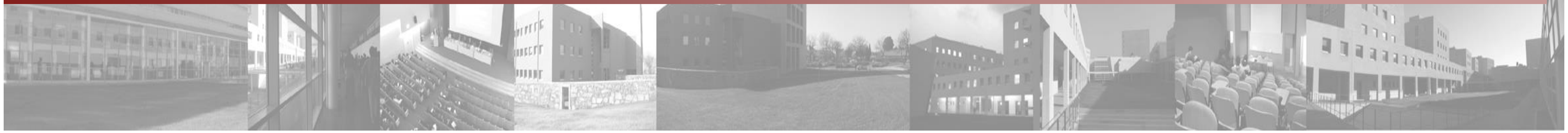

SISTEMAS ROBÓTICOS AUTÓNOMOS

Mestrado em Engenharia Eletrotécnica e de Computadores de Faculdade de Engenharia da Universidade do Porto

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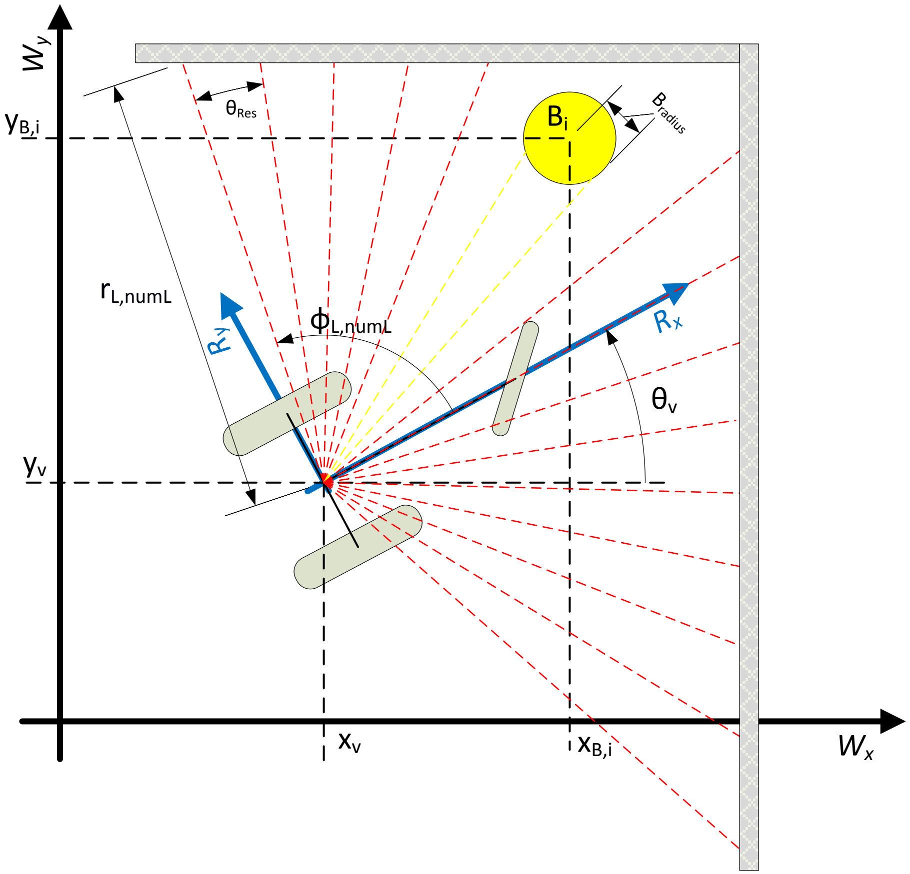
EKF LOCALIZATION – REFLECTIVE BEACONS

Observations– distance, angle and reflection rate in reflective beacons:

$$Z_L(k) = \left\{ Z_{L,i}(k) = \left[r_{L,i}(k) \quad \phi_{L,i}(k) \right]^T, c_{L,i}(k) : i \in [1 \quad numL] \right\}$$



EKF LOCALIZATION – REFLECTIVE BEACONS

State transition model (motion model):

$$X_v(k+1) = \begin{bmatrix} x_v(k+1) \\ y_v(k+1) \\ \theta_v(k+1) \end{bmatrix} = f\left(X_v(k), u(k)\right) + N\left(0, Q(k)\right)$$

with:

$$f\left(X_v(k), u(k)\right) = \begin{bmatrix} x_v(k) + \Delta d(k)\cos\left(\theta(k) + \dfrac{\Delta\theta(k)}{2}\right) \\ y_v(k) + \Delta d(k)\sin\left(\theta(k) + \dfrac{\Delta\theta(k)}{2}\right) \\ \theta(k) + \Delta\theta(k) \end{bmatrix}$$

