

数字信号处理B

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HW1

Exercise 1

$$\begin{aligned}x(nT_s) &= e^{-nT_s}, \quad n = 0, 1, \dots \\r_x(mT_s) &= \sum_{n=0}^{\infty} x(nT_s)x((n+m)T_s) \\&= \sum_{n=0}^{\infty} e^{-(2n+m)T_s} \\&= e^{mT_s} \sum_{n=0}^{\infty} e^{-2nT_s} \\&= \frac{e^{mT_s}}{1 - e^{-2T_s}}\end{aligned}$$

Exercise 2

性质1

$$\begin{aligned}r_x(-m) &= \sum_{n=-\infty}^{\infty} x(n)x(n-m) \\&= \sum_{n=-\infty}^{\infty} x(n+m)x(n) \\&= \sum_{n=-\infty}^{\infty} x(n)x(n+m) \\&= r_x(m)\end{aligned}$$

性质2

$$\begin{aligned}r_x^*(-m) &= \left(\sum_{n=-\infty}^{\infty} x^*(n)x(n-m) \right)^* \\&= \sum_{n=-\infty}^{\infty} x(n)x^*(n-m) \\&= \sum_{n=-\infty}^{\infty} x(n+m)x^*(n) \\&= \sum_{n=-\infty}^{\infty} x^*(n)x(n+m) \\&= r_x(m)\end{aligned}$$

