第三章 简单体系的薛定谔方程(12学时)

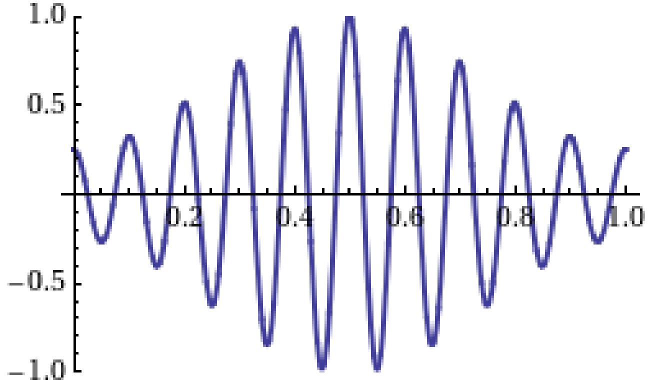
- 3.1 自由电子及平面波
- 3.2 量子阱、量子线与量子点
- 3.3 谐振子与氢原子简介

3.1 自由电子及平面波

1. 自由粒子定态薛定谔方程的解
   ① 解薛定谔方程的一般方法及定态薛定谔方程的含义
   ② 自由粒子定态薛定谔方程的平面波解：一维，三维
   ③ 平面波的时间依赖关系与自由粒子波函数的演化
   ④ 通过对易关系找定态薛定谔方程的解
2. 守恒力学量
   ① 动量的期望值及其随时间的依赖关系：用general公式推导；用测量几率密度的含义推导
   ② 与哈密顿算符对易的力学量是守恒量：直接推导；共同本征函数推导

3.1 自由电子及平面波

3. 平面波的色散关系
   ① 一维情况的色散关系
   ② 相速度
4. 波包与群速度
   ① 波包
   ② 群速度是包络的运动速度，相速度是载波的运动速度
   ③ 自由电子的运动速度：群速度



3.1 自由电子及平面波

5. 有限空间中的平面波

① 周期边界条件：当电子运动范围很大时，边界条件的选取对大多数结论不重要，所以可以选择周期性边界条件（Born-Von Karman boundary condition)

② 一维情况

$\psi(x=0)=\psi(x=L)$，从而$k_n=\frac{2\pi}{L}n$，$E_n=\frac{\hbar^2 k_n^2}{2m}$，$n$取值所有整数

③ 三维情况

$\boldsymbol{k}_{m,n,l}=(\frac{2\pi}{L_x}m,\frac{2\pi}{L_y}n,\frac{2\pi}{L_z}l)$，$E_n=\frac{\hbar^2\left|\boldsymbol{k}_{m,n,l}\right|^2}{2m}$，$(m,n,l)$取值所有整数

3.1 自由电子及平面波

6. 泡利不相容原理
   每个独立的量子状态只能被一个电子占据
   内禀的量子关联，与库伦相互作用等无关
7. 自由电子气的费米能和费米波矢

$$E_F = \frac{\hbar^2 k_F^2}{2m_e}$$

3.1 自由电子及平面波

7. 自由电子气的费米能和费米波矢

   1D: $k_F = \frac{1}{2}\pi n$

   2D: $k_F = \sqrt{2\pi n}$

   3D: $k_F = (3\pi^2 n)^{1/3}$

8. 自由电子气的态密度(DOS)

   1D: $D(E) = \frac{1}{\pi\hbar}\sqrt{\frac{2m}{E}}$

   2D: $D(E) = \frac{m}{\pi\hbar^2}$

   3D: $D(E) = \frac{m}{\pi^2\hbar^3}\sqrt{2mE}$

3.2 量子阱、量子线与量子点

1. 一维无限深势阱问题

$$V(x) = \begin{cases} 0 & 0 \le x \le W \\ \infty & otherwise \end{cases}$$

$\psi_n(x) = \sin(k_n x)$，$k_n = \frac{\pi}{W} n$，$E_n = \frac{\hbar^2 k_n^2}{2m_e} = \frac{\hbar^2 \pi^2}{2m_e W^2} n^2$，$n$取值所有正整数

驻波：$\psi_n(x,t) = \sin(k_n x) \exp(-iE_n t/\hbar)$

零点能：动量为零要求位置无限扩展

3.2 量子阱、量子线与量子点

2. 量子阱

① x方向受限，yz方向“自由”

$\psi_{n,k_y,k_z}(\boldsymbol{r}) = \sin\left(\frac{n\pi}{W}x\right)\exp(ik_y y)\exp(ik_z z)$，$E_{n,k_y,k_z} = \frac{\hbar^2\pi^2}{2m_e W^2}n^2 + \frac{\hbar^2(k_y^2+k_z^2)}{2m_e}$，$n$ 取值所有正整数

