Midterm

Introduction to Quantum Information Science
Tuesday, October 23rd

Your Name and EID: ______________________________

1.	/20
2.	/30
a)	/6
b)	/4
c)	/4
d)	/4
e) (Extra Credit)	/5
f)	/6
3.	/25
a)	/3
b)	/2
c)	/3
d)	/2
e)(Extra Credit)	/5
f)	/5
g)	/5
h)	/5
4.	/25
a)	/3
b)	/5
c)	/5
d)	/5
e)	/5
f) (Extra Credit)	/5
Total	/100

1. True or false? Write your answer on the provided line. **[20 Points, 2 Points per Part]**

_____ **a)** The states $(1/\sqrt{2})(|0\rangle + i\,|1\rangle)$ and $1/\sqrt{2}(|0\rangle - i\,|1\rangle)$ are orthogonal.

_____ **b)** If the state ρ_{AB} is entangled between Alice and Bob, then Alice's reduced state is necessarily mixed (i.e., not pure).

_____ **c)** Given ρ_{AB}, if Alice's reduced state is mixed, then she necessarily shares entanglement with Bob.

_____ **d)** Superdense coding requires Alice and Bob to share entanglement.

_____ **e)** Quantum key distribution requires Alice and Bob to share entanglement.

_____ **f)** Quantum teleportation requires Alice and Bob to share entanglement.

_____ **g)** Although perfect cloning is impossible, given an unknown 1-qubit state $|\psi\rangle$, it's possible to produce a state *arbitrarily close* to $|\psi\rangle\,|\psi\rangle$.

_____ **h)** While Alice and Bob can't use entanglement to send instantaneous signals, they *can* instantaneously check whether they've won the CHSH game.

_____ **i)** Every k-qubit mixed state can be written as a mixture of at most 2^k pure states.

_____ **j)** It's possible to "drag" a qubit from the $|+\rangle$ state to the $|-\rangle$ state, with arbitrarily high success probability, purely by making a sequence of two-outcome measurements.

2. [20 Points Total]

a) Which of the following views about quantum mechanics necessarily lead to predictions for experiments that are *different* from the predictions of standard QM? (Here, we're talking only about "standard physics experiments" that one could do in a lab just like one does the double-slit experiment, and whose interpretations don't depend on any beliefs about consciousness or anything of that kind.) **[6 Points]**

_____The Many-Worlds Interpretation
_____The Copenhagen Interpretation
_____GRW (Ghirardi-Rimini-Weber) dynamical collapse
_____Penrose gravitational collapse
_____Local hidden variables
_____Nonlocal hidden variables (including Bohmian mechanics)

For each of the following states, say whether it's separable or entangled. If entangled, briefly justify your answer. If separable, give a factorization or a decomposition that proves separability.

b) [4 Points]

$$\frac{1}{2}(|00\rangle + i\,|01\rangle + i\,|10\rangle - |11\rangle)$$

c) [4 Points]

$$\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle - |11\rangle)$$

d) [**4 Points**]

$$\frac{1}{3}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

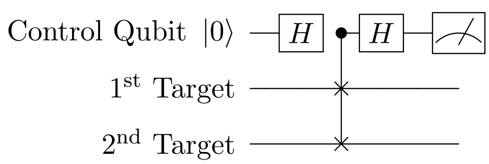
e) [**Extra Credit, 5 Points**]

$$\frac{1}{4}\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}$$

f) Draw a quantum circuit, using standard 1- and 2-qubit gates, that maps the state $|000\rangle$ to the GHZ state $(|000\rangle + |111\rangle)/\sqrt{2}$, while also mapping $|100\rangle$ to $(|000\rangle - |111\rangle)/\sqrt{2}$.[**6 points**]

g) Draw a quantum circuit, again using standard 1- and 2-qubit gates, that tests whether the input state is $(|000\rangle + |111\rangle)/\sqrt{2}$ or $(|000\rangle - |111\rangle)/\sqrt{2}$, promised that one of those is the case. [**6 points**]

