Simulating Star Systems

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We simulate the orbits of the planets in a one-star system and the motion of one planet in a binary star system. The binary system exhibits chaotic behaviour, while the one-star system is stable. In the one-star system, we model the star movement and simulate observation data as obtained by a distant system with angle of inclination 90 degrees. From this data, one can construct a clear velocity curve for the star, but deciphering the planets' radii from a light curve is impossible due to excessive noise.

I. METHOD

Table I. Mass, radius and semi-major axis of system objects

Object	**Earth masses**	**Earth radii**	**AU**
Star	$227 \cdot 10^3$	93.2	-
Planet 1	$3.46 \cdot 10^{-2}$	.319	.11
Planet 2	$2.18 \cdot 10^{-1}$	.604	.27
Planet 3	1.92	1.22	.46
Planet 4	1.40	1.17	.56
Planet 5	$1.95 \cdot 10^{-2}$	.266	1.1
Planet 6	$1.60 \cdot 10^{-3}$	.157	2.0
Planet 7	9.03	4.42	4.2

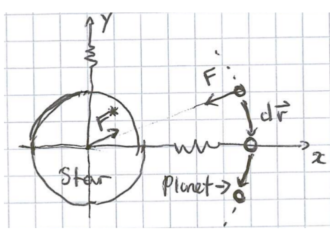

Figure 1. *Planet orbiting star, here drawn in three time-steps. F is the force from the star on the planet giving a centripetal acceleration. F* is the counter force on the star from the planet, of equal magnitude but opposite direction. Thus, we need only calculate one of these forces, as the other is its additive inverse. Note that the objects are not to scale.*

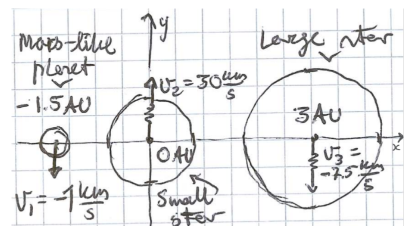

Figure 2. *Unpertubated initial conditions for the three body problem, as found in [1]. The planet has mass equal to Mars (given in [3]), the small star has equal mass to the Sun and the large star has mass four times that of the Sun. Note that the objects are not to scale.*

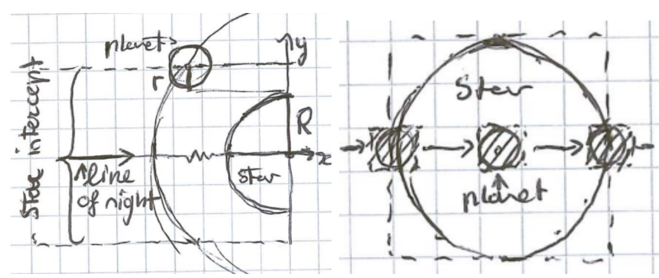

Figure 3. *Illustration of an eclipse, on the left a planet barely intercepting the sun as viewed from a distant system of inclination 90 degrees from the left (negative x-direction). R is the radius of the star and r the radius of the planet (Values can be found in table I), so that the planet intersects the star while its absolute y coordinate is less than R+r. On the right, we show a head on view of the eclipse, demonstrating our square approximation for simple overlap calculations. The apparent luminosity of the star to the distant observers is proportional to its non-covered area, disregarding gravitational lensing. Thus, as the planet moves in between the star and the observers, the luminosity drops. This can be seen clearly in the reduced noise light curves of figure 11. Note that the objects are not to scale.*

II. RESULTS

We have plotted the planet orbits over 11 years (Figure 4), and the path of the star over 30 years (Figure 5). Notice the difference in scale on the axes of these figures; the star moves very little, as will be discussed in the next section. The orbits are realistic, with low eccentricities resembling the inner orbits around the Sun.