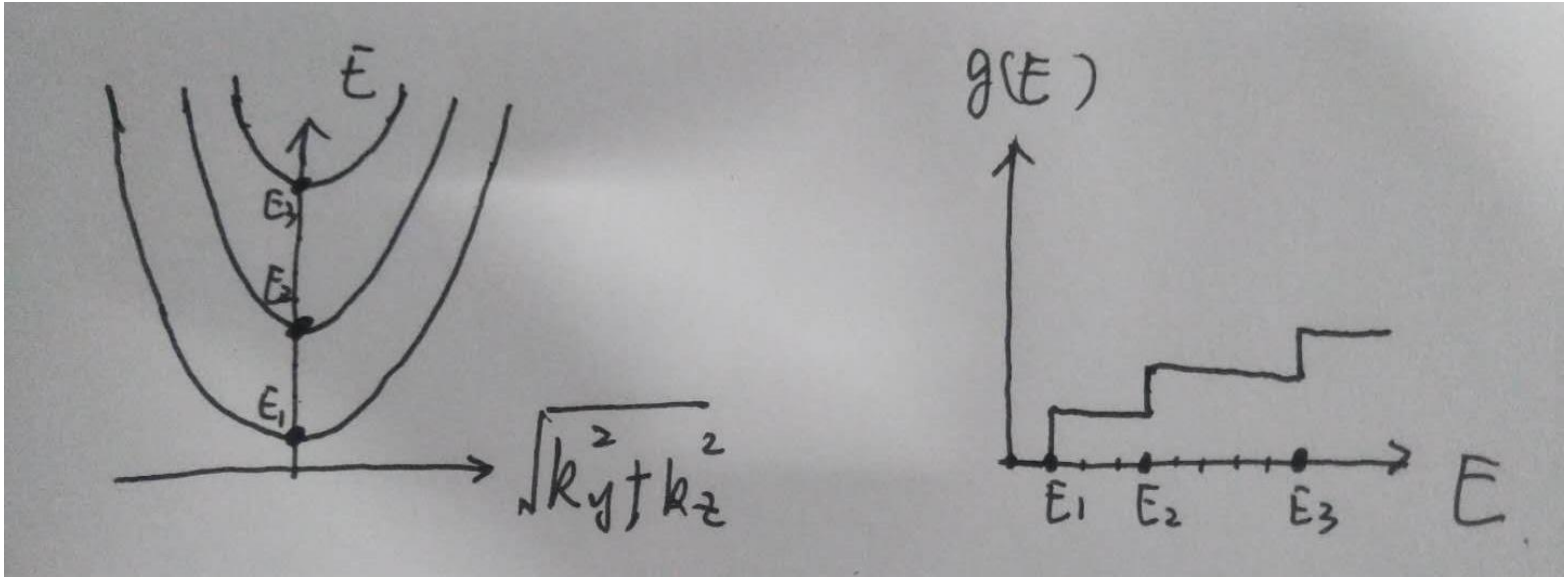
② 能带的概念

3.2 量子阱、量子线与量子点

2. 量子阱

③ 二维自由电子气（第一个能带）：$k_F = (2\pi n)^{1/2}$，$E_F = \hbar^2 k_F^2 / 2m_e$，$D(E) = \frac{m_e}{\hbar^2 \pi}$

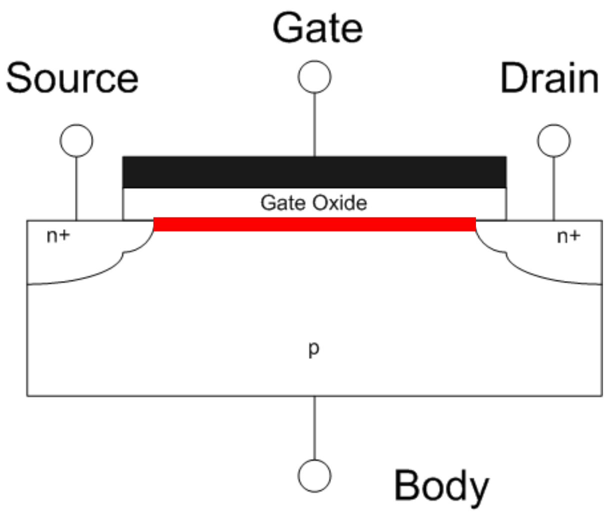
④ 考虑多个能带的情况


E
E3
E2
E1
$\sqrt{k_y^2+k_z^2}$
g(E)
E1 E2 E3
E

3.2 量子阱、量子线与量子点

2. 量子阱

⑤ 实际体系一：MOSFET



VOLUME 45, NUMBER 6 PHYSICAL REVIEW LETTERS 11 AUGUST 1980

New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance

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(Received 30 May 1980)

Measurements of the Hall voltage of a two-dimensional electron gas, realized with a silicon metal-oxide-semiconductor field-effect transistor, show that the Hall resistance at particular, experimentally well-defined surface carrier concentrations has fixed values which depend only on the fine-structure constant and speed of light, and is insensitive to the geometry of the device. Preliminary data are reported.

PACS numbers: 73.25.+i, 06.20.Jr, 72.20.My, 73.40.Qv

In this paper we report a new, potentially high-accuracy method for determining the fine-structure constant, α. The new approach is based on the fact that the degenerate electron gas in the inversion layer of a MOSFET (metal-oxide-semiconductor field-effect transistor) is fully quantized when the transistor is operated at helium temperatures and in a strong magnetic field of order 15 T.[1] The inset in Fig. 1 shows a schematic diagram of a typical MOSFET device used in this work. The electric field perpendicular to the surface (gate field) produces subbands for the motion normal to the semiconductor-oxide interface, and the magnetic field produces Landau quantization of motion parallel to the interface. The density of states $D(E)$ consists of broadened δ functions[2]; minimal overlap is achieved if the magnetic field is sufficiently high. The number of states, N_L, within each Landau level is given by

$$N_L = eB/h, \tag{1}$$

where we exclude the spin and valley degeneracies. If the density of states at the Fermi energy, $N(E_F)$, is zero, an inversion layer carrier cannot be scattered, and the center of the cyclotron orbit drifts in the direction perpendicular to the electric and magnetic field. If $N(E_F)$ is finite but small, an arbitrarily small rate of scattering cannot occur and localization produced by the long lifetime is the same as a zero scattering rate, i.e., the same absence of current-carrying states occurs.[3] Thus, when the Fermi level is between

