电路基本理论

二端口网络

Huarui Yin

中国科学技术大学
电子工程与信息科学系
Hefei, Anhui, 230027

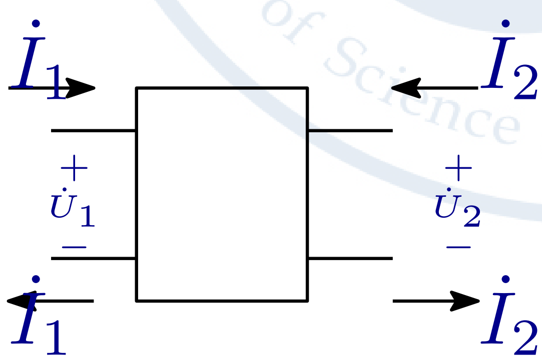
内容简介

- 介绍二端口的各种参数方程，重点包括阻抗参数方程，导纳参数方程，传输参数方程和混合参数方程。
- T等效和 Π等效
- 二端口网络和电源、负载的相互连接及特性

P299 10.2 10.3 10.5 10.7 10.12 10.13 10.15

二端口网络

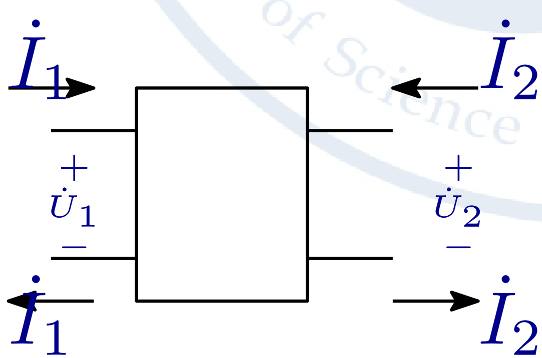
- 端口：对于一个电路给定的一对端子，其中一个端子流入的电流等于从另一个端子流出的电流
- 二端口网络：一个模块或者元件输入输出满足 2 个端口的约束



$\dot{U}_1$: input voltage
$\dot{I}_1$: input current
$\dot{U}_2$: output voltage
$\dot{I}_2$: output current

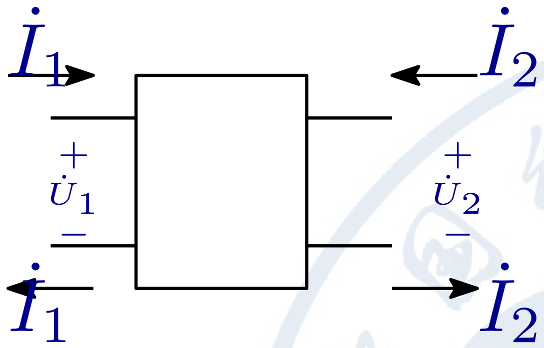
二端口网络

- 端口：对于一个电路给定的一对端子，其中一个端子流入的电流等于从另一个端子流出的电流
- 二端口网络：一个模块或者元件输入输出满足 2 个端口的约束
- 可以将一个电路或者元件利用其端口特性进行隔离
- 隔离的电路成为一个黑盒子，Black Box



$\dot{U}_1$: input voltage
$\dot{I}_1$: input current
$\dot{U}_2$: output voltage
$\dot{I}_2$: output current

导纳参数方程

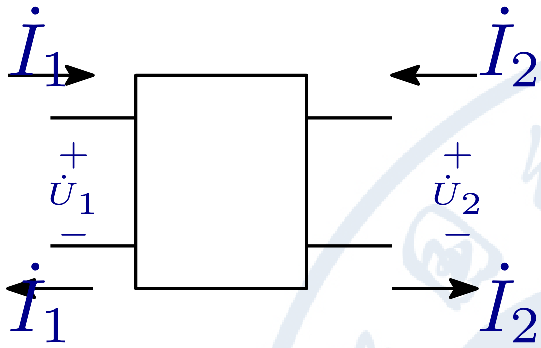


输入：$\dot{U}_1, \dot{U}_2$，输出：$\dot{I}_1, \dot{I}_2$

$$\dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2$$
$$\dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2$$

$$\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix}$$

导纳参数方程



输入：$\dot{U}_1, \dot{U}_2$， 输出：$\dot{I}_1, \dot{I}_2$

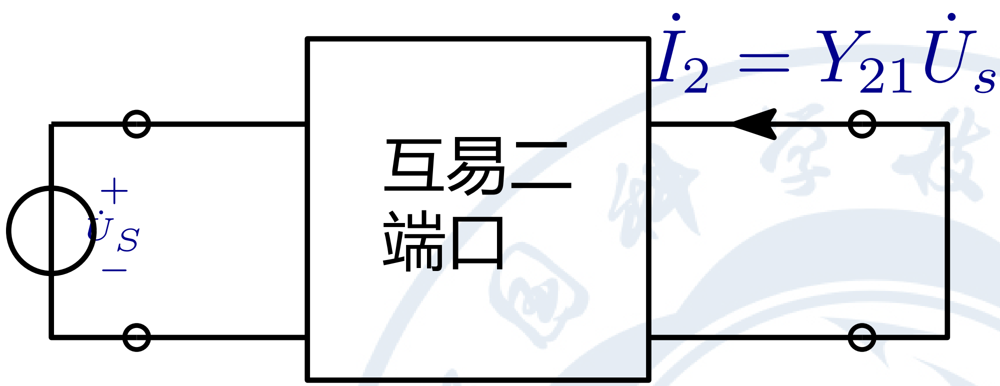
$$\dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2$$
$$\dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2$$

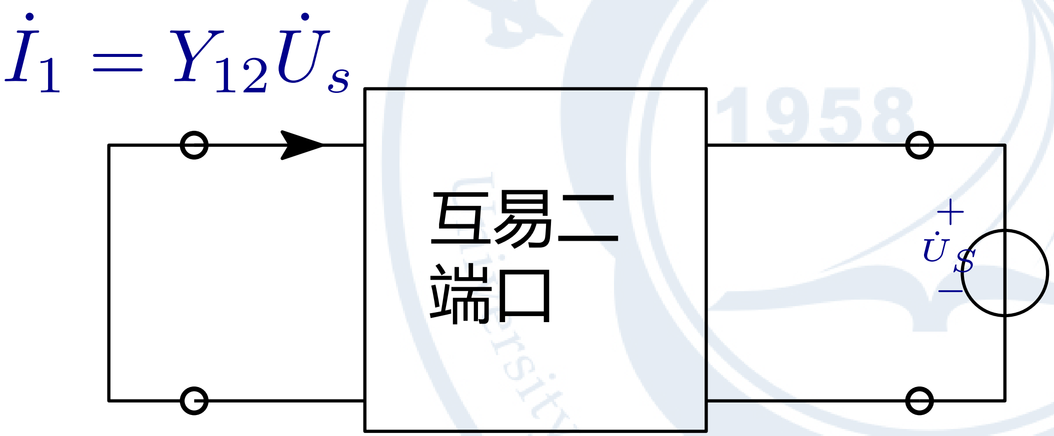
$$\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix} = \boxed{\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}} \begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix}$$

导纳参数矩阵，也称为 Y 参数矩阵

$$Y_{11} = \frac{\dot{I}_1}{\dot{U}_1}\Big|_{\dot{U}_2=0} \quad Y_{12} = \frac{\dot{I}_1}{\dot{U}_2}\Big|_{\dot{U}_2=1} \quad Y_{21} = \frac{\dot{I}_2}{\dot{U}_1}\Big|_{\dot{U}_2=0} \quad Y_{22} = \frac{\dot{I}_2}{\dot{U}_2}\Big|_{\dot{U}_1=0}$$

导纳参数方程（Y参数矩阵)