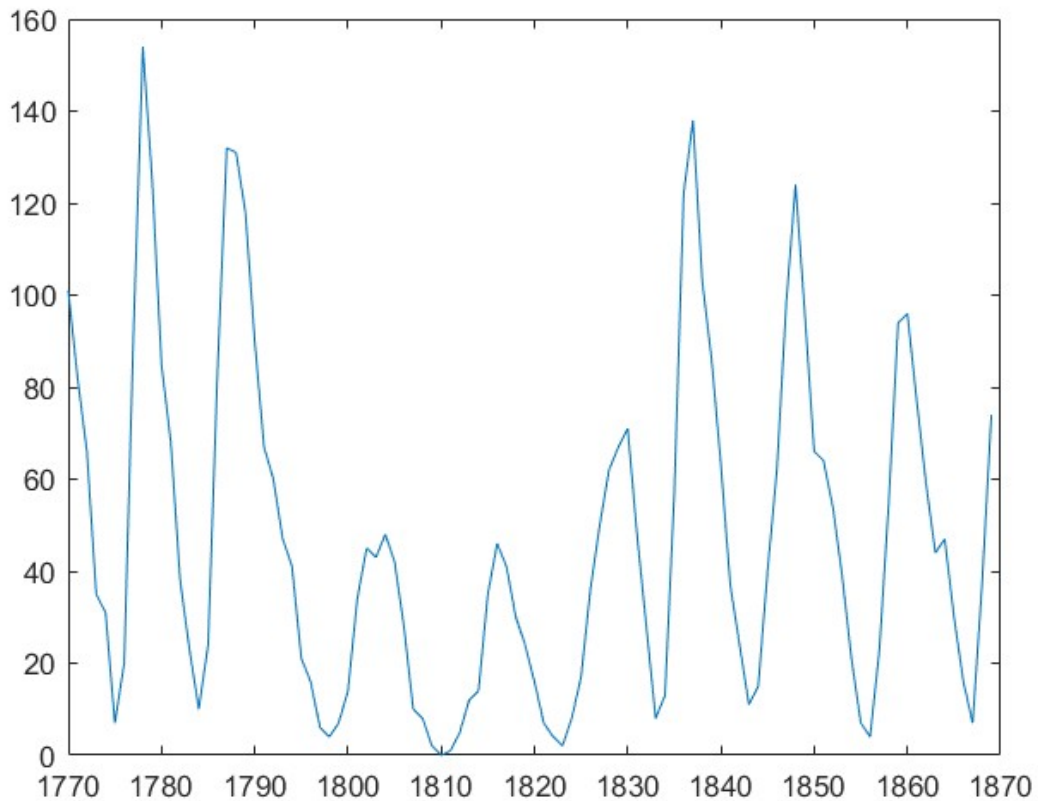
Exercise 3

$$\begin{aligned}
 x(n) &= A_1 \cos(2\pi f_1 n T_s) + A_2 \sin(2\pi f_2 n T_s) \\
 r_x(m) &= \sum_{\langle T \rangle} [A_1 \cos(2\pi f_1 n T_s) + A_2 \sin(2\pi f_2 n T_s)] \cdot [A_1 \cos(2\pi f_1 (n+m) T_s) + A_2 \sin(2\pi f_2 (n+m) T_s)] \\
 &= \sum_{\langle T \rangle} A_1^2 \cos(2\pi f_1 n T_s) \cos(2\pi f_1 (n+m) T_s) + A_1 A_2 \cos(2\pi f_1 n T_s) \sin(2\pi f_2 (n+m) T_s) \\
 &\quad + A_1 A_2 \sin(2\pi f_2 n T_s) \cos(2\pi f_1 (n+m) T_s) + A_2^2 \sin(2\pi f_2 n T_s) \sin(2\pi f_2 (n+m) T_s) \\
 &= \sum_{\langle T \rangle} \frac{A_1^2}{2} [\cos(2\pi f_1 (2n+m) T_s) + \cos(2\pi f_1 m T_s)] \\
 &\quad + \frac{A_1 A_2}{2} [\sin(2\pi f_1 (2n+m) T_s) - \sin(2\pi (f_1 n - f_2 n - f_2 m) T_s)] \\
 &\quad + \frac{A_1 A_2}{2} [\sin(2\pi (f_2 n + f_1 n + f_1 m) T_s) + \sin(2\pi (f_2 n - f_1 n - f_1 m) T_s)] \\
 &\quad + \frac{A_2^2}{2} [\cos(2\pi f_2 m T_s) - \cos(2\pi (f_1 n + f_2 n + f_2 m) T_s)] \\
 &= \sum_{\langle T \rangle} \frac{A_1^2}{2} \cos(2\pi f_1 m T_s) + \frac{A_2^2}{2} \cos(2\pi f_2 m T_s) \\
 &= \frac{A_1^2}{2} \cos(2\pi f_1 m T_s) + \frac{A_2^2}{2} \cos(2\pi f_2 m T_s)
 \end{aligned}$$

Exercise 4

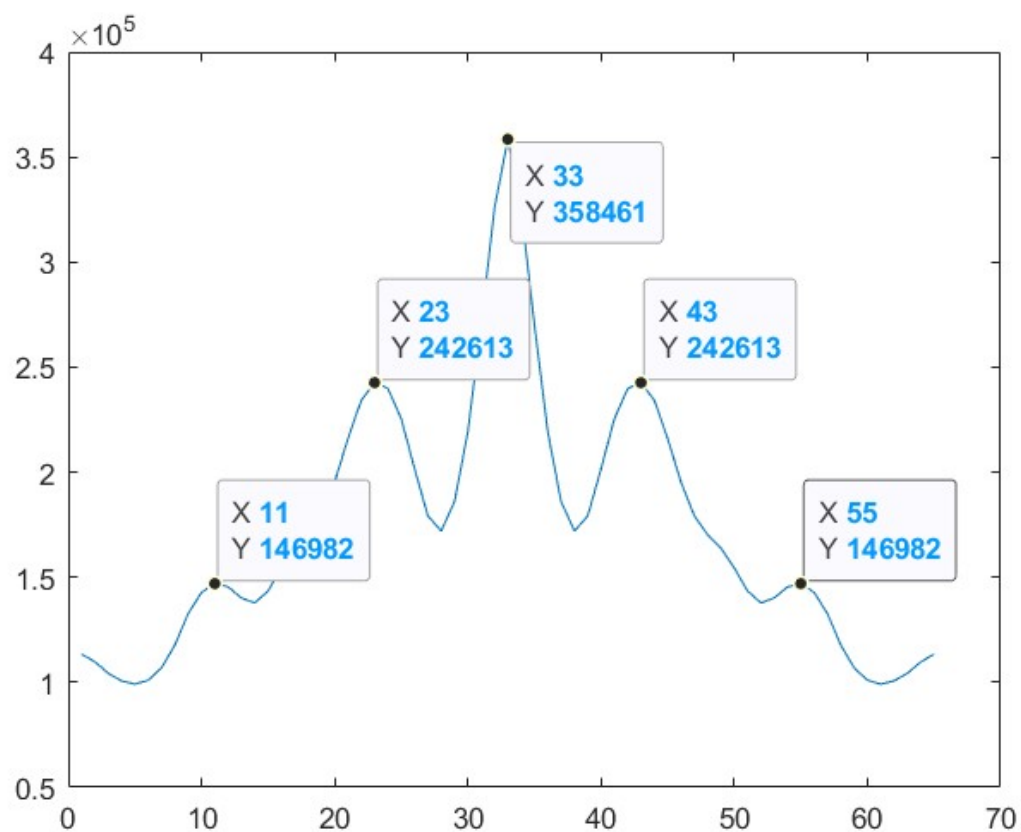
(1)

太阳黑子图像:



(2)

根据相关函数图可以得出，太阳黑子的周期为11年



(3)

蓝线为原相关函数，红线为新相关函数，可以看出：减去平均值，相关函数整体往下平移，但是形状不变