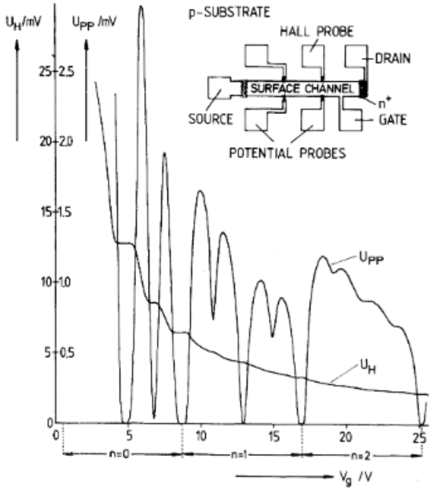
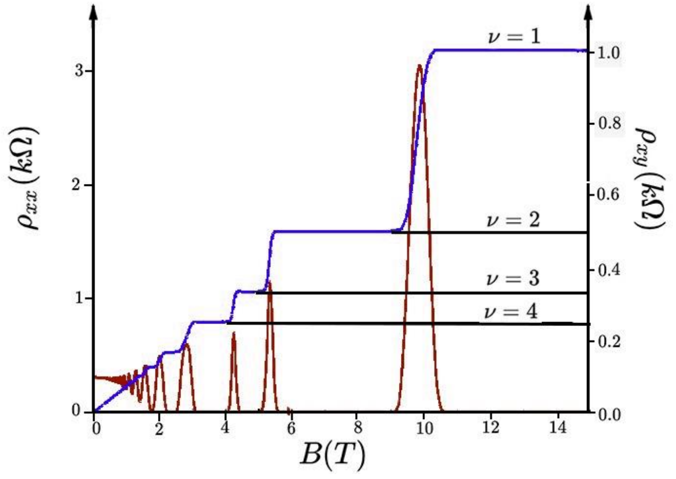


FIG. 1. Recordings of the Hall voltage U_H, and the voltage drop between the potential probes, U_{pp}, as a function of the gate voltage V_g at $T = 1.5$ K. The constant magnetic field (B) is 18 T and the source drain current, I, is 1 μA. The inset shows a top view of the device with a length of $L = 400$ μm, a width of $W = 50$ μm, and a distance between the potential probes of $L_{pp} = 130$ μm.

量子霍尔效应，1980，Klaus von Klitzing

量子效应显现的条件：
低缺陷、低温



The Nobel Prize in Physics 1985

Photo from the Nobel Foundation archive.

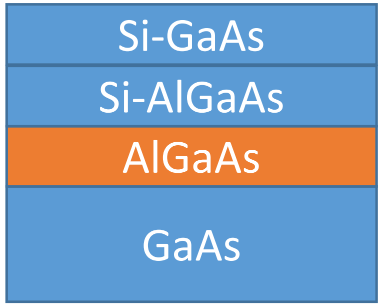
Klaus von Klitzing

Prize share: 1/1

3.2 量子阱、量子线与量子点

2. 量子阱

⑥ 实际体系二：AlGaAs异质结



The Nobel Prize in Physics 1998

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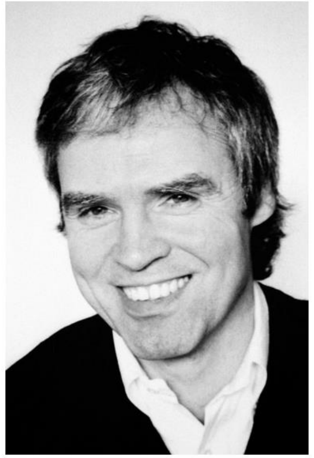
Robert B. Laughlin



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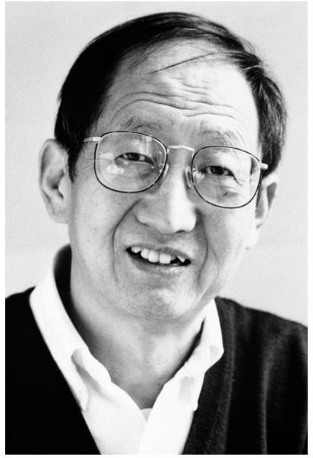
Horst L. Störmer



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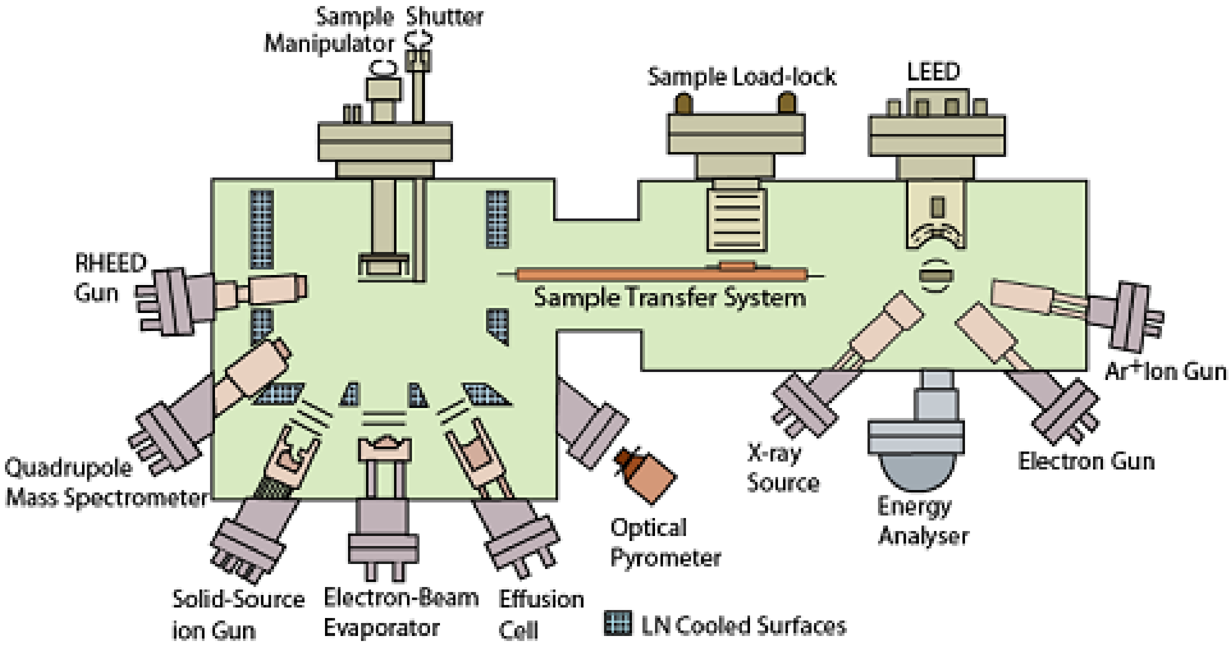
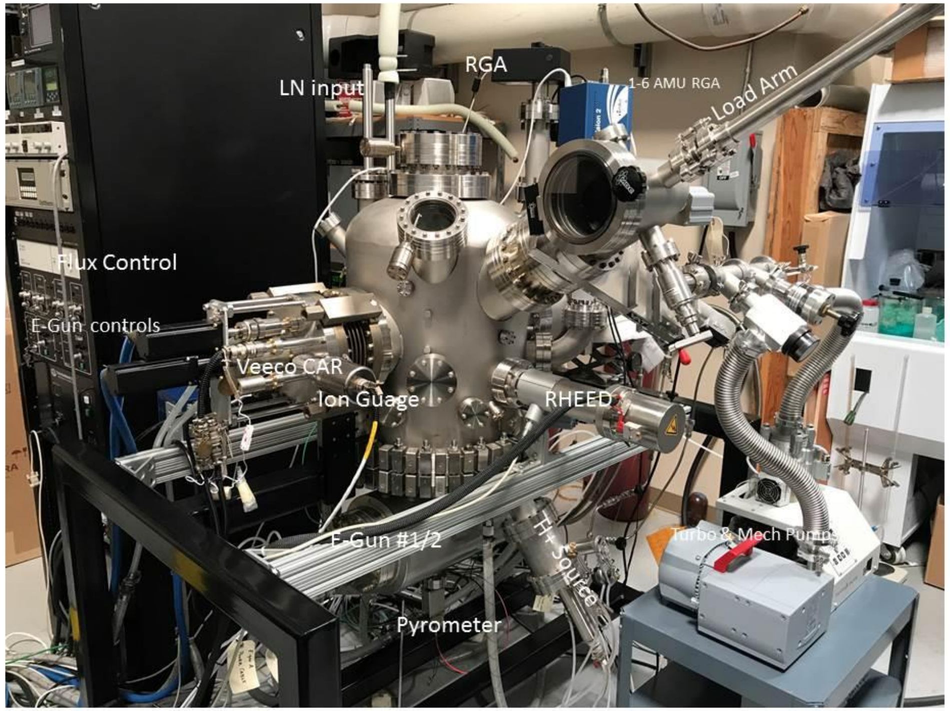
分数量子霍尔效应，1982


