hat{k}^t\cdot\vec{r}}$$

$$\vec{H}^t = \frac{E_0^t}{\eta_2}(-\hat{x}\cos\theta_t + \hat{z}\sin\theta_t)e^{-jk_2\hat{k}^t\cdot\vec{r}}$$

$$\bar{\vec{S}} = \frac{1}{2\eta}\left|\dot{\vec{E}}\right|^2\hat{k} \Rightarrow \begin{cases} \bar{\vec{S}}^i\cdot(-\hat{n}) = \dfrac{1}{2\eta_1}\left|\dot{\vec{E}}^i\right|^2\cos\theta_i \\ \bar{\vec{S}}^r\cdot\hat{n} = \dfrac{1}{2\eta_1}\left|\dot{\vec{E}}^r\right|^2\cos\theta_i \\ \bar{\vec{S}}^t\cdot(-\hat{n}) = \dfrac{1}{2\eta_2}\left|\dot{\vec{E}}^t\right|^2\cos\theta_t \end{cases}$$

$$\Rightarrow \boxed{R = \left|\frac{\dot{E}^r}{\dot{E}^i}\right|^2}$$

$$\Rightarrow \boxed{T = \frac{\eta_1\cos\theta_t}{\eta_2\cos\theta_i}\left|\frac{\dot{E}^t}{\dot{E}^i}\right|^2} = \frac{n_2\cos\theta_t}{n_1\cos\theta_i}\left|\frac{\dot{E}^t}{\dot{E}^i}\right|^2$$

$\mu_1 = \mu_2 = \mu_0$

边界条件:

$$\boxed{E_{1t} = E_{2t}} \Rightarrow E_0^i + E_0^r = E_0^t$$

$$\boxed{H_{1t} = H_{2t}} \Rightarrow -\frac{E_0^i}{\eta_1}\cos\theta_i + \frac{E_0^r}{\eta_1}\cos\theta_i = -\frac{E_0^t}{\eta_2}\cos\theta_t$$

$$R^\perp = \frac{E_0^r}{E_0^i} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = \frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t} = \frac{\sin(\theta_t - \theta_i)}{\sin(\theta_i + \theta_t)}$$

$\mu_1 = \mu_2 = \mu_0$ $\qquad$ $\dfrac{n_1}{n_2} = \dfrac{\sin\theta_t}{\sin\theta_i}$

$$T^\perp = \frac{E_0^t}{E_0^i} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = \frac{2n_1\cos\theta_i}{n_1\cos\theta_i + n_2\cos\theta_t} = \frac{2\sin\theta_t\sin\theta_i}{\sin(\theta_i + \theta_t)} = 1 + R^\perp$$

全反射时，功率反射与传输系数:

$$R = \left|R^{/\!/}\right|^2 = \left|R^\perp\right|^2 = 1 \qquad T = 1 - R = 0$$

✓ 场的反射系数模值=1，全反射。但媒质2中有场！

✓ 功率传输系数 T=0 说明：没有能量进入媒质2（在周期平均意义上）。

$$R = 1, T = 0$$

因此，在两种介质分界面上（法线方向），有：

$$\bar{\vec{S}} = \bar{\vec{S}}^i\cdot(-\hat{n}) + \bar{\vec{S}}^r\cdot(\hat{n}) + \bar{\vec{S}}^t\cdot(-\hat{n})$$

$$= \left(-\frac{1}{2\eta_1}\left|\dot{\vec{E}}^i\right|^2\cos\theta_i\right) + \frac{1}{2\eta_1}\left|\dot{\vec{E}}^r\right|^2\cos\theta_i - \frac{1}{2\eta_2}\left|\dot{\vec{E}}^t\right|^2\cos\theta_t$$

