22

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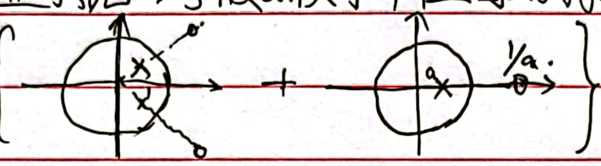
~~相位~~ 因果 与系统

大前提.
① 稳定系统：极点全在圆内.
② 实系统：零极点共轭是一对对关于实轴对称的

③ 线性相位系统(FIR)：$H(z)$零点 = $H(z^{-1})$零点.

④ 全通系统 $H_{ap}(z)$：零极点关于单位圆对称. $P(z) = |H(z)|^2 = H(z)H(z^{-1})$.



$\Rightarrow$ N个零极点对 (N阶).
$\varphi(w=\pi) = -N\pi$

⑤ 最小相位系统：零极点全在单位圆内. $\varphi(w=\pi) = 0$

⑥ 最大相位：零全在圆外 ~~极在圆内~~.

⑦ 混合相位：零有在圆外也有在圆内.

最小相位 $\xrightarrow{+全通}$ 混合相位 $\xrightarrow{+全通}$ 最大相位.

有逆系统.

⑧ 谱分解. 已知 $P(z) = \underbrace{H(z)}_{最小相位}\ \underbrace{H(z^{-1})}_{最大相位}$ 求 $H(z)$. $H(z)$为FIR系统.

$$\cos w = \frac{e^{jw}+e^{-jw}}{2} = \frac{z+z^{-1}}{2}.$$

① 分子分母化为 z 的多项式.
② 因式分解. ③ 将最小相位化为 $(1-az^{-1})$ 最大相位化为 $(1-az)$.

eg. $$\frac{2(1-\frac{2}{3}z^{-1})(1-\frac{3}{2}z^{-1})}{(1-\frac{1}{3}z^{-1})(1-3z^{-1})} = 2\,\frac{\cancel{(-\frac{3}{2}z^{-1})}(1-\frac{2}{3}z^{-1})(1-\frac{2}{3}z)}{\cancel{(-3z^{-1})}(1-\frac{1}{3}z^{-1})(1-\frac{1}{3}z)}.$$

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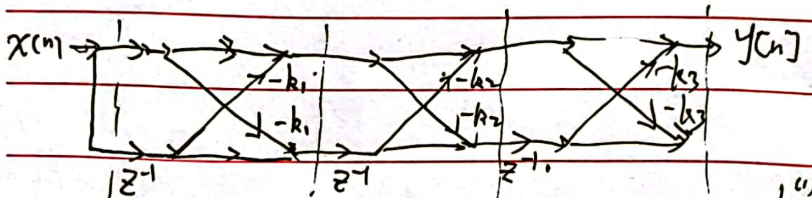
数字信号处理B.

①Lattice网. $H(z)=1+b_3^{(1)}z^{-1}+b_3^{(2)}z^{-2}+b_3^{(3)}z^{-3}$.

$b_3^{(1)}$, $b_3^{(2)}$, $b_3^{(3)}$ $\downarrow$ $k_3=-b_3^{(3)}$.

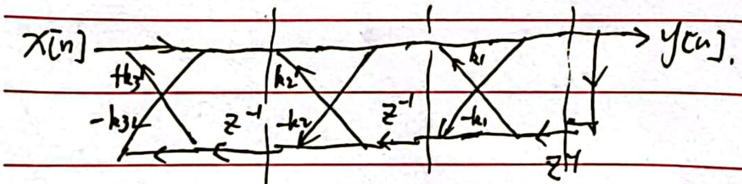
$$\begin{cases} b_2^{(1)}=\dfrac{b_3^{(1)}+k_3b_3^{(2)}}{1-k_3^2} \\ b_2^{(2)}=\dfrac{b_3^{(2)}+k_3b_3^{(1)}}{1-k_3^2} \\ \downarrow \\ k_2=-b_2^{(2)} \end{cases}$$

$$\begin{cases} b_1^{(1)}=\dfrac{b_2^{(1)}+k_2b_2^{(1)}}{1-k_2^2} \\ \downarrow \\ k_1=-b_1^{(1)} \end{cases}$$



