6. **COL703: Major-Q4 11:30-11:55, late submissions accepted till 12:00, 2+4+4+5 = 15 marks**
   Consider the following argument.

   ϕ_1. *For every set x there is a set y whose cardinality is greater than the cardinality of x.*

   ϕ_2. *If x is contained in y, the cardinality of x is no more than the cardinality of y.*

   ϕ_3. *Every set is contained in the universe.*

   Therefore

   ϕ_4. *The universe is not a set.*

   (a) Define a minimal signature Σ to which the statements of the above argument apply.

   $\Sigma =$

   (b) Provide an intuitively faithful translation of the sentences in the above argument.

   $\phi_1 \equiv$

   $\phi_2 \equiv$

   $\phi_3 \equiv$

   $\phi_4 \equiv$

   (c) Transform each predicate ϕ_i $(1 \leq i \leq 4)$ in the argument into a formula in Skolem Conjunctive Normal Form (SCNF).

   $sko(\phi_1) \equiv$

   $sko(\phi_2) \equiv$

   $sko(\phi_3) \equiv$

   $sko(\phi_4) \equiv$

   (d) Prove the above argument using first-order resolution refutation.