COL703: Logic for Computer Science

Sat 16 Oct 2021 **Quiz 5** 20+5+5 minutes Max marks 10

Instructions:
1. Download the paper.
2. Write your name and entry number in the designated space on top and *do not forget to sign the honour statement below*.
3. Answer the question(s) in the appropriate space provided starting from this page.
4. Scan the paper with your completed answer.
5. Upload it on Gradescope 2001-COL703 page within the given time. *Make sure the first page with your name, entry no and signature is also the first page of your uploaded file*
6. Late submissions (within 2 minutes of submission deadline) on the portal will attract a penalty of 2 marks out of 10.
7. Email submissions after the closing of the portal will not be evaluated (You get a 0).
8. Uploads without the first page details (including signature) will be awarded 0 marks.

I abide by the Honour code that I have signed on my admission to IIT Delhi. I have neither given any help to anybody nor received any help from anybody in solving the question(s) in this paper.

Signature: **Date:**

10 marks

1. Prove the analogue of lemma 24.1 (Alpha conversion) for existential quantification i.e. prove that

   Lemma 0.1 *For every formula ϕ for which $\{y/x\}$ is admissible* $\exists x[\phi] \dashv\vdash_{\mathscr{H}_1} \exists y[\{y/x\}\phi]$

   You are allowed to use the rule $\exists\mathsf{E}$ and any other rule of the $\mathscr{H}_1$ system that is mentioned in the Hypernotes.

2. Combine the proofs above to obtain the derived rule

   $$\exists \equiv_\alpha . \ \frac{}{\exists x[X] \leftrightarrow \exists y[\{y/x\}X]}, \{y/x\} \text{ admissible for } X$$