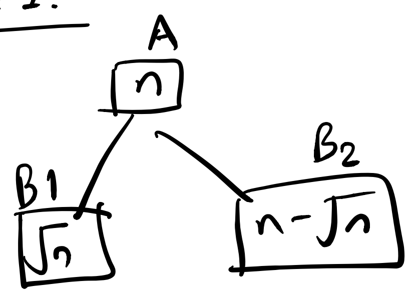
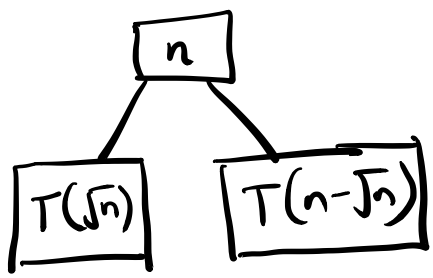
COL351: Tutorial 1:

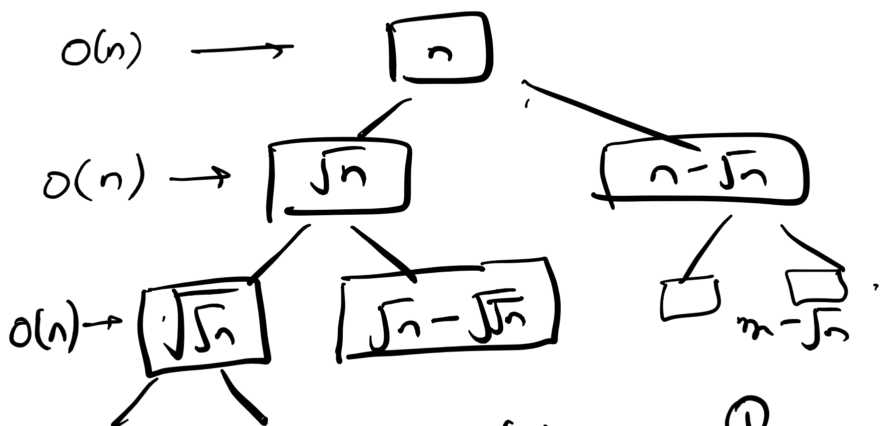
Q1.

MS(B_1)
MS(B_2)
merge



$$T(n) = T(\sqrt{n}) + T(n-\sqrt{n}) + O(n)$$





$$T(n) = n \cdot H(n) \quad \text{— } 1$$

$n - \sqrt{n} > \sqrt{n}$ (for large n)

$H(n-\sqrt{n})$ becomes dominant.

$$H(n) = 1 + H(n-\sqrt{n})$$

$$H(n) < 2\sqrt{n}$$

$$T(n) < 2n\sqrt{n} = O(n\sqrt{n})$$

Claim: $H(n) \le O(2\sqrt{n})$:

$$H(n) = 1 + H(\overbrace{n - \sqrt{n}}^{n'})$$
$$= 1 + \left(1 + H(n' - \sqrt{n'})\right)$$

$H(n) =$ #steps it takes to reduce n to 1.

$k \le 2\sqrt{n}$. $\qquad k \cdot \omega = n$

$2\sqrt{n} \cdot \omega \geq n$

$\Rightarrow \omega \geq \sqrt{n}/2$.

$$\sqrt{n} > \sqrt{n}/2$$
$$\sqrt{n'} = \sqrt{n - \sqrt{n}} > \sqrt{n}/2$$

$$\boxed{H(n)} \to \boxed{H(n')} \to H(n'') \to \boxed{H(n/2)}$$

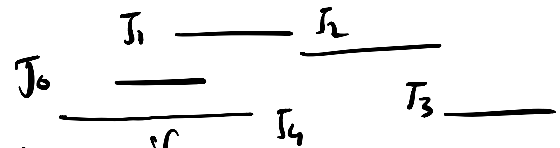
$-\sqrt{n}/2$

$$H(n/2)$$
$$= 1 + H(n/2 - \sqrt{n/2})$$

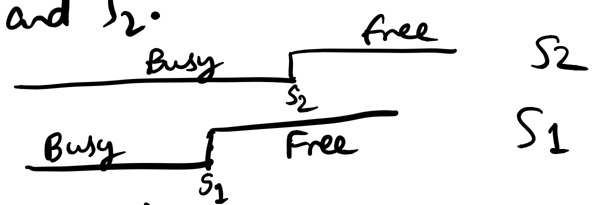
steps: $\dfrac{n - n/2}{\sqrt{n}/2}$ $\left\{ \begin{array}{l} \therefore \text{Each step from } n \text{ to } n/2 \text{ reduces atleast } \sqrt{n}/2. \\ \text{From } n/2 \text{ to } n/4 \to \cdots \end{array} \right.$