B. $H(z)=\dfrac{1}{1+b_3^{(1)}z^{-1}+b_3^{(2)}z^{-2}+b_3^{(3)}z^{-3}}$

$\begin{rcases} b_3^{(1)} \\ b_3^{(2)} \\ b_3^{(3)} \end{rcases} \Rightarrow \begin{cases} k_1 \\ k_2 \\ k_3. \end{cases}$



C. $H(z)=\dfrac{b_0+b_1z^{-1}+b_2z^{-2}+b_3z^{-3}}{1+a_3^{(1)}z^{-1}+a_3^{(2)}z^{-2}+a_3^{(3)}z^{-3}}$

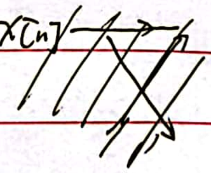
$\begin{rcases} a_3^{(1)} \\ a_3^{(2)} \\ a_3^{(3)} \end{rcases} \begin{matrix} k_1 & a_2^{(1)} \\ k_2 & a_2^{(2)}, \\ k_3 & a_1^{(1)} \end{matrix}$

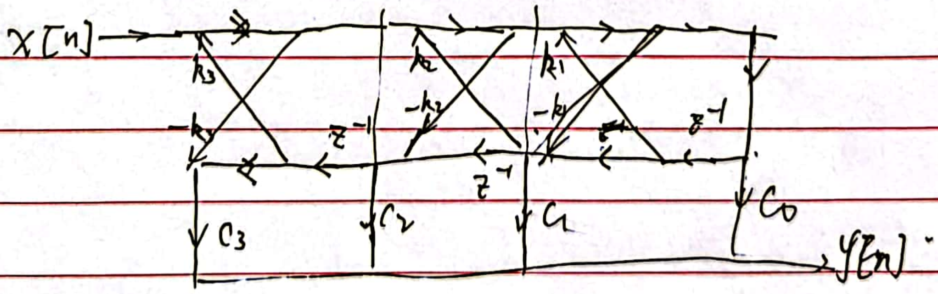
$c_3=b_3$

$c_2=b_2-c_3a_3^{(1)}$

$c_1=b_1-c_2a_2^{(1)}-c_3a_3^{(2)}$

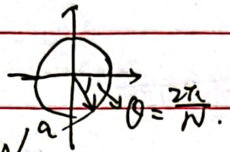
$c_0=b_0-c_1a_1^{(1)}-c_2a_2^{(2)}-c_3a_3^{(3)}$.





