COL774: Assignment 3

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Contents

1 Random Forests

1.1 Grid Search

The best model obtained is for the following parameters:

- **n_estimators**: 450

- **max_features**: 0.3
- **min_samples_split**: 6

1.2 Paramter Sensitivity Analysis

1.2.1 n_estimators

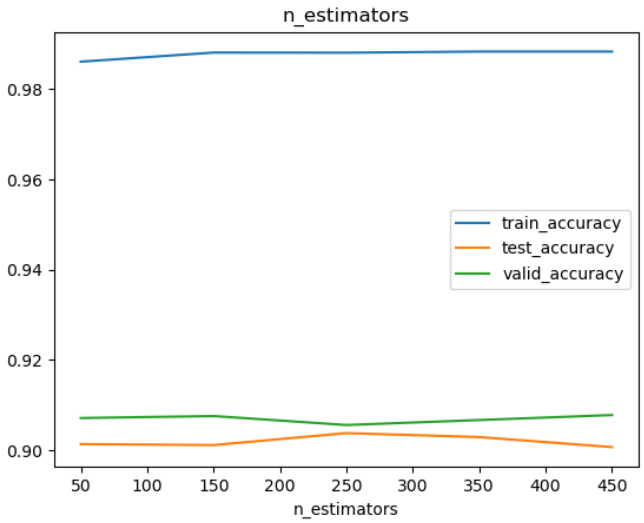


Figure 1: Plot of the effect of n_estimators on the accuracy of the model

The validation accuracy increases on increasing the number of estimators. However the test accuracy drops slightly. However the training accuracy increases with increasing the number of estimators. Therefore, the model overfits with larger number of estimators.

1.2.2 max_features

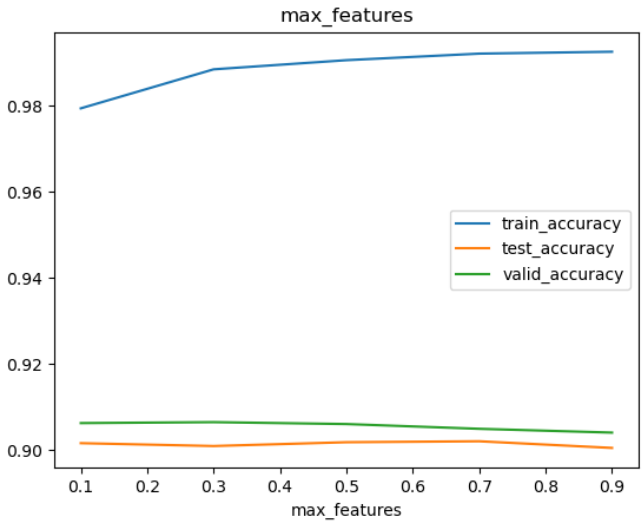


Figure 2: Plot of the effect of max_features on the accuracy of the model

The accuracy decreases with the increase in max_features. However, for small increase, the accuracy improves slightly. Therefore, smaller values of max_features are preferred. The training accuracy increases for larger values of max_features. Therefore, larger values overfit the model.