Sample Manipulator
Shutter
Sample Load-lock
LEED
RHEED Gun
Sample Transfer System
Ar+Ion Gun
Quadrupole Mass Spectrometer
X-ray Source
Energy Analyser
Electron Gun
Optical Pyrometer
Solid-Source ion Gun
Electron-Beam Evaporator
Effusion Cell
LN Cooled Surfaces

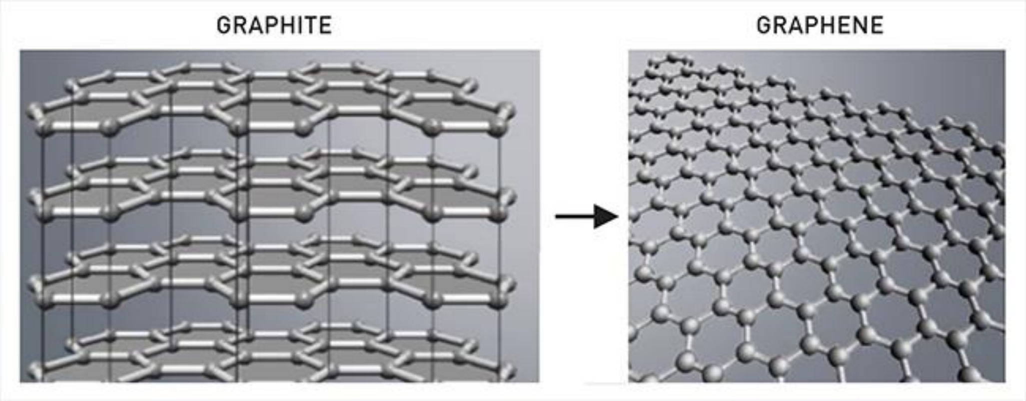

LN input
RGA
1-6 AMU RGA
Load Arm
Flux Control
E-Gun controls
Veeco CAR
Ion Guage
RHEED
E-Gun #1/2
TH Source
Turbo & Mech Pumps
Pyrometer

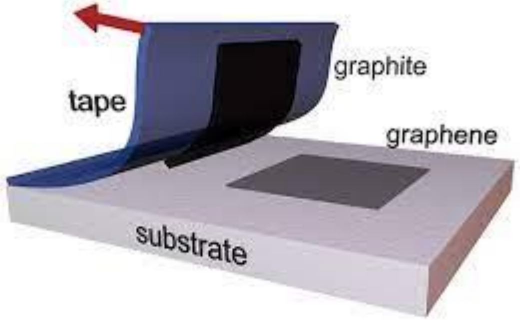
分子束外延

3.2 量子阱、量子线与量子点

2. 量子阱

⑦ 实际体系三：二维材料





The Nobel Prize in Physics 2010

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Andre Geim

Prize share: 1/2

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Konstantin Novoselov

Prize share: 1/2

3.2 量子阱、量子线与量子点

3. 量子线

① xy方向受限，z方向“自由”

$$\psi_{m,n,k_z}(\boldsymbol{r}) = \sin\left(\frac{m\pi}{W_x}x\right)\sin\left(\frac{n\pi}{W_y}y\right)\exp(ik_z z),$$

$E_{m,n,k_z} = \frac{\hbar^2\pi^2}{2m_e W_x^2}m^2 + \frac{\hbar^2\pi^2}{2m_e W_y^2}n^2 + \frac{\hbar^2 k_z^2}{2m_e}$，$m,n$取值所有正整数

