Ics assignment 7

Basic things to remember-

The mathematical foundation of a CPU is Boolean algebra.

- Boolean logic is also the foundation of systems that behave as if they were intelligent, i.e., systems that exhibit artificial intelligence. Logic (not just Boolean logic) is a sub-field of artificial intelligence that enables programs to reason about their environment, solve planning problems, or provide powerful search facilities on large amounts of formalized knowledge.
- Boolean variables can either be True or False and are stored as 16-bit (2-byte) values. A Boolean function is a function that operates on binary values (typically denoted as 0 and 1) and returns binary values. These functions form the basis of logical operations in digital circuits and computing. To understand Boolean functions, let's break it down step by step:

  1. Boolean Variables:
    - These are the simplest form of Boolean functions. They are typically denoted by letters like A, B, C, ... etc.
    - A Boolean variable can only take two values: 0 or 1.

  2. Basic Boolean Operations:
    - **NOT (¬)**: This operation inverts the value of a Boolean variable.
      - (¬A) is 1 if A is 0 and 0 if A is 1.
    - **AND (∧): The result is 1 if and only if both inputs are 1.
      - \(A ∧ B \) is 1 only if both A and B are 1.
    - **OR (∨)**: The result is 1 if at least one of the inputs is 1.
      - \(A ∨ B \) is 1 if A is 1 or B is 1 or both are 1.

  3. Truth Tables:
    - A truth table lists all possible values of the input variables and the corresponding results of the function.
    - It's a comprehensive way to describe a Boolean function. For instance, the truth table for AND operation is:

A	B	A∧B
0	0	0
0	1	0
1	0	0
1	1	1

  4. Combining Boolean Operations:
    - You can form more complex Boolean functions by combining basic operations.
    - Example: (F(A, B) = A ∧ (¬B). This function outputs 1 only if A is 1 and B is 0.

  5. Other Common Operations:
    - NOR: The output is 1 if and only if both inputs are 0.
    - NAND: The output is 1 unless both inputs are 1.

  - XOR (Exclusive OR): The output is 1 if exactly one of the inputs is 1.
  - XNOR Exclusive NOR*: The output is 1 if both inputs are equal.

6. Properties:
  - Boolean functions have various properties rooted in logic, such as:
    - Commutative: ($A \land B = B \land A$) and ($A \lor B = B \lor A$)
    - Associative: ($A \land (B \land C) = (A \land B) \land C$ \) and ($A \lor (B \lor C) = (A \lor B) \lor C$)
    - Distributive: ($A \land (B \lor C) = (A \land B) \lor (A \land C)$ \) and vice versa.

7. Applications:
  - Boolean functions are fundamental in designing digital circuits, computer algorithms, and many aspects of computer science and engineering.
  - They serve as the building blocks for logical gates in electronics, which in turn form more complex circuits and components like adders, multipliers, memory units, and processors.

Problem 7.1

1. Implementing AND ($\land$) using NOR ($\overline{\lor}$):

The AND function returns TRUE (1) only when both inputs are TRUE (1). We can implement this using NOR as follows:

Explanation:
The AND function returns TRUE (1) only when both inputs are TRUE (1). We can use NOR ($\overline{\lor}$) to implement AND by taking the NOR of the two inputs and then negating the result. When we NOR two inputs and then NOR the result with itself ($A \overline{\lor} B \overline{\lor}$), we obtain the behaviour of AND, as it produces 1 only when both A and B are 1.

Problem 7.2

Given:
$F(X, Y, Z) = (((X \land Y) \lor (X \land \neg Z)) \lor (Z \land \neg 0))$

simplify this:
$G(X, Y, Z) = (X \lor Z)$

simplification:
Step 1: Apply Identity Law
$Z \land \neg 0 = Z$
because $A \land 1 = A$
$F(X, Y, Z) = ((X \land Y) \lor (X \land \neg Z)) \lor Z$
Step 2: Distributive Law
We'll factor out the common term X:
$F(X, Y, Z) = X \land (Y \lor \neg Z) \lor Z$
Step 3: Absorption Law
Using the fact that $A \land \neg B \lor A = A$ (where A = X and B = Z in our equation):

$F(X, Y, Z) = X \lor Z$

Thus, we've derived:
$G(X, Y, Z) = X \lor Z$

In summary:
1. Used Identity Law to simplify $Z \land \neg 0$ to Z.
2. Used the Distributive Law to factor out the common term X.
3. Applied the Absorption Law to simplify to $X \lor Z$.