Lecture 31

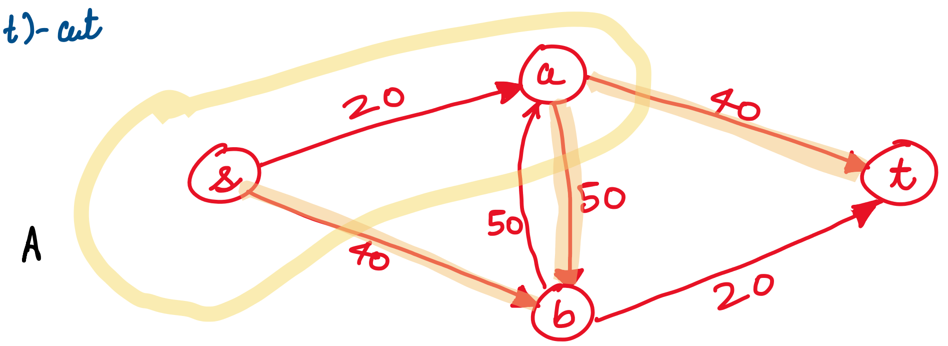
Wednesday, 27 October 2021 8:45 AM

Def^n: (s,t) - cut

For any $\boxed{A} \subsetneq V$ with $(s,t) \in (A, \bar{A})$

$$\text{cut}(A,\bar{A}) := \{(x,y) \in E \mid x \in A,\ y \in \bar{A}\}$$

Eg. (s,t)-cut



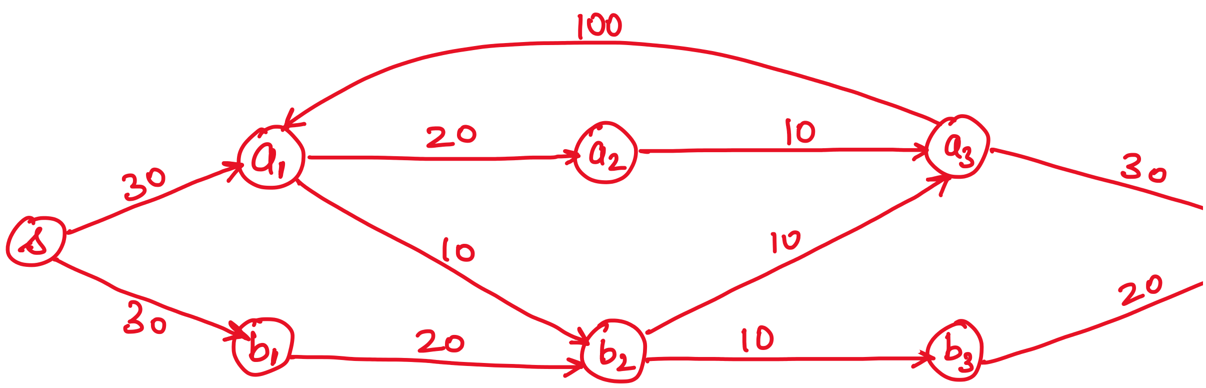
Capacity of (s,t)-cut

For a cut $(A,\bar{A})$ with $s \in A$ and $t \in \bar{A}$

$$c(A,\bar{A}) = \sum_{(x,y) \in \text{cut}(A,\bar{A})} c(x,y)$$

Q. Which (s,t)-cut has minimum capacity?

Eg.



