COL 351 : Analysis and Design of Algorithms

Tutorial Sheet - 9

Question 1 Let $G = (V, E, c : E \to \mathbb{Z}^+)$ be a directed graph with source $s \in V$, sink $t \in V$, and integer edge capacities $c(e) \geq 1$. Let $R = \max_{e \in E} c(e)$. Present an $O(mnR)$ algorithm to decide whether G has a unique minimum (s, t)-cut.

Question 2 Let $G = (V, E)$ be a directed graph, and (s, t) be a vertex pair. Two paths from s to t are said to be *internally-vertex-disjoint* if they do not share any vertex except end-points s and t. Present an $O(mn)$ algorithm to compute the maximum number of vertex disjoint paths from s to t.

Question 3 There are n clients $(c_1, \ldots, c_n)$ who want to be connected to one of the k mobile towers $(m_1, \ldots, m_k)$ in a town. You are given the (x,y) coordinates of each client and each tower, a distance parameter d, and a load parameter L. Design a polynomial time algorithm to decide if every client can be connected simultaneously to some mobile tower subject to the following constraints.

1. Each client is connected with exactly one of the mobile towers, and a client can only be connected to tower that is within distance d.

2. No more than L clients can be connected to any single mobile tower.

Question 4 Let $X = (x_{ij})$ be a square matrix of size n storing positive real numbers. It is given that the sum of elements of each column as well as each row is a positive integer. Prove that elements of X can be replaced by integers without changing any column sum or row sum.

Question 5 Provide an extension of hashing based pattern searching algorithm covered in Lecture 28 (Rabin-Karp algorithm) for searching a pattern of $k \times k$ matrix in an $n \times n$ binary matrix. What is the time complexity of your algorithm?

Question 6 Let $U = [1, M]$ be a universe of M elements, p be a prime in range $[M + 1, 2M]$, and $S \subseteq [1, M]$ be a set of size n $(<< M)$. Let r, c be uniformly chosen random numbers in $[2, p - 1]$ interval that are independent of S. Consider the hash function:

$$H_{r,c}(x) := ((rx + c) \mod p) \mod n$$

Prove that for any distinct $x, y \in [1, M]$, $Prob[H_{r,c}(x) = H_{r,c}(y)] \leq \frac{1}{n}$.