3. **COL703: Minor-Q3 10:10-10:35, late submission accepted till 10:40, 11 marks**

   Let A be an infinite alphabet and let $S \subseteq A^*$ be a set satisfying the following properties.

   - S is prefix-closed i.e. if $x \in S$ then every prefix of x is also a member of S
   - If $x \in S$ then there are only a finite number of words $xa \in S$, $a \in A$.
   - There is <u>no infinite chain</u> of proper prefixes in S i.e. there is no set $C \subseteq S$ such that $C = \{x_i \in S \mid |x_i| = i \in \mathbb{N}, x_i <_{pre} x_{i+1}\}$.

   Prove that S is finite.