CSC-257
Theory Of Computation
(BSc CSIT, TU)

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Context Free Grammar(CFG)

- Context free grammar is a formal grammar which is used to generate all possible strings in a given formal language.
- A context free grammar is defined by 4-tuples (V, T, P, S) where

  V = set of variables

  T = set of terminal symbols

  P = set of rules and productions

  S = start symbol and S ∈ V

  Note : Variables are always represented in capital letters

- Eg.

  $S \rightarrow \varepsilon$, $S \rightarrow 0S1$

  Where V = { S }, T = {ε, 0, 1 }, P = {$S \rightarrow \varepsilon$, $S \rightarrow 0S1$ } and S = { S }

- This is a CFG defining the grammar of all the strings with equal no of 0's followed by equal no of 1's.

CFG Vs RE

- The CFG are more powerful than the regular expressions as they have more expressive power than the regular expression.
- Generally, regular expressions are useful for describing the structure of lexical constructs as identical keywords, constants etc. But they do not have the capability to specify the recursive structure of the programming constructs.
- However, the CFG are capable to define any of the recursive structure also. Thus, CFG can define the languages that are regular as well as those languages that are not regular.

Context Free Grammar Notation

- CFG representing the language over Σ = {a, b} which is palindrome language

  $S \rightarrow \varepsilon$

  $S \rightarrow a$

  $S \rightarrow b$

  $S \rightarrow aSa$

  $S \rightarrow bSb$

- This language can also be written in compact notation as :

  $S \rightarrow \varepsilon \mid a \mid b \mid aSa \mid bSb$

Meaning of context free

- Consider an example

  $S \rightarrow aMb$

  $M \rightarrow A \mid B$

  $A \rightarrow \varepsilon \mid aA$

  $B \rightarrow \varepsilon \mid bB$

  where $V = \{ S, M, A, B \}$, $T = \{ \varepsilon, a, b \}$, $P = \{S \rightarrow aMb, M \rightarrow A \mid B, A \rightarrow \varepsilon \mid aA, B \rightarrow \varepsilon \mid bB \}$ and $S = \{ S \}$

- A formal grammar is considered as "context free" when its production rules can be applied regardless of the context of a nonterminal.
- No matter which symbols surround it, the single nonterminal on the left hand side can always be replaced by the right hand side

Examples of CFG

- Construct the CFG for the language having any number of a's over the set ∑= {a}.

  Grammar is

  $S \rightarrow aS,\ S \rightarrow \varepsilon$

- Construct a CFG for the regular expression (0+1)*

  Grammar is

  $S \rightarrow 0S \mid 1S,\ S \rightarrow \varepsilon$

- Construct a CFG for the language $L = a^n b^{2n}$ where n>=1

  The string that can be generated for a given language is {abb, aabbbb, aaabbbbbb....}

  Grammar is

  $S \rightarrow aSbb \mid abb$