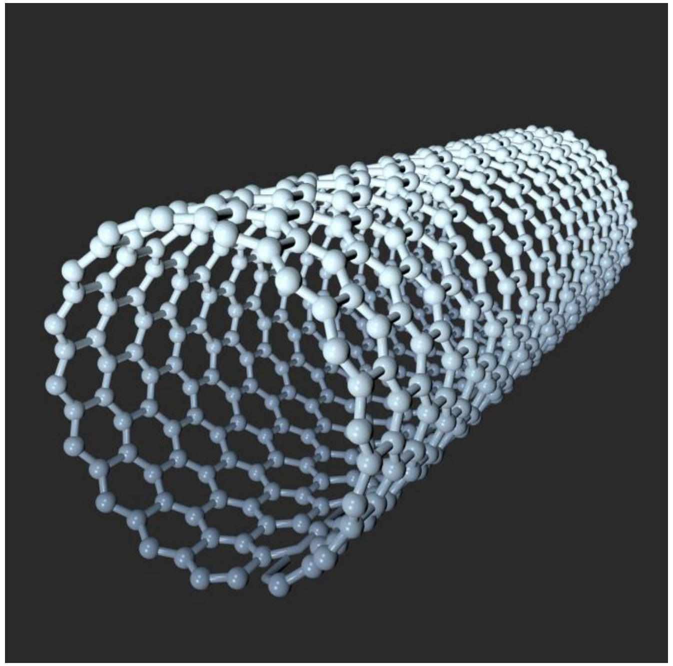
② 一维电子气（第一个能带）： $k_F = \pi n/2$， $E_F = \hbar^2 k_F^2/2m_e$，

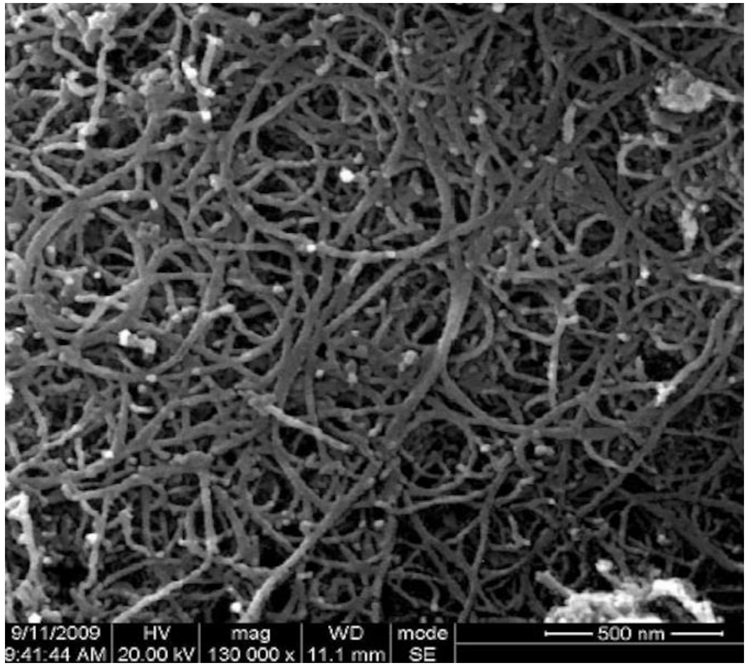
$$D(E) = \frac{2m_e}{\hbar\pi\sqrt{2m_e E}}$$

