COL352
Problem Sheet 1

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Question. *For any string $w = w_1 w_2 \dots w_n$ the reverse of w written w^R is the string $w_n \dots w_2 w_1$. For any language A, let $A^R = \{w^R | w \in A\}$. Show that if A is regular, then so is A^R. In other words, regular languages are closed under the reverse operation.*

Proof. Consider the DFA for $D_A = (Q, \Sigma, \delta, q_0, F)$. Construct the following NFA $N_A = (Q', \Sigma', \delta', q', F')$:

$$\begin{aligned} Q' &= Q \cup q' \\ \Sigma' &= \Sigma \\ \delta'(q_i, a) &= q_j \iff \delta(q_j, a) = q_i \\ \delta'(q', \epsilon) &= F \\ F' &= \{q_0\} \end{aligned} \tag{1}$$

Claim 5.1. *N_A recognises the language A^R, i.e., every word in A^R is accepted by N_A and every word not in A^R is rejected by N_A*

Proof. Consider any word w in A, it is recognised by D_A (from construction). Consider the sequence of states visited during the run of acceptance of w. Let it be $S = \{q_0 = q_{w_1}, q_{w_2}, \dots, q_{w_n}\}$. Now consider the following run in N_A of w^R:

$$\{q', q_{w_n}, \dots, q_{w_2}, q_0 = q_{w_1}\} \tag{2}$$

The above is a valid run since each transition is a valid one from the construction of N_A (the first one is an ϵ-transition and the remaining are fixed transitions). This proves that every word in A is recognised by N_A.
To prove the converse, assume by contradiction that a word $w^R = w_n \dots w_2 w_1 \notin A$ is accepted by N_A. Let the sequence of states be $S' = \{q', q_{w_n}, \dots, q_{w_2}, q_{w_1} = q_0\}$. Now, from the construction of N_A, the following sequences of states should be accepted by D_A:

$$\{q_{w_1} = q_0, q_{w_2}, \dots, q_{w_n}\} \tag{3}$$

This corresponds to accepting the string w. However, D_A accepts only strings in A. This is a contradiction to our assumption. Therefore, N_A rejects every string not in A.
Therefore, A^R is recognised by N_A which is an NFA. Therefore, A^R is a regular language as well. Thus, regular languages are closed under the reverse operation. Hence, proved. □

□

6 Question 6

Question 6

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