A distant star system with line of sight along the x-axis with an inclination of 90 degrees and peculiar velocity $2\frac{km}{s}$ would observe the velocity curve of figure 8. With subpar measuring equipment, they would observe the very noisy - and thus not at all illuminating - light curve of figure 10, which is taken over one year. Without noise, this light curve would look like figure 9, clearly showing the four most massive planets as spikes of differing length. If the distant aliens had better equipment and could reduce the standard deviation of the noise in figure 10 by a factor of about 1000, they could measure the radius of the greatest planet from figure 11, though such measurement precision is unfortunately beyond the current ability of our extraterrestrial friends.

In figure 6 we show the modelled movement of the three body system, consisting of a Mars-like planet, a Sun-like star and a star of four times the mass of the Sun, in two dimensions with slightly differing initial conditions, all based around those described in figure 2. Clearly, the movement is chaotic, that is, highly sensitive to initial conditions, so that the planet climate is completely unpredictable. In fact, the chaotic nature of the system means that it is not solvable on longer time scales with normal numerical methods, since even small numerical inaccuracies may lead to vastly differing results, not to mention measurement inaccuracies which make it impossible to set up the initial conditions exactly. Such a planet could surely not support life as we know it, due to the unpredictable variations in temperature.

Finally, figure 7 shows plots of the same system with the same initial conditions of figure 2 but now also with noise in the z-direction. Without noise, the initial conditions guarantee planar motion by conservation of angular momentum, since there is no torque, so all z-motion is due to noise.

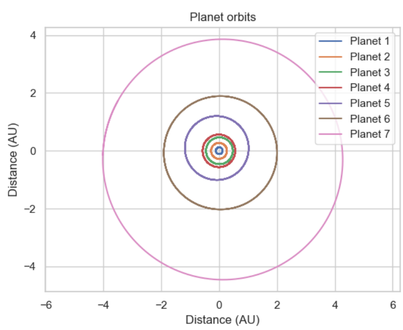

Figure 4. *Planet orbits, plotted over 11 years disregarding the star motion, though including it makes no visible difference. See table I for the semi-major axes, and note the low eccentricities similar to the inner planets of our Solar system.*

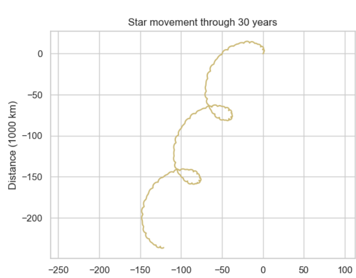

Figure 5. *The path of the star over 30 years, experiencing the gravitational pull of the four most massive planets in the system, from a reference frame where it is initially at rest in the origin. The reference frame is not moving with the system centre of mass and thus the star drifts down and to the left; still, we see that the star is orbiting. Note that the axes are not in astronomical units since the star moves so little.*

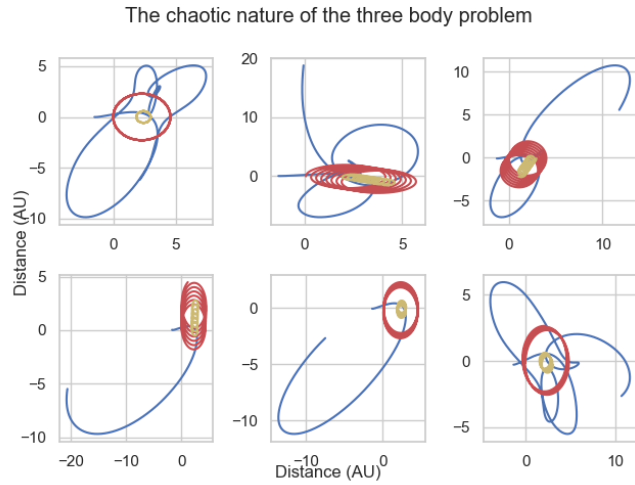

Figure 6. *Six plots of the three body problem with slightly different initial conditions over about twelve and a half years. The blue line denotes the trajectory of a Mars-like planet, the red line describes the movement of a Sun-like star, and the yellow line that of a star four times more massive than the Sun. The upper left plot has no noise, with initial conditions as drawn in Figure 2. The other plots have small noise in both initial position and velocity. Note that the axes have different scales, but that the stars orbit relatively stably. The planet movement seems entirely unpredictable.*

