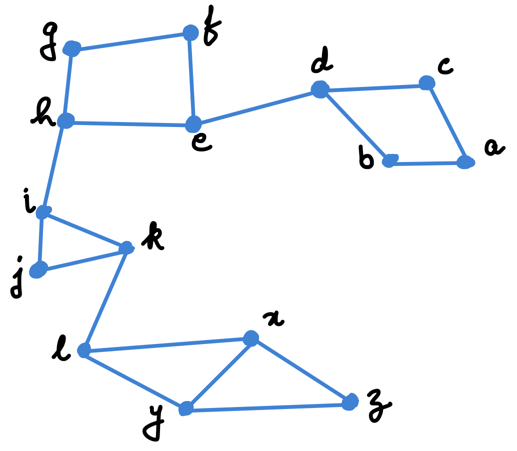
DFS Application: Finding ALL bridges

Bridge Edge:

An edge ***(x,y)*** is said to be a bridge edge in *G* if ***x*** and ***y*** are disconnected in ***G\(x,y)***.



(d,c)
(i,h)
(k,l)
> Three bridges

Check (x,y) is bridge edge or not

Trivial way – Is x,y dis-connected in G\(x,y)

Naive – $O(\underbrace{m}_{\text{edges}} * \underbrace{m}_{\text{time per edge}})$

Ancestors of x in a tree T:
All vertices lying on root to ***x*** path in T, including both root and ***x***.

r, a, b, c, x

Proper-Ancestors of x in a tree T:
Ancestors of ***x*** in T other than itself.

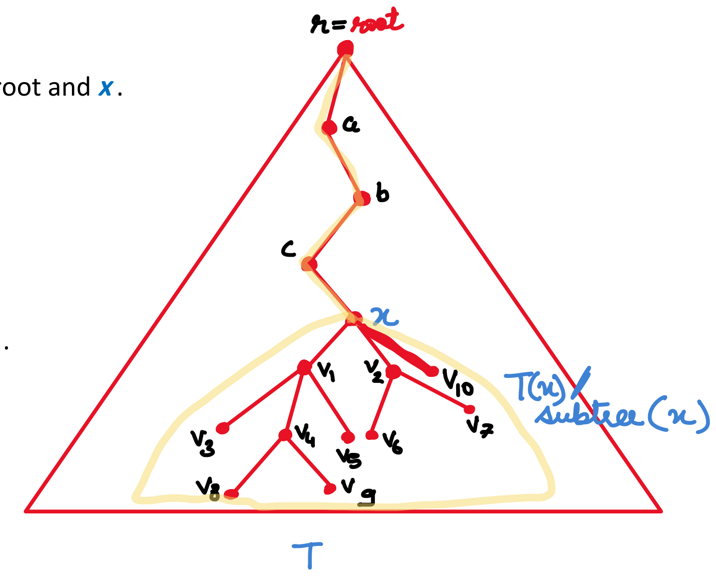
r, a, b, c

Descendants of x in a tree T:
All vertices lying in subtree of ***x***, including the vertex ***x***.

x, v_1, ..., v_{10}

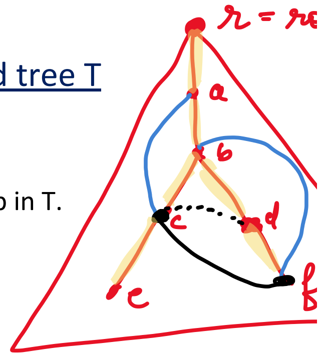
Proper-Descendants of x in a tree T:
Descendants of ***x*** in T other than itself.

v_1 ... v_{10}



Classification of edges of undirected G with respect to Arbitrary rooted tree T

- Tree edges - Edges parts of tree (6 edges)
- Back edges - Non tree Edges whose endpoint have ancestor descendant relationship in T.
  (a,c) and (b,f)
- Cross edges - Edges whose endpoint have NO ancestor descendant relationship.
  (c,f) and (c,d)



Position of edges of G.

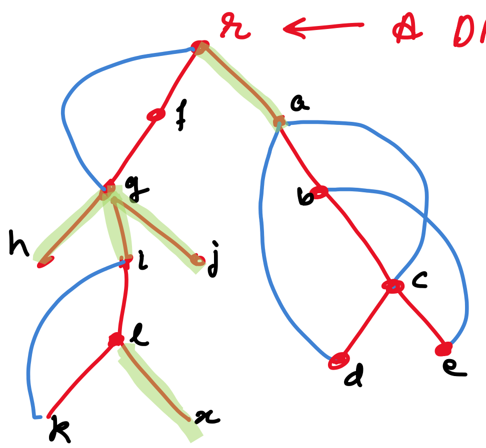
$G = (V, E)$

r, a, b, c, d, e, f

(r,a) (a,b

⋮

Lemma (last class): *Let T be a DFS tree of G=(V,E), then all non-tree edges in G are back edges.*



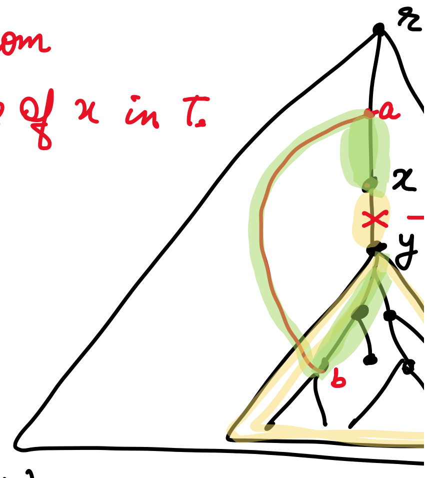
5 back edges.

