

5.21 1) 该连续时间 LTI 系统的频率响应为 $H(\omega) = \begin{cases} 1, & |\omega| < 2\pi \times 10^3 \\ 0, & |\omega| > 2\pi \times 10^3 \end{cases}$

a) (2) $y(t) = 1 + 0.5 \cos(0.75\pi \times 10^3 t + \frac{\pi}{4}) + 0.25 \cos(1.5\pi \times 10^3 t + \frac{\pi}{2})$

(3) $y(t) = 0.5 + (2/\pi) \cos(1.5\pi \times 10^3 t)$

2) 该离散时间 LTI 系统的频率响应为 $\tilde{H}(\Omega) = \begin{cases} 1, & |\Omega| < \pi/3 \\ 0, & \pi/3 < |\Omega| \leq \pi \end{cases}$

b) $y[n] = 0.125 + 0.25 \cos(\pi n/4)$

d) $y[n] = 1 + \sin(\pi n/4)$

5.26 1) $X[0] = 2$, $X[1] = -1 - j$, $X[2] = 4$, $X[3] = -1 + j$

2) $x[0] = 1$, $x[1] = 0$, $x[2] = 2$, $x[3] = -1$

5.34-2 $F(s) = \frac{1}{s+a} + \frac{1}{s+b}$, ROC 为 $\text{Re}\{s\} > \max(-a, -b)$

$\because a, b > 0$, $\therefore s = j\omega$ 在 ROC 内, 存在傅里叶变换

$$F(\omega) = \frac{1}{j\omega + a} + \frac{1}{j\omega + b}$$

5.34-4 $F(s) = -\frac{1}{s+a} - \frac{1}{s+b}$, ROC 为 $\text{Re}\{s\} < \min(-a, -b) = -b$

$\because a, b < 0$, $\therefore s = j\omega$ 在 ROC 内, 存在傅里叶变换

$$F(\omega) = -\frac{1}{j\omega + a} - \frac{1}{j\omega + b}$$

5.34-5 $F(s) = \frac{1}{s+a} (1 - e^{-(s+a)T})$, ROC 为整个 S 平面, 零点 $z_k = -\frac{j2k\pi}{T} - a$

当 $a > 0$ 时, 存在傅里叶变换, $F(\omega) = \frac{1}{j\omega + a} (1 - e^{-(j\omega + a)T})$

5.35 2) $f[n] = a^n \{u[n] - u[n-N]\}$

$$F(z) = \sum_{n=-\infty}^{+\infty} f[n]z^{-n} = \sum_{k=0}^{N-1} a^k z^{-k} = \frac{1 - (az^{-1})^N}{1 - az^{-1}}$$

ROC: $|z| > 0$

(另若 $N=1$ 则 $f[n] = \delta[n]$, $F(z) = 1$, 收敛域为整个 Z 平面, 无极点)
收敛域包含单位圆, 存在傅里叶变换:

$$\tilde{F}(\Omega) = \frac{1 - a^N e^{-jN\Omega}}{1 - a e^{-j\Omega}}$$

4) $f[n] = a^n u[n] - b^{-n} u[-n-1]$ $|b| > 1 > |a|$

$$F(z) = \frac{1}{1 - az^{-1}} + \frac{1}{1 - b^{-1}z^{-1}}$$

$$\text{ROC: } \left\{ \{ |z| > |a| \} \cap \left\{ |z| < \frac{1}{|b|} \right\} \right\}$$

若 $|a| < \frac{1}{|b|}$ 则收敛域为 $|a| < |z| < \frac{1}{|b|}$, 且无傅里叶变换; 否则收敛域为空集, 无拉普拉斯变换。

$$5) f[n] = a^{-n}u[-n] - b^{-n}u[-n] \quad |b| > 1 > |a|$$

$$\begin{aligned} F(z) &= \sum_{n=-\infty}^{+\infty} f[n]z^{-n} = \sum_{n=-\infty}^0 a^{-n}z^{-n} - \sum_{n=-\infty}^0 b^{-n}z^{-n} \\ &= \sum_{n=0}^{\infty} a^n z^n - \sum_{n=0}^{\infty} b^n z^n \\ &= \frac{1}{1-az} - \frac{1}{1-bz} \end{aligned}$$

$$ROC: \left\{ \left\{ |z| < \frac{1}{|a|} \right\} \cap \left\{ |z| < \frac{1}{|b|} \right\} \right\} = \left\{ |z| < \frac{1}{|b|} \right\}$$