$= \sqrt{n}/2 + \sqrt{n/4} + \cdots$

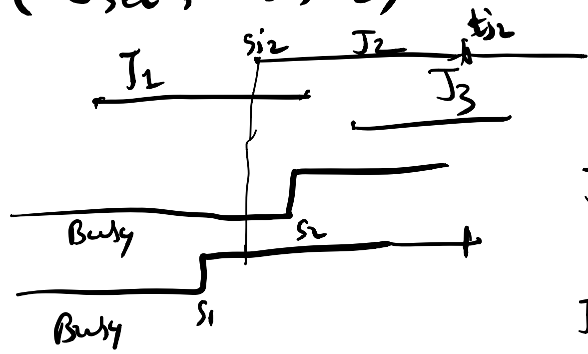
Q2.



Need to see if
there is atleast one free server.
Servers s_1 and s_2.



Schedule(J_{set} , s_1, s_2)



J_1 : can't be scheduled

J_2 : can be scheduled on S_1.

J_3 : can be schedule on both s_1 & s_2

Select J with the earliest end time, out of the set of jobs starting after min(s_1, s_2)

$\underline{s_1 < s_2}$.

schedule: if J's start time < s_2 :
schedule on s_1
return $\{J\} \cup$ Schedule($J_{set} \setminus \{J\}$, t_j , s_2)

o/w schedule on s_2

Schedule (J_{set}, s_1, s_2) $\rightarrow$ A, B ← set of scheduled jobs.

A : jobs scheduled at S_1
B : ———— s_2.

$A \cap B = \emptyset$ (no job can be scheduled on two servers)

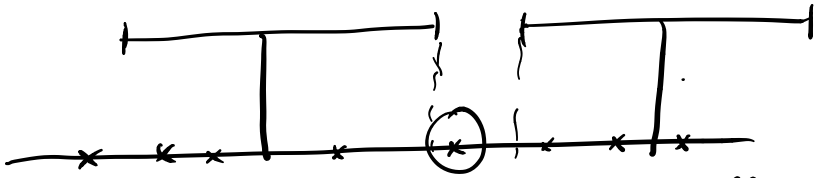
A start times $> s_1$; A's are non-overlapping

B start times $> s_2$ ————

$|A \cup B|$ is the maximum.

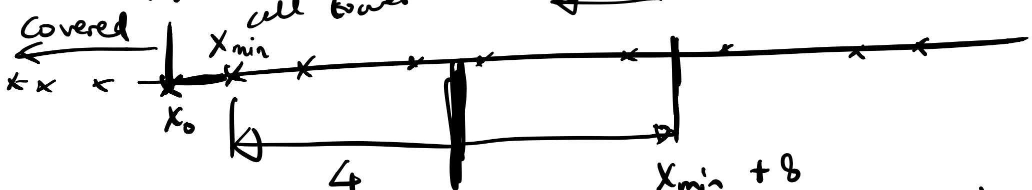
$$OPT(A \cup B, s_1, s_2) = OPT(A' \cup B', s_1', s_2') + 1$$

Q3.



Min # of cell towers are installed & you cover each house

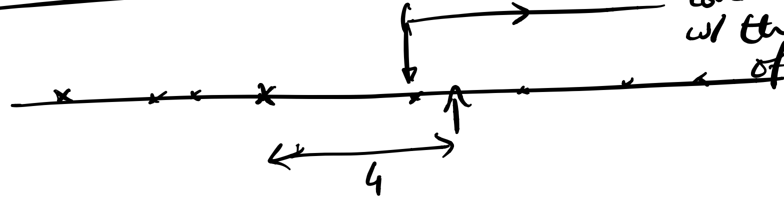
General case problem: suppose all houses with $x_i < x_0$ are covered. Find the position of the next cell tower.



X_{min} : location of leftmost house to the right of x_0.

