中国科学技术大学

6.12.

1.根据三维紧束缚近似，简立方晶体s能带的色散关系为$E(k)=E_0+2t\cdot[\cos(k_xa)+\cos(k_ya)+\cos(k_za)]$，其中$-\frac{\pi}{a}\le k_{x,y,z}\le\frac{\pi}{a}$. 请计算不同$\vec{k}$处布洛赫电子的群速度$\vec{v_g}$和有效质量矩阵$[m]$.

解:

$$\vec{v_g}=\frac{1}{\hbar}\nabla_k E(\vec{k})$$

故$\vec{k}=(k_x,k_y,k_z)$状态下，电子的群速度为

$$\vec{v_g}=-\frac{2at}{\hbar}[\sin(k_xa)\cdot\vec{e_x}+\sin(k_ya)\cdot\vec{e_y}+\sin(k_za)\cdot\vec{e_z}]$$

$$\left[\frac{1}{m^*}\right]=\frac{1}{\hbar^2}\left[\frac{\partial^2E}{\partial\vec{k}\partial\vec{k}}\right]=-\frac{2a^2t}{\hbar^2}\begin{bmatrix}\cos(k_xa)&0&0\\0&\cos(k_ya)&0\\0&0&\cos(k_za)\end{bmatrix}$$

$$[m^*]=\left[\frac{1}{m^*}\right]^{-1}=-\frac{\hbar^2}{2a^2t}\begin{bmatrix}\frac{1}{\cos(k_xa)}&0&0\\0&\frac{1}{\cos(k_ya)}&0\\0&0&\frac{1}{\cos(k_za)}\end{bmatrix}$$

6.14.

1.考虑一个由三个近独立全同粒子组成的体系，并且已知该体系的单粒子本征函数只有三个:$\psi_1(\vec{r})$, $\psi_2(\vec{r})$, $\psi_3(\vec{r})$，分别针对该粒子为玻色子和费米子两种情况，写出这三个粒子体系所有可能的本征波函数(不要求归一化).

解:

玻色子: $\psi_{111}=\psi_1(\vec{r_1})\psi_1(\vec{r_2})\psi_1(\vec{r_3})$

$\psi_{112}=\psi_1(\vec{r_1})\psi_1(\vec{r_2})\psi_2(\vec{r_3})+\psi_1(\vec{r_2})\psi_1(\vec{r_3})\psi_2(\vec{r_1})+\psi_1(\vec{r_3})\psi_1(\vec{r_1})\psi_2(\vec{r_2})$

$\psi_{113}=\psi_1(\vec{r_1})\psi_1(\vec{r_2})\psi_3(\vec{r_3})+\psi_1(\vec{r_2})\psi_1(\vec{r_3})\psi_3(\vec{r_1})+\psi_1(\vec{r_3})\psi_1(\vec{r_1})\psi_3(\vec{r_2})$

$$\psi_{122}=\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)\psi_2(\vec{r}_3)+\psi_1(\vec{r}_2)\psi_2(\vec{r}_3)\psi_2(\vec{r}_1)+\psi_1(\vec{r}_3)\psi_2(\vec{r}_1)\psi_2(\vec{r}_2)$$

$$\psi_{123}=\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)\psi_3(\vec{r}_3)+\psi_1(\vec{r}_2)\psi_2(\vec{r}_3)\psi_3(\vec{r}_1)+\psi_1(\vec{r}_3)\psi_2(\vec{r}_2)\psi_1(\vec{r}_1)$$
$$+\psi_1(\vec{r}_1)\psi_2(\vec{r}_3)\psi_3(\vec{r}_2)+\psi_1(\vec{r}_2)\psi_2(\vec{r}_1)\psi_3(\vec{r}_3)+\psi_1(\vec{r}_3)\psi_2(\vec{r}_1)\psi_3(\vec{r}_2)$$

$$\psi_{133}=\psi_1(\vec{r}_1)\psi_3(\vec{r}_2)\psi_3(\vec{r}_3)+\psi_1(\vec{r}_2)\psi_3(\vec{r}_3)\psi_3(\vec{r}_1)+\psi_1(\vec{r}_3)\psi_3(\vec{r}_1)\psi_3(\vec{r}_2)$$

$$\psi_{222}=\psi_2(\vec{r}_1)\psi_2(\vec{r}_2)\psi_2(\vec{r}_3)$$

$$\psi_{223}=\psi_2(\vec{r}_1)\psi_2(\vec{r}_2)\psi_3(\vec{r}_3)+\psi_2(\vec{r}_2)\psi_2(\vec{r}_3)\psi_3(\vec{r}_1)+\psi_2(\vec{r}_3)\psi_2(\vec{r}_1)\psi_3(\vec{r}_2)$$

$$\psi_{233}=\psi_2(\vec{r}_1)\psi_3(\vec{r}_2)\psi_3(\vec{r}_3)+\psi_2(\vec{r}_2)\psi_3(\vec{r}_3)\psi_3(\vec{r}_1)+\psi_2(\vec{r}_3)\psi_3(\vec{r}_1)\psi_3(\vec{r}_2)$$

$$\psi_{333}=\psi_3(\vec{r}_1)\psi_3(\vec{r}_2)\psi_3(\vec{r}_3)$$

费米子:

$$\psi_{123}=\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)\psi_3(\vec{r}_3)-\psi_1(\vec{r}_1)\psi_2(\vec{r}_3)\psi_3(\vec{r}_2)$$
$$+\psi_1(\vec{r}_2)\psi_2(\vec{r}_3)\psi_3(\vec{r}_1)-\psi_1(\vec{r}_2)\psi_2(\vec{r}_1)\psi_3(\vec{r}_3)$$
$$+\psi_1(\vec{r}_3)\psi_2(\vec{r}_1)\psi_3(\vec{r}_2)-\psi_1(\vec{r}_3)\psi_2(\vec{r}_2)\psi_3(\vec{r}_1)$$

$$\sim\begin{vmatrix}\psi_1(\vec{r}_1) & \psi_1(\vec{r}_2) & \psi_1(\vec{r}_3)\\ \psi_2(\vec{r}_1) & \psi_2(\vec{r}_2) & \psi_2(\vec{r}_3)\\ \psi_3(\vec{r}_1) & \psi_3(\vec{r}_2) & \psi_3(\vec{r}_3)\end{vmatrix}$$