1.2.3 min_samples_split

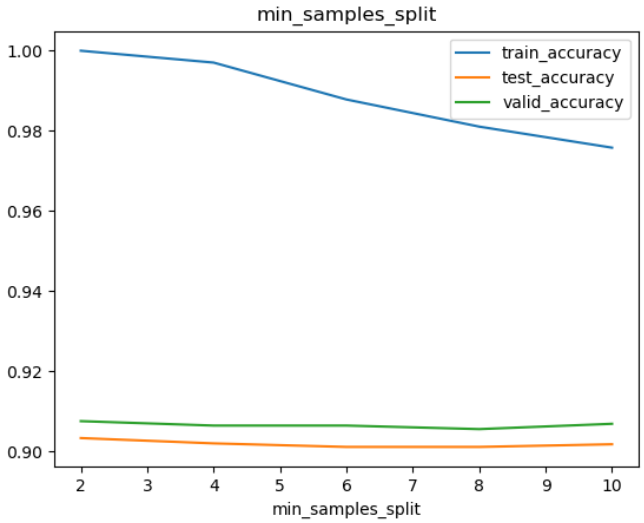


Figure 3: Plot of the effect of min_samples_split on the accuracy of the model

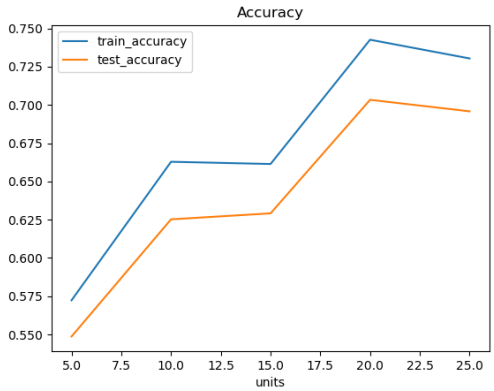
The training accuracy decreases with the increase in min_samples_split. However, the validation and testing accuracy increases with the increase in min_samples_split. Therefore, the model overfits

with smaller values of min_samples_split.

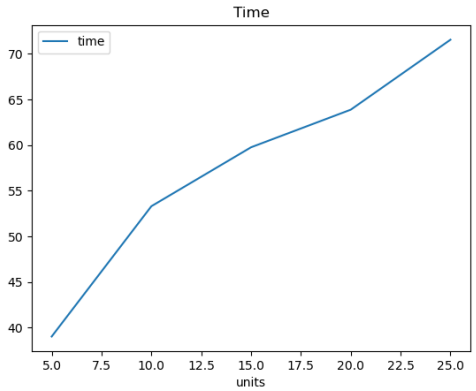
2 Neural Networks

2.1 Sigmoid Activation using a Fixed Learning Rate

The stopping criteria used is to reach a change in the average cost function across two epochs less than 10^{-8} or 1000 epochs, whicheve happens earlier. When learning, it was observed that for the more complicated neural networks, the epoch criteria was met earlier.
The plots and confusion matrices are shown below. It is observed that the model improves when using larger number of perceptrons, however, it slightly drops on using 25 perceptrons. This is because 1000 epochs is too less to converge.
Additionally, we notice that no model learns the classes $2-9$ at all. This is inferred from the zeroes in all the columns other than the first two in the confusion matrix.



(a) Plot of the training and test accuracy



(b) Plot of the training time

2.1.1 5 Hidden Units

$$\text{training accuracy} = 0.5723310675729708$$

$$\text{test accuracy} = 0.548671$$

$$\text{confusion matrix} = \begin{pmatrix} 377431 & 123778 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 251258 & 171240 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 21594 & 26028 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 7397 & 13724 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2007 & 1878 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1746 & 250 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 352 & 1072 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 30 & 200 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{1}$$

$$\text{training time} = 39.02067995071411$$

2.1.2 10 **Hidden Units**

$$\text{training accuracy} = 0.6628548580567772$$

$$\text{test accuracy} = 0.625259$$

$$\text{confusion matrix} = \begin{pmatrix} 400963 & 100246 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 198202 & 224296 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 11667 & 35955 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4343 & 16778 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2129 & 1756 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1747 & 249 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 141 & 1283 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 224 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{2}$$

$$\text{training time} = 53.303688526153564$$

2.1.3 15 **Hidden Units**

$$\text{training accuracy} = 0.6614154338264694$$

$$\text{test accuracy} = 0.629156$$

$$\text{confusion matrix} = \begin{pmatrix} 395803 & 105406 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 189145 & 233353 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 11336 & 36286 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2809 & 18312 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1754 & 2131 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1806 & 190 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 116 & 1308 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 229 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{3}$$

$$\text{training time} = 59.76480436325073$$

2.1.4 20 Hidden Units

$$\text{training accuracy} = 0.7426229508196721$$

$$\text{test accuracy} = 0.703346$$

$$\text{confusion matrix} = \begin{pmatrix} 416637 & 84572 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 135789 & 286709 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3786 & 43836 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 942 & 20179 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2269 & 1616 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1787 & 209 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1423 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 230 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{4}$$