A

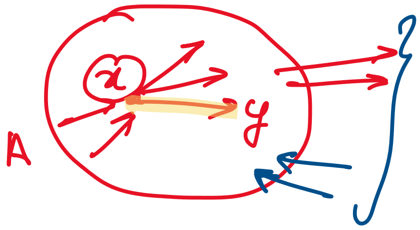
$C(A,\bar{A}) = 10+10+10 = 30$ $A = \{ s$

$c(A,\bar{A}) = 20+20+10 = 50$ $A = \{ s$

(*) $\underline{f_{out}(A)} := \sum_{\substack{(x,y)\in E \\ x\in A \\ y\in \bar{A}}} f(x,y)$ (*) $f_{in}(A) :=$

(*) Lemma from lec30: For each $A \subseteq V$, and a flow f

$$f_{out}(A) - f_{in}(A) = \sum_{\underline{x\in A}} \Big(f_{out}(x) - f$$

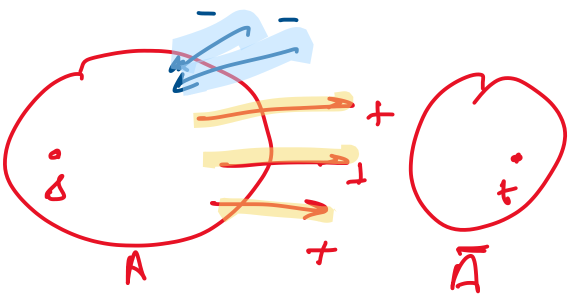


will not be co

Corollary of Lemma:

For any cut $(A,\bar{A})$ with $\boxed{s\in A}$ and $\boxed{t\in \bar{A}}$

$$\underbrace{f_{out}(A) - f_{in}(A)}_{LHS} = f_{out}(s$$



||| val(f)

Proof.

$LHS = \sum_{x\in}$

$= f$

$= .$

$= v$

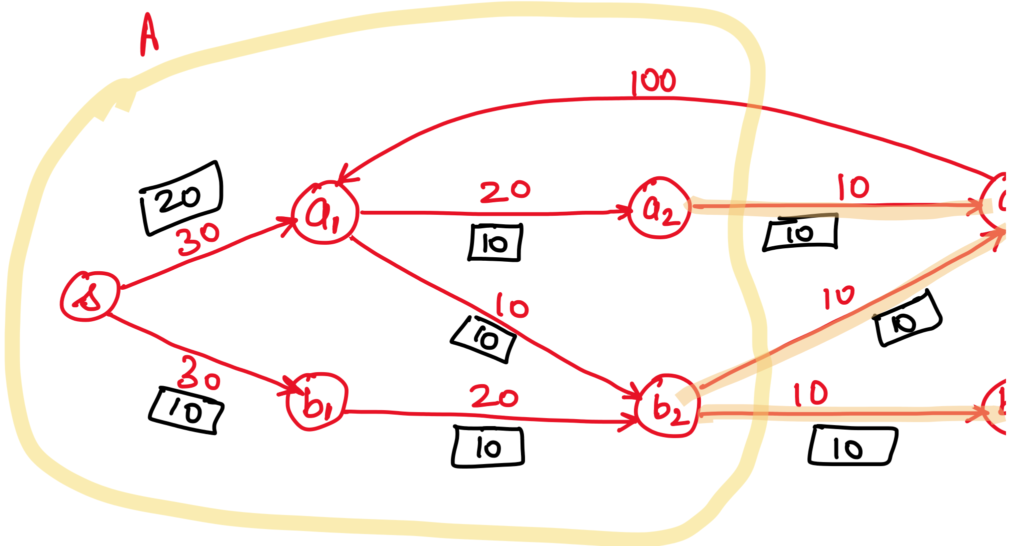
Observation 1

For any cut $(A, \bar{A})$ $\quad (s,t) \in A \times \bar{A}$

$$\mathrm{val}(f) = f_{out}(A) - f_{in}(A) \leq f_{out}(A) \leq$$

Theorem 1: $\underbrace{(s,t)\text{-max-flow value}} \overset{?}{\leq} \min_{\substack{A \subseteq V \\ (s,t) \in}}$

Eg:



