数字信号处理B

PB21511897 李霄奕

HW5

Exercise 1

自制fft函数:

```
function y = MATLAB_ditfft(x)
    m=nextpow2(length(x));
    N=2^m;
    if length(x)<N
        x=[x,zeros(1,N-length(x))];
    end
    %nxd=bin2dec(fliplr(dec2bin([1:N]-1,m)))+1;
    nxd0=dec2bin([1:N]-1,m);
    nxd1=fliplr(nxd0);
    nxd=bin2dec(nxd1)+1;
    y=x(nxd);
    for mm=1:m
        Nmr=2^mm;
        u=1;
        WN=exp(-1i*2*pi/Nmr);
        for j=1:Nmr/2
            for k=j:Nmr:N
                kp=k+Nmr/2;
                t=y(kp)*u;
                y(kp)=y(k)-t;
                y(k)=y(k)+t;
            end
            u=u*WN;
        end
    end
end
```

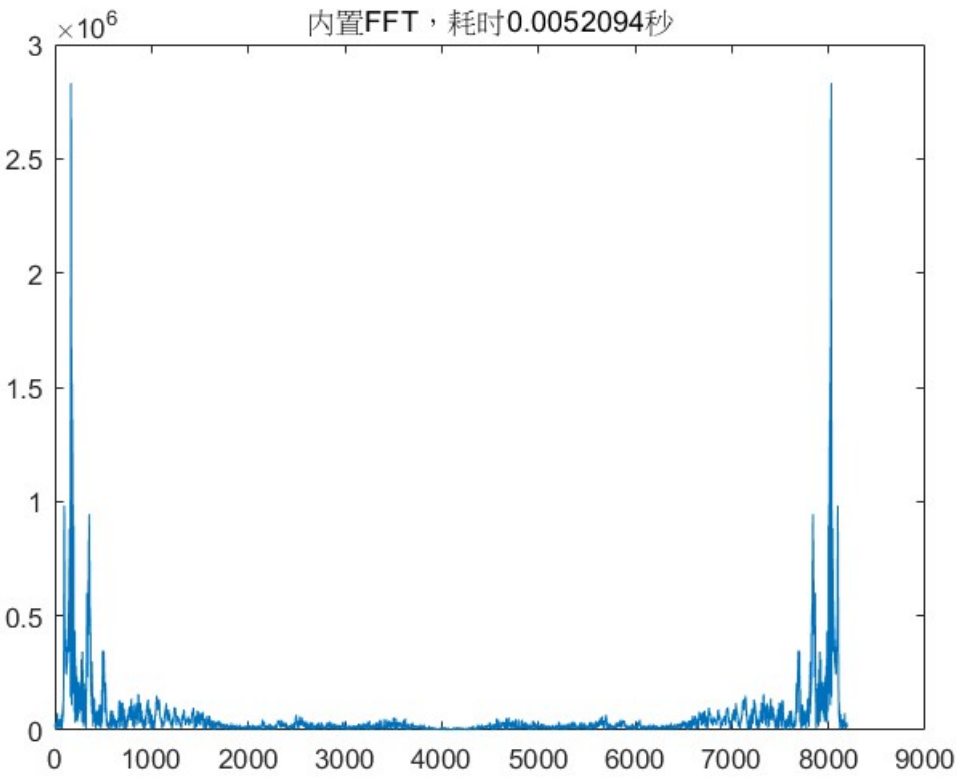
自制完全未优化的DFT函数:

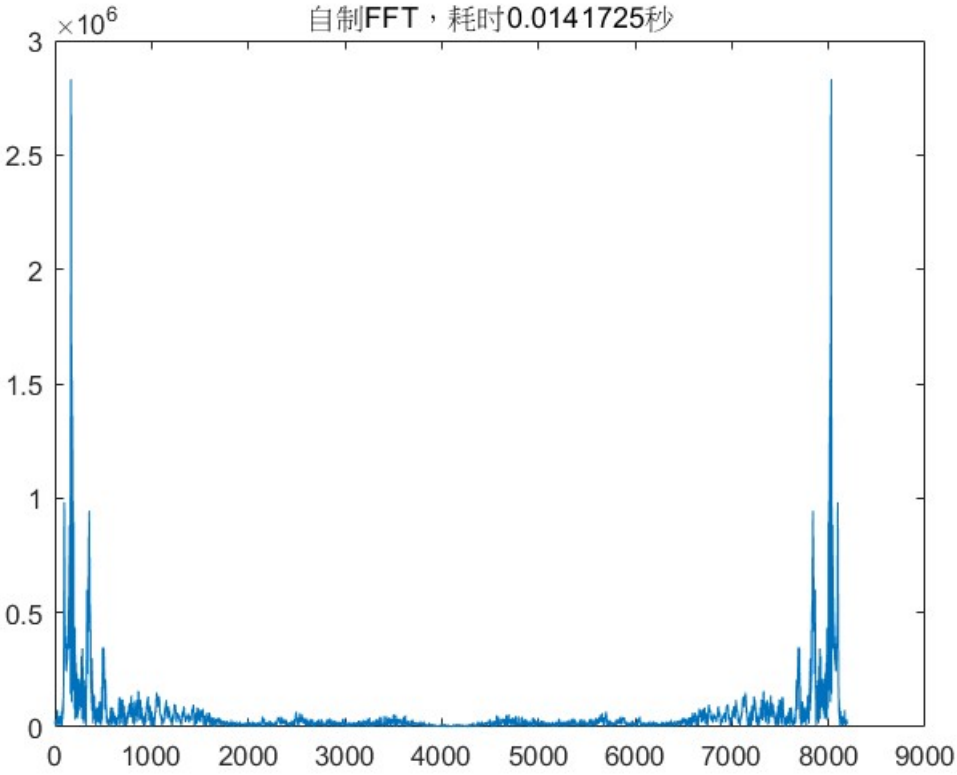
```
function y=MATLAB_dft(x)
    N=length(x);
    x=x';
    A=zeros(N,N);
    W=exp(-1i*2*pi/N);
    for i=0:N-1
        for j=0:N-1
            A(i+1,j+1)=W^(i*j);
        end
    end
    y=A*x;
end