$$\text{training time} = 63.88574767112732$$

2.1.5 25 Hidden Units

$$\text{training accuracy} = 0.7303878448620552$$

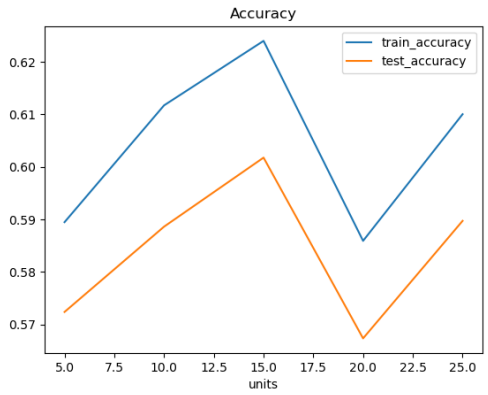
$$\text{test accuracy} = 0.695839$$

$$\text{confusion matrix} = \begin{pmatrix} 435371 & 65838 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 162030 & 260468 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6315 & 41307 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2124 & 18997 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2619 & 1266 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1857 & 139 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 20 & 1404 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 228 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 11 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{5}$$

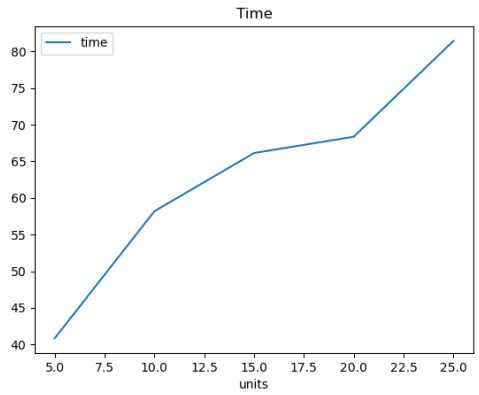
$$\text{training time} = 71.55454468727112$$

2.2 Sigmoid Activation using an Adaptive Learning Rate

The stopping criteria used is the same as the previous section. The plots and confusion matrices are given below. It is observed that the accuracy is higher for 5 units of th hidden layer. However the accuracy is smaller for the other cases. Additionally, the accuracy is lesser for 20 and 25 hidden units compares to $10, 15$ hidden units. This is because using the adaptive learning rate slows down the learning and therefore 1000 epochs is not enough to converge. The training time increases as expected.



(a) Plot of the training and test accuracy



(b) Plot of the training time

2.2.1 5 Hidden Units

$$\text{training accuracy} = 0.589484206317473$$

$$\text{test accuracy} = 0.572393$$

$$\text{confusion matrix} = \begin{pmatrix} 387031 & 114178 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 237136 & 185362 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 19466 & 28156 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5737 & 15384 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1329 & 2556 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1678 & 318 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 251 & 1173 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 18 & 212 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{6}$$

$$\text{training time} = 40.8355929851532$$

2.2.2 10 Hidden Units

$$\text{training accuracy} = 0.6117153138744502$$

$$\text{test accuracy} = 0.588626$$

$$\text{confusion matrix} = \begin{pmatrix} 381880 & 119329 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 215752 & 206746 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 16304 & 31318 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4216 & 16905 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2739 & 1146 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1761 & 235 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 210 & 1214 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 221 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 11 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{7}$$

$$\text{training time} = 58.18630528450012$$

2.2.3 15 Hidden Units

$$\text{training accuracy} = 0.6239904038384646$$

$$\text{test accuracy} = 0.601769$$

$$\text{confusion matrix} = \begin{pmatrix} 385665 & 115544 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 206394 & 216104 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 14607 & 33015 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4136 & 16985 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2545 & 1340 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1792 & 204 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 180 & 1244 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10 & 220 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{8}$$