$$= \left(-\frac{1}{2\eta_1}\left|\dot{\vec{E}}^i\right|^2\cos\theta_i\right) + \frac{1}{2\eta_1}R\left|\dot{\vec{E}}^i\right|^2\cos\theta_i - \frac{1}{2\eta_2}\frac{n_1\cos\theta_i T}{n_2}\left|\dot{\vec{E}}^i\right|^2$$

$$= 0$$

1）证明：无耗传输线上距离λ/4的任意两点处阻抗的乘积均等于传输线特性阻抗的平方

无耗传输线任意一点z处波阻抗为Z_{in}

$$Z_{in}(z)=Z_0\frac{Z_L+jZ_0\tan\beta z}{Z_0+jZ_L\tan\beta z}$$

$$Z_{in}\left(z\pm\frac{\lambda}{4}\right)=Z_0\frac{Z_L+jZ_0\tan\beta\left(z\pm\frac{\lambda}{4}\right)}{Z_0+jZ_L\tan\beta\left(z\pm\frac{\lambda}{4}\right)}=Z_0\frac{Z_L+jZ_0\tan\left(\beta z\pm\frac{\pi}{2}\right)}{Z_0+jZ_L\tan\left(\beta z\pm\frac{\pi}{2}\right)}=Z_0\frac{Z_L-jZ_0\cot(\beta z)}{Z_0-jZ_L\cot(\beta z)}$$

$$Z_{in}(z)\cdot Z_{in}\left(z\pm\frac{\lambda}{4}\right)=Z_0\frac{Z_L+jZ_0\tan\beta z}{Z_0+jZ_L\tan\beta z}\cdot Z_0\frac{Z_L-jZ_0\cot(\beta z)}{Z_0-jZ_L\cot(\beta z)}=Z_0^2\frac{Z_L^2+jZ_0Z_L\tan\beta z-jZ_0Z_L\cot(\beta z)+Z_0^2}{Z_0^2+jZ_0Z_L\tan\beta z-jZ_0Z_L\cot(\beta z)+Z_L^2}=Z_0^2$$

2）设一无耗传输线，终端接有负载$Z_L=40-j30(\Omega)$

问①要使传输线上驻波比最小，则该传输线的特性阻抗应取多少？

②此时最小的反射系数及驻波比各为多少？

③离终端最近的波节点位置在何处

④画出特性阻抗与驻波比的关系曲线

① $\rho=\dfrac{1+|\Gamma_L|}{1-|\Gamma_L|}$

$$\Gamma_L=\frac{Z_L-Z_0}{Z_L+Z_0}=\frac{40-j30-Z_0}{40-j30+Z_0}=\frac{(40-Z_0)-j30}{(40+Z_0)-j30}\qquad |\Gamma_L|^2=\frac{(40-Z_0)^2+30^2}{(40+Z_0)+30^2}$$

求导为0得 $Z_L=50\Omega$ 时 $|\Gamma_L|^2$ 取最小值，ρ取最小值

② $Z_L = 50\Omega$ 时

$$\Gamma_L = \frac{(40-50)-j30}{(40+50)-j30} = \frac{-10-j30}{90-j30} = \frac{-1-j3}{9-j3} = -\frac{j}{3} = \frac{1}{3}e^{-j\frac{\pi}{2}}$$

$$\rho = \frac{1+\frac{1}{3}}{1-\frac{1}{3}} = 2$$

注意反射系数的相位

③

$$|U(z)| = |A_1|\sqrt{1+|\Gamma_L|^2 + 2|\Gamma_L|\cos(\varphi_L - 2\beta z)}$$

PPT CH8-1 P35

电压的波节点为电压幅度最小$\cos(\phi_L - 2\beta z) = -1$ $\longrightarrow$ $z = \frac{\lambda}{4\pi}\varphi_L + (2n\pm1)\frac{\lambda}{4}$

又由②知$\varphi_L = -\frac{\pi}{2}$ $\longrightarrow$ $z = (2n\pm1)\frac{\lambda}{4} - \frac{\lambda}{8}$ $\quad z_{\min} = \frac{\lambda}{8}$