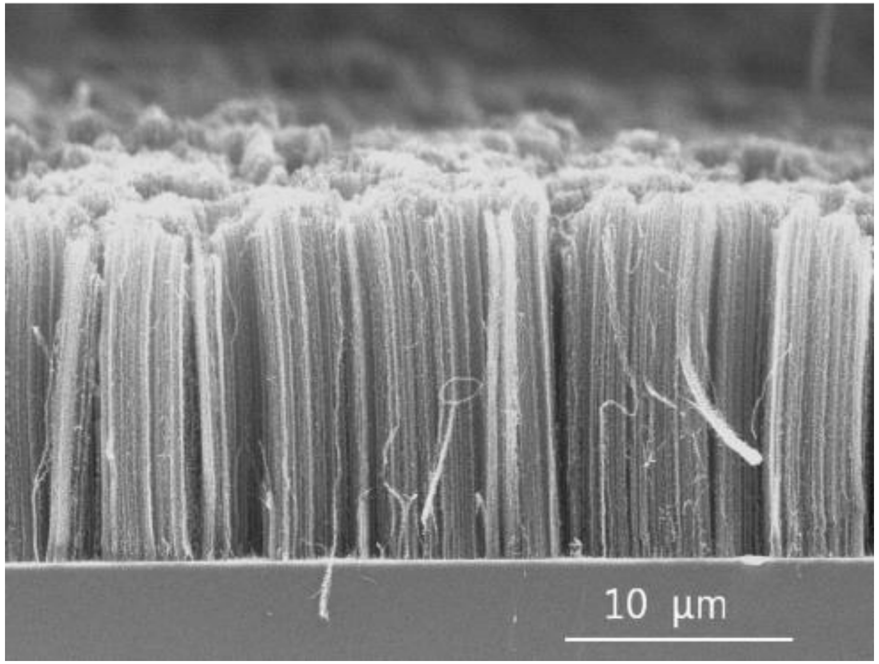
3.2 量子阱、量子线与量子点

3. 量子线

③ 实际体系一：碳纳米管





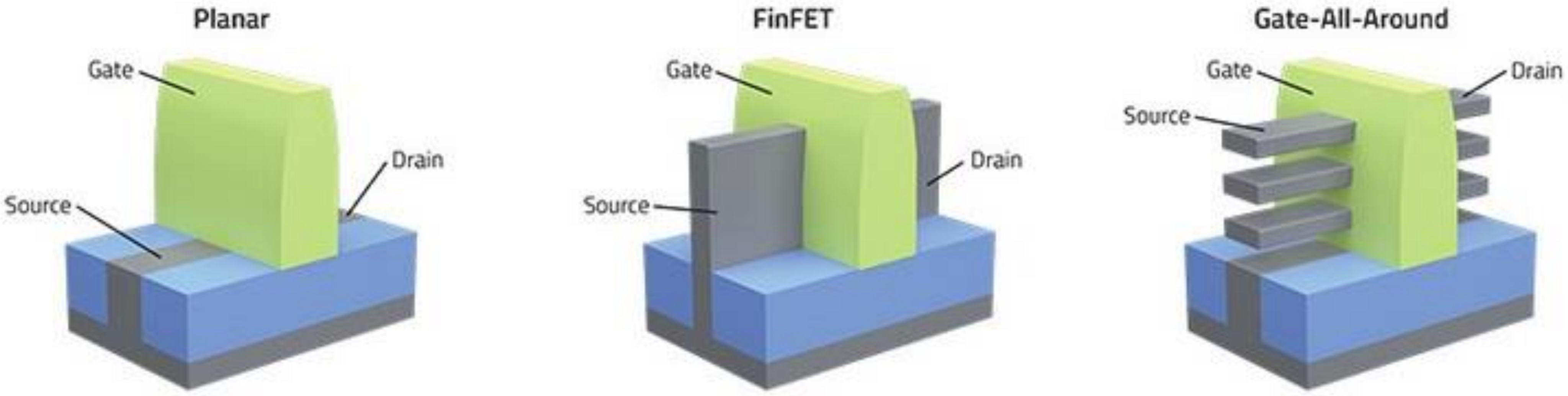


机械特性好、弹道输运

3.2 量子阱、量子线与量子点

3. 量子线

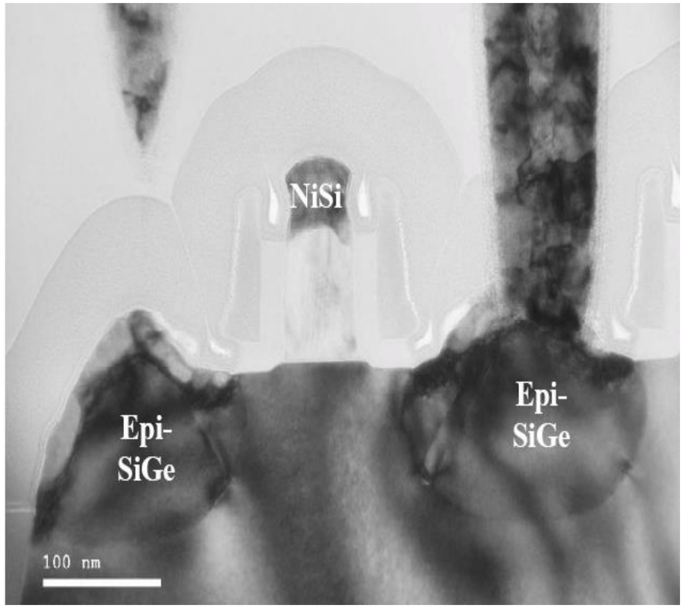
④ 实际体系二： GAA

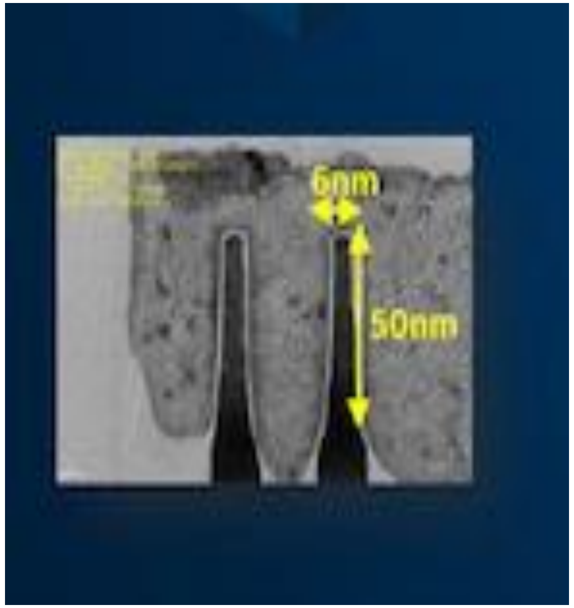


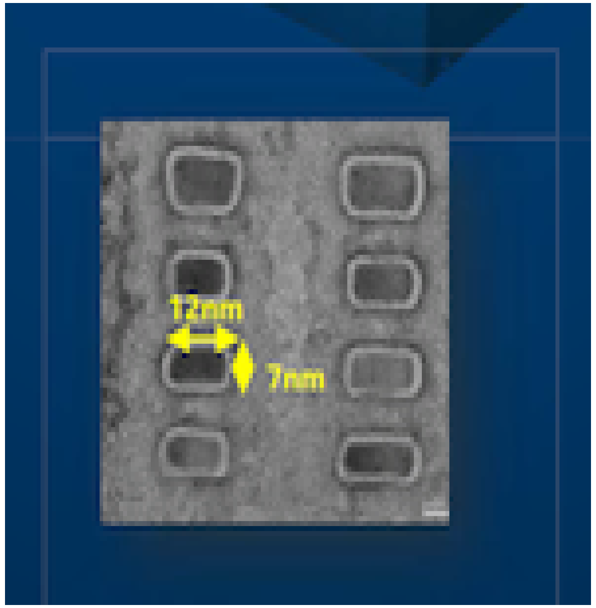
3.2 量子阱、量子线与量子点

3. 量子线

④ 实际体系二： GAA







3.2 量子阱、量子线与量子点

3. 量子线

④ 实际体系二： GAA

Gate-All-Around Transistor Deployment			
AnandTech	**Name**	**Process**	**Timeframe**
Intel	RibbonFET	20A	2024
		18A	2025
TSMC	GAAFET	N2 / 2nm	EoY 2023?
Samsung	MBCFET	3GAE	2022
		3GAP	2023