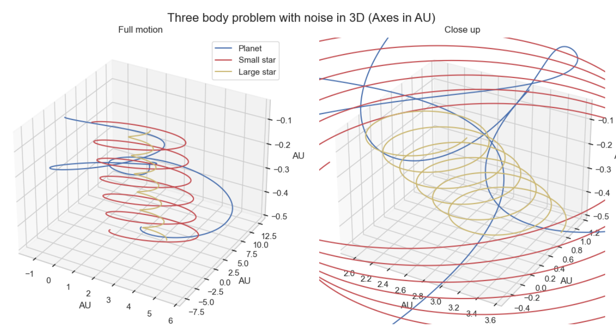

Figure 7. *Binary star system with planet of similar mass to Mars, small star of similar mass to the Sun and large star with mass four times the Sun's, here modelled over about twelve and a half years. The initial conditions are still based the planar situation of figure 2, but we have some noise also in the third dimension.*

III. DISCUSSION

Our solar system is small, with the outermost planet just over 4 AU away from the star - which is closer than Jupiter is to the Sun, at about 5 AU [2] - meaning that one whole revolution of the outermost planet takes less than 11 years. This is also the only gas planet, and has by far the greatest mass of about 9 earth masses (Table I). The star has a mass of 0.68 sun masses and a radius

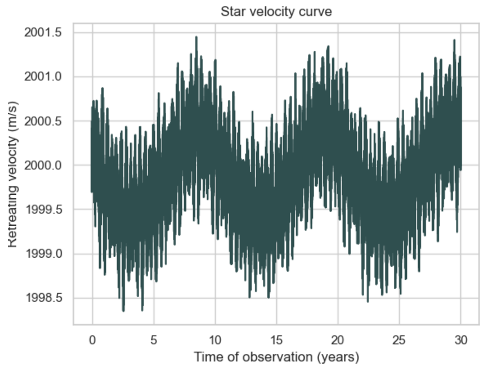

Figure 8. *Velocity curve as observed from a distant system with line of sight along the x-axis with inclination 90 degrees and peculiar velocity* $2\frac{km}{s}$ *with noise.*

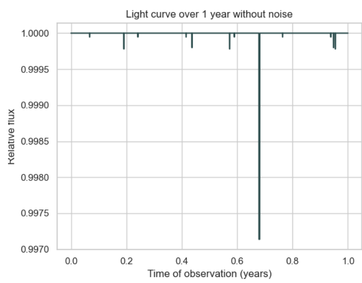

Figure 9. *Light curve with no noise over one year. We clearly see the eclipses from the four most massive planets as spikes of differing heights. Note that planets 3 and 4 are of similar size and give very similar spikes.*

of .85 sun radii; these numbers are given in earth masses in table I for comparison to the planets. Still, we see that the planets are, even as compared to the star, less massive than the planets orbiting the Sun, as Jupiter exceeds 300 earth masses [2]. This might explain that the star moves so little, with a maximal radial velocity of about $0.5m/s$, and that the planet orbits are unaffected by the inclusion of the star's motion (Figure 4).

In the calculations underlying these plots, we disregarded the three lightest planets, but their mass is so small (Table I) that their exclusion makes no difference to the star path nor the velocity curve. However, this is the reason why there are only four distinct types of spikes in figure 9. Either way, the smaller planets would make tiny spikes which would be overcrowded by the slightest amount of noise; even reducing the noise one thousand-fold leaves only the biggest planet visible (Figure 11). Still, we have included the option in our code to include all planets at a tiny performance cost.