$$\text{training time} = 66.1306664943695$$

2.2.4 20 Hidden Units

$$\text{training accuracy} = 0.5859256297481008$$

$$\text{test accuracy} = 0.567364$$

$$\text{confusion matrix} = \begin{pmatrix} 392936 & 108273 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 248070 & 174428 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 19182 & 28440 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6580 & 14541 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2806 & 1079 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1809 & 187 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 273 & 1151 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 22 & 208 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{9}$$

$$\text{training time} = 68.35846948623657$$

2.2.5 25 Hidden Units

$$\text{training accuracy} = 0.6100359856057577$$

$$\text{test accuracy} = 0.589734$$

$$\text{confusion matrix} = \begin{pmatrix} 410562 & 90647 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 243326 & 179172 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 19587 & 28035 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4789 & 16332 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3881 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1760 & 236 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 291 & 1133 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 221 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{10}$$

$$\text{training time} = 81.43693113327026$$

2.3 Multiple Hidden Layers: Comparing ReLU and Sigmoid

The same stopping criteria is used for this part as well.

2.3.1 ReLU

$$\text{training accuracy} = 0.9233106757297082$$

$$\text{test accuracy} = 0.922804$$

$$\text{confusion matrix} = \begin{pmatrix} 500667 & 542 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 361 & 422137 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 47622 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 21121 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3721 & 164 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1984 & 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1424 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 230 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{11}$$

$$\text{training time} = 195.58691668510437$$

Even though the model still doesn't learn classes $2 - 9$, it does learn the classes 0 and 1 highly accurately. In the given training data, about 92% of the training data is in class 0 and 1 and the training accuracy is larger than 92%, most of which has contribution from these two classes. Therefore, ReLU is a very good choice for the given problem.

2.3.2 Sigmoid

$$\text{training accuracy} = 0.5003998400639744$$

$$\text{test accuracy} = 0.4974$$

$$\text{confusion matrix} = \begin{pmatrix} 482380 & 18829 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 407478 & 15020 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 46043 & 1579 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 20476 & 645 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3707 & 178 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1879 & 117 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1389 & 35 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 225 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{12}$$

$$\text{training time} = 77.64794874191284$$

The accuracy drops to 50% in this case. The stopping criteria reached in this case was of change in cost function becoming smaller than 10^{-8}. Thus, we can conclude that the sigmoid derivative is not a very good choice since the backpropagation is smaller.

2.4 MLPClassifier (Scikit-Learn Library)

Models were trained for both ReLU and sigmoid activation functions.

2.4.1 ReLU

$$\text{training accuracy} = 0.9999200319872051$$

$$\text{test accuracy} = 0.989527$$

$$\text{confusion matrix} = \begin{pmatrix} 501126 & 5 & 0 & 0 & 75 & 3 & 0 & 0 & 0 & 0 \\ 107 & 422383 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 565 & 706 & 46328 & 23 & 0 & 0 & 0 & 0 & 0 & 0 \\ 924 & 6 & 163 & 20028 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3694 & 1 & 0 & 0 & 190 & 0 & 0 & 0 & 0 & 0 \\ 1929 & 0 & 0 & 0 & 0 & 67 & 0 & 0 & 0 & 0 \\ 442 & 0 & 94 & 365 & 0 & 0 & 521 & 2 & 0 & 0 \\ 0 & 0 & 0 & 228 & 0 & 0 & 2 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 1 & 3 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{13}$$

$$\text{training time} = 9.497213125228882$$

The model almost perfectly learns the traning data and performs very well on the test data. The training also happens almost instantly and many folds faster than our training (our model was infact incomplete since the criteria of 1000 epochs was reached).

2.4.2 Sigmoid

$$\text{training accuracy} = 0.11095561775289885$$

$$\text{test accuracy} = 0.102564$$

$$\text{confusion matrix} = \begin{pmatrix} 378353 & 122856 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 320188 & 102310 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 36359 & 11263 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 16278 & 4843 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2865 & 1020 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1384 & 612 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1096 & 328 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 172 & 58 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{14}$$

$$\text{training time} = 11.053844213485718$$

The accuracy is terrible in this case and much worse than what is learnt by our model. This further proves our claim that ReLU is a much better activation function for the given problem.