④

$$|\Gamma_L|^2 = \frac{(40-Z_0)^2 + 30^2}{(40+Z_0)^2 + 30^2} = 1 - \frac{160}{80 + (\frac{50^2}{Z_0} + Z_0)}$$

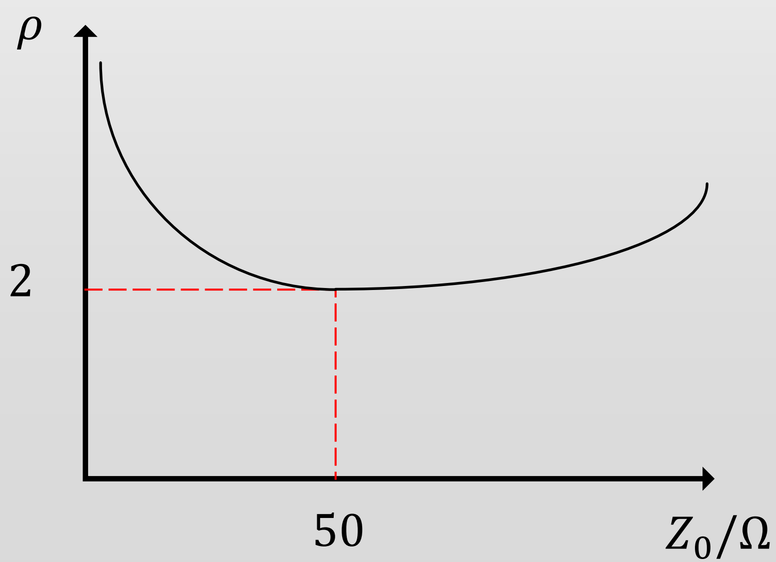
$$\rho = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \frac{1}{80}\left[\frac{50^2}{Z_0} + Z_0 + \sqrt{\left(\frac{50^2}{Z_0} + Z_0\right)^2 - 80^2}\right]$$

无耗传输线耗，Z_0为实数

$$Z_0 \to 0, \Gamma_L \to 1, \rho \to \infty$$

$$Z_0 \to \infty, \Gamma_L \to -1, \rho \to \infty$$

$$Z_0 = 50, |\Gamma_L| \to \frac{1}{3}, \rho \to 2$$



8-1设一矩形波导截面的尺寸为a=86.4mm,b=43.2mm,当频率f_1=3GHz和f_2=5GHz时，该波导能传播哪几种模式？

$\lambda < \lambda_c$ 时才能传播 $\lambda_1 = \frac{c}{f_1} = 100mm$ $\lambda_2 = \frac{c}{f_2} = 60mm$

$TE_{10}: \lambda_c = 2a = 172.8mm$ $TE_{01}: \lambda_c = 2b = 86.4mm$

$TE_{20}: \lambda_c = a = 86.4mm$

$$TE_{11}: \lambda_c = \frac{2}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}} = 77mm \qquad TM_{11}: \lambda_c = 77mm$$

$$TE_{21}: \lambda_c = \frac{2}{\sqrt{\left(\frac{2}{a}\right)^2 + \left(\frac{1}{b}\right)^2}} = 61mm \qquad TM_{21}: \lambda_c = 61mm$$

更高模式：$\lambda_c < 60mm$

综上，当频率f_1=3GHz时，该波导仅能传播 TE_{10}

当频率f_2=5GHz时，该波导仅能传播 $TE_{10}, TE_{01}, TE_{20}, TE_{11}, TM_{11}, TE_{21}, TM_{21}$