FFT

$W_N = e^{-j\frac{2\pi}{N}}$

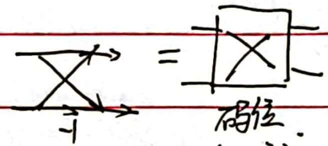


$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk}$

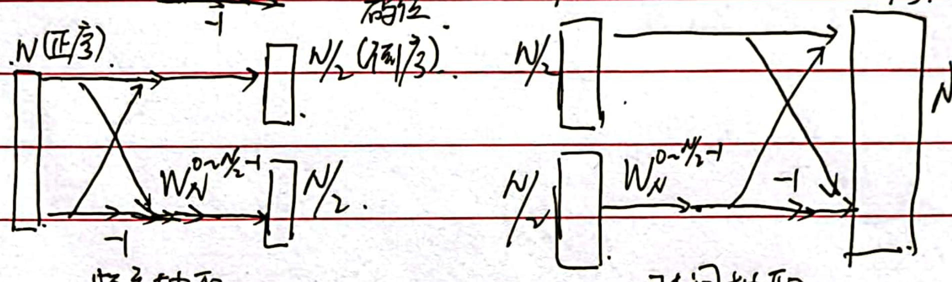
$W_N = \cancel{W_{N/a}}\, W_{aN}^{a}$

$W_2^1 = -1$

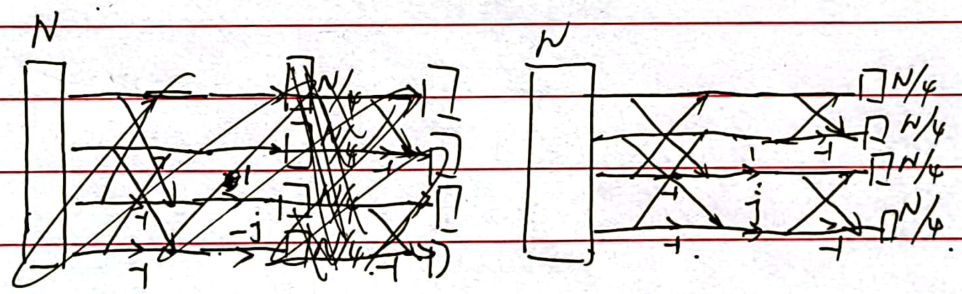
两点FFT:



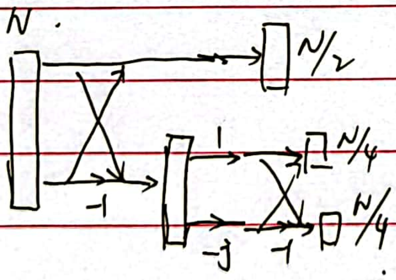
① 基2



② 基4



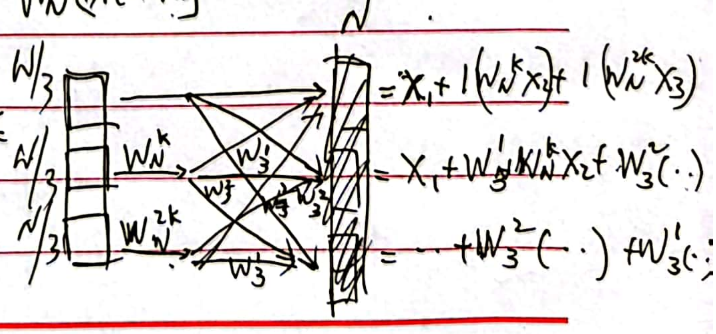
③ 分裂基



④ 注意公式推导中. $X(k) = FFT[x(2r)] + W_N^k[x(2r+1)]$

$X(k+N/2)$

若分割成三段呢? 推导三



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滤波器设计（模拟滤波）

input

① 归一化变量.（$\Omega_p, \Omega_s, \alpha_p, \alpha_s$）.

$p = j\lambda$.

$s = j\Omega$.

$\lambda_p = \frac{\Omega_p}{\Omega_p} = 1$

$\lambda_s = \frac{\Omega_s}{\Omega_p}$

以dB为单位

② 求阶数参数（$\lambda_p, \lambda_s, \alpha_p, \alpha_s$）（$N$）.

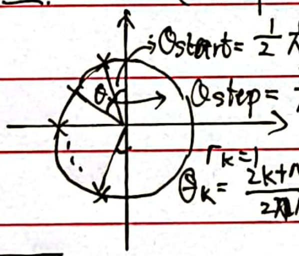
$\varepsilon = 1 \quad \overset{\alpha_p \neq 3dB}{=} \quad \sqrt{10^{\alpha_p/10} - 1}$

Butterworth: $N = \frac{\lg\sqrt{10^{\alpha_s/10}-1}}{\lg \lambda_s} \quad \overset{\alpha_p \neq 3dB}{=} \quad \frac{\lg\sqrt{\frac{\cdots}{\varepsilon}}}{\lg \lambda_s}$

ChebshevI: $N = \frac{\cosh^{-1}\sqrt{10^{\alpha_s/10}-1}}{\cosh^{-1}\lambda_s} \quad \overset{\alpha_p \neq 3dB}{=} \quad \frac{\cosh^{-1}\sqrt{\frac{\cdots}{\varepsilon}}}{\cosh^{-1}\lambda_s}$

③ 标准模型.（N）.（$G(p)$）.

Butterworth:



$\theta_{start} = \frac{1}{2N}\pi$

$\theta_{step} = \frac{\pi}{N}$.

$r_k = 1$

$\theta_k = \frac{2k+N-1}{2N}\pi$

$G_2(p) = \frac{1}{p^2+\sqrt{2}p+1}$

$G_3(p) = \frac{1}{(p+1)(p^2+p+1)} = \frac{1}{p^3+2p^2+2p+1}$

$G_4(p) = \frac{1}{(p^2+2\cos\frac{\pi}{8}p+1)(p^2+2\sin\frac{\pi}{8}p+1)}$

ChebshevI: $p = j\lambda$. $G_n(\lambda) = \frac{1}{1+\varepsilon^2 C_n^2(\lambda)}$.

