



# CSC-257

# Theory Of Computation

(BSc CSIT, TU)

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### 3. Eliminating Unit Production

- A unit production is a production of the form  $A \rightarrow B$ , where  $A$  and  $B$  are both variables(Non terminals)
- Here, if  $A \rightarrow B$ , we say  $B$  is  $A$ -derivable and  $B \rightarrow C$ , we say  $C$  is  $B$ -derivable
- Thus if both of two  $A \rightarrow B$  and  $B \rightarrow C$ , then  $A \rightarrow^* C$ , hence  $C$  is also  $A$ -derivable
- Here pairs  $(A, B)$ ,  $(B, C)$  and  $(A, C)$  are called the unit pairs.
- To eliminate the unit productions, first find all of the unit pairs
- $(A, A)$  is a unit pair for any variable  $A$  as  $A \rightarrow^* A$
- If we have  $A \rightarrow B$  then  $(A, B)$  is unit pair
- If  $(A, B)$  is unit pair i.e.  $A \rightarrow B$ , and if we have  $B \rightarrow C$  then  $(A, C)$  is also a unit pair

### 3. Eliminating Unit Production

- Suppose a grammar  $G = (V, T, P, S)$
- To eliminate unit productions from  $G$ , we have to find another grammar  $G' = (V, T, P', S)$  with no unit productions. For this, we may workout as below:
  - Initialize  $P' = P$
  - For each  $A \in V$ , find a set of  $A$ -derivable variables
  - For every pair  $(A, B)$  such that  $B$  is  $A$ -derivable and for every non-unit production  $B \rightarrow \alpha$ , we add production  $A \rightarrow \alpha$  in  $P'$  if it is not in  $P'$  already
  - Delete all unit productions from  $P'$

### 3. Eliminating Unit Production

- Example 1: Remove the unit production for grammar G defined by productions

$$P = \{ \begin{array}{l} S \rightarrow S+T \mid T \\ T \rightarrow T*F \mid F \\ F \rightarrow (S) \mid a \end{array} \}$$

- Solution : Initialize

$$P' = \{ \begin{array}{l} S \rightarrow S+T \mid T \\ T \rightarrow T*F \mid F \\ F \rightarrow (S) \mid a \end{array} \}$$

### 3. Eliminating Unit Production

- Now, find unit pairs as :

Here,  $S \rightarrow T$  So,  $(S, T)$  is unit pair

$T \rightarrow F$  So,  $(T, F)$  is unit pair

Also,  $S \rightarrow T$  and  $T \rightarrow F$  So,  $(S, F)$  is unit pair

- Now, add each non-unit productions of the form  $B \rightarrow \alpha$  in  $P'$  for each pair  $(A, B)$  as :

$P' = \{$

$S \rightarrow S+T \mid T * F \mid (S) \mid a \mid T$

$T \rightarrow T * F \mid (S) \mid a \mid F$

$F \rightarrow (S) \mid a$

$\}$

### 3. Eliminating Unit Production

- Now delete unit productions of the form  $A \rightarrow B$  from  $P'$  and final production will be as :

$$P' = \{ \begin{array}{l} S \rightarrow S+T \mid T * F \mid (S) \mid a \\ T \rightarrow T * F \mid (S) \mid a \\ F \rightarrow (S) \mid a \end{array} \}$$

### 3. Eliminating Unit Production

- Exercise 1 : Remove unit productions from the following

$$P = \{ \\ S \rightarrow XY, X \rightarrow a, Y \rightarrow Z \mid b, \\ Z \rightarrow M, M \rightarrow N, N \rightarrow a \\ \}$$

- Exercise 2 : Remove unit productions from the following

$$P = \{ \\ S \rightarrow 0A \mid 1B \mid C \\ A \rightarrow 0S \mid 00 \\ B \rightarrow 1 \mid A \\ C \rightarrow 01 \\ \}$$