Lecture 11

DFS trees in digraphs.

Timer = 0

DFS(x)

  Set Start-time(x) = timer, and timer := timer + 1

  Mark x as visited

  For each out-neighbor y of x:

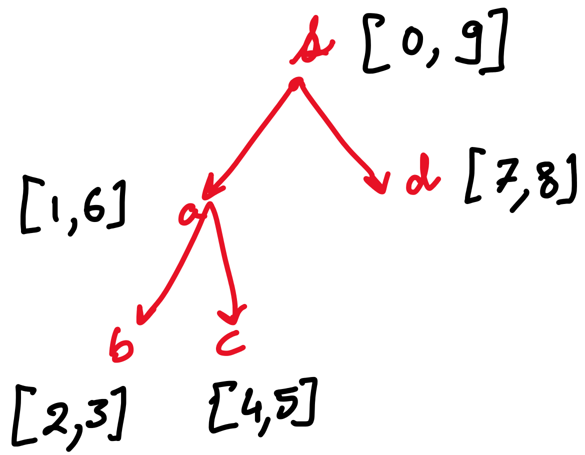
    If y is unvisited then

      Invoke DFS(y),

      Set y as child of x in tree T

  Set Finish-time(x) = timer, and timer = timer + 1

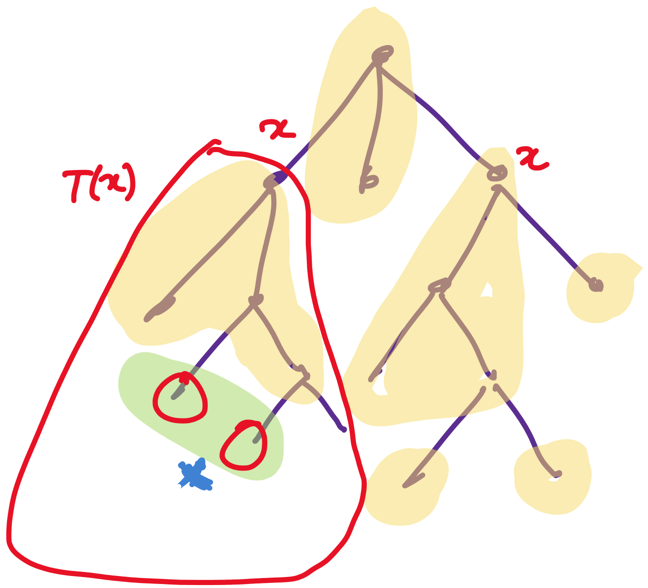
$n = 5$



- Start time (x) = Time when DFS(x) is invoked
- Finish time (x) = Time when DFS(x) is exited

Obs 1: If x, y are in an SCC "S", then any path $x \rightsquigarrow y$ lies in S.

Lemma: Let T be a DFS tree of G, and S be an SCC, then $T[S]$ is a contigous subtree.



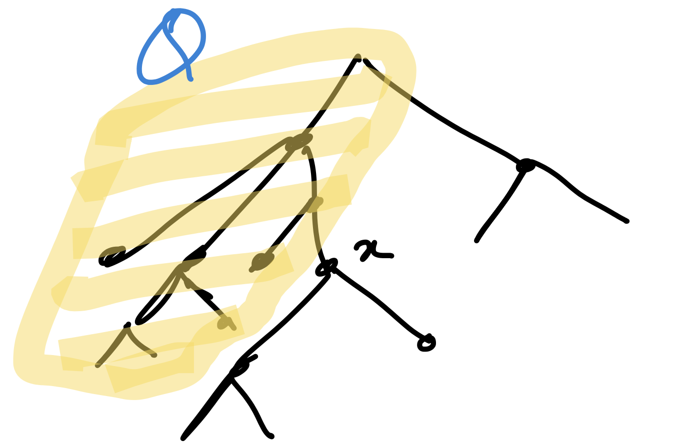
Proof:

$x \leftarrow$ vertex in S that is visited first.

Claim 1: S lies in tree $T(x)$

Q = Vertices in G visited before x.

R = Vertices reachable from x in $G \setminus Q$



relation b/w R and $T(x)$?

(i) ⊛ $T(x) = R$ (Think why?)

Now

Now,

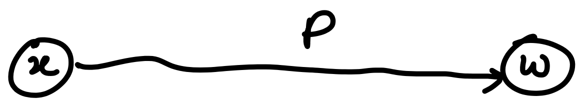
(*) $(S \cap Q) = \emptyset$

(by def^n of x, x = first vertex of S visited by DFS)

(ii)

(*) Take a vertex $w \in S$.