8-5证明矩形波导中,单一模式的TE波或TM波其电场与磁场互相垂直

TE_{mn}波：

$$\begin{cases} H_z = H_{mn}\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-j\beta z} \\ E_x = \frac{jw\mu}{k_c^2}\frac{n\pi}{b}H_{mn}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-j\beta z} \\ E_y = \frac{-jw\mu}{k_c^2}\frac{m\pi}{a}H_{mn}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-j\beta z} \\ H_x = \frac{j\beta}{k_c^2}\frac{m\pi}{a}H_{mn}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-j\beta z} \\ H_y = \frac{j\beta}{k_c^2}\frac{n\pi}{b}H_{mn}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-j\beta z} \end{cases}$$

$$\vec{E} = E_x\hat{x} + E_y\hat{y} \qquad \vec{H} = H_x\hat{x} + H_y\hat{y} + H_z\hat{z}$$

$$\vec{E}\cdot\vec{H} = E_xH_x + E_yH_y = 0$$

TM_{mn}波：同理

10-1 天线的方向性系数D定义为辐射图中波印亭矢量的最大数值与波印亭矢量在整个球面上的平均值之比，即

$$D=\frac{S_{\max}}{\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}S\sin\theta d\theta d\varphi}$$

证明电偶极子和磁偶极子的方向性系数是1.5。

电偶极子远区

$$\begin{cases}E_\theta=j\dfrac{\eta_0 Idl\sin\theta}{2\lambda r}e^{-jkr}\\ H_\phi=j\dfrac{Idl\sin\theta}{2\lambda r}e^{-jkr}\end{cases}$$

$$\bar{\vec{S}}=\frac{1}{2}\mathrm{Re}\left(\vec{E}\times\vec{H}^*\right)=\frac{1}{2}\mathrm{Re}\left(j\frac{\eta_0 Idl\sin\theta}{2\lambda r}e^{-jkr}\hat{\theta}\times\left(-j\frac{Idl\sin\theta}{2\lambda r}e^{jkr}\hat{\varphi}\right)\right)=\frac{\eta_0\left(Idl\right)^2}{8\lambda^2}\frac{\left(\sin\theta\right)^2}{r^2}\hat{r}$$

$$S_{\max}=S\,|_{\theta=\frac{\pi}{2}}=\frac{\eta_0\left(Idl\right)^2}{8\lambda^2r^2}$$

$$\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}S\sin\theta d\theta d\varphi=\frac{1}{4\pi}\times 2\pi\times\int_0^{\pi}\frac{\eta_0\left(Idl\right)^2}{8\lambda^2}\frac{\left(\sin\theta\right)^3}{r^2}d\theta=\frac{\eta_0\left(Idl\right)^2}{16\lambda^2r^2}\left(\frac{\cos\theta^3}{3}-\cos\theta\right)\Big|_0^{\pi}=\frac{\eta_0\left(Idl\right)^2}{12\lambda^2r^2}$$

$$D=\frac{S_{\max}}{\frac{1}{4\pi}\int_\pi^{2\pi}\int_0^{\pi}S\sin\theta d\theta d\varphi}=\frac{\frac{\eta_0\left(Idl\right)^2}{8\lambda^2r^2}}{\frac{\eta_0\left(Idl\right)^2}{12\lambda^2r^2}}=1.5$$

