Lecture 15 (GDA)

1 Preliminiaries

$$\prod P(x_i, y_i; \theta) = \prod P(y_i; \theta) P(x_i | y_i; \theta)$$

We assume $x|y$ to be a normal distribution

Covariance matrix is:

$$\Sigma_{ij} = cov(X_i, X_j) = E\left[(X - E[X])(X - E[X])^T\right]$$

Now, we consider X to be normally distributed as $N(\mu_X, \Sigma)$. Also note that Σ will be symmetric and positive semi-definitive

$$P(x = z) = \frac{1}{\sqrt{2\pi|\Sigma|}} \exp\left(-\frac{(X-\mu)^T \Sigma^{-1} (X-\mu)}{2}\right)$$

2 Gaussian Discriminant Analysis

The idea is to generate the contour of $x|y = 0$ and $x|y = 1$ using μ_0, Σ_0 and μ_1, Σ_1 respectively.

$$\Theta = (\phi, \mu_0, \Sigma_0, \mu_1, \Sigma_1)$$

Θ is the set of parameters of our model. Now, $LL(\Theta)$$ is given by

$$LL(\Theta) = \sum_{i=1}^{m} y_i \log(\phi) + (1 - y_i)\log(1 - \phi) + y_i \left(\log \frac{1}{\sqrt{2\pi|\Sigma_0|}} - \frac{(x_i - \mu_0)^T \Sigma_0^{-1} (x_i - \mu_0)}{2}\right)$$

$$+(1 - y_i)\left(\log \frac{1}{\sqrt{2\pi|\Sigma_1|}} - \frac{(x_i - \mu_1)^T \Sigma_1^{-1} (x_i - \mu_1)}{2}\right)$$

Now, $\nabla_\Theta LL(\Theta) = 0$ gives,

$$\phi = \frac{\mu_Y}{m}$$

$$\mu_0 = \frac{\sum_{i=1}^{m} (1 - y_i) x_i}{\sum_{i=1}^{m} 1 - y_i}$$

$$\Sigma_0 = \frac{\sum_{i=1}^{m} (1 - y_i)(x_i - \mu_0)(x_i - mu_0)^T}{\sum_{i=1}^{m} 1 - y_i}$$

$$\mu_1 = \frac{\sum_{i=1}^{m} y_i x_i}{\sum_{i=1}^{m} y_i}$$

$$\Sigma_1 = \frac{\sum_{i=1}^{m} y_i(x_i - \mu_1)(x_i - mu_1)^T}{\sum_{i=1}^{m} y_i}$$