P = a path from x to w



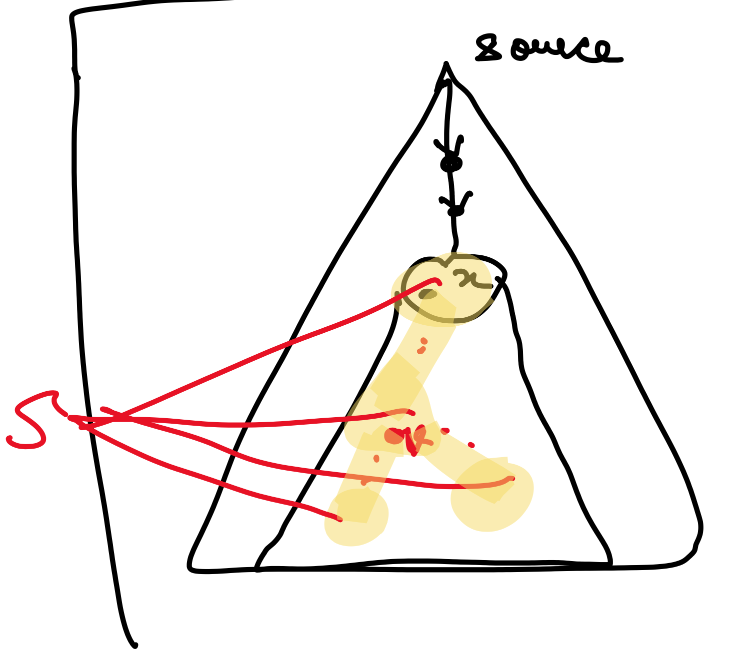
vertices of P lie in S by Obs 1.

P lies in $G \setminus Q$.

$\Rightarrow$ w lies in R.

$S \subseteq R$.

What we have shown in Claim 1



(i) & (ii) S lies in $T(x)$.

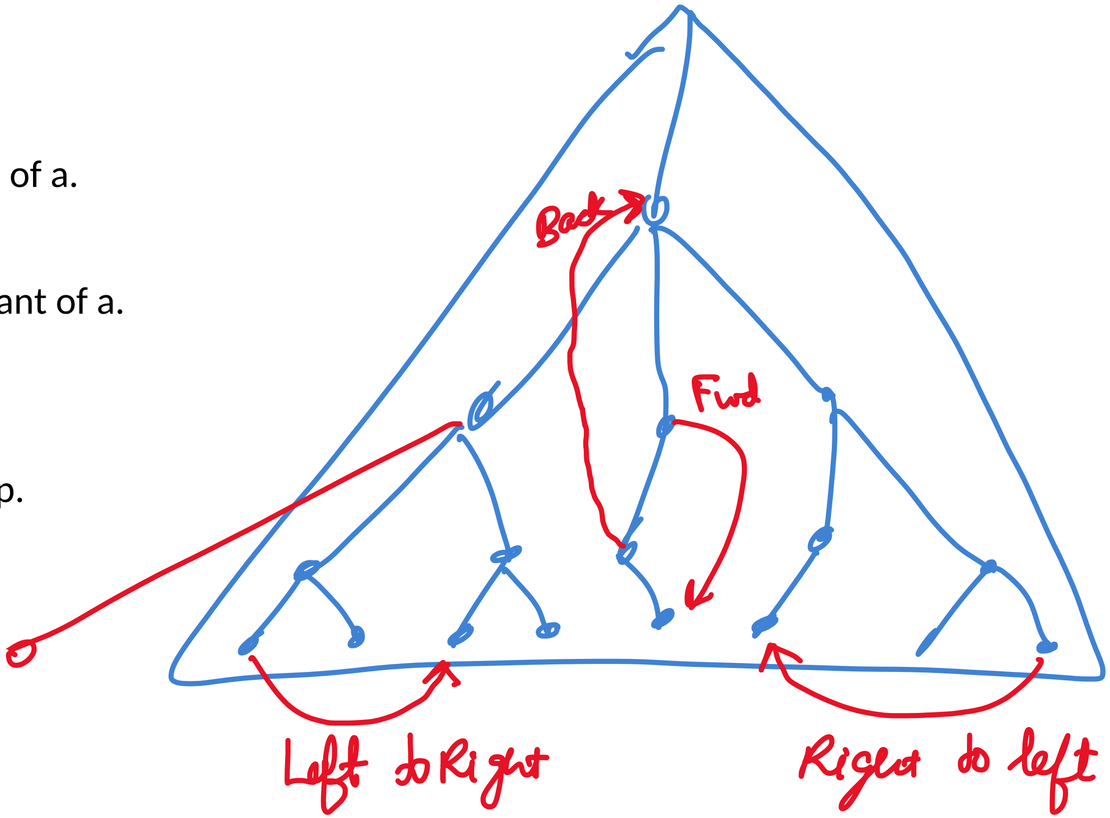
Claim 2: For any vertex $v \in S$, treepath(x, v) lies in S.

Proof : By Obs 1

By Claim 1 & Claim 2 we get T[S] is one single entity.

Classification of edges of **Directed-graph** with respect to a Rooted tree

- **Tree edges -**
  Edges parts of tree
- **Back edges** -
  Non-tree edge(a,b) s.t. b is ancestor of a.
- **Forward edges** -
  Non-tree edge(a,b) s.t. b is descendant of a.
- **Cross edges** -
  Edges whose endpoints have
  NO ancestor-descendant relationship.
  - Left --> Right
  - Right --> Left



CLAIM: **In directed graphs there can be no Left--> Right cross edges in DFS tree (or the forest obtained by DFS traversal)**

Proof: **H. W.**