COL703: Logic for Computer Science

Sat 21 Aug 2021 **Quiz 2** 20+5+5 minutes Max marks 10

Instructions:
1. Download the paper.
2. Write your name and entry number in the designated space on top and *do not forget to sign the honour statement below*.
3. Answer the question(s) in the appropriate space provided starting from this page.
4. Scan the paper with your completed answer.
5. Upload it on Gradescope 2001-COL703 page within the given time. *Make sure the first page with your name, entry no and signature is also the first page of your uploaded file*
6. Late submissions (within 2 minutes of submission deadline) on the portal will attract a penalty of 2 marks out of 10.
7. Email submissions after the closing of the portal will not be evaluated (You get a 0).
8. Uploads without the first page details (including signature) will be awarded 0 marks.

I abide by the Honour code that I have signed on my admission to IIT Delhi. I have neither given any help to anybody nor received any help from anybody in solving the question(s) in this paper.

Signature: **Date:**

4+6=10 marks

Let $\mathbb{T}_\Omega(V)$ be any term algebra such that Ω has at most a countable number of constants.

1. Prove that $\mathbb{T}_\Omega$ is empty if Ω has no constants.
2. Prove that $\mathbb{T}_\Omega(V)$ is countable if Ω is at most countable and has at least one constant and at least one function symbol of arity $n > 0$.

Solution.

1. Since $\mathbb{T}_\Omega$ is free of all variables, the basis of construction is empty. Following (7) (page 118 §-3 of the hyper-notes) we have that $\mathbb{T}_\Omega = A_\infty$. From (8) (page 118 §-3 of the hyper-notes) it follows that the basis $B = A_0 = \emptyset$. By induction for each $k \geq 0$ if $A_k = \emptyset$ then $A_{k+1} = \emptyset$ for each $f \in K$. and hence their union $A_\infty = \bigcup_{i\geq 0} A_i = \emptyset = \mathbb{T}_\Omega$.
2. Since the set of variables V is countable, and Ω is at most countable there cannot be more than a countable numbe rof constant symbols in Ω. Hence the basis $B = A_0 \supseteq V$ is countable. Again it is easy to show since there is at least one function symbol of arity > 0, that each set A_k consisting of construction sequences of length at most k is also countable. By theorem -4.5 (page 89 of the Hyper-notes) the countable union of countable sets is also countable. Hence $A_\infty = \bigcup_{i\geq 0} A_i = \mathbb{T}_\Omega(V)$ is countable.