3.2 量子阱、量子线与量子点

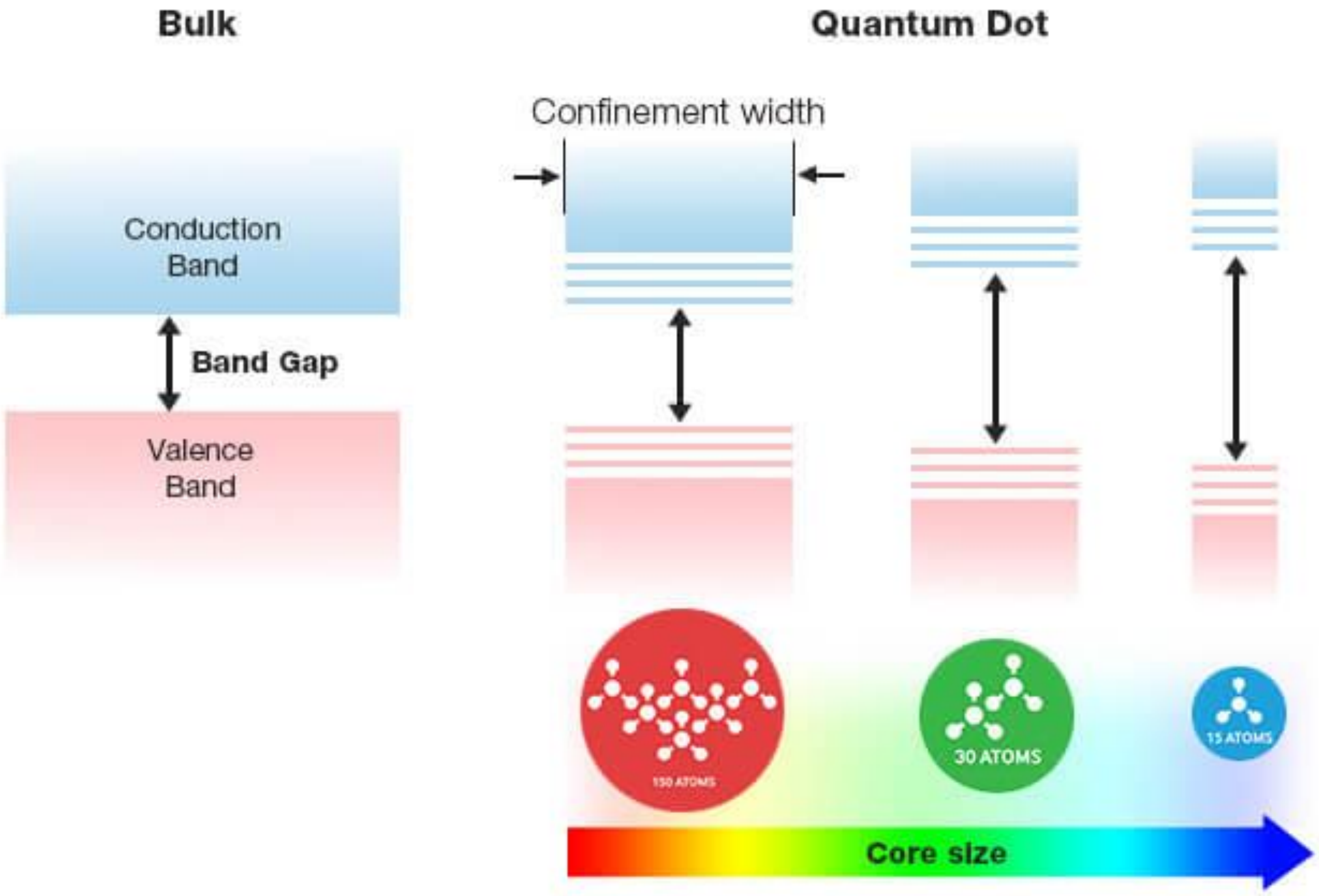
4. 量子点

① xyz方向均受限

$$\psi_{m,n,l}(\boldsymbol{r}) = \sin\left(\frac{m\pi}{W_x}x\right)\sin\left(\frac{n\pi}{W_y}y\right)\sin\left(\frac{l\pi}{W_z}z\right),$$

$$E_{m,n,l} = \frac{\hbar^2\left[\left(\frac{m\pi}{W_x}\right)^2+\left(\frac{n\pi}{W_y}\right)^2+\left(\frac{l\pi}{W_z}\right)^2\right]}{2m_e}，$$ m,n,l取值所有正整数