```

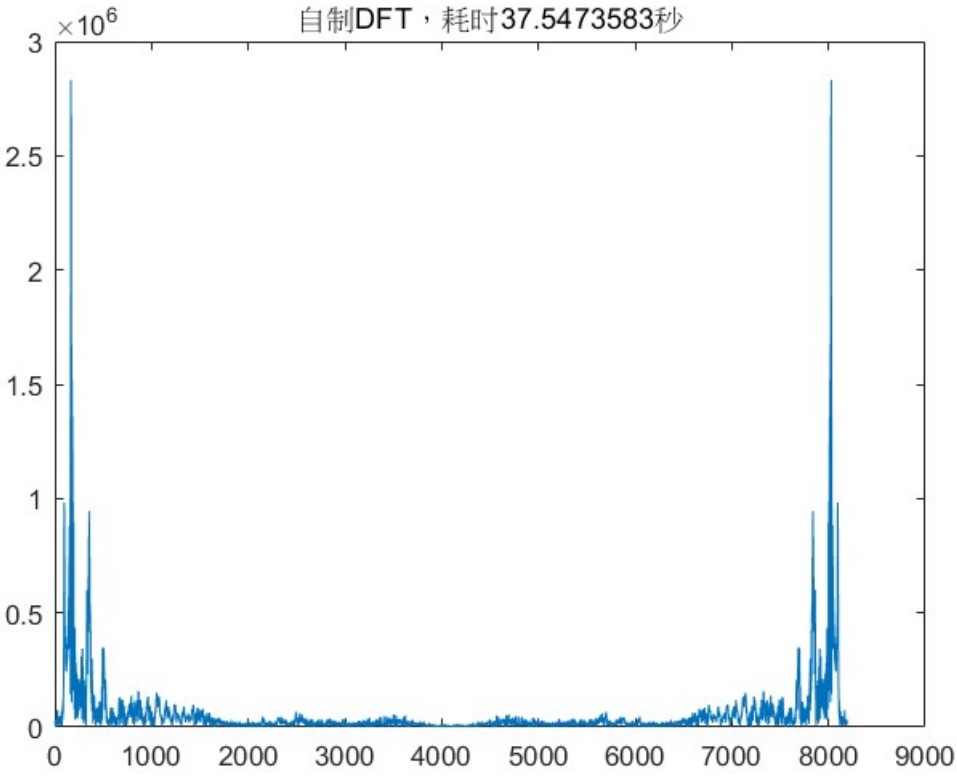
验证代码：

```
[y0,fs]=audioread("bluesky1.wav");
y=y0(1:8192,1);
y=y';
y=y.*32768;
tic;
Y1=fft(y);
t1=toc
tic;
Y2=MATLAB_ditfft(y);
t2=toc
tic;
Y3=MATLAB_dft(y);
t3=toc

plot(abs(Y1));
title("内置FFT，耗时"+mat2str(t1)+"秒");
plot(abs(Y2));
title("自制FFT，耗时"+mat2str(t2)+"秒");
plot(abs(Y3));
title("自制DFT，耗时"+mat2str(t3)+"秒");
```

结果：







从运行耗费时间上来说，自制DFT>>自制FFT>内置FFT。

从输出结果来看，三幅图片完全相同，所以，DFT和FFT算法都是正确的。

Exercise 2

由Nyquist采样定理，抽样频率f_s应当大于信号频率的2倍，所以

$$
\begin{gathered}
f_s \geqslant 2f_h = 4kHz \\
\Delta f \leqslant 2Hz \\
T \geqslant 1/\Delta f = 0.5s \\
N \geqslant T \cdot f_s = 2000 \\
N_{min} = 2^{11} = 2048 \\
T_{min} = N/f_s = 0.512s
\end{gathered}
$$

Exercise 3

$$
\begin{aligned}
X(k) &= \sum_{n=0}^{23} x(n) W_{24}^{nk} \\
&= \sum_{n=0}^{7} x(3n) W_{24}^{3nk} + \sum_{n=0}^{7} x(3n+1) W_{24}^{3nk+k} + \sum_{n=0}^{7} x(3n+2) W_{24}^{3nk+2k} \\
&= \sum_{n=0}^{7} x(3n) W_{8}^{nk} + W_{24}^{k} \cdot \sum_{n=0}^{7} x(3n+1) W_{8}^{nk} + W_{12}^{k} \cdot \sum_{n=0}^{7} x(3n+2) W_{8}^{nk} \\
&= FFT_8(x(3n)) + W_{24}^{k} \cdot FFT_8(x(3n+1)) + W_{12}^{k} \cdot FFT_8(x(3n+2))
\end{aligned}
$$