Lecture 10

Tuesday, 31 August 2021 5:30 AM

DFS Application: Finding Strongly Connected Components (SCCs)

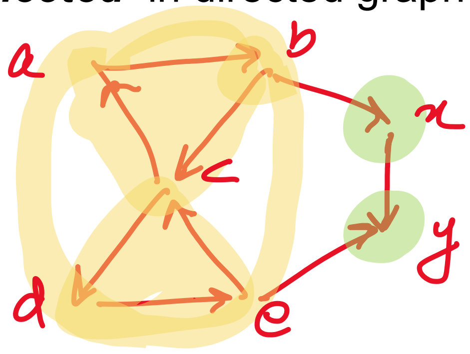
Directed Graph:

A graph G=(V,E) in which edges have direction.

Strong-Connectivity:

A pair of vertices ***(x,y)*** is said to be a ***strongly-connected*** in directed graph *G* if

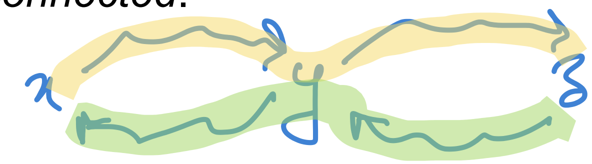
- There is x to y path in G, and
- There is y to x path in G.



$\underbrace{a, b, c, d, e}_{\text{strongly connected}}$

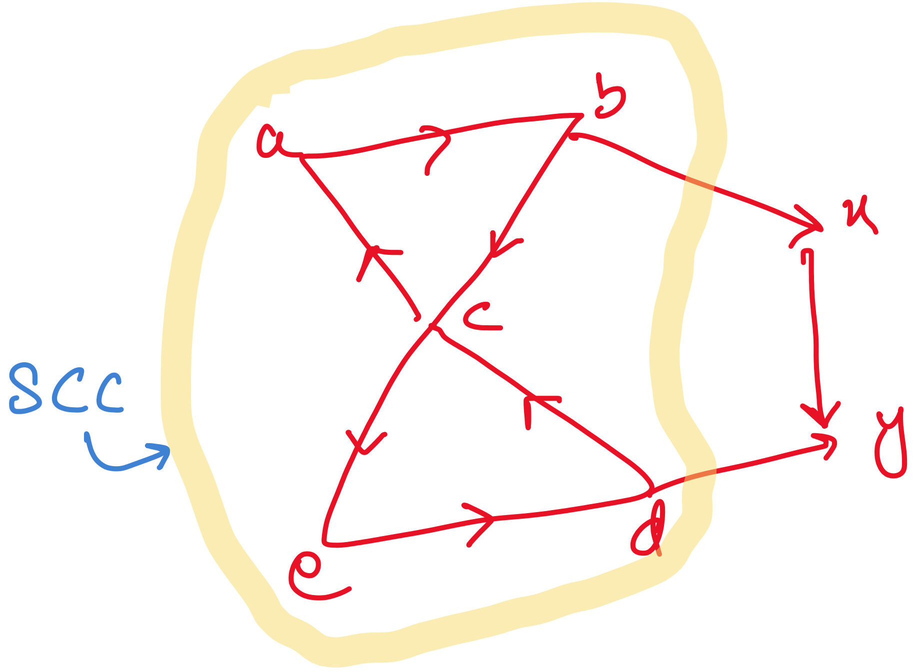
Transitivity Property:

If ***x*** and ***y*** are *strongly-connected,* and ***y*** and ***z*** are *strongly-connected,* then ***x*** and ***z*** are also *strongly-connected*.



Strong-Connected-Component(SCC):

A maximal subset ***S*** of *G* that all the vertices in ***S*** are *strongly-connected* to each other.