Ques 1: If (x,y) is bridge edge in a connected graph G.
(x,y) is a tree-edge

Proof: There is a path from x to y in tree, and we know by def^n of bridge such a path is just edge (x,y).

Ques 2: A tree edge (x,y), with $x = \text{parent}(y,T)$, we h
(x,y) is bridge edge if
"There is no back edge from $T(y)$ to ancestors of x"

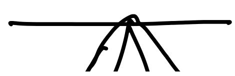
Proof: (i) If there is a back edge (b,a) from $b \in \text{subtree}(x)$ to 'a' - ancestor of x in T.
Then (x,y) is not a bridge edge.



Proof of (i)

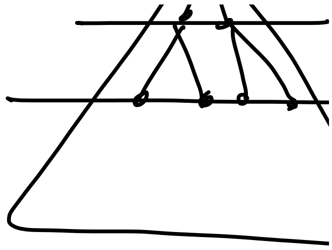
$\text{treepath}(x,a) \cdot (a,b) \cdot \text{treepath}(b,y)$

is a path from x to y in $G \setminus (x,y)$.



(ii) Converse

H.W.



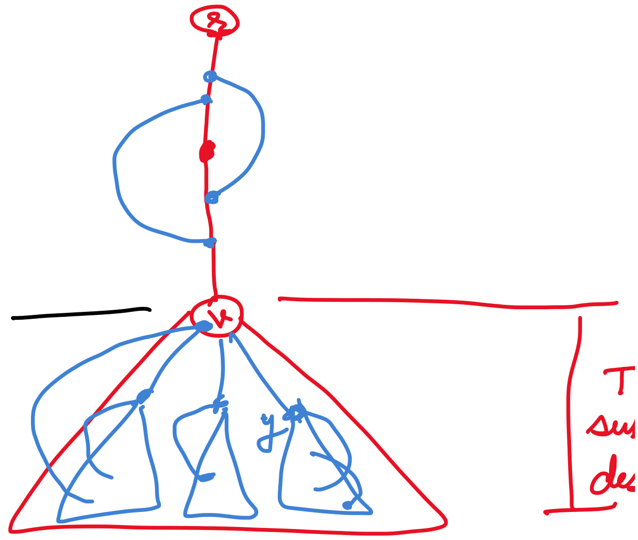
High-point(*v*):

The level of the *highest ancestor* of ***v*** to which there is a back edge from descendants of ***v*** (is such a back edge exists).

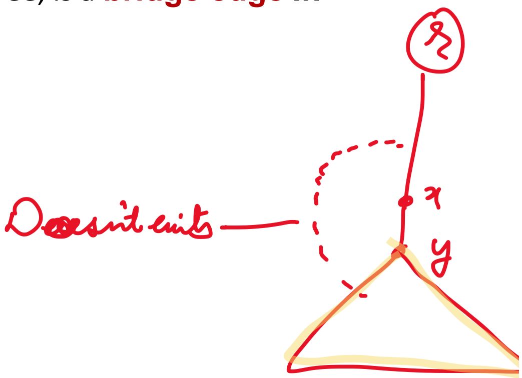
Otherwise set it to be just level(v).

High point(v)

= level(v).



Theorem : A tree edge (***x,y***), with x being parent of y in DFS tree, is a **bridge edge iff** **High-point**(**y**) = **Level**(**y**).



$\underbrace{\text{Level}(v),\ \forall v}$

can be computed in $O(n)$ time

if we have already DFS tree.

Ques: How to compute High-point for all vertices.

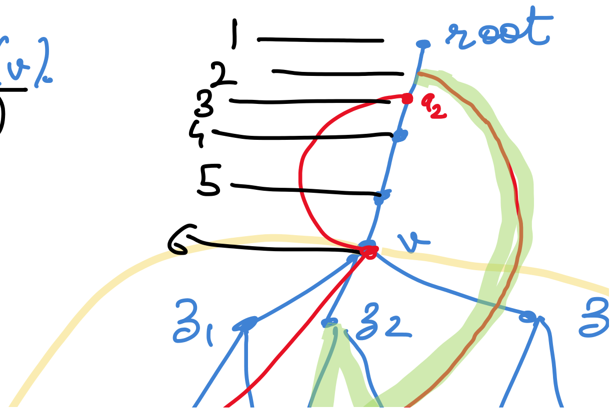
Take a vertex v. Let $z_1 \ldots z_k$ be children of v.

① Initialize High-point(v) to be $\underbrace{\text{level}(v)}_{6}$.

② Scan all non-tree edges (a,v):

if level(a) < level(v).

then high-point(v) =



$\min\{\text{highpoint}(v), \text{level}(a)\}$

③ For $i = 1$ to k :

if high-point (3_i) < high-point (v)

then set high-point (v) to be high-point (3_i).

$$\text{Total time} = \sum_{v \in V(G)} \deg(v) = O(m)$$

① Find DFS & levels

② Compute high points

③ $\forall\, v$ compare high-point & level (v).

All bridges can be reported in $O(m)$ time a connected g