3. The Swap Test [**26 Points**] The"swap test" is one of the most important subroutines in quantum algorithms and protocols, used to test whether two quantum states are close or far from one another. Given two one-qubit states (which we call "target qubits"), the swap test is given by the following quantum circuit:



Where the measurement of the control qubit is in the $\{|0\rangle, |1\rangle\}$ basis. Here, the middle gate swaps the two target qubits if and only if the control qubit is $|1\rangle$. We say the test "accepts" if and only if the measurement outcome is $|0\rangle$.

a) Suppose the target qubits are both $|0\rangle$. With what probability does the swap test accept? [**3 Points**]

b) More generally, suppose the target qubits are both $|v\rangle$. With what probability does the swap test accept? [**2 Points**]

c) Suppose one target qubit is $|0\rangle$ and the other is $|1\rangle$. With what probability does the swap test accept? **[3 Points]**

d) More generally, suppose one target qubit is $|v\rangle$ and the other is $|w\rangle$, for any orthogonal $|v\rangle, |w\rangle$. With what probability does the swap test accept? **[2 Points]**

e) Suppose one target qubit is $|v\rangle$ and the other is $|w\rangle$, for some arbitrary $|v\rangle$ and $|w\rangle$. With what probability does the swap test accept, as a function of the inner product $\langle v|w\rangle$? **[Extra Credit, 5 Points]**

f) Here you'll investigate whether your answer to part (b) generalizes to the case where the two identical states being prepared are mixed rather than pure. Suppose the target qubits are both in the maximally mixed state $I/2$ (and not entangled with each other). With what probability does the swap test accept? *[Hint: Remember that the "1-qubit maximally mixed state" is just a fancy term for the uniform distribution over 0 and 1.]* **[5 Points]**

g) Here you'll investigate why the swap test can't be used to counterfeit Wiesner's quantum money, or otherwise violate the No-Cloning Theorem, by simply swapping an unknown state against candidate states like $|0\rangle, |1\rangle, |+\rangle, |-\rangle$ until we find a match. Suppose we run the swap test with $|0\rangle$ and $|1\rangle$ as the target qubits, and suppose it accepts. What then is the post-measurement state of the target qubits? [**5 Points**]

h) A physicist claims that "every photon in the universe is identical to every other one, apart from the known properties in which they differ like position, energy, and polarization." A philosopher replies, "logically, you can never *know* anything of that kind—for who's to say that photons don't have little RFID tags that differentiate them, which you simply haven't discovered yet?" Assuming quantum mechanics is true, how might something analogous to the swap test be used to address this dispute? [**5 Points**]

4. Hardy's Paradox [**25 Points**]

Consider the state $|\psi\rangle = (|00\rangle + |01\rangle + |10\rangle)/\sqrt{3}$, shared by Alice and Bob.

a) Suppose Alice and Bob both measure their qubits in the $\{|0\rangle, |1\rangle\}$ basis. What is the probability that they both see $|1\rangle$? [**3 Points**]

b) Suppose Alice and Bob both measure their qubits in the $\{|+\rangle, |-\rangle\}$ basis. What is the probability that they both see $|-\rangle$? **[5 Points]**

c) Suppose one of the qubits is measured in the $\{|0\rangle, |1\rangle\}$ basis, and the outcome is $|0\rangle$. If the other qubit is now measured in the $\{|+\rangle, |-\rangle\}$ basis, what is the probability that the outcome will be $|-\rangle$? **[5 Points]**

d) Consider the following "paradox." If Alice's qubit is $|0\rangle$, then Bob must see $|+\rangle$. Likewise, if Bob's qubit is $|0\rangle$, then Alice must see $|+\rangle$. Yet at least one of the two qubits must be $|0\rangle$, which means that either Alice or Bob *must* see $|+\rangle$. How can you reconcile this reasoning with your answer from part (b)? **[5 Points]**

e) Calculate Alice's (or equivalently, Bob's) reduced density matrix. **[5 Points]**

f) Calculate the entanglement entropy of $|\psi\rangle$, by extracting the eigenvalues from the relevant matrix. (You don't need to simplify your answer further or numerically estimate it.) [**Extra Credit, 5 Points**]