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The Jacobian matrix of f(.) with respect to the state, will be:

$$\nabla f_x\left(X_v(k), u(k)\right) = \begin{bmatrix} \dfrac{\partial x_v(k+1)}{\partial x_v(k)} & \dfrac{\partial x_v(k+1)}{\partial y_v(k)} & \dfrac{\partial x_v(k+1)}{\partial \theta_v(k)} \\ \dfrac{\partial y_v(k+1)}{\partial x_v(k)} & \dfrac{\partial y_v(k+1)}{\partial y_v(k)} & \dfrac{\partial y_v(k+1)}{\partial \theta_v(k)} \\ \dfrac{\partial \theta_v(k+1)}{\partial x_v(k)} & \dfrac{\partial \theta_v(k+1)}{\partial y_v(k)} & \dfrac{\partial \theta_v(k+1)}{\partial \theta_v(k)} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\Delta d(k)\sin\left(\theta_v(k) + \dfrac{\Delta\theta(k)}{2}\right) \\ 0 & 1 & \Delta d(k)\cos\left(\theta_v(k) + \dfrac{\Delta\theta(k)}{2}\right) \\ 0 & 0 & 1 \end{bmatrix}$$

EKF LOCALIZATION – REFLECTIVE BEACONS

Assuming that the variances of the noise associated with the inputs of the state transition model:

$$u(k) = \begin{bmatrix} \Delta d(k) & \Delta\theta(k) \end{bmatrix}^T$$

respectively are given by (there are other possibilities):

$$\left(\sigma_{\min,\Delta d} + \alpha_1 \Delta d(k) + \alpha_2 \Delta\theta(k)\right)^2$$

and

$$\left(\sigma_{\min,\Delta\theta} + \alpha_3 \Delta d(k) + \alpha_4 \Delta\theta(k)\right)^2$$

EKF LOCALIZATION – REFLECTIVE BEACONS

The Jacobian matrix of f(.) with respect to the inputs, will be:

$$\nabla f_u\left(X_v(k),u(k)\right)=\begin{bmatrix}\dfrac{\partial x_v(k+1)}{\partial \Delta d(k)} & \dfrac{\partial x_v(k+1)}{\partial \Delta\theta(k)}\\[2ex] \dfrac{\partial y_v(k+1)}{\partial \Delta d(k)} & \dfrac{\partial y_v(k+1)}{\partial \Delta\theta(k)}\\[2ex] \dfrac{\partial \theta_v(k+1)}{\partial \Delta d(k)} & \dfrac{\partial \theta_v(k+1)}{\partial \Delta\theta(k)}\end{bmatrix}=\begin{bmatrix}\cos\left(\theta_v(k)+\dfrac{\Delta\theta(k)}{2}\right) & -\dfrac{1}{2}\Delta d(k)\sin\left(\theta_v(k)+\dfrac{\Delta\theta(k)}{2}\right)\\[2ex] \sin\left(\theta_v(k)+\dfrac{\Delta\theta(k)}{2}\right) & \dfrac{1}{2}\Delta d(k)\cos\left(\theta_v(k)+\dfrac{\Delta\theta(k)}{2}\right)\\[2ex] 0 & 1\end{bmatrix}$$

And the covariance matrix Q(k):

$$Q(k)=\nabla f_u\left(X_v(k),u(k)\right)*\begin{bmatrix}\left(\sigma_{\min,\Delta d}+\alpha_1\Delta d(k)+\alpha_2\Delta\theta(k)\right)^2 & 0\\ 0 & \left(\sigma_{\min,\Delta\theta}+\alpha_3\Delta d(k)+\alpha_4\Delta\theta(k)\right)^2\end{bmatrix}*\ \nabla f_u^{\,T}\left(X_v(k),u(k)\right)$$