$\cancel{G_1(\lambda) = \frac{1}{1+\lambda^2} = \frac{1}{1-p^2}}$

$\cancel{G_2(\lambda) = \frac{1}{1+(2\lambda^2-1)^2} = \frac{1}{1+(2p^2+1)^2}}$

$\cancel{G_3(\lambda) = \frac{1}{1+(4\lambda^3-3\lambda)^2}}$???

[illegible]

$$\begin{cases} p_k = -\sin\varphi_1 \sinh\varphi_2 + j\cos\varphi_1 \cosh\varphi_2 \\ \varphi_1 = \frac{2k-1}{2N}\pi \quad k=1,\cdots N. \\ \varphi_2 = \frac{1}{n}\operatorname{arcsinh}\left(\frac{1}{\varepsilon}\right) = \frac{0.88137}{n}. \end{cases}$$

④ 回代参数.（$G(p)$）（$G(s)$）.

$G(s) = G(p = \frac{s}{\Omega_p})$.

⑤ 分解因式.（$G(s)$）.

$G(s) = A \cdot \prod \frac{\beta_i}{(s-\alpha_i)^2+\beta_i^2} \prod \frac{1}{(s-\gamma_i)}$.

4

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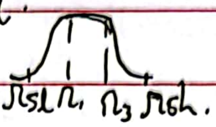
滤波器设计(模拟高通带通带阻)

A. 高通: ① 归一化 ② 转化. ③ 计算 ④ 回代 ⇒ $G(s)$.

$(\Omega_p,\Omega_s)(\lambda_p,\lambda_s)$. $\eta_p=1$ $\eta_s=\frac{\Omega_s}{\Omega_p}$ $\lambda=\frac{1}{\eta}$ $p=j\lambda=\frac{j}{\eta}=\frac{j\Omega_p}{\Omega}=\frac{\Omega_p}{s}$

B. 带通: $(\Omega_{sl},\Omega_1,\Omega_3,\Omega_{sh})$



① 归一化.

带宽: $\Omega_{BW}=\Omega_3-\Omega_1$

中心频率: $\Omega_2=\sqrt{\Omega_1\Omega_3}$

$\eta=\frac{\Omega}{\Omega_{BW}}$ ② $\lambda=\frac{\eta^2-\eta_2^2}{\eta}$ ~~其中λ取较小者~~

③ 计算 ④ 回代, $p=\frac{s^2+\Omega_1\Omega_3}{s(\Omega_3-\Omega_1)}$.

C. 带阻.

① $\Omega_{BW}=\Omega_3-\Omega_1$ ② $\lambda=\frac{\eta}{\eta^2-\eta_2^2}$ ③ . . ④ : . $p=\frac{s(\Omega_3-\Omega_1)}{s^2+\Omega_1\Omega_3}$.

$\Omega_2=\sqrt{\Omega_1\Omega_3}$

$\eta=\frac{\Omega}{\Omega_{BW}}$.

A.

滤波器设计(模→数 冲激响应不变法).

① $\omega=\Omega T_s$. $\Omega=\frac{\omega}{T_s}$.

② 模拟得 $G(s)=A\prod\frac{\beta_i}{(s-\alpha_i)^2+\beta_i^2}\prod\frac{1}{s-\alpha_i}\cancel{\prod\frac{s-\alpha_i}{(s-\alpha_i)^2+\beta_i^2}}$ $\frac{z}{z-e^{\alpha T_s}}$

③ $H(z)=A\prod\frac{e^{\alpha T_s}\sin(\beta T_s)\,z}{z^2-[e^{\alpha T_s}\cos(\beta T_s)]2z+(e^{\alpha T_s})^2}\prod$

B. 双线性 z 变换法.

① $\Omega=\tan\left(\frac{\omega}{2}\right)$.

② $G(s)$.

③ $H(z)=G\left(s=\frac{z-1}{z+1}\right)$

FIR滤波器设计(窗函数法) 长度=N. M=N-1.