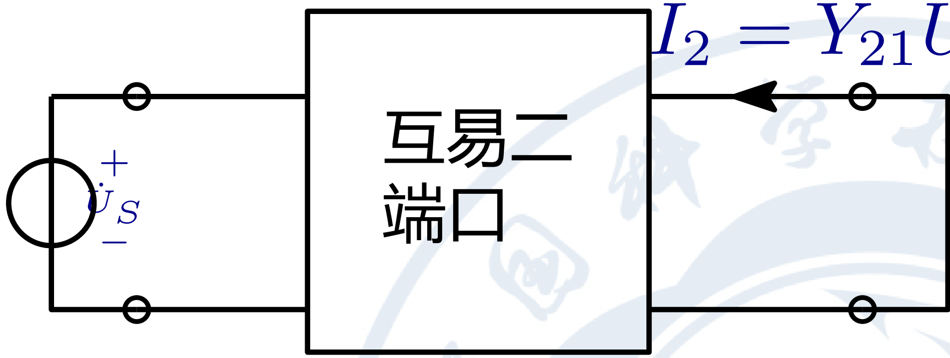


互易条件：$Y_{12} = Y_{21}$

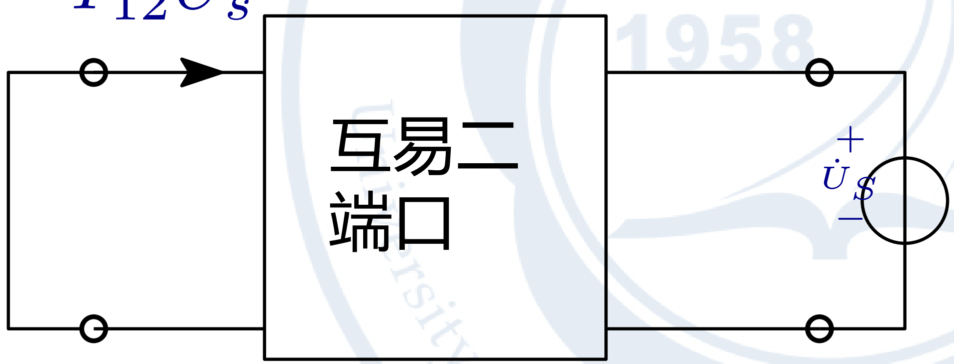
对称条件：$Y_{12} = Y_{21}, Y_{11} = Y_{22}$

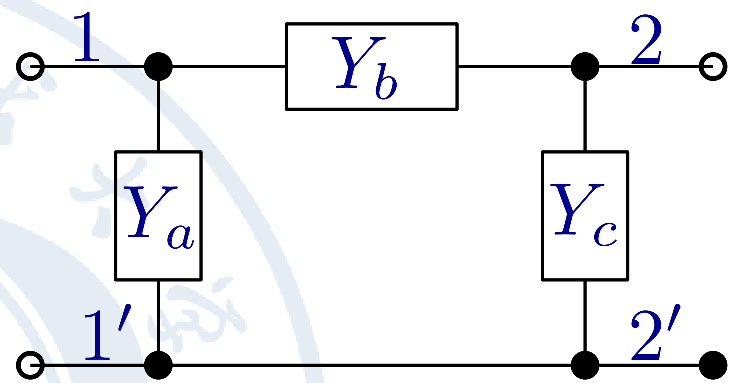
导纳参数方程（Y参数矩阵）

$\dot{I}_2 = Y_{21}\dot{U}_s$



$\dot{I}_1 = Y_{12}\dot{U}_s$





$$Y = \begin{bmatrix} Y_a + Y_b & -Y_b \\ -Y_b & Y_b + Y_c \end{bmatrix}$$

互易条件：$Y_{12} = Y_{21}$

对称条件：$Y_{12} = Y_{21}, Y_{11} = Y_{22}$

传输阻抗参数（z-Parameters)

$$\begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix}$$

where

$$Z_{11} = \frac{\dot{U}_1}{\dot{I}_1}\Big|_{\dot{I}_2=0}, \quad Z_{12} = \frac{\dot{U}_1}{\dot{I}_2}\Big|_{\dot{I}_1=0}$$

$$Z_{21} = \frac{\dot{U}_2}{\dot{I}_1}\Big|_{\dot{I}_2=0}, \quad Z_{22} = \frac{\dot{U}_2}{\dot{I}_2}\Big|_{\dot{I}_1=0}$$

Z_{11}: 开路输入阻抗;

Z_{22}: 开路输出阻抗

Z_{21}, Z_{12}: 开路转移阻抗

传输阻抗参数（z-Parameters)

$$\begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix}$$

where

$$Z_{11} = \frac{\dot{U}_1}{\dot{I}_1}\Big|_{\dot{I}_2=0}, \quad Z_{12} = \frac{\dot{U}_1}{\dot{I}_2}\Big|_{\dot{I}_1=0}$$

$$Z_{21} = \frac{\dot{U}_2}{\dot{I}_1}\Big|_{\dot{I}_2=0}, \quad Z_{22} = \frac{\dot{U}_2}{\dot{I}_2}\Big|_{\dot{I}_1=0}$$

Z_{11}: 开路输入阻抗;
Z_{22}: 开路输出阻抗
Z_{21}, Z_{12}: 开路转移阻抗

短路导纳矩阵 (Y) 和开路阻抗矩阵（Z) 的关系：

$$Z = Y^{-1}$$

阻抗参数方程

互易条件:

$$Y_{12} = Y_{21} \to Z_{12} = Z_{21}$$

对称条件:

$$Y_{12} = Y_{21}, Y_{11} = Y_{22} \to Z_{12} = Z_{21}, Z_{11} = Z_{22}$$

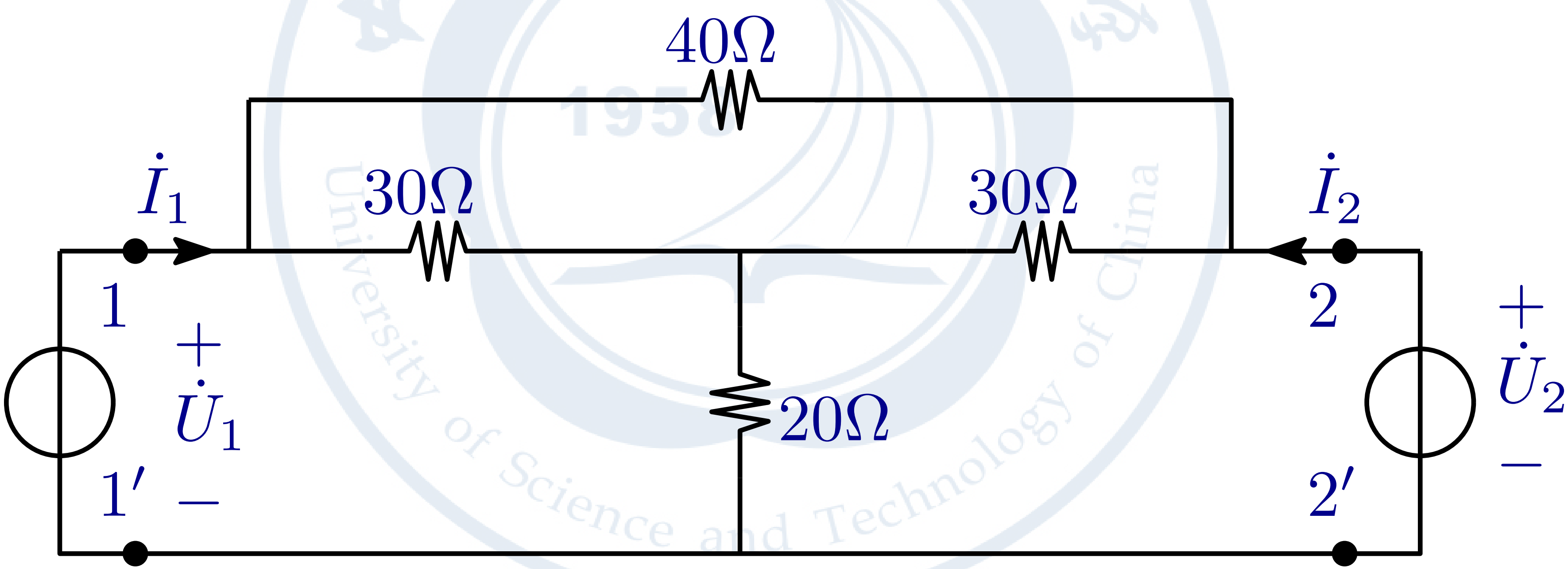
阻抗参数方程

互易条件:

$$Y_{12} = Y_{21} \to Z_{12} = Z_{21}$$

对称条件:

$$Y_{12} = Y_{21}, Y_{11} = Y_{22} \to Z_{12} = Z_{21}, Z_{11} = Z_{22}$$



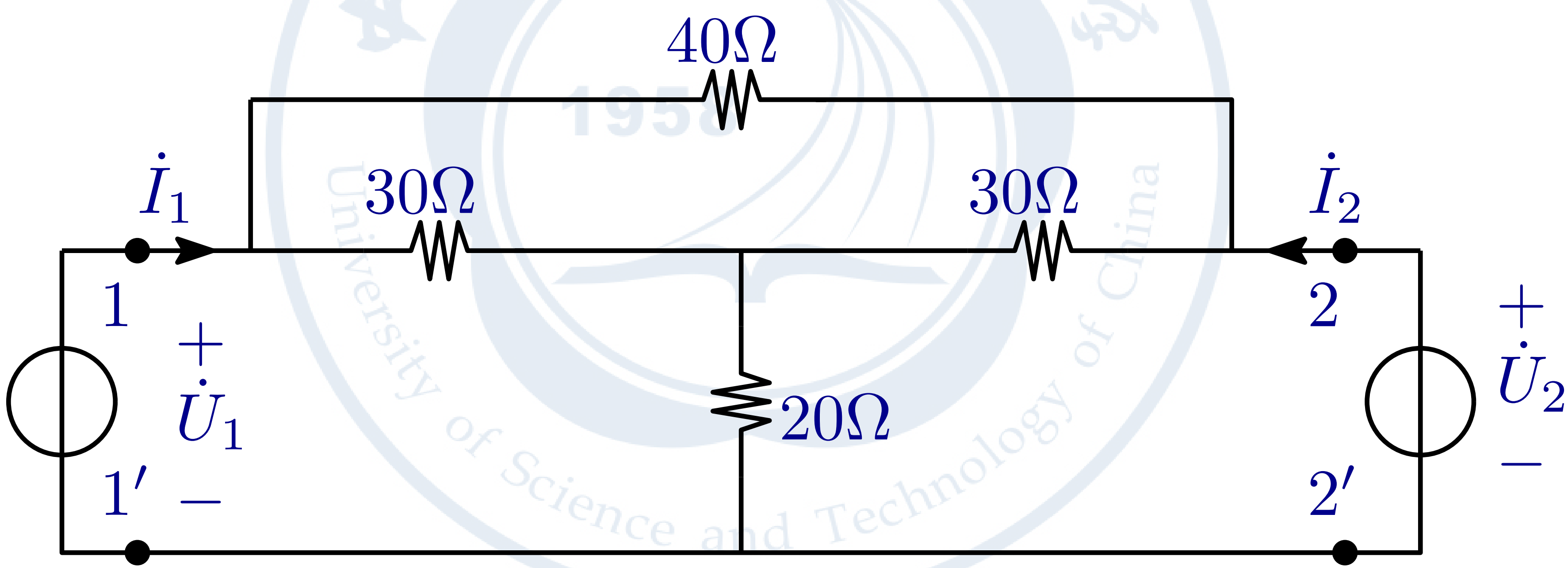
阻抗参数方程

互易条件:

$$Y_{12} = Y_{21} \to Z_{12} = Z_{21}$$

对称条件:

$$Y_{12} = Y_{21}, Y_{11} = Y_{22} \to Z_{12} = Z_{21}, Z_{11} = Z_{22}$$



$$Z = \begin{bmatrix} 41 & 29 \\ 29 & 41 \end{bmatrix} \Omega$$

对称二端口，互易二端口

混合参数方程，Hybrid-Parameters

输入：$(\dot{I}_1)$，$(\dot{U}_2)$ 输出：$(\dot{I}_2)$，$(\dot{U}_1)$

$$\begin{aligned}\dot{U}_1 &= H_{11}\dot{I}_1 + H_{12}\dot{U}_2 \\ \dot{I}_2 &= H_{21}\dot{I}_1 + H_{22}\dot{U}_2\end{aligned} \Rightarrow \begin{bmatrix}\dot{U}_1 \\ \dot{I}_2\end{bmatrix} = \boxed{\begin{bmatrix}H_{11} & H_{12} \\ H_{21} & H_{22}\end{bmatrix}}\begin{bmatrix}\dot{I}_1 \\ \dot{U}_2\end{bmatrix}$$

混合参数方程，Hybrid-Parameters

输入：$(\dot{I}_1)$，$(\dot{U}_2)$ 输出：$(\dot{I}_2)$，$(\dot{U}_1)$

$$\begin{array}{l} \dot{U}_1 = H_{11}\dot{I}_1 + H_{12}\dot{U}_2 \\ \dot{I}_2 = H_{21}\dot{I}_1 + H_{22}\dot{U}_2 \end{array} \Rightarrow \begin{bmatrix} \dot{U}_1 \\ \dot{I}_2 \end{bmatrix} = \boxed{\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}} \begin{bmatrix} \dot{I}_1 \\ \dot{U}_2 \end{bmatrix}$$

混合参数矩阵：$H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}$

混合参数方程，Hybrid-Parameters

输入：($\dot{I}_1$)，($\dot{U}_2$) 输出：($\dot{I}_2$)，($\dot{U}_1$)

$$\begin{aligned}\dot{U}_1 &= H_{11}\dot{I}_1 + H_{12}\dot{U}_2 \\ \dot{I}_2 &= H_{21}\dot{I}_1 + H_{22}\dot{U}_2\end{aligned} \Rightarrow \begin{bmatrix}\dot{U}_1 \\ \dot{I}_2\end{bmatrix} = \boxed{\begin{bmatrix}H_{11} & H_{12} \\ H_{21} & H_{22}\end{bmatrix}} \begin{bmatrix}\dot{I}_1 \\ \dot{U}_2\end{bmatrix}$$

混合参数矩阵：$H = \begin{bmatrix}H_{11} & H_{12} \\ H_{21} & H_{22}\end{bmatrix}$

利用短路导纳矩阵计算混合参数矩阵：

$$H_{11} = Y_{11}^{-1}, H_{12} = -\frac{Y_{12}}{Y_{11}}$$

$$H_{21} = \frac{Y_{21}}{Y_{11}}, H_{22} = \frac{Y_{11}Y_{22} - Y_{21}Y_{12}}{Y_{11}}$$

混合参数方程，Hybrid-Parameters

输入：$(\dot{I}_1)$，$(\dot{U}_2)$ 输出：$(\dot{I}_2)$，$(\dot{U}_1)$

$$\begin{aligned}\dot{U}_1 &= H_{11}\dot{I}_1 + H_{12}\dot{U}_2 \\ \dot{I}_2 &= H_{21}\dot{I}_1 + H_{22}\dot{U}_2\end{aligned} \Rightarrow \begin{bmatrix} \dot{U}_1 \\ \dot{I}_2 \end{bmatrix} = \boxed{\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}} \begin{bmatrix} \dot{I}_1 \\ \dot{U}_2 \end{bmatrix}$$

混合参数矩阵：$H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}$

利用短路导纳矩阵计算混合参数矩阵：

$$H_{11} = Y_{11}^{-1}, H_{12} = -\frac{Y_{12}}{Y_{11}}$$

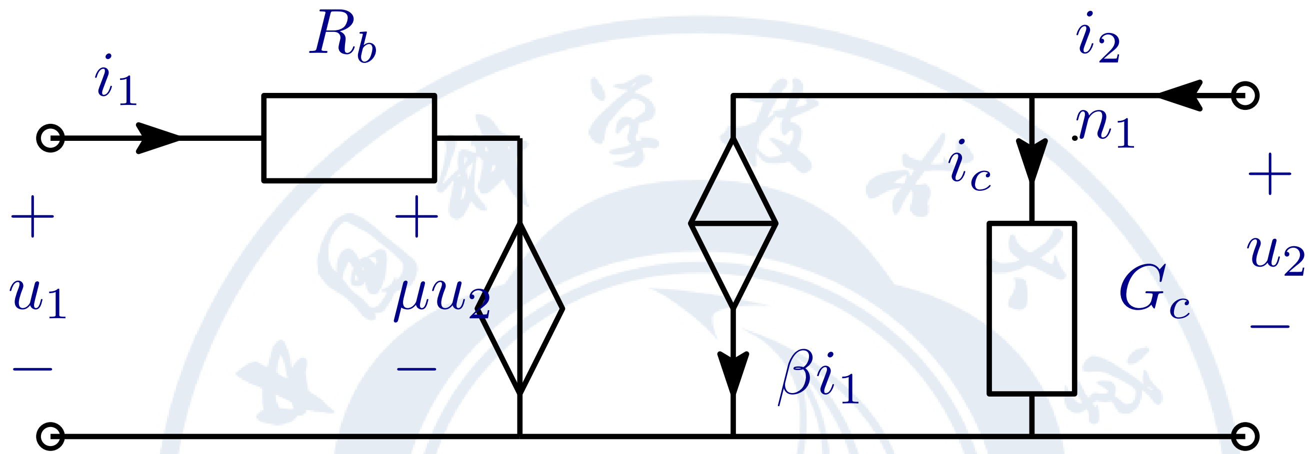
$$H_{21} = \frac{Y_{21}}{Y_{11}}, H_{22} = \frac{Y_{11}Y_{22} - Y_{21}Y_{12}}{Y_{11}}$$