EKF LOCALIZATION – REFLECTIVE BEACONS

The prediction of the next state given the current estimate of the state and the values for the model inputs, will be:

$$\widehat{X}(k+1|k) = f\left(\widehat{X}(k|k), u(k)\right)$$

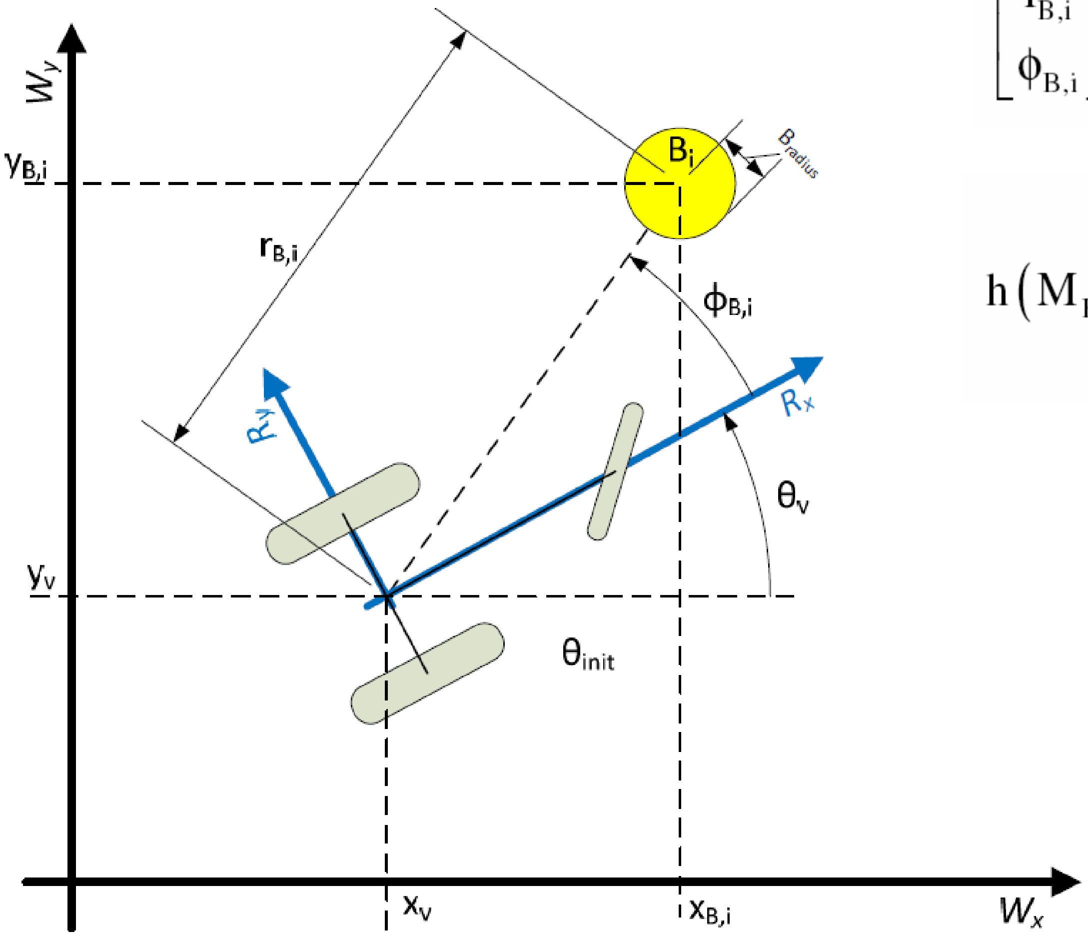
And the uncertainty of this estimate given by the covariance matrix:

$$P(k+1|k) = \nabla f_X\left(\widehat{X}(k|k), u(k)\right) * P(k\,|\,k) * \nabla f_X{}^T\left(\widehat{X}(k|k), u(k)\right) + Q(k)$$

The Jacobians are calculated on present estimates of the state and present values of inputs.

EKF LOCALIZATION – REFLECTIVE BEACONS

Observation model:



$$\begin{bmatrix} r_{B,i} \\ \phi_{B,i} \end{bmatrix} = h\left(M_{B,i}, X_v(k)\right) + N(0,R)$$

$$h\left(M_{B,i}, X_v(k)\right) = \begin{bmatrix} \sqrt{\left(x_{B,i} - x_v\right)^2 + \left(y_{B,i} - y_v\right)^2} \\ a\tan 2(y_{B,i} - y_v, x_{B,i} - x_v) - \theta_v(k) \end{bmatrix}$$

Measurements noise covariance:

$$R = \begin{bmatrix} \sigma_r^{\,2} & 0 \\ 0 & \sigma_\phi^{\,2} \end{bmatrix}$$