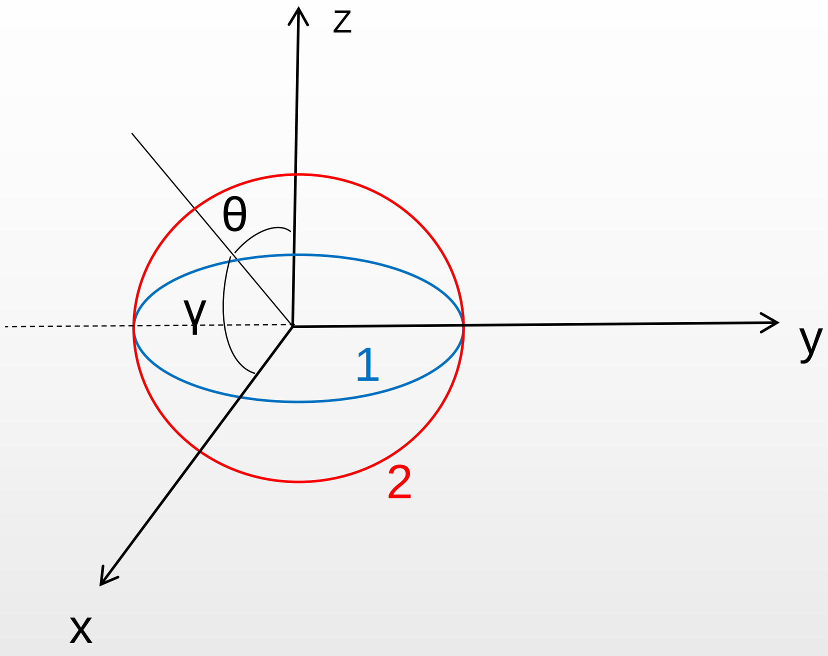
磁偶极子远区 $\bar{\vec{S}}=\frac{1}{2}\mathrm{Re}\left(\vec{E}\times\vec{H}^*\right)=\frac{\mu_0 w^4\left|m_m\right|^2}{32\pi c^3r^2}\left(\sin\theta\right)^2\hat{r}$ 书P295（10-45）

$$S_{\max}=\frac{\mu_0 w^4\left|m_m\right|^2}{32\pi c^3r^2}$$

$$\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}S\sin\theta d\theta d\varphi=\frac{1}{4\pi}\times 2\pi\times\frac{\mu_0 w^4\left|m_m\right|^2}{32\pi c^3r^2}\int_0^{\pi}\left(\sin\theta\right)^3d\theta=\frac{\mu_0 w^4\left|m_m\right|^2}{48\pi c^3r^2}$$

$$D=\frac{\frac{\mu_0 w^4\left|m_m\right|^2}{32\pi c^3r^2}}{\frac{\mu_0 w^4\left|m_m\right|^2}{48\pi c^3r^2}}=1.5$$

10-2两个磁偶极子互相垂直，直径相同，证明：如果一个偶极子比另一个相位超前了 $(\pi/2)rad$，则在垂直于它们的公共直径的平面内，辐射图（振幅对θ的函数关系）是一个圆。



设磁偶极子1在xoy平面，磁偶极子2在yoz平面

则公共直径为y轴，垂直公共直径的平面为xoz面

由磁偶极子的远区电场公式 $E_\varphi = \eta_0 \dfrac{\pi IS \sin\theta}{\lambda^2 r} e^{-jkr}$ 知

对磁偶极子1有 $E_1 = \eta_0 \dfrac{\pi IS \sin\theta}{\lambda^2 r} e^{-jkr}$

对磁偶极子2有 $E_2 = j\eta_0 \dfrac{\pi IS \sin\gamma}{\lambda^2 r} e^{-jkr}$

总电场 $E = E_1 + E_2 = \eta_0 \dfrac{\pi IS}{\lambda^2 r} e^{-jkr} \left(\sin\theta + j\sin\gamma\right)$

$$= \eta_0 \frac{\pi IS}{\lambda^2 r} e^{-jkr} \left(\sin\theta + j\sin\left(\frac{\pi}{2} - \theta\right)\right)$$

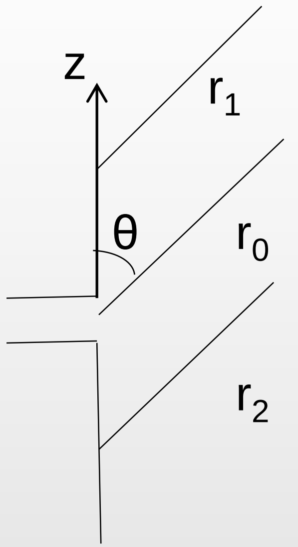
$$= \eta_0 \frac{\pi IS}{\lambda^2 r} e^{-jkr} \left(\sin\theta + j\cos(\theta)\right)$$