When plotting the orbits, we disregard inter-planetary attraction. A glance at the orders of magnitude of the masses in table I, together with Newton's law of gravita-

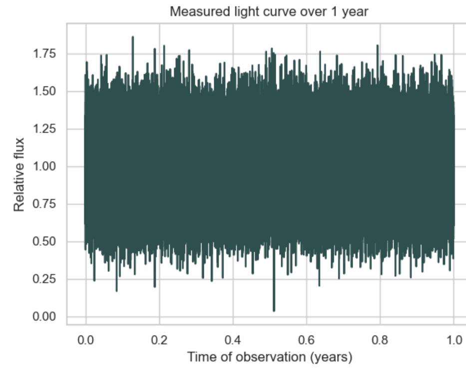

Figure 10. *Observed light curve from the distant system over one year. Clearly, the noise is way to big for any planets to be visible through eclipses; in fact, even if* half the star *was blocked out temporarily, we might not even notice.*

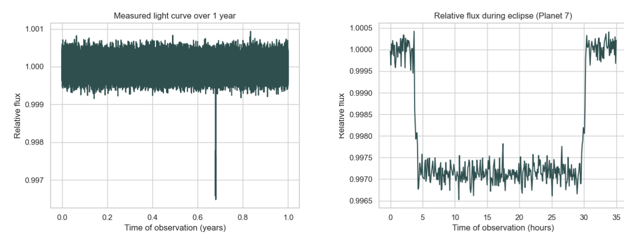

Figure 11. *Light curve with reduced noise. On the left, the full curve over one year, where the planet 7 eclipse is easily distinguishable. On the right, we zoom in, showing only the 35 hours around the eclipse. There are about 5 minutes and 16 seconds between measurements.*

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$$F = -G\frac{Mm}{r^2},$$

should convince you that this is a reasonable assumption, since it significantly simplifies the calculations.

However, there is still room for improvement. In the inner orbits of figure 4, one can just about see the thickening of the orbit line, indicating that the planet's semi-major axis is changing. This is a numerical artifact due to our time-step being too large, but going beyond 10^6 time-steps to cover the eleven years required is resource intensive. Instead, one should improve the solving algorithm. We use the Euler-Cromer, but a higher order method, such as a fourth order Runge-Kutta, could dramatically improve the stability of the orbits. This is even more relevant for figures 6 and 7, showing solutions of the three body problem, where the results could potentially change dramatically with increased accuracy, and we find that our results are not stable to changes in time-step length. For the velocity curve and the light curve, potential numerical inaccuracies are overshadowed by the noise so that 10^5 time-steps with Euler-Cromer is sufficient.

We obtain the parameters of the solar system from the ast2000tools package. Here, the solar system is planar, so that every orbit is in the xy-plane. This is probably a reasonable assumption, but if 3D data becomes available, it is a simple matter to add this to our programs.

Our simulation of noise in the simulated observation data is however not very accurate. For the velocity curve, we use normal noise of standard deviation one fifth of the maximal actual amplitude. This means that, even though star velocity relative to its system center of mass is low, the graph is crisp. To increase the relevance of our findings, the noise term should probably be based on the current measuring accuracy of Doppler-shifts or the noise conditions between the observer and the system, but this is beyond the scope of this report.

In the light curve noise term, we make a very conservative estimate of the technological abilities of the observers, but we do use a noise term independent of the actual measurements. Due to this noise term, our approximation using square cross sectional areas for the star and planets is greatly overshadowed and is no cause for inaccuracy. The assumption of a square star is reasonable even if doing calculations to high precision, since it is so large as compared to the planets, but one may consider using circular cross sections for the planets in such a case.

IV. CONCLUSION

We have simulated the orbits of planets and the star in a typical solar system, and created velocity and light curves as would be observed from another distant system. We demonstrate that extraterrestrials in this distant system would, with their current methods, be unable to use this light curve to observe planets due to excessive noise. We also simulate a binary star system, demonstrating the instability of the orbiting planet's motion under perturbations to initial conditions. Here, future studies may improve our simulation by using a higher order method to solve the equations of motion governing the three body problem.

[1] Hansen, F. K. (2017): Lecture note 1B for the course AST2000 at UiO.
[2] Williams, D. R. (2024): Jupiter Fact Sheet from NASA.
[3] Williams, D. R. (2024): Mars Fact Sheet from NASA.