互易条件：$H_{12} = -H_{21}$

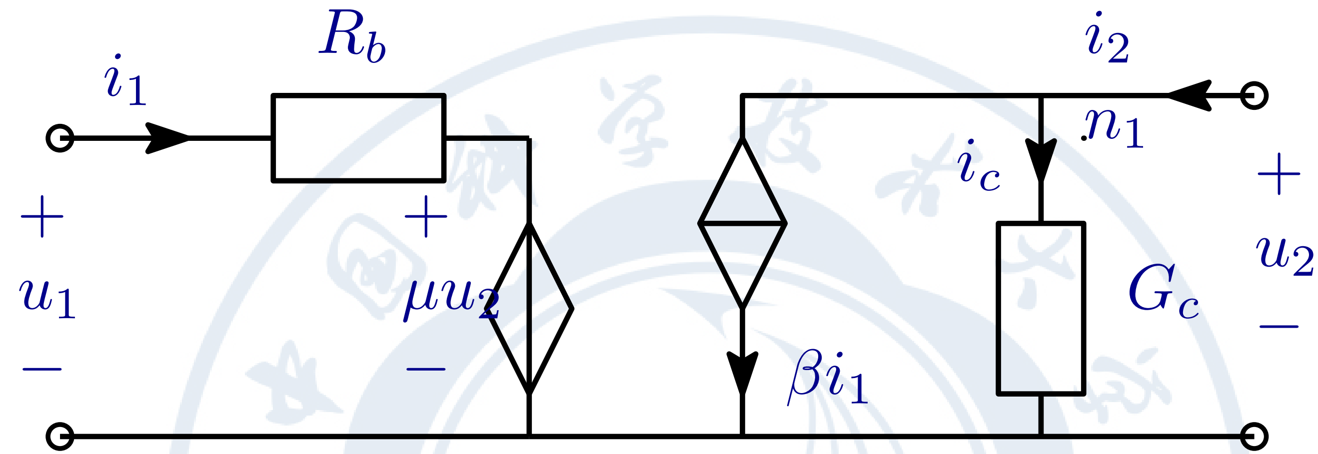
对称条件：

$\Delta_H = H_{11}H_{22} - H_{12}H_{21} = 1$，$H_{12} = -H_{21}$

混合参数方程，Hybrid-Parameters

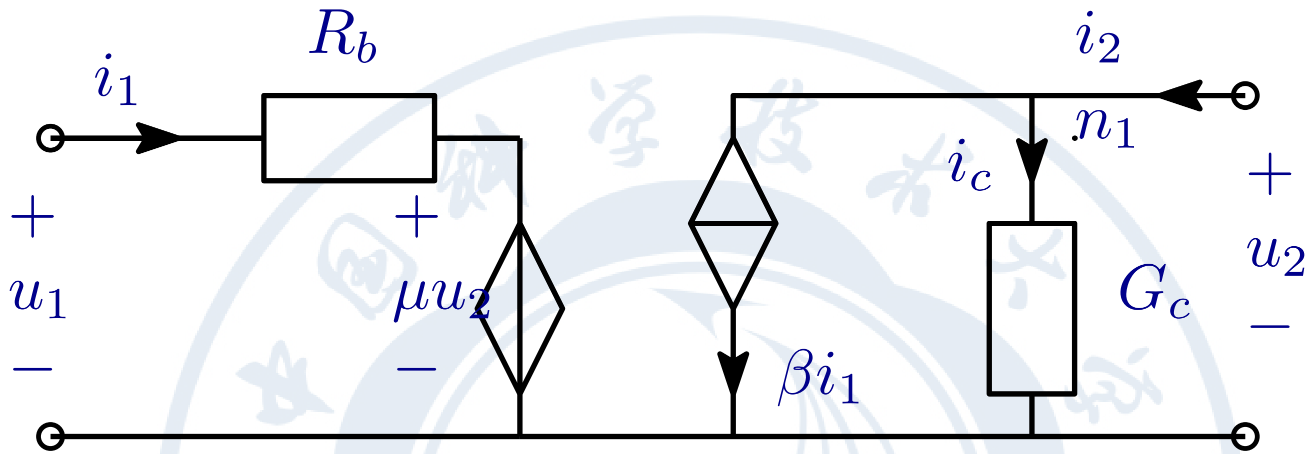


混合参数方程，Hybrid-Parameters



输入端使用 KVL： $u_1 = R_b i_1 + \mu u_2$

混合参数方程，Hybrid-Parameters

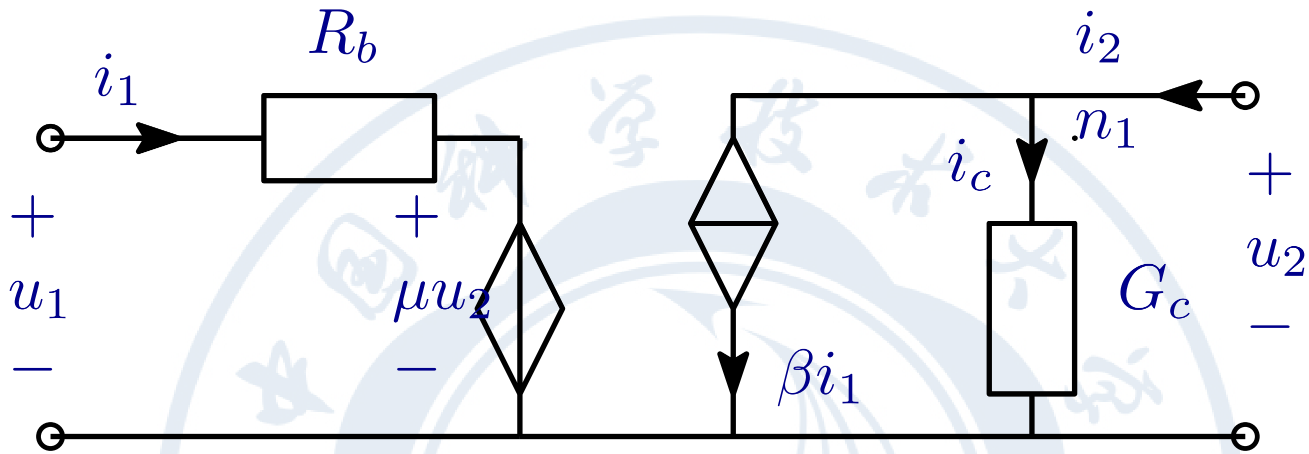


输入端使用 KVL： $u_1 = R_b i_1 + \mu u_2$

节点 n2 使用 KCL: $i_2 = \beta i_1 + G_c u_2$

$$H = \begin{bmatrix} R_b & \mu \\ \beta & G_c \end{bmatrix}$$

混合参数方程，Hybrid-Parameters



输入端使用 KVL： $u_1 = R_b i_1 + \mu u_2$

节点 n2 使用 KCL: $i_2 = \beta i_1 + G_c u_2$

$$H = \begin{bmatrix} R_b & \mu \\ \beta & G_c \end{bmatrix}$$

传输参数方程及矩阵 (A参数)

$$\begin{bmatrix} \dot{U}_1 \\ \dot{I}_1 \end{bmatrix} = \boxed{\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}} \begin{bmatrix} \dot{U}_2 \\ -\dot{I}_2 \end{bmatrix}$$

传输参数 A利用次级变量 $\dot{U}_2, -\dot{I}_2$表征初级参数 $\dot{U}_1, \dot{I}_1$

传输参数方程及矩阵 (A参数)

$$\begin{bmatrix} \dot{U}_1 \\ \dot{I}_1 \end{bmatrix} = \boxed{\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}} \begin{bmatrix} \dot{U}_2 \\ -\dot{I}_2 \end{bmatrix}$$

传输参数 A利用次级变量 $\dot{U}_2, -\dot{I}_2$表征初级参数 $\dot{U}_1, \dot{I}_1$

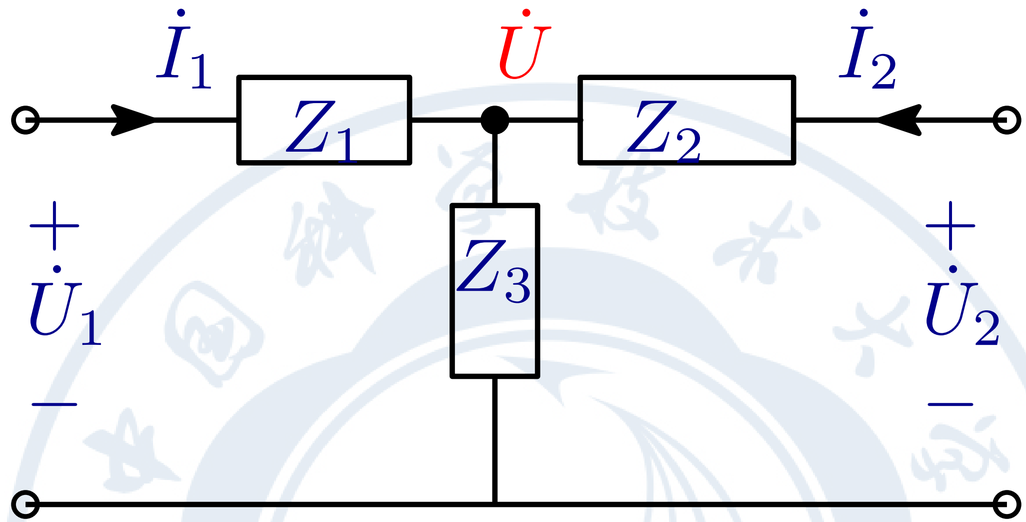
互易条件:

$$\Delta_A = A_{11}A_{22} - A_{12}A_{21} = \frac{Y_{12}}{Y_{21}} = 1$$

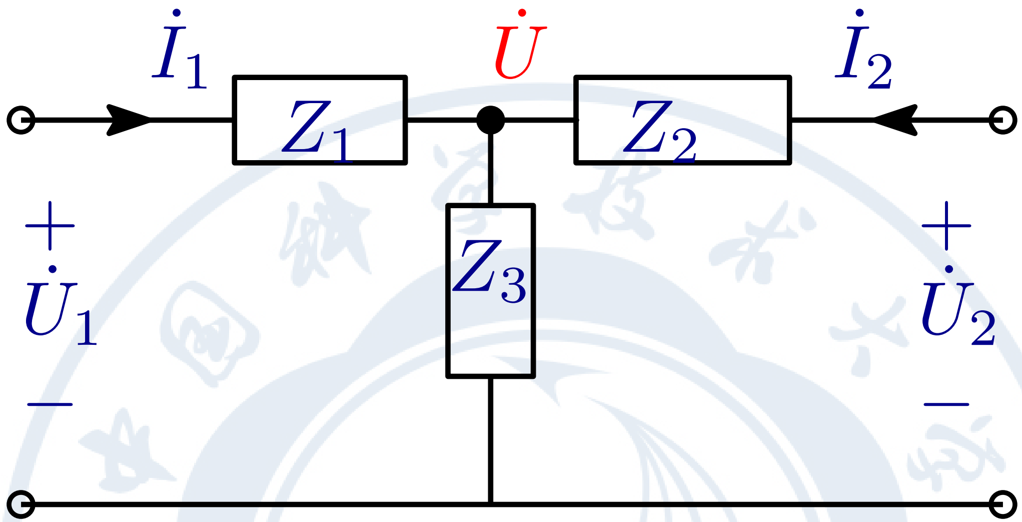
对称条件:

$$\Delta_A = 1 \text{ and } A_{11} = A_{22}$$

传输参数方程及矩阵 (A参数)



传输参数方程及矩阵 (A参数)



$$A = \begin{bmatrix} 1 + \frac{Z_1}{Z_3} & Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3} \\ \frac{1}{Z_3} & 1 + \frac{Z_2}{Z_3} \end{bmatrix}$$

二端口网络等效

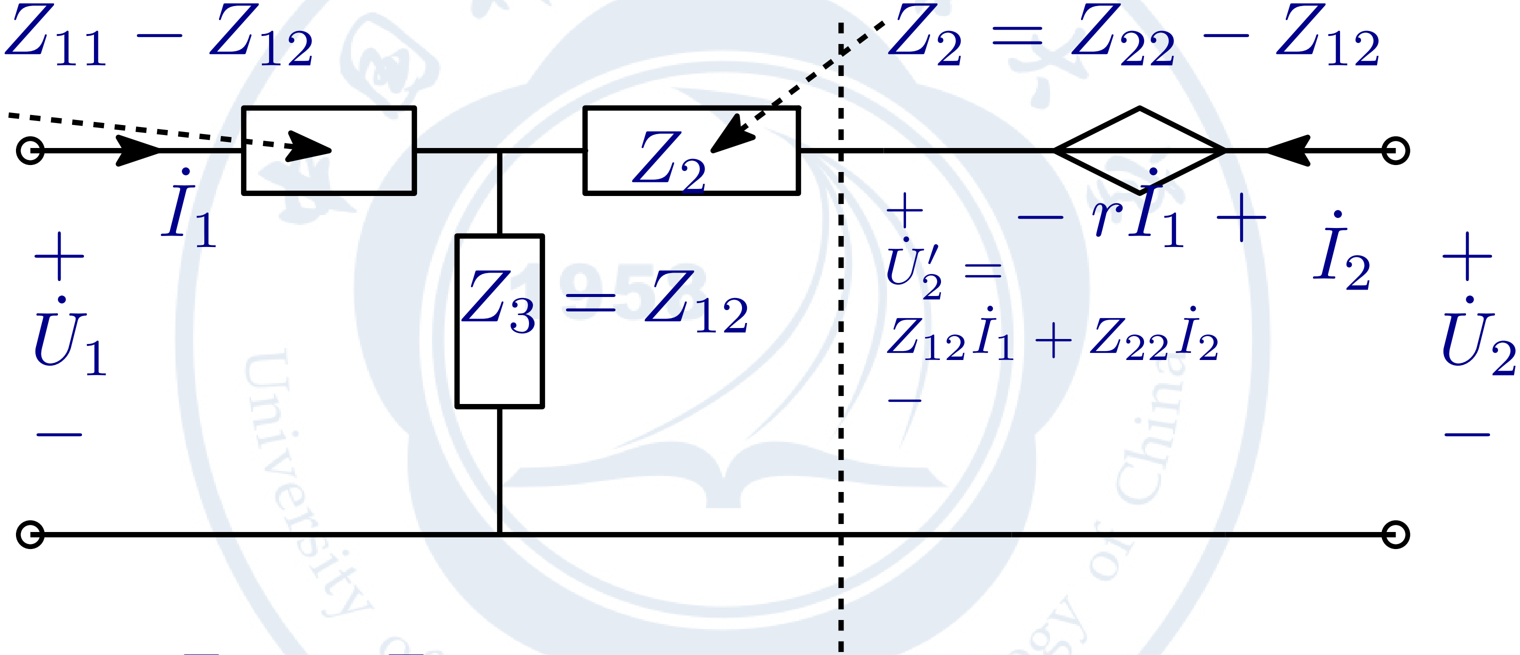
$$\begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix}$$