EKF LOCALIZATION – REFLECTIVE BEACONS

Jacobian matrix of funtion h(.):

$$
\nabla h\left(M_{B,i}, X_v(k)\right) = \begin{bmatrix} \dfrac{\partial r_i}{\partial x_v(k)} & \dfrac{\partial r_i}{\partial y_v(k)} & \dfrac{\partial r_i}{\partial \theta_v(k)} \\[2ex] \dfrac{\partial \phi}{\partial x_v(k)} & \dfrac{\partial \phi}{\partial y_v(k)} & \dfrac{\partial \phi}{\partial \theta_v(k)} \end{bmatrix}
$$

$$
= \begin{bmatrix} -\dfrac{x_{B,i} - x_v(k)}{\sqrt{\left(x_{B,i} - x_v(k)\right)^2 + \left(y_{B,i} - y_v(k)\right)^2}} & -\dfrac{y_{B,i} - y_v(k)}{\sqrt{\left(x_{B,i} - x_v(k)\right)^2 + \left(y_{B,i} - y_v(k)\right)^2}} & 0 \\[3ex] \dfrac{y_{B,i} - y_v(k)}{\left(x_{B,i} - x_v(k)\right)^2 + \left(y_{B,i} - y_v(k)\right)^2} & -\dfrac{x_{B,i} - x_v(k)}{\left(x_{B,i} - x_v(k)\right)^2 + \left(y_{B,i} - y_v(k)\right)^2} & -1 \end{bmatrix}
$$

EKF LOCALIZATION – REFLECTIVE BEACONS

Correction phase taking into account the observations

Estimation of measurements taking into account the current estimation of the state and the detected beacon:

$$\hat{Z}_{B,i}(k) = h\left(M_{B,i}, \hat{X}\left(k+1|k\right)\right)$$

with covariance:

$$S_i(k) = \nabla h\left(M_{B,i}, \hat{X}\left(k+1|k\right)\right) * P\left(k+1|k\right) * \left[\nabla h\left(M_{B,i}, \hat{X}\left(k+1|k\right)\right)\right]^T + R$$

EKF LOCALIZATION – REFLECTIVE BEACONS

Correction phase taking into account the observations

Kalman Filter gain:

$$K_i(k) = P(k+1|k) * \left[\nabla h\left(M_{B,i}, \hat{X}(k+1|k)\right)\right]^T * \left[S_i(k)\right]^{-1}$$

State correction:

$$\widehat{X}(k+1|k+1) = \widehat{X}(k+1|k) + K_i(k) * \left[Z_{B,i}(k) - \hat{Z}_{B,i}(k)\right]$$

Update of the covariance of the estimated state:

$$P(k+1|k+1) = \left[I - K_i(k) * \nabla h\left(M_{B,i}, \widehat{X}(k+1|k)\right)\right] * P(k+1|k)$$

EKF LOCALIZATION – REFLECTIVE BEACONS

Correction step (*update*) with variable number of observations

The state estimation and its covariance are updated in the prediction step:

$$\widehat{X}(k+1 \mid k+1) = \widehat{X}(k+1 \mid k) \text{ e } P(k+1 \mid k+1) = P(k+1 \mid k)$$

For each observation the estimation of the state and its covariance are updated repeatedly in accordance with the following equations:

$$S_i(k) = \nabla h\left(M_{B,i}, \hat{X}(k+1|k+1)\right) * P(k+1|k+1) * \left[\nabla h\left(M_{B,i}, \hat{X}(k+1|k+1)\right)\right]^T + R$$

$$K_i(k) = P(k+1|k+1) * \left[\nabla h\left(M_{B,i}, \hat{X}(k+1|k+1)\right)\right]^T * \left[S_i(k)\right]^{-1}$$

$$\widehat{X}(k+1 \mid k+1) = \widehat{X}(k+1 \mid k+1) + K_i(k) * \left[Z_{B,i} - \widehat{Z}_{B,i}(k)\right]$$

$$P(k+1 \mid k+1) = \left[I - K_i(k) * \nabla h\left(B_{j=C_{B,i}(k)}, \widehat{X}(k+1|k+1)\right)\right] * P(k+1 \mid k+1)$$

EKF LOCALIZATION – REFLECTIVE BEACONS

VIDEOS:

PRODUTECH project demonstration at ADIRA:

https://youtu.be/aUM5B6sZQVk

https://youtu.be/oNGMu-gQqeA