COL 351 : Analysis and Design of Algorithms

Tutorial Sheet - 3

Question 1 A graph $G = (V, E)$ is said to be **bipartite** if vertex set V can be partitioned into two sets X and Y such that every edge $e \in E$ connects a vertex in X to one in Y.

i) Show that any graph having an odd length cycle cannot be bipartite.

ii) Let $G = (V, E)$ be a connected graph and T be a BFS tree of G. Show that G is bipartite iff for each edge $(a, b) \in E$, $|Level(a) - Level(b)| = 1$.

iii) Devise an $O(n+m)$ time algorithm to check if a connected/unconnected graph G is bipartite.

Question 2 The girth of the graph is the length of the shortest cycle. Give an $O(mn)$ time algorithm for finding the girth of an undirected graph, where m and n are the number of edges and vertices respectively.
(Hint: For each vertex $v \in V$, use BFS(v) to find out the smallest cycle containing v.)

Question 3 A vertex x in an undirected graph is said to be *cut-vertex* if there are vertices u, v different from x, such that u and v are disconnected in $G \setminus x$. Let T be a DFS tree of G.

i) Show that a leaf node of T cannot be a cut-vertex.

ii) Show that root of T is cut-vertex iff it has at least two children.

iii) Prove that an internal node x is cut vertex iff it has a child, say y, in DFS tree T satisfying $High\text{-}point(y) \geq Level(x)$.

iv) Devise an $O(n + m)$ time algorithm to find all cut-vertices of a graph G.

Question 4 Let $G = (V, E)$ be a directed graph. Let $C_1, C_2, \ldots, C_k$ denote the strongly connected components of G (each C_i is a subset of vertices).

We construct another directed graph H from G as follows: There are k vertices in H, namely, $v_1, \ldots, v_k$. There is a directed edge (v_i, v_j) in H iff there is an edge in G from a vertex in C_i to a vertex in C_j.

i) Give a linear time algorithm to construct H from G.

ii) Prove that H is acyclic.
(Hint: Show that if there is a cycle in H, say on the first L vertices, i.e. $(v_1, \ldots, v_L)$ is a cycle in H, then vertices in set $C_1 \cup \cdots \cup C_L$ must be strongly connected to each other. This is a contradiction.)