# Variants of Differential Privacy

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## Basic Definitions

The following all admit the Gaussian mechanism with the specified noise variance $\sigma^2$.

| Definition | Gaussian mech. | Seq. Comp. | Advanced comp. | Conv. to $(\epsilon, \delta)$-DP |
|---|---|---|---|---|
| $(\epsilon, \delta)$-DP | $\sigma^2 = \frac{2\Delta^2 \log(1.25/\delta)}{\epsilon^2}$ | $(\epsilon_1 + \epsilon_2, \delta_1 + \delta_2)$ | $(2\epsilon\sqrt{2k\log(1/\delta')}, k\delta + \delta')$ | n/a |
| Moments acct. | (same as DP) | (same as DP) | $(4\epsilon\sqrt{2k\log(1/\delta)}, \delta)$ | n/a |
| $(\alpha, \epsilon)$-RDP | $\sigma^2 = \frac{\Delta^2\alpha}{(2\epsilon)}$ | $(\alpha, \epsilon_1 + \epsilon_2)$ | n/a | $(\epsilon + \frac{\log(1/\delta)}{\alpha-1}, \delta)$ |
| $\rho$-zCDP | $\sigma^2 = \frac{\Delta^2}{(2\rho)}$ | $\rho_1 + \rho_2$ | n/a | $(\rho + 2\sqrt{\rho\log(1/\delta)}, \delta)$ |

## tCDP

The *arsinh* mechanism for a query $q$ with $L_2$ sensitivity $\Delta$ provides $(16\rho, \frac{A}{8\Delta})$-tCDP:

$$M(x) \leftarrow q(x) + A\,\mathrm{arsinh}(\frac{1}{A}\,\mathcal{N}(\frac{\Delta^2}{2\rho}))$$

where $\mathrm{arsinh}(x) = \log(x + \sqrt{x^2+1})$.

## Amplification by Subsampling

Here, we uniformly sample a size-$n$ subset of a size-$N$ dataset. We let $s = n/N$.

| Definition | Sampling bound |
|---|---|
| $(\epsilon, \delta)$-DP | $(\log(1 + s(e^\epsilon - 1)), s\delta)$-DP |
| $(\alpha, \epsilon)$-RDP | see below |
| $\rho$-zCDP | N/A |
| $(\rho, \omega)$-tCDP | $(13s^2\rho, \frac{\log(1/s)}{4\rho})$-tCDP |

For $(\alpha, \epsilon(\alpha))$-RDP, when the Gaussian mechanism is used, a not-quite-tight bound is $(\alpha, \epsilon'(\alpha))$, where:

$$\epsilon'(\alpha) = \frac{1}{\alpha - 1}\log\left(1 + \sum_{j=2}^{\alpha} 2s^j\binom{\alpha}{j}e^{(j-1)\epsilon(j)}\right)$$

The tight bound is available in Wang et al. (2018).