收敛域不包含单位圆，不存在傅里叶变换。

5.36 (1) (a) 包含 $Re\{s\} = 1$ (b) $Re\{s\} > a$ (c) $Re\{s\} < a$

(2) (a) ROC: $-2 < Re\{s\} < 2$, 两边时间函数

ROC: $Re\{s\} > 2$, 右边时间函数

ROC: $Re\{s\} < -2$, 左边时间函数

(c) ROC: $Re\{s\} > -2$, 右边时间函数

ROC: $Re\{s\} < -2$, 左边时间函数

5.37 (1) (a) 包含 $|z| = 2/3$ (b) $|z| > a$ (c) $|z| < a$

(2) (a) ROC: $|z| < 0.5$, 左边时间函数

ROC: $|z| > 2$, 右边时间函数

ROC: $0.5 < |z| < 2$, 两边时间函数

(c) ROC: $|z| \neq 0$

5.40 2) $F(s) = \frac{s+1}{s^2+5s+6} \quad Re\{s\} < -3$

$$F(s) = -\frac{1}{s+2} + \frac{2}{s+3}$$

由收敛域可知信号在时域上是左边信号:

$$f(t) = e^{-2t}u(-t) - 2e^{-3t}u(-t)$$

4) $F(s) = \frac{2s}{s^2-1} \quad -1 < Re\{s\} < 1$

$$F(s) = \frac{2s}{s^2-1} = \frac{2s}{(s-1)(s+1)} = \frac{1}{s-1} + \frac{1}{s+1}$$

$$f(t) = e^{-t}u(t) - e^t u(-t)$$

5.41 (1) $f[n] = -\frac{5}{3} \left(\frac{1}{2} \right)^n u[n] - \frac{8}{3} 2^n u[-n-1]$

6.1-2

$$f(t) = e^{-2t}[u(t+2) - u(t-2)]$$

$$\because e^{-2t}u(t) \Leftrightarrow \frac{1}{2+j\omega}$$

$$\therefore e^{-2t}u(t+2) = e^4 e^{-2(t+2)}u(t+2) \Leftrightarrow \frac{e^4 e^{j\omega 2}}{2+j\omega}$$

$$e^{-2t}u(t-2) = e^{-4} e^{-2(t-2)}u(t-2) \Leftrightarrow \frac{e^{-4} e^{-j\omega 2}}{2+j\omega}$$

$$\begin{aligned} \Rightarrow F(\omega) &= \frac{e^4 e^{j\omega 2}}{2+j\omega} - \frac{e^{-4} e^{-j\omega 2}}{2+j\omega} \\ &= \frac{e^4 e^{j\omega 2} - e^{-4} e^{-j\omega 2}}{2+j\omega} \end{aligned}$$

6.1-8

$$f(t) = e^{-a|t-2|}, a > 0$$

$$\because e^{-a|t|}, a > 0 \Leftrightarrow \frac{2a}{\omega^2 + a^2}$$

$$\therefore e^{-a|t-2|}, a > 0 \Leftrightarrow \frac{2a}{\omega^2 + a^2} e^{-j\omega 2}$$

$$\Rightarrow F(\omega) = \frac{2a}{\omega^2 + a^2} e^{-j\omega 2}$$

6.1-20

$$f(t) = \sin \pi t [2u(t) - u(t+1) - u(t-1)]$$

$$F(j\omega) = \frac{2\pi(1 + \cos \omega)}{\pi^2 - \omega^2}$$

6.3-11

$$x_0(t) = \cos(\pi t)[u(t) - u(t-1)]$$

$$x(t) = \sum_{k=0}^{+\infty} x_0(t-2k)$$

$$X_0(s) = \frac{s}{s^2 + \pi^2} + \frac{e^{-s}s}{s^2 + \pi^2} = \frac{(1 + e^{-s})s}{s^2 + \pi^2}$$

$$X(s) = \sum_{k=0}^{+\infty} X_0(s) e^{-2ks} = \frac{(1 + e^{-s})s}{s^2 + \pi^2} \frac{1}{1 - e^{-2s}}$$

6.3-12

$$x[n] = u[n] + u[n-4] + u[n-8] + \dots = \sum_{k=0}^{\infty} u[n-4k]$$

$$u[n-4k] \Leftrightarrow \frac{1}{1-z^{-1}} z^{-4k}$$

$$\therefore X(z) = \sum_{k=0}^{\infty} \frac{1}{1-z^{-1}} z^{-4k} = \frac{1}{(1-z^{-1})(1-z^{-4})}, |z^{-4}| < 1$$

7.4

