Discrete Mathematics Quiz 1

2025-4-21

Name ______________ Student Number ________________ 序号 ______________

by 5dbwat4

1. ***(35%)*** Determine whether the following statements are true or false.
   (**5 points** for a correct answer, **0 points** for a blank answer, **-2 points for an incorrect answer**)
   a) If x is not occurring in A, then $\exists x(P(x) \to A) \equiv \forall x P(x) \to A$. ( )
   b) If A, B, and C are sets, then $A - (B \cap C) = (A - B) \cup (A - C)$. ( )
   c) If n is integer, then $n = \left\lceil \frac{n}{2} \right\rceil + \left\lfloor \frac{n}{2} \right\rfloor$. ( )
   d) Suppose $P(x, y)$ is a predicate and the universe for the variables x and y is $\{1,2,3,4\}$. Suppose $P(1,3)$, $P(2,1)$, $P(2,4)$, $P(3,2)$, $P(3,4)$, $P(4,1)$, $P(4,4)$ are true, and $P(x, y)$ is false otherwise. Then the statement $\forall y \exists x\big((x \le y) \land P(x, y)\big)$ is true. ( )
   e) $n^{0.01}$ is $O(\log_{1.01} n)^{99999}$. ( )
   f) The set of positive real numbers less than 1 with decimal representations consisting only of 0s and 1s is countable. ( )
   g) $2025^{2026} \equiv 1 \pmod{2027}$. ( )

2. ***(12%)*** Write a proposition equivalent to $p \oplus q$,
   a) using only p, q, $\neg$, and the connective $\land$.
   b) using only p, q, and the connective $|$.("$|$" represents NAND 与非.)

3. ***(9%)*** Find the full conjunctive normal form of $(p \oplus q) \lor r$.

4. ***(8%)*** Build all the functions from $A = \{1,2\}$ to $B = \{a, b\}$ and point out which is bijection, and which is surjection.

5. ***(9%)*** If all the positive integers that are relatively prime with 77 are arranged into a strictly increasing sequence, find the 600*th* term of this sequence.

6. ***(9%)*** Use the construction in the proof of the Chinese remainder theorem to find all solutions to the system of congruences $x \equiv 1 \pmod 3$, $x \equiv 2 \pmod 5$, and $x \equiv 3 \pmod 8$.

7. ***(9%)*** Prove that the distributive law $A_1 \cup (A_2 \cap \cdots \cap A_n) = (A_1 \cup A_2) \cap \cdots \cap (A_1 \cup A_n)$ is true for all $n > 2$.

8. ***(9%)*** Prove that every positive integer ($n > 2$) can be expressed as the sum of different Fibonacci numbers.