Lecture 22 (Probabilistic Reasoning)

1 Uncertainity in AI

1. Guess values of unobserved variables from observed variables (evidence)
2. Generate a model to obtain the unobserved variables using probabilistic reasoning

2 Random Variables

1. Used to represent the uncertain variable
2. It can take values from a domain

3 Joint Distribution

1. Combined distribution of a set of variables
$$P(X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n) = P(x_1, x_2, \ldots, x_n)$$
2. They obey the following conditions:
$$P(x_1, x_2, \ldots, x_n) \geq 0$$
$$\sum_{(x_1, x_2, \ldots, x_n)} P(x_1, x_2, \ldots, x_n) = 1$$
3. However, this is not feasible since the size of the table will be $O(d^n)$

4 Events

An event E is a set of outcomes.

$$P(E) = \sum_{(x_1, x_2, \ldots, x_n) \in E} P(x_1, x_2, \ldots, x_n)$$

5 Marginalization

Reduce (marginalize) variables by collapsing rows by adding liklihoods

$$P(x_1, x_2, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n) = \sum_{x_i} P(x_1, x_2, \ldots, x_n)$$

6 Conditional Probabilities

Probability of some variables given fixed values of others

$$P(x_1, x_2, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n | x_i) = \frac{P(x_1, x_2, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)}{P(x_i)}$$

7 Product Rule

$$P(y)P(x|y) = P(x, y)$$

8 Chain Rule

$$\prod_{i=1}^{n} P(X_i | X_1, X_2, \ldots, X_{i-1})$$

9 Bayes Rule

$$P(cause|effect) = \frac{P(effect|cause)P(cause)}{P(effect)}$$

10 Independence

$$X \perp\!\!\!\perp Y \iff P(x, y) = P(x)P(y)$$

11 Bayesian Network

1. It is also called as **Probabilistic Graphical Networks**
2. Encodes how variables locally infulence each other
3. Helps in describing complex joint distributions
4. The involve *entities* (RVs) and *interactions* (dependence)
5. They are DAGs
6. They are represented as a conditional probability table for each node wrt its parents

$$P(X|(a_1, a_2, \ldots, a_n))$$

7. They implicitly encode joint distributions
8. The complete joint distribution is given as:

$$P(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} P(x_i | parents(X_i))$$