Introduction to Quantum Information Science — UT Austin Fall 2023
CS358H/PHY341/M375T/ECE379K

Midterm Exam

Due 6:00pm Friday October 20, 2023

Do not download, print, or open the exam until you are ready to take it.

Rules

- You **may not** communicate with any other person about the contents of the exam until after 6pm on Friday. Don't post anything about the exam on Ed or anywhere else. If there are urgent questions about the exam you can email the course instructor (nickrhj@utexas.edu).
- **The time limit is 2 hours**. Once you open the pdf and start looking at the problems, your allotted 2 hours begins. The exam must be completed in a **single sitting**.
- During the exam, you may **only** refer to your personal course notes, the course lectures notes on canvas, your HWs for the class, the HW solutions on canvas, and the QIS lecture notes or other textbooks mentioned in the syllabus. You **may not** refer to any other sources of information, specifically, you may not use the internet, mathematical software, other references online, etc.
- You must adhere to all of these instructions regarding this exam. Any violation constitutes an act of academic dishonesty. If you are unclear about any of these instructions, please immediately ask for clarification.

Submission

Please submit the exam by uploading to gradescope before **6pm on Friday**.

Please let me know if there are any questions. Otherwise, good luck!

The exam starts on the next page.
Once you scroll/read past here, your 2 hours begin.

0) Rules. Make sure you read the rules.

1) Short conceptual questions. (14pts, 2pts each)
No need for lengthy calculations or detailed explanations, simply answer yes or no.

a) Does there exist an experiment that can measure *global* phases of quantum states? (Yes or no)

b) Is it possible to construct an EPR state from the $|00\rangle$ state with a 2-qubit circuit consisting *only* of single qubit gates? (Yes or no)

c) Can every two-outcome measurement (e.g. measurement of a qubit) be simulated by a unitary transformation and then a measurement in the computational ($\{|0\rangle, |1\rangle\}$) basis? (Yes or no)

d) Can a linear combination of entangled states also be a product state? (Yes or no)

e) If Alice and Bob share an EPR pair, then by simply measuring her qubit, can Alice instantaneously transmit information to Bob? (Yes or no)

f) In Elitzur-Vaidman bomb testing, if we were to measure our qubit in the computational basis after each step (i.e. upon getting it back from the package), would we still be able to successfully detect the bomb (distinguishing the identity from a measurement)? (Yes or no)

g) In Wiesner's quantum money scheme, if the bank only permits you to *successfully* verify your quantum notes, i.e. doesn't return bills which fail their test, then is the scheme still secure? (Yes or no)

Slightly longer questions. (12pts, 4pts each)

h) The $|+\rangle$ state is uniform over measurement outcomes when measured in the computational basis, i.e. it gives 0 and 1 with equal probability. Write down a single-qubit state which gives uniformly random outcomes when measured in *either* the Hadamard basis or the computational basis.

i) Which of the following three protocols requires Alice and Bob to share entanglement: (choose all that apply)
BB84 quantum key distribution, superdense coding, quantum teleportation.

j) Consider the following two-qubit states:

$$\frac{1}{\sqrt{2}}\big(\,|00\rangle - i\,|11\rangle\,\big) \qquad \text{and} \qquad \frac{1}{2}\big(\,|00\rangle + |01\rangle - |10\rangle - |11\rangle\,\big)$$

Are either of them entangled states? How can you tell?

2) Sequential measurements. (14pts)

Consider two orthonormal qubit bases, the α-basis spanned by basis states $|\alpha_1\rangle$ and $|\alpha_2\rangle$, and the β-basis spanned by basis states $|\beta_1\rangle$ and $|\beta_2\rangle$. It turns out these basis states of these two bases are related as

$$|\alpha_1\rangle = \frac{1}{\sqrt{5}}\big(|\beta_1\rangle + 2|\beta_2\rangle\big) \quad \text{and} \quad |\alpha_2\rangle = \frac{1}{\sqrt{5}}\big(2|\beta_1\rangle - |\beta_2\rangle\big)\,.$$

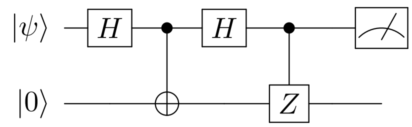
a) (2pts) Say we start out in some unknown qubit state $|\phi\rangle$, we then measure in the β-basis and obtain the measurement outcome β_2. What is the state of the system after measurement?

b) (4pts) If our qubit starts out in the basis state $|\beta_1\rangle$, and we measure in the α-basis, what is the probability of getting a measurement outcome of α_1?

c) (8pts) Again, assume our qubit starts out in $|\beta_1\rangle$. If we measure in the α-basis and then we measure in the β-basis, what is the probability of obtaining the value β_1 after the second measurement? (Hint: you should account for each possible outcome in the first measurement)

3) Two-qubit quantum circuits. (24pts)

Let $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$ be a single qubit state. Consider the following circuit



where H is the Hadamard gate, the first two-qubit gate is the CNOT gate, and the last two-qubit gate is the controlled-Z gate (both controlled on the first qubit). The measurement is taken in the computational basis.

a) (12pts) What is the state of the two qubits right before the measurement? (Feel free to use circuit identities from HW3 without proof if you'd like, but you don't have to)

b) (8pts) After the measurement of the first (i.e. top-most) qubit, what is the state $|\phi\rangle$ of the second qubit? Does the state $|\phi\rangle$ depend on the measurement outcome?

c) (4pts) Is there a single-qubit operation which relates $|\phi\rangle$ (the final state of the second qubit) and the state $|\psi\rangle$?

4) EPR states. (20pts)

Consider the following two-qubit EPR state

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}\big(\,|01\rangle - |10\rangle\,\big)\,.$$

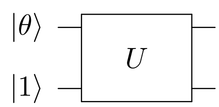
a) (2pts) If the first qubit is measured in the computational basis, and we observe 0, what is the state of the second qubit?

b) (8pts) Consider the following two single-qubit basis states

$$|\theta\rangle = \cos(\theta)\,|0\rangle + \sin(\theta)\,|1\rangle \quad \text{and} \quad |\theta'\rangle = -\sin(\theta)\,|0\rangle + \cos(\theta)\,|1\rangle$$

where θ is an angle ($\theta \in \mathbb{R}$) and $\{|\theta\rangle\,,|\theta'\rangle\}$ is just a rotated basis. Given the 2-qubit EPR state $|\Psi^-\rangle$, if we measure both qubits in the $\{|\theta\rangle\,,|\theta'\rangle\}$ basis, what is the probability that we observe θ for the first qubit and θ' for the second qubit?

c) (10pts) Write down a two-qubit quantum circuit which constructs the $|\Psi^-\rangle$ EPR state starting from the $|\theta\rangle \otimes |1\rangle$ state, with $|\theta\rangle$ defined in part **b)**, using the CNOT gate and single qubit gates. Such a circuit should look like



where the 2-qubit unitary U is a circuit built from a CNOT gate and single qubit gates and the final state of the circuit is the EPR state $|\Psi^-\rangle$.
(Hint: remember we already know how to build certain EPR states from the $|00\rangle$ state)