即 $|E| = const$ 对任意θ 均成立，辐射图是一个圆

10-6 如图是一个半波天线，其上的电流分布为 $I = I_m \cos kz(-l/2 < z < l/z)$

（1）求证当 $r_0 >> l$ 时，P点的矢量磁位为

$$A_z = \frac{I_m e^{-jkr_0}}{2\pi k r_0} \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$$



沿z方向上电流在空间中任意一点产生的磁矢位(PPT Ch9 P7)为

$$A_z = \frac{\mu_0}{4\pi}\int_{-l/2}^{l/2}\frac{I_z e^{-jkr}}{r}dz$$

当 $r_0 >> l$ 时 $\begin{cases} r_1 \approx r_0 - z\cos\theta \\ r_2 \approx r_0 + z\cos\theta \end{cases}$ $\frac{1}{r_1} \approx \frac{1}{r_2} \approx \frac{1}{r_0}$

$$A_z = \frac{\mu_0}{4\pi}\int_{-l/2}^{l/2}\frac{I_z e^{-jkr}}{r}dz = \frac{\mu_0 I_z}{4\pi}\left(\int_0^{l/2}\frac{\cos(kz)e^{-jkr_1}}{r_0}dz + \int_{-l/2}^{0}\frac{\cos(kz)e^{-jkr_2}}{r_0}dz\right)$$

$$= \frac{\mu_0 I_m}{4\pi}\left(\int_0^{l/2}\frac{\cos(kz)e^{-jk(r_0 - z\cos\theta)}}{r_0}dz + \int_{-l/2}^{0}\frac{\cos(kz)e^{-jk(r_0 + z\cos\theta)}}{r_0}dz\right)$$

$$= \frac{\mu_0 I_m}{4\pi r_0}e^{-jkr_0}\int_0^{l/2}2\cos(kz)\cos(kz\cos\theta)dz$$

$$= \frac{\mu_0 I_m}{4\pi r_0}e^{-jkr_0}\int_0^{l/2}\frac{\cos(kz(1+\cos\theta)) + \cos(kz(1-\cos\theta))}{2}dz$$

$$= \frac{\mu_0 I_m}{4\pi k r_0}e^{-jkr_0}\left(\frac{(1-\cos\theta)\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta} + \frac{(1+\cos\theta)\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\right) = \frac{\mu_0 I_m}{2\pi k r_0}e^{-jkr_0}\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$$

（2）求远区的磁场和电场

$$\vec{H}=\frac{1}{\mu_0}\nabla\times\vec{A}=\frac{1}{\mu_0 r^2\sin\theta}\begin{vmatrix}\hat{r} & r\hat{\theta} & r\sin\theta\hat{\varphi}\\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial\theta} & \dfrac{\partial}{\partial\varphi}\\ A_r & rA_\theta & r\sin\theta A_\varphi\end{vmatrix}\qquad \begin{cases}A_r=A_z\cos\theta\\ A_\theta=-A_z\sin\theta\\ A_\varphi=0\end{cases}$$