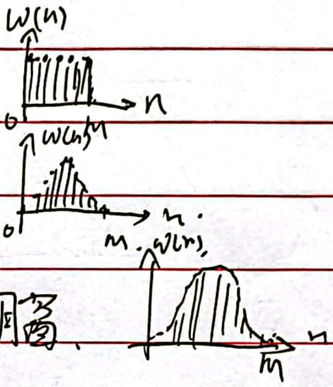
① 得 $H_d(e^{j\omega})$.

② $h_d[n]=\frac{1}{2\pi}\int_{-\pi}^{\pi}H_d(e^{j\omega})e^{j\omega n}d\omega$.

注意讨论 $n=0$ 情况.

③ $h[n]=h_d[n-M/2]w[n]$.

$$w[n]=\begin{cases}1, & n=0,\dots,M. \text{ 矩形窗} \\ \begin{cases}2n/N. & n=0\dots N/2 \\ w(N-n) & n=N/2\dots M\end{cases} & \text{三角窗} \\ 0.5-0.5\cos(\frac{2\pi n}{N}), & n=0\dots M \text{ 汉宁窗} \\ 0.54-0.46\cos(\frac{2\pi n}{N}) & n=0\dots M \text{ 汉明窗}\end{cases}$$



FIR滤波器设计(频率抽样法).

① $H_d(e^{j\omega})$

② $H_{d_k}[k]=H_d(\omega=\frac{2\pi}{N}k)e^{-jk\frac{M}{N}\pi}$

③ $h[n]=\frac{1}{N}\sum_{k=0}^{N-1}H_d(k)e^{j\frac{2\pi}{N}nk}$. IDFT.

1/4

Fourier Transform,
离散 ↔ 周期
连续 ↔ 非周期,

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1. 频率转换:

时域分辨率: $\Delta t = T_s$.
频域分辨率: $\Delta \omega = \frac{2\pi}{N}$
$\Delta f = \frac{F_s}{N}$

f(Hz): 数字频率

ω(rad): 数字单位角度(1单位内转角度),

Ω(rad/s): 模拟角频率

关系. $f \longrightarrow \omega \longrightarrow \Omega$.

$F_s = \frac{1}{T_s}$. $[0, F_s]$. $[0, 2\pi]$ $[0, 2\pi F_s]$

$\omega = \frac{2\pi}{F_s} f$. $\Omega = \omega / T_s = 2\pi f$.

CFS: $X(k\Omega_0) = \frac{1}{T}\int_{\langle T \rangle} x(t) e^{-jk\Omega_0 t} dt$.

$x(t) = \sum_{k=-\infty}^{\infty} X(k\Omega_0) e^{jk\Omega_0 t}$

CFT: $X(j\Omega) = \int_{-\infty}^{\infty} x(t) e^{-j\Omega t} dt$.

$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\Omega) e^{j\Omega t} d\Omega$.

DTFS: $\tilde{X}(k) = \frac{1}{N}\sum_{n=0}^{N-1} \tilde{x}[n] e^{-j\frac{2\pi}{N}kn}$

$\tilde{x}[n] = \sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi}{N}kn}$.

DTFT: $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$

$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$

卷积; 相关系数:

$r_{xy}(m) = \sum_{n=-\infty}^{\infty} x^*(n) y(n+m) = x(-m) * y(m)$

$r_x(m) = r_{xx}(m) = \sum_{n=-\infty}^{\infty} x^*(n) x(n+m) = x(-m) * x(m)$

$h[n] = x[n] * y[n] = \sum_{k=-\infty}^{\infty} x[k] y[n-k]$

L: $X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$

Z: $X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$.

$e^{-j\Omega t} = e^{-st} = z^{-n}$.

$s = j\omega$

$z = e^{-j\omega}$.

线性卷积, 循环卷积实现有限长卷积: 补0至

$L_1 * L_2 = L_1 + L_2 - 1$ 长度 $N_1 + N_2 - 1$

Z变换: $\sum_{n=0}^{\infty} a^n$ 收敛: $|a| < 1$.

$\leftarrow \frac{1}{1-a}$.

长卷积 $\begin{cases} \text{重叠相加法} \\ \text{重叠舍去法} \end{cases}$

相加