② 量子点显示



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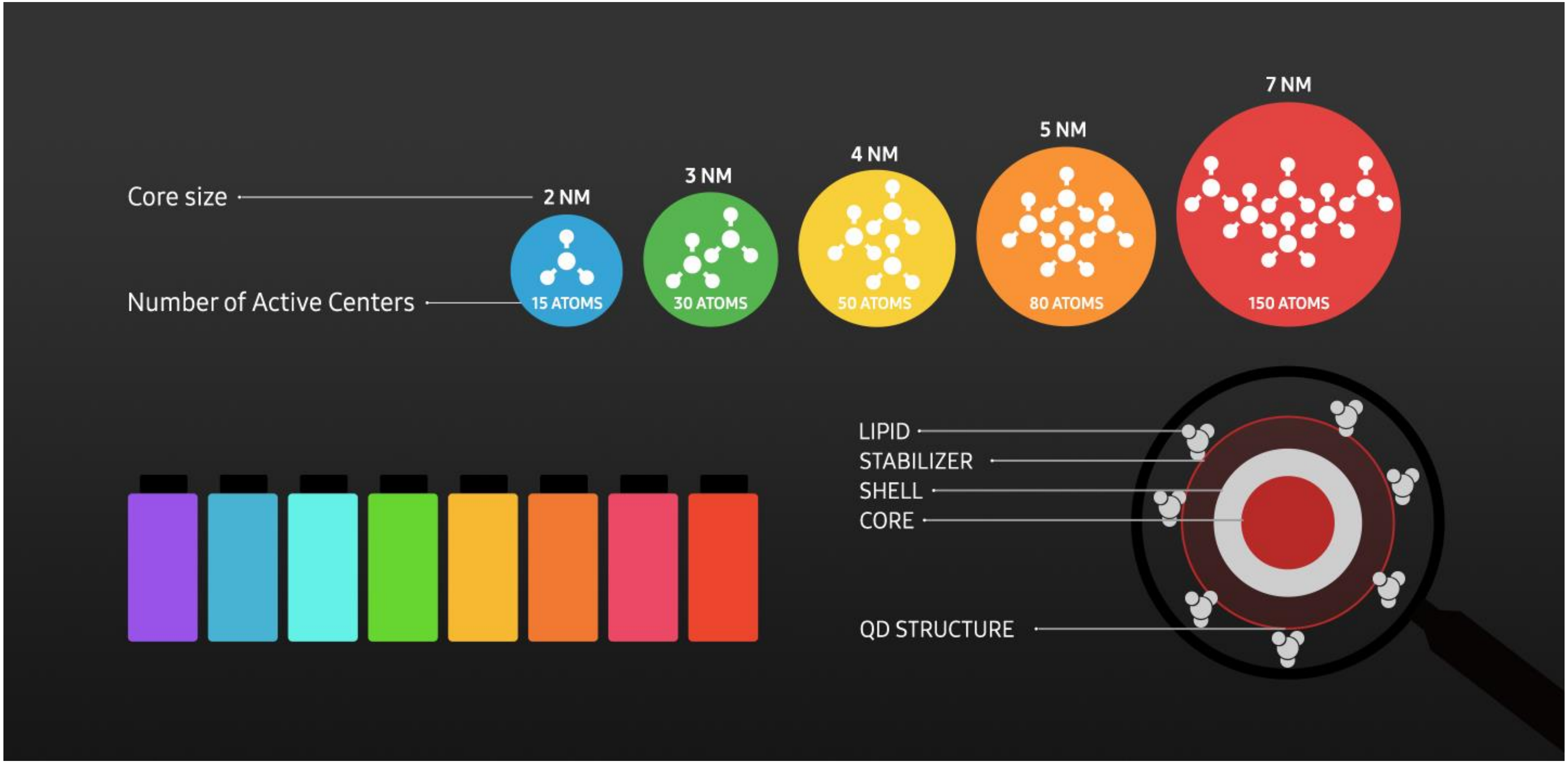

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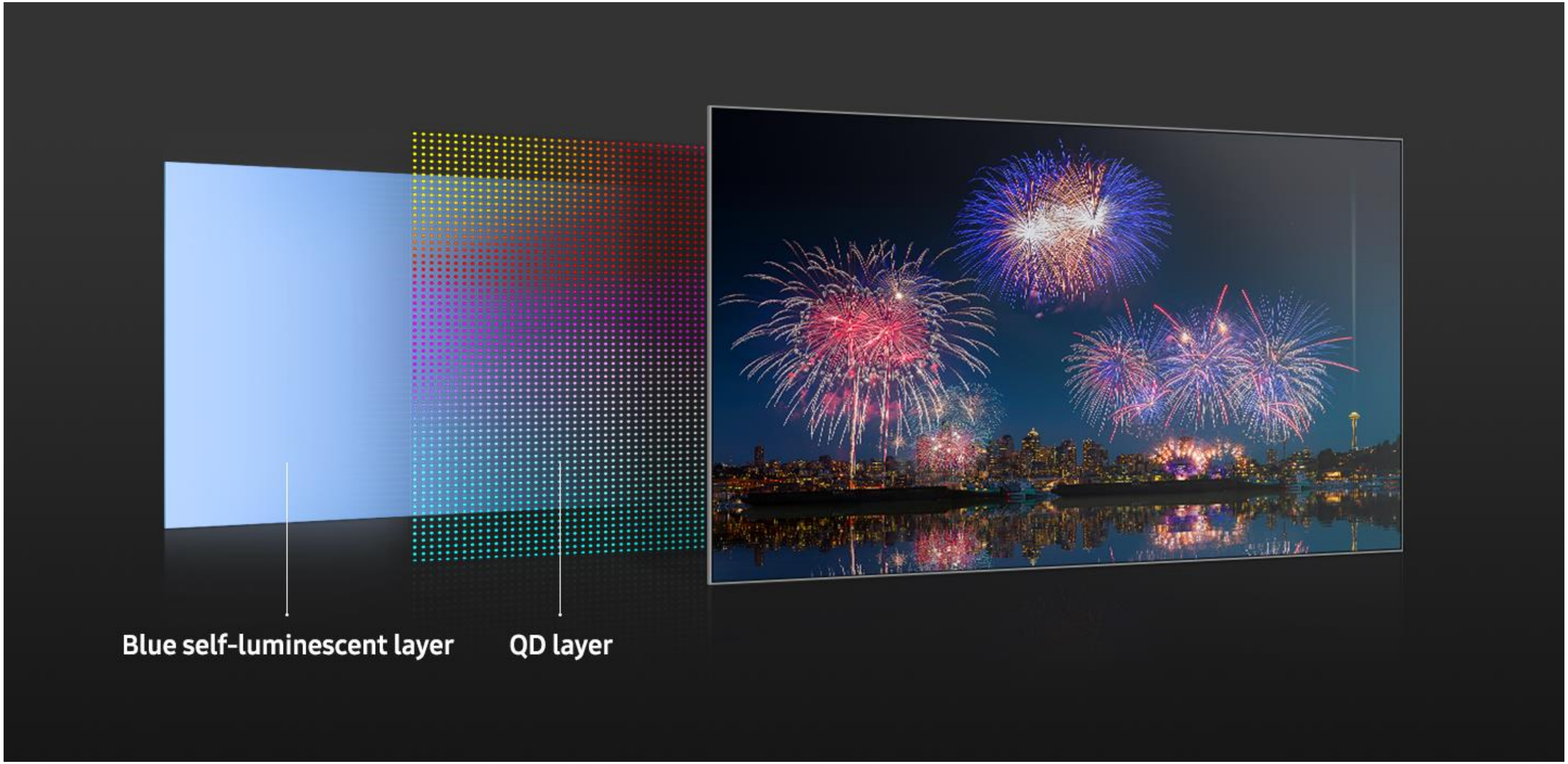

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