二端口网络等效

$$\begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix}$$

$Z_1 = Z_{11} - Z_{12}$

$Z_2 = Z_{22} - Z_{12}$



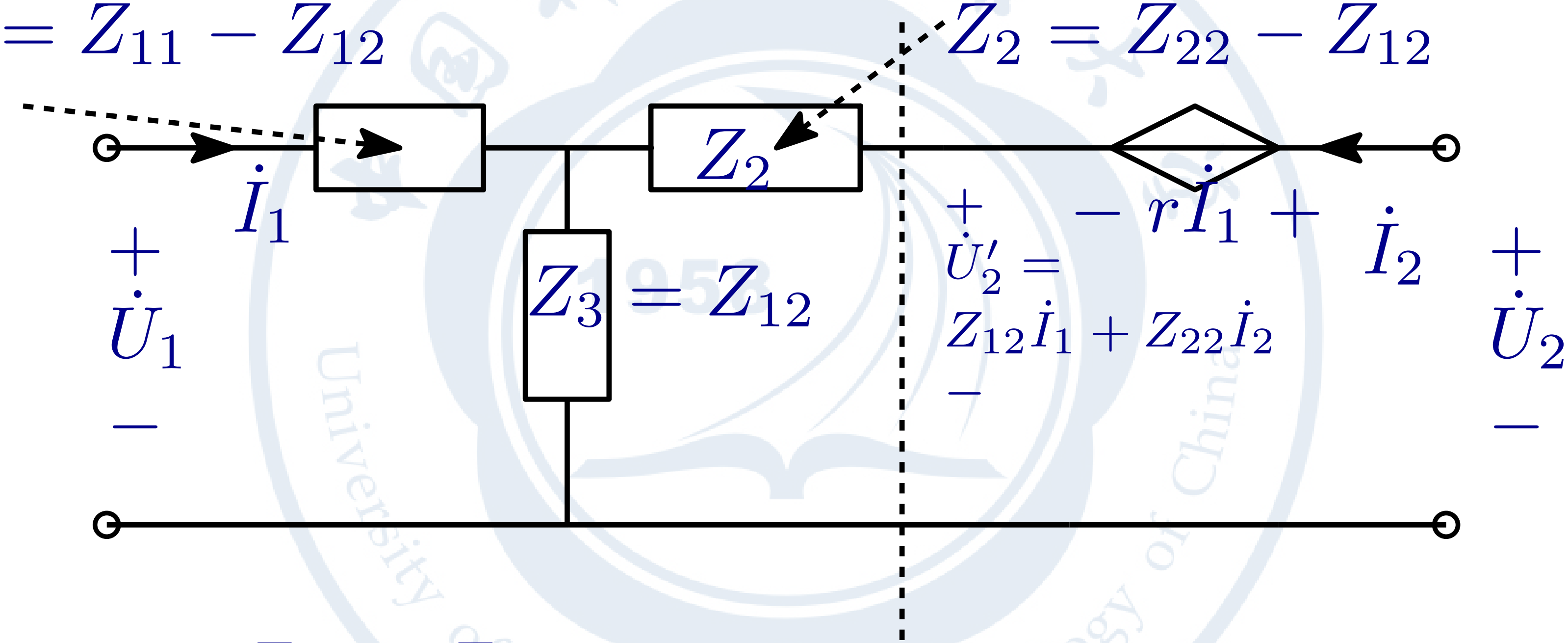
$r = Z_{21} - Z_{12}$

二端口网络等效

$$\begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix}$$

$Z_1 = Z_{11} - Z_{12}$

$Z_2 = Z_{22} - Z_{12}$



$r = Z_{21} - Z_{12}$

对于互易二端口我们可以有 $Z_{21} = Z_{12}$, 此时电流控制电压源 (CCVS) 不再需要，我们可以利用 T型网络实现互易 Z参数网络

二端口等效

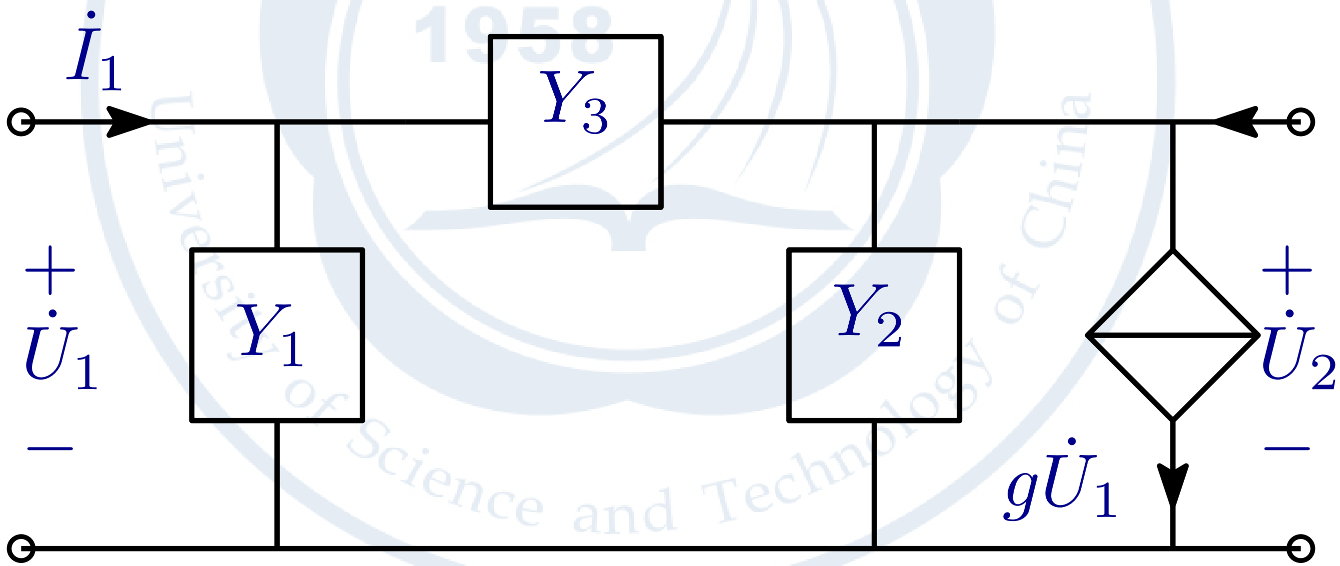
$$\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix}$$

二端口等效

$$\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix}$$

$$Y_1 = Y_{11} + Y_{12} \qquad Y_3 = -Y_{12}$$

$$Y_2 = Y_{22} + Y_{12} \qquad g = Y_{12} - Y_{21}$$

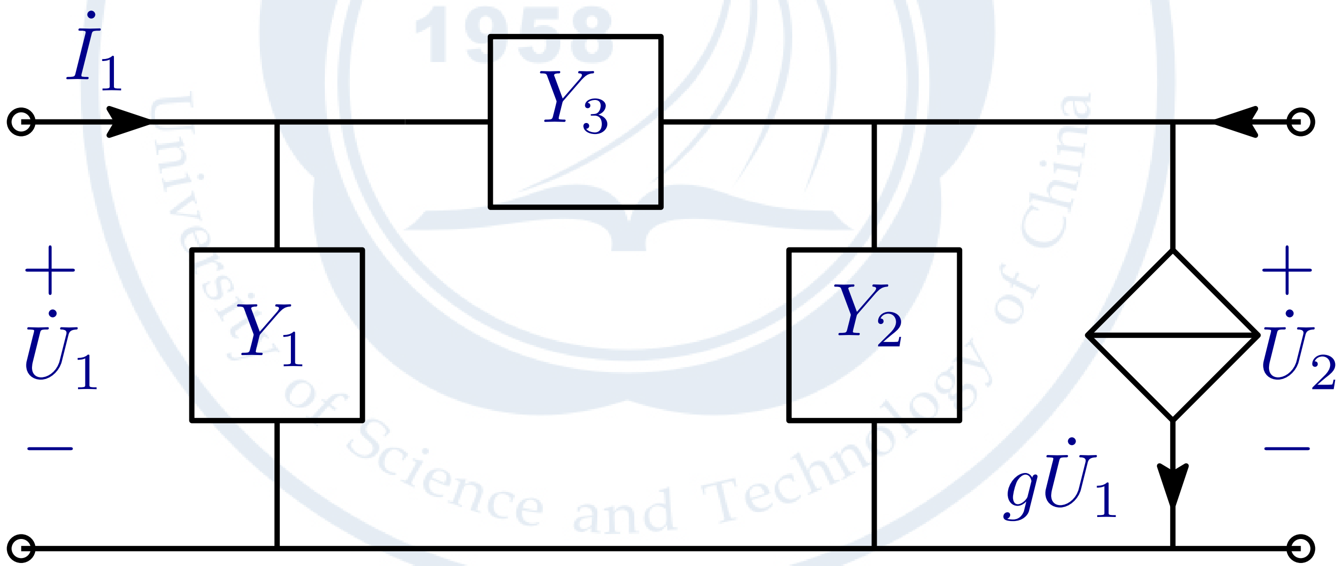


二端口等效

$$\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix}$$

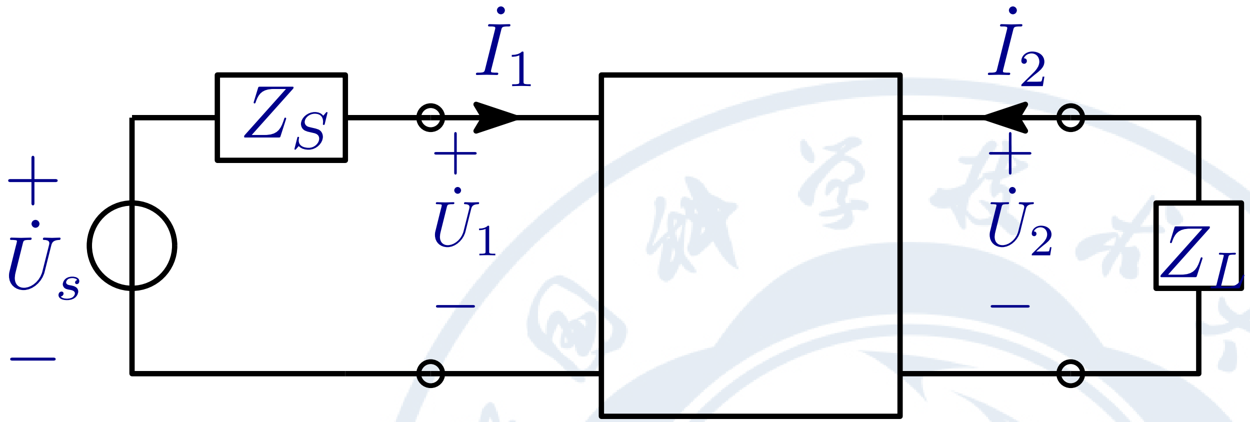
$$Y_1 = Y_{11} + Y_{12} \qquad Y_3 = -Y_{12}$$