$\underbrace{(s,t)\text{-flow} = 30}$ $\qquad c($

This is max-flow by Theorem

~~Lemma: Suppose f is a flow that cannot be Ford-Fulkerson algorithm further~~

Lemma. Let f be flow s.t. $\nexists\ s \to t$ path i

Let $A :=$ set of vertices reachable f

Then,

$$\underbrace{val(f)}_{\substack{\| \\ f_{out}(A) - \underbrace{f_{in}(A)}_{=0}}} = c(A, \bar{A}).$$

Proof

$E_1 = \{(x,y) \in E(G) \mid x \in A, y \in \bar{A}\}$

For $e=(x,y) \in E_1$, $f(e) = c(e)$

bcoz o/w $c_r(x,y) \neq 0$ and $y \in A$

E_1 is fully saturated

$E_2 = \{(x,y) \in E$

For (x,y)

bcoz

E_2 has

$\Rightarrow f_{out}(A) - f_{in}(A) = c(A, \bar{A}) - 0$

$\Rightarrow val(f) = c(A, \bar{A})$

$\Rightarrow$ Flow f is a (s,t) - max - flo

$\Longrightarrow$ Flow f computed from F.F. algo

Theorem 2 : (s,t) - max-flow value =

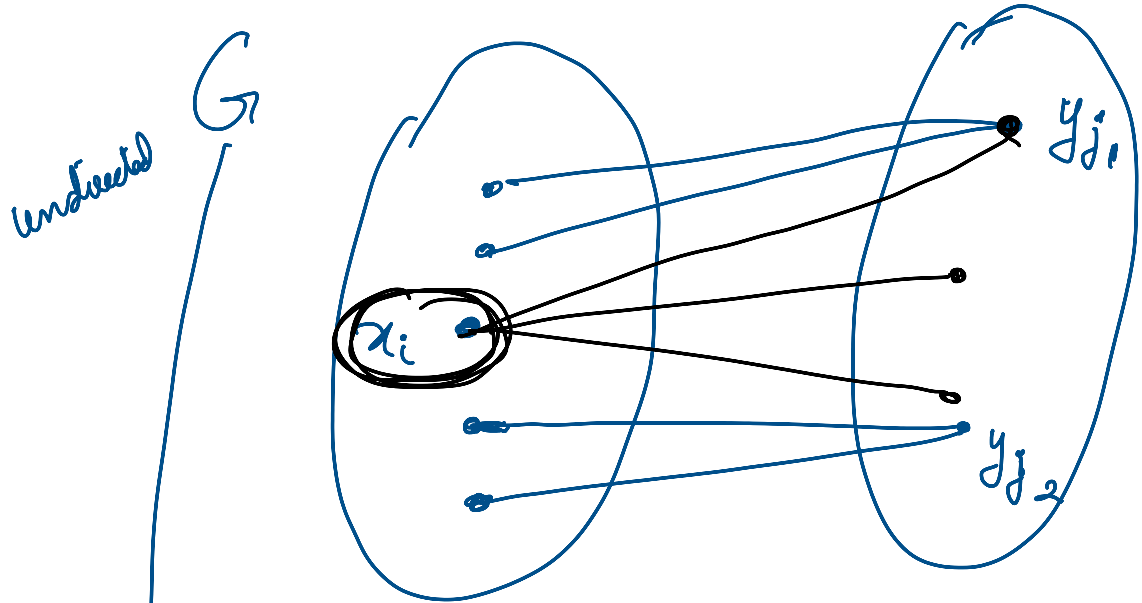
(s

$\boxed{\text{Max Flow Min cut Theorem}}$

Applications

① **Bi-partite Matching**

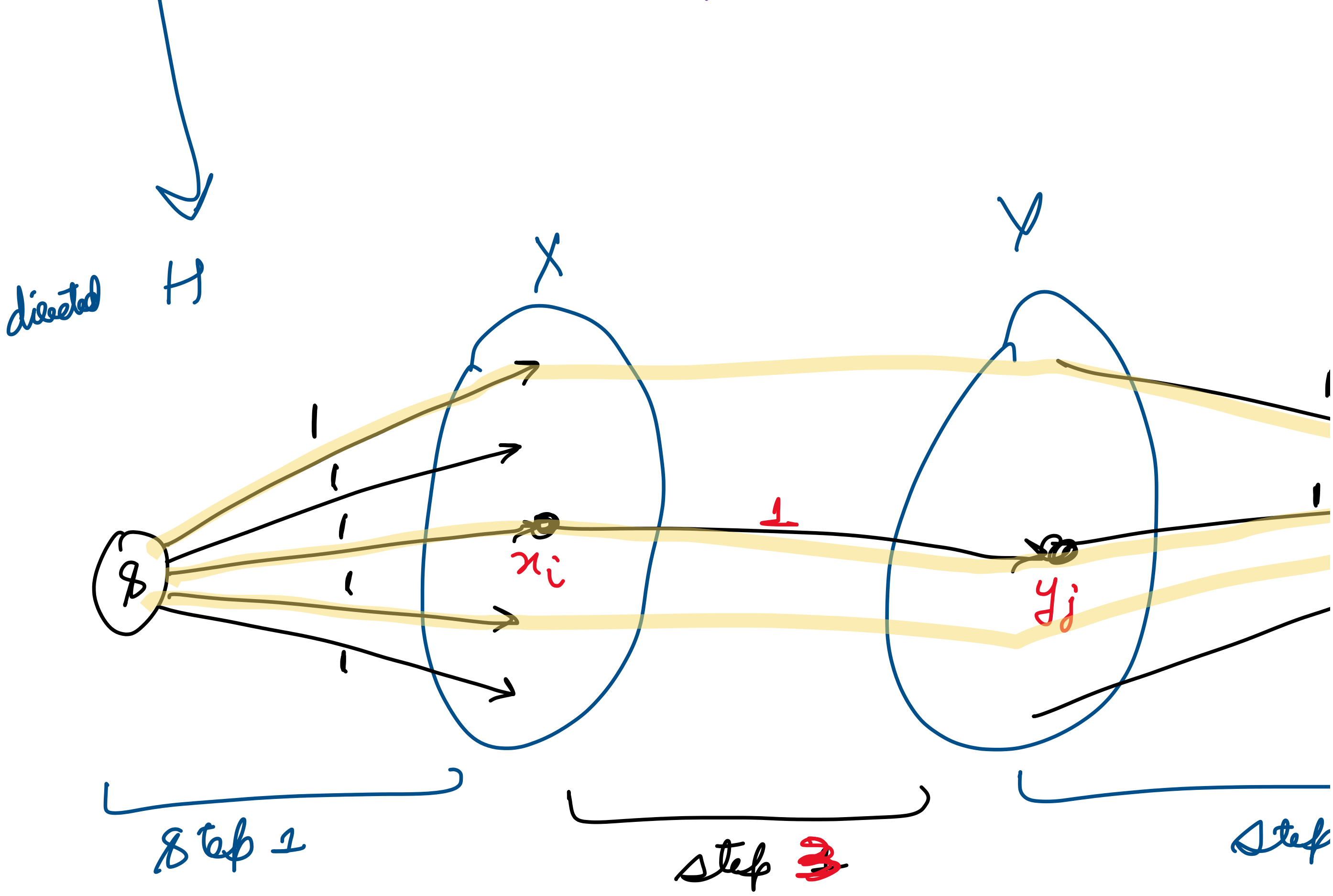
Set $M \subseteq E(G)$
s.t.
two edges $e_1, e_2 \in M$
have a
common vertex.



X
(Faculty @ CSE)

Y
COL 351 students.

Question: To find a matching M ⊆



Integral-(s,t)-max-flow → edges from X

val(s,t - max-flow) ≡ No of .

Time complexity to find M.M. = $\underbrace{\text{Time to compute new graph}}_{O(m+n)}$ -

$$= O\big((m+n)\, n \big)$$

$$= O(m \cdot n)$$