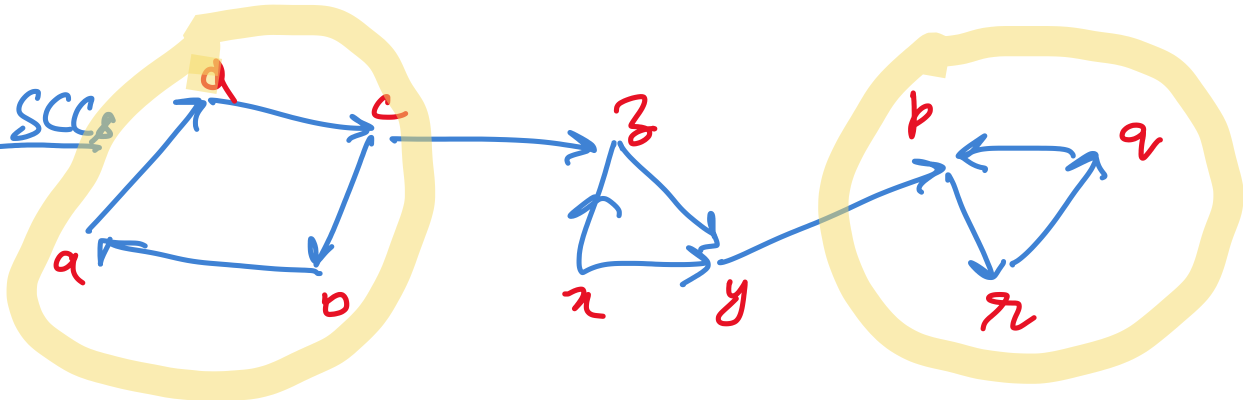


{a, b, c}
not an SCC

b coz not maximal

How many SCCs

Two.



To compute ALL SCCs

Trivial way: Take vertex $w \in V$, and find vertices

- reachable from w (BFS(w))
- Having a path to w (BFS in G^{rev})

Intersection

$O(m)$ per vertex

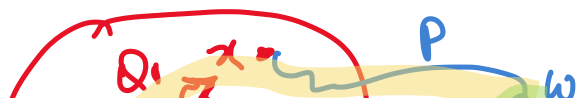
Repeat This for all vertices

n

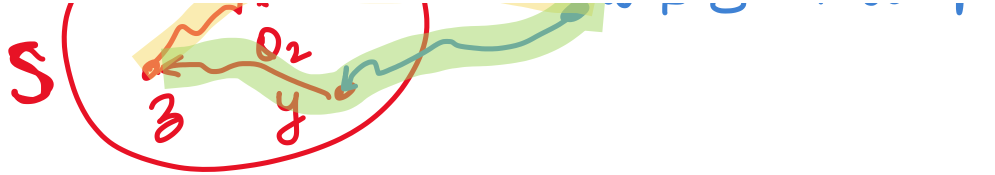
Total time $= O(mn)$.

Observation 1.

Let S be an SCC in G and x,y be vertices in S. Then, any path from x to y must entirely lie in S.



$w \notin S$ not possible.



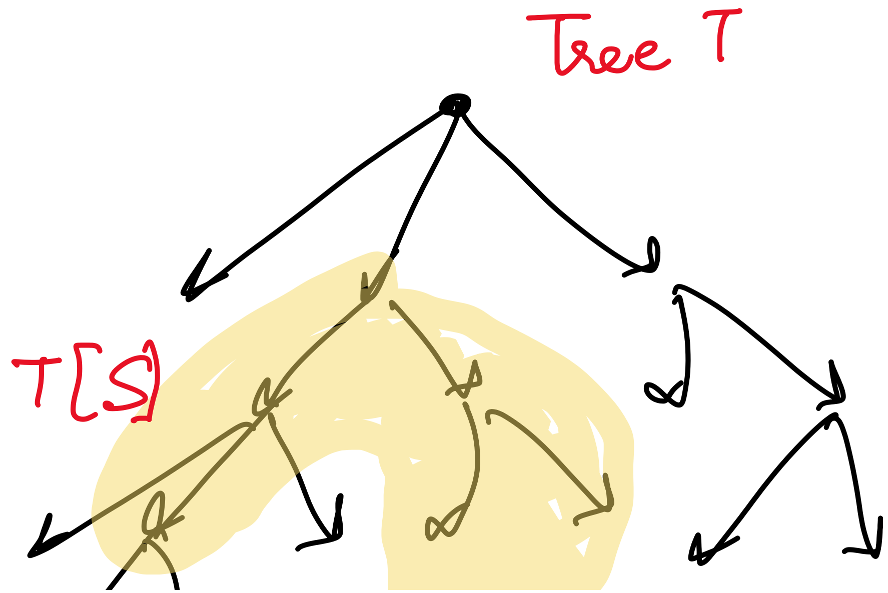
Proof:

Take z, $Q_1 = z$ to x path $\Rightarrow Q_1 \circ P[x,w]$ path from z to w

$Q_2 = y$ to z path

$\Rightarrow P[w,y] \circ Q_2$ is a w to z path

$\Rightarrow w \in S$



Lemma (Important).

*Let T be a DFS tree of directed graph G=(V,E), and **S** be an SCC of G. Then, the subgrph T[S] is a contiguous subtree of T.*

Proof:

Next class