$$Y_2 = Y_{22} + Y_{12} \qquad g = Y_{12} - Y_{21}$$



对于**互易二端口**$Y_{12} = Y_{21}$，电压控制电流源对于 Π型等效不需要

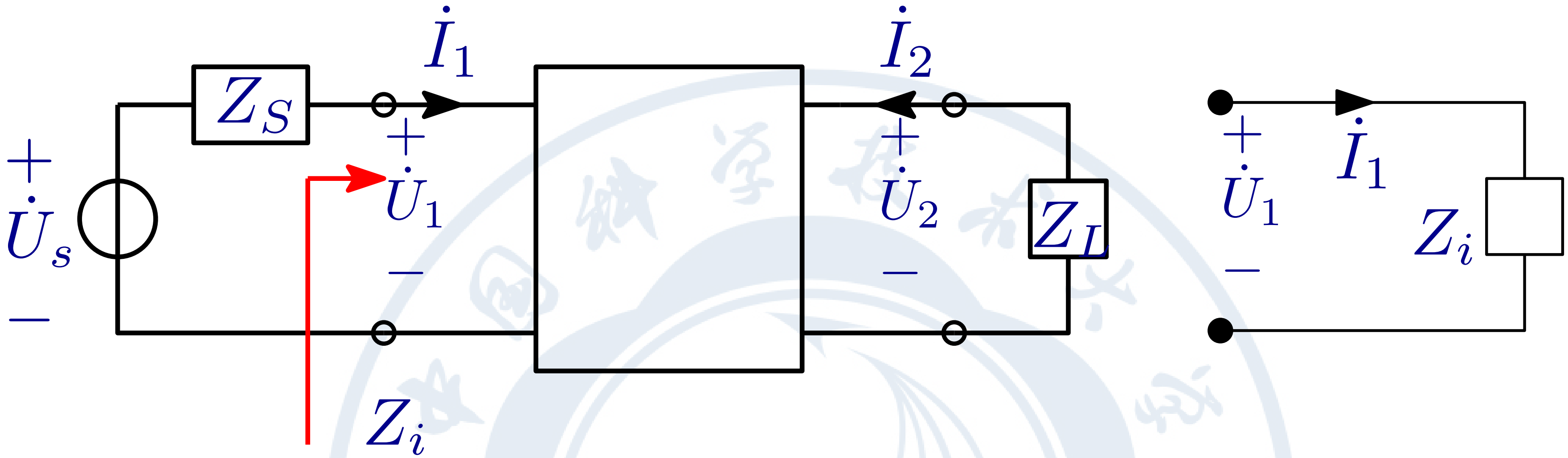
二端口网络连接



$$\dot{U}_1 = A_{11}U_2 + A_{12}(-\dot{I}_2) \qquad \dot{U}_s = \dot{I}_1 Z_s + \dot{U}_1$$

$$\dot{I}_1 = A_{21}\dot{U}_2 + A_{22}(-\dot{I}_2) \qquad \dot{U}_2 = (-\dot{I}_2)Z_L$$

二端口网络连接



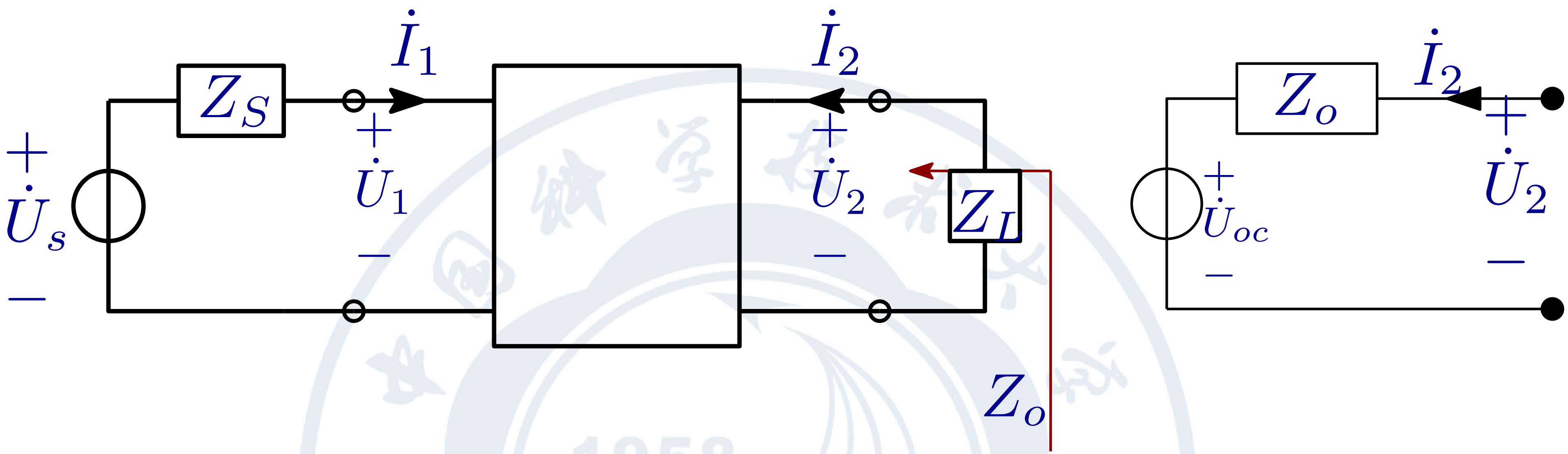
$$\dot{U}_1 = A_{11}\dot{U}_2 + A_{12}(-\dot{I}_2) \qquad \dot{U}_s = \dot{I}_1 Z_s + \dot{U}_1$$

$$\dot{I}_1 = A_{21}\dot{U}_2 + A_{22}(-\dot{I}_2) \qquad \dot{U}_2 = (-\dot{I}_2) Z_L$$

输入阻抗:

$$Z_i = \frac{\dot{U}_1}{\dot{I}_1} = \frac{A_{11}\dot{U}_2 - A_{12}\dot{I}_2}{A_{21}\dot{U}_2 - A_{22}\dot{I}_2} = \frac{A_{11}Z_L + A_{12}}{A_{21}Z_L + A_{22}}$$

二端口网络连接



$$\dot{U}_1 = A_{11}U_2 + A_{12}(-\dot{I}_2) \qquad \dot{U}_s = \dot{I}_1 Z_s + \dot{U}_1$$

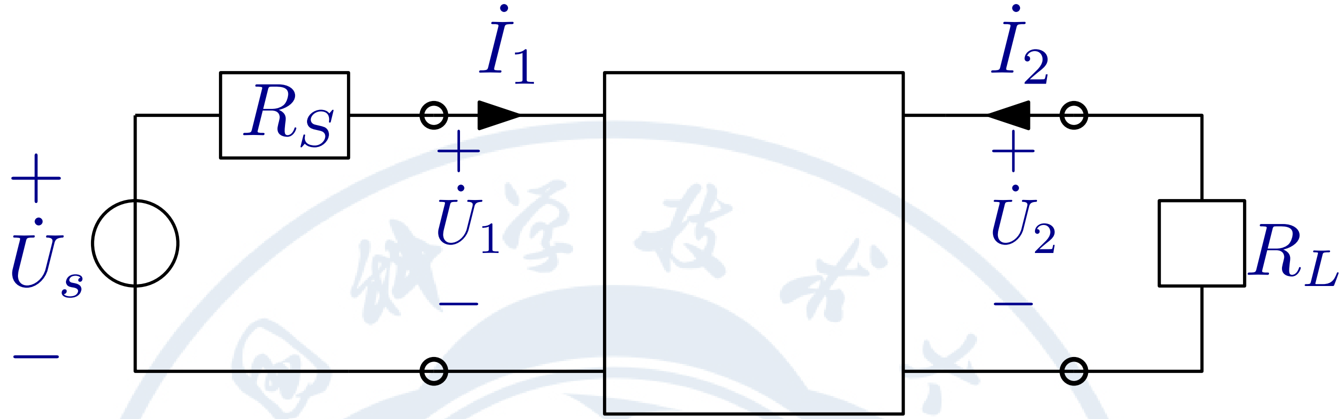
$$\dot{I}_1 = A_{21}\dot{U}_2 + A_{22}(-\dot{I}_2) \qquad \dot{U}_2 = (-\dot{I}_2)Z_L$$

输出阻抗:

$$\dot{U}_{oc} = \frac{\dot{U}_s}{A_{21}Z_s + A_{11}}$$

$$\dot{Z}_o = \frac{\dot{U}_2}{\dot{I}_2} = \frac{A_{22}Z_2 + A_{12}}{A_{21}Z_s + A_{11}}$$