Recurse on the problem for $x_0 = x_{min} + 8$.

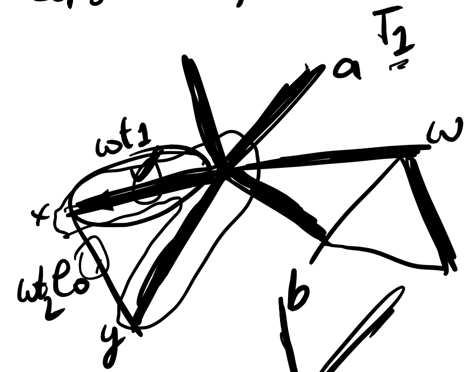
Proof of correctness:

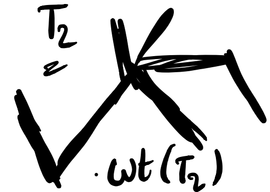


till house x_i, I have covered them w/ the min # of towers.

If have a tower within 4,
If not, then cannot cover the house w/o adding new tower.

Q5.





$wt(T_2) < wt(T_1)$

$wt_2 < wt_1$

w.

Spanning tree:

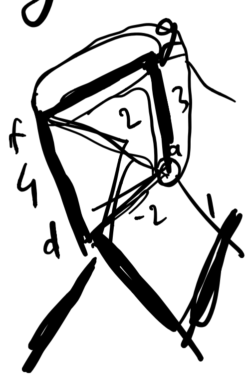
spanning ✓

Tree:

- connected
- # edges = n-1. ✓

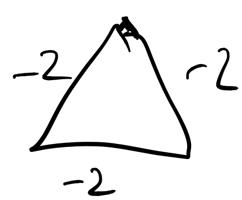
edge xy connects the tree.

Q6.



af : You can replace fd w/ some other edge in the MST.

~~path from a to f~~



-4, -6.

Q6.(a)

Observe if there exists a path connecting nodes d, f consisting only of edges with weights strictly less than the weight of the edge (d, f).

Then by Q6(b) below, we can conclude that edge (d, f) does not lie in any MST.

We see that the path d - a - f is one such path.

Q6.(b):

Claim 1: If an edge $e_0 = (x, y) \in E$ does not lie in any MST of G then there exists a path connecting x and y consisting only of edges with weights strictly less than $wt(e_0)$.

Pf (by contradiction): If there doesn't exist such a path then show that e_0 must lie in some MST of G.

x and y are not connected besides via e_0:

Case I: Since a spanning tree must span all nodes, e_0 must be part of any spanning tree, and in particular, a MST.

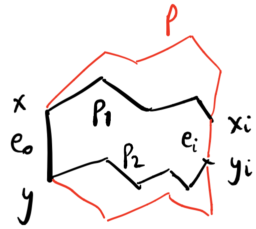
Case II: x and y are connected via a path $[e_1, e_2, \ldots, e_k]$ s.t. there exists an edge e_i with $wt(e_i) \geq wt(e_0)$

If e_0 does not lie on the MST, then (by Problem 5) replacing e_i with e_0 would result in another spanning tree containing e_0. Further, this would be an MST because $wt(e_0) \leq wt(e_i)$.

$\square$

Claim 2: If there exists a path connecting x and y consisting only of edges with weights strictly less than $wt(e_0)$ then $e_0 = (x, y) \in E$ does not lie in any MST of G.

Pf (by contradiction): If such a path P exists and also e_0 lies in an MST of G, then show that it leads to a contradiction. (P is denoted in red)



Since $e_0 \in$ MST, the MST cannot contain all edges in P because otherwise we would observe a cycle in the MST.

$\therefore$ There must exist an edge $e_i = (x_i, y_i) \in P$ s.t. $e_i \notin$ MST.

Call the path connecting x to x_i in the MST by P_1 and that connecting y to y_i in the MST by P_2. We have that path $P_1 \cup \{e_0\} \cup P_2$ lies in the MST.

Now we can apply Problem 5 on edge e_i and replace e_0 by e_i in path $P_1 \cup \{e_0\} \cup P_2$ in the MST.

Doing this, the overall weight of the "MST" would reduce further because $wt(e_i)$ is strictly less than $wt(e_0)$.

$\Rightarrow$ Contradiction.

$\square$.