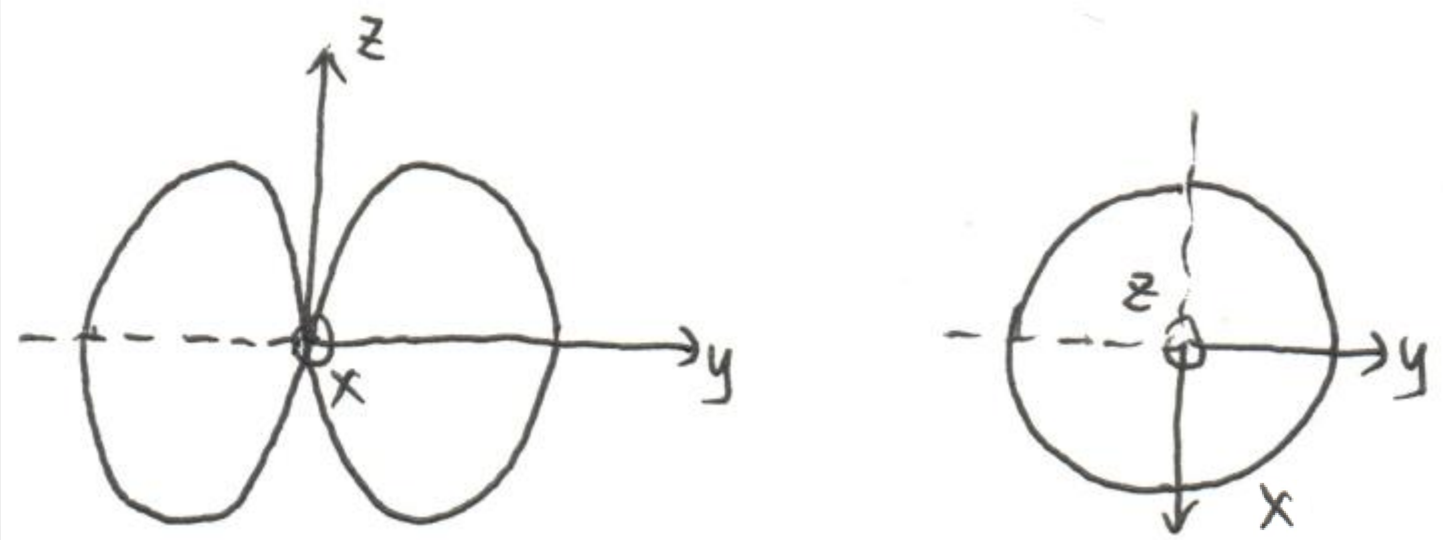
得$H_\varphi=j\dfrac{I_m e^{-jkr_0}}{2\pi r_0}\dfrac{\cos\left(\dfrac{\pi}{2}\cos\theta\right)}{\sin\theta}$

$$\vec{E}=\frac{1}{jw\varepsilon_0}\nabla\times\vec{H}$$

得$E_\theta=\eta_0 H_\varphi=j\dfrac{\eta_0 I_m e^{-jkr_0}}{2\pi r_0}\dfrac{\cos\left(\dfrac{\pi}{2}\cos\theta\right)}{\sin\theta}$

（3）用极坐标画出方向图 $\quad f(\theta)=\dfrac{\cos\left(\dfrac{\pi}{2}\cos\theta\right)}{\sin\theta}$

E面：“∞”形，H面：“○”



（4）求波印亭矢量

$$\vec{S}=\vec{E}\times\vec{H}=-\frac{\eta_0 I_m^2}{8\pi^2 r_0^2}e^{-2jkr_0}\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\hat{r}$$

（5）求辐射电阻

$$R_r=\frac{2P_r}{I_m^2}$$

$$P_r=\oint\bar{\vec{S}}\cdot d\vec{S}=\int_0^{2\pi}\int_0^{\pi}Sr^2\sin\theta d\theta d\varphi=\int_0^{2\pi}\int_0^{\pi}\frac{\eta_0 I_m^2}{8\pi^2 r_0^2}\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}r_0^2\sin\theta d\theta d\varphi=\frac{\eta_0 I_m^2}{4\pi}\int_0^{\pi}\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}d\theta$$

$$R_r=\frac{2P_r}{I_m^2}=\frac{\eta_0}{2\pi}\int_0^{\pi}\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}d\theta=73\Omega$$

（6）求方向性系数

$$D=\frac{S_{\max}}{\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}S\sin\theta d\theta d\varphi}=\frac{2}{\int_0^{\pi}\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}d\theta}=1.64=2.15dB$$