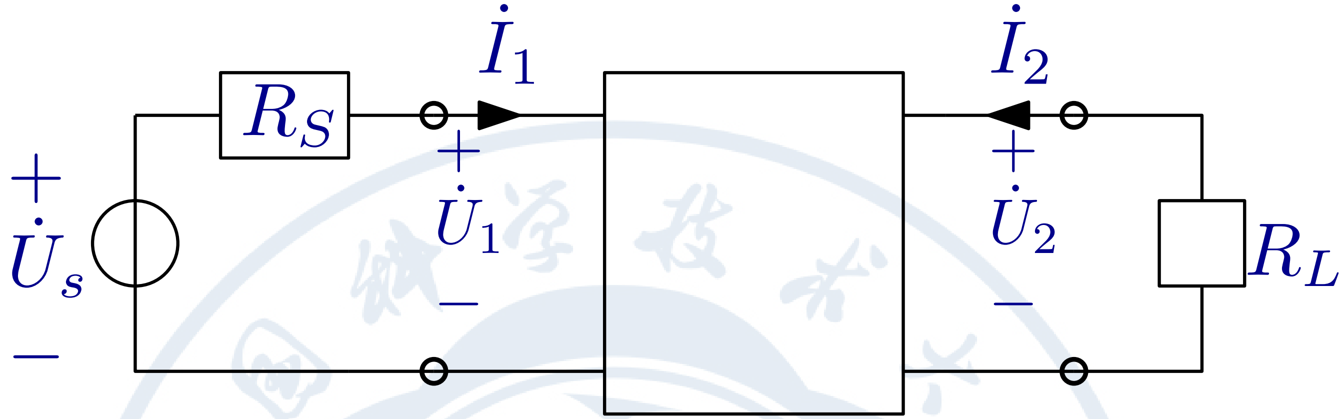
二端口网络连接举例



根据下述的传输参数矩阵。回答 $R_L = ?$ 我们可以得到最大的 U_1和 I_1。

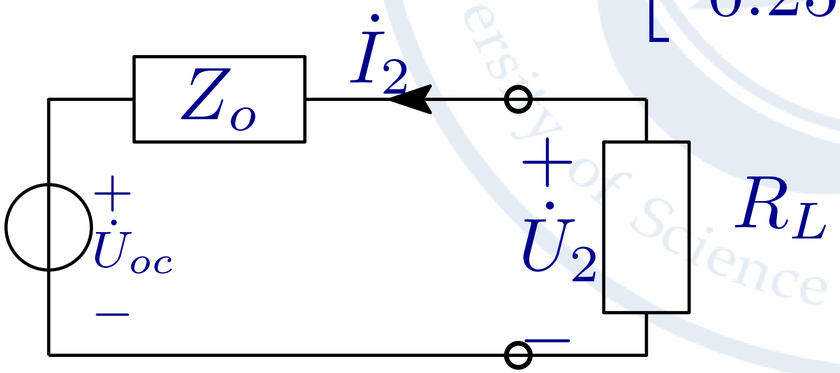
$$A = \begin{bmatrix} 1.5 & 5\Omega \\ 0.25S & 1.5 \end{bmatrix}$$

二端口网络连接举例



根据下述的传输参数矩阵。回答 $R_L = ?$ 我们可以得到最大的 U_1和 I_1。

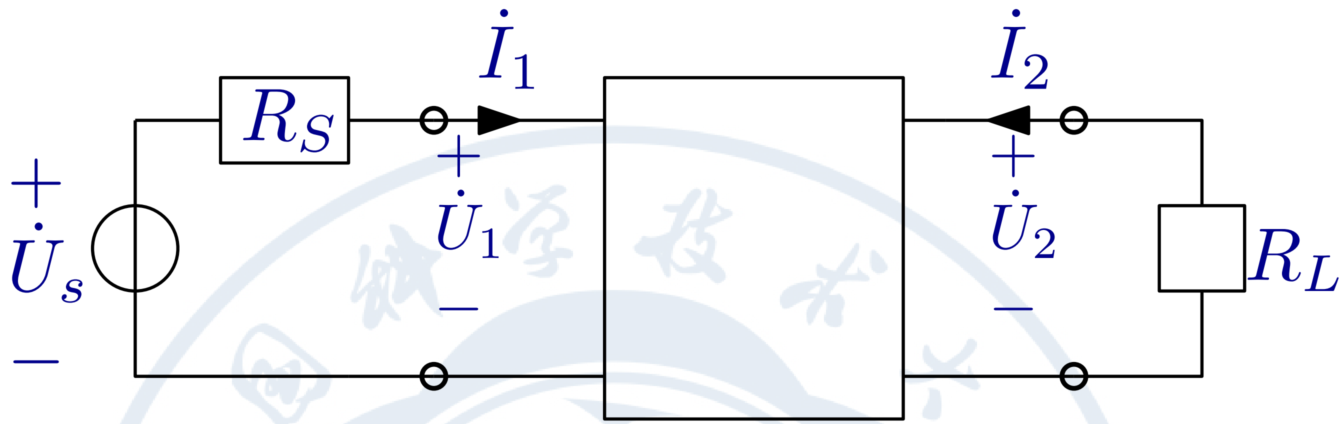
$$A = \begin{bmatrix} 1.5 & 5\Omega \\ 0.25S & 1.5 \end{bmatrix}$$



$$U_{oc} = \frac{U_s}{A_{21}R_s + A_{11}} = 12V$$

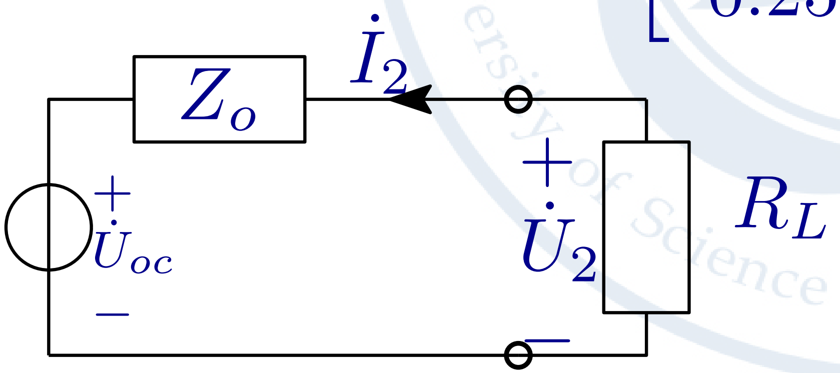
$$R_o = \frac{U_2}{I_2} = \frac{A_{22}R_s + A_{12}}{A_{21}R_s + A_{11}} = 4\Omega$$

二端口网络连接举例



根据下述的传输参数矩阵。回答 $R_L = ?$ 我们可以得到最大的 U_1和 I_1。

$$A = \begin{bmatrix} 1.5 & 5\Omega \\ 0.25S & 1.5 \end{bmatrix}$$



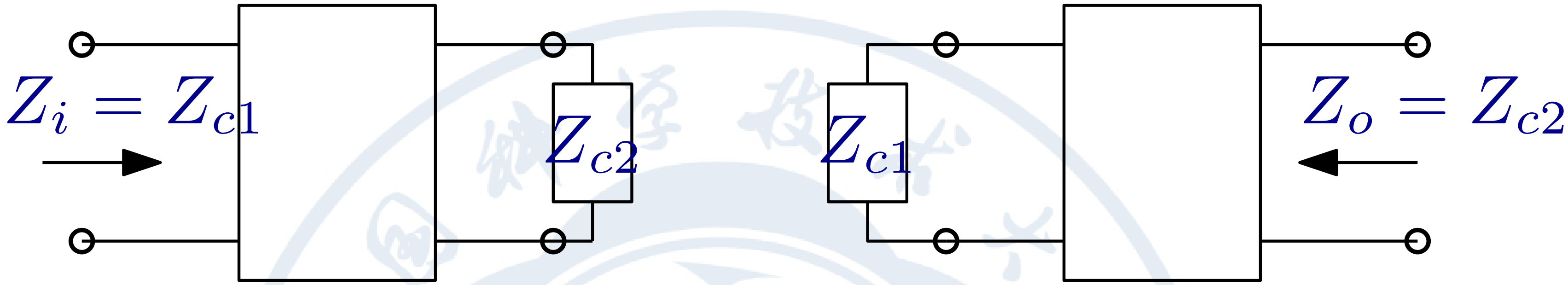
$$U_{oc} = \frac{U_s}{A_{21}R_s + A_{11}} = 12V$$

$$R_o = \frac{U_2}{I_2} = \frac{A_{22}R_s + A_{12}}{A_{21}R_s + A_{11}} = 4\Omega$$

When $R_o = R_L = 4\Omega$, R_L gets the maximal power:

$$P_{max} = \frac{U_{oc}^2}{4R_o} = 9W$$

特性阻抗



$$Z_i = Z_{c1} = \frac{A_{11}Z_{c2}+A_{12}}{A_{21}Z_{c2}+A_{22}} \qquad Z_o = Z_{c2} = \frac{A_{22}Z_{c1}+A_{12}}{A_{21}Z_{c1}+A_{11}}$$

输入端口的特性阻抗:

$$Z_{c1} = \sqrt{\frac{A_{11}A_{12}}{A_{21}A_{22}}}$$

输出端口的特性阻抗:

$$Z_{c2} = \sqrt{\frac{A_{22}A_{12}}{A_{11}A_{21}}}$$