L06: Adequacy and Functional Completeness

$\{f \mid f : A \to B\} = B^A$ $\qquad |B^A| = b^a$ $\qquad$ Finite sets

$|A| = a \qquad |B| = b$

$\cancel{\{\mathbb{N} \to 2}$ $\qquad |\{f \mid f : \mathbb{N} \to 2\}|$ is uncountable

$= |2^{\mathbb{N}}|$

$\quad = [0,1]$

$X \subseteq \mathbb{N}$

$f_X : \mathbb{N} \to 2$

$$f_X(n) = \begin{cases} 0 & \text{if } n \notin X \\ 1 & \text{if } n \in X \end{cases}$$

With a finite number of operators it is impossible to express all boolean-valued functions using the given finite set O of operations.

functions defined as expressions over this finite set of operators $= O$

$f \in O, \qquad f : \mathbb{N}^{\alpha(f)} \to 2$

$|A| = a$ $\quad |B| = b$ $\quad |B^A| = b^a$

$\| \; 2^k$ $\quad \| \; 2$

$\overline{\wedge}$ or $\overline{\vee}$

$\langle 2, \{ \bar{\ }, +, \cdot \}, \{ \leq, = \} \rangle$

$$\mathbb{U} = \boxed{\bigcup_{k \geq 0} 2^{2^k}}$$

the set of all boolean-valued functions of arbitrary finite arities over 2.

is functionally complete.

(Every function in the set may be expressed as a finite expression involving only the constants 0, 1 and operators $\bar{\ }, +, \cdot$

$O = \{0, 1, +, \cdot, \bar{\ }\}$

(is adequate to express all functions in $\mathbb{U}$.

Functional completeness with a finite set of operators is impossible if either domain or co-domain is ~~fin~~ infinite

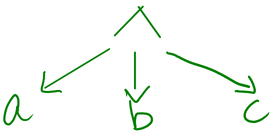
Functional completeness may be possible only if both domain and co-domain are finite

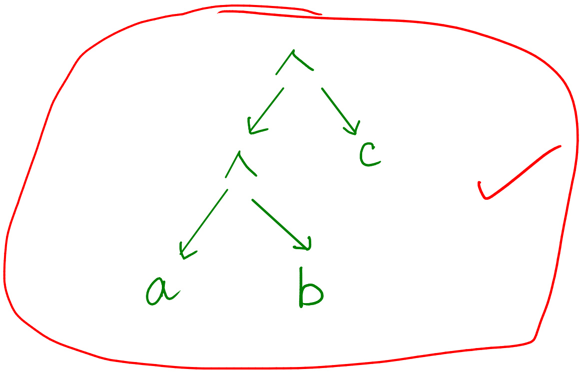
$\mathcal{B} = \{0, 1, 2\}$

$\bigcup_{k \geq 0} \mathcal{B}^{\mathcal{B}^k}$ was only recently shown to have a functionally complete set of operators

$((a\wedge b)\wedge c) \equiv$

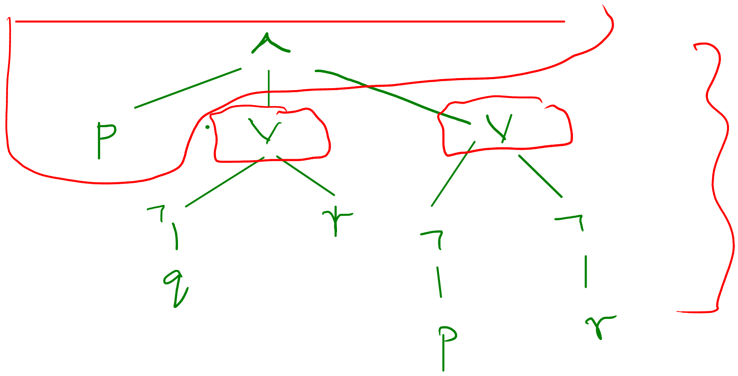
$a \wedge b \wedge c$





= AST

$p \wedge (\neg q \vee r) \wedge (\neg p \vee \neg r)$



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