CSC-257
Theory Of Computation
(BSc CSIT, TU)

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Module Structure

- Semester : IV
- Nature of the Course
  - Theory + Lab
- Full Marks : 60 + 20 + 20
- Pass Marks : 24 + 8 + 8
- Credit Hours : 3
- Total Teaching Hours : 45

Mathematical Preliminaries

- **Sets**
  - A set is a collection of well defined objects.
  - Usually the element of a set has common properties.
  - e.g. all the student who enroll for a course “theory of computation” make up a set.
- **Examples**
  - The set of even positive integer less than 20 can be expressed by
  - $E = \{2,4,6,8,10,12,14,16,18\}$ Or
  - $E = \{x|x \text{ is even and } 0<x<20\}$
- **Finite and Infinite Sets**
  - A set is finite if it contains finite number of elements.
  - And, infinite otherwise.
  - The empty set has no element and is denoted by ϕ.

Mathematical Preliminaries

- **Cardinality of a set**
  - It is a number of element in a set. The cardinality of set E is |E|=9.
- **Subset**
  - a set A is subset of a set B if each element of A is also element of B
  - and is denoted by $A \subseteq B$.
  - A = { 1, 3, 5 } and B = { 1, 2, 3, 4, 5 } then $A \subseteq B$
- **Union**
  - The union of two sets A and B is the collection of elements of A, B and both
  - and denoted by A U B . Here, A U B = { 1, 2, 3, 4, 5 }
- **Intersection**
  - The intersection of two sets A and B is the collection of elements of A and B which are common in both sets and denoted by $A \cap B$. Here, $A \cap B$ = { 1, 3, 5 }

Mathematical Preliminaries

- **Difference**
  - The difference of two sets A and B, denoted by A-B, is the set of all elements that are in the set A but not in the set B.
  - Here, $A - B = \{\} / \phi$
  - $B - A = \{ 2, 4 \}$

- **Sequence**
  - A sequence of objects is a list of objects in some order.
  - For example, the sequence 7,4,17 would be written as (7,4,17).
  - In set, order does not matter but in sequence it does.
  - Also, repetition is not permitted in a set but is allowed in a sequence.
  - Like set, sequence may be finite or infinite

Mathematical Preliminaries

- **Types of Set**
  - Empty set: A set, which has no element, is called as empty set or null set or void set. It is denoted by ϕ.
  - Singleton set: A set, which has single element, is called as singleton set.
  - Disjoint sets: Two or more sets are said to be disjoints, if there are no common elements among.
  - Overlapping sets: Two or more sets are said to be disjoints, if there are at least one common element among them.
  - Finite sets: A set having specified number of elements is called as a finite set.
  - Infinite sets: A set is called infinite set, if it is not finite set.
  - Universal set: The set of all objects or things under consideration in discussion is called the universal set.

Mathematical Preliminaries

- **Relation**
  - A relation is a correspondence between two sets (called the domain and the range) such that to each element of the domain, there is assigned one or more elements of the range
  - Eg. R = { (2, –3), (4, 6), (3, –1), (6, 6), (2, 3) }
  - In the relation R above, set of all x’s is the domain and set of all y’s is the range.
  - Domain : { 2, 3, 4, 6 } and Range : { -3, -1, 3, 6 }
  - This is a relation because every element of domain is associated with at least one element of range.

Types of Relation

- **Identity Relation**
  - In an identity relation "R", every element of the set “A” is related to itself only.
  - Note the conditions conveyed through words “every” and “only”.
  - The word “every” conveys that identity relation consists of ordered pairs of element with itself - all of them.
  - The word “only” conveys that this relation does not consist of any other combination
  - Consider a set A={1,2,3} Then,
  - its identity relation is: R={(1,1), (2,2),(3,3)}

Types of Relation

- **Reflexive Relation**
  - In reflexive relation, "R", every element of the set “A” is related to itself.
  - The definition of reflexive relation is exactly same as that of identity relation except that it misses the word “only” in the end of the sentence.
  - The implication is that this relation includes identity relation and permits other combination of paired elements as well.
  - Consider a set A={1,2,3} Then,
  - one of the possible reflexive relations can be: R={(1,1), (2,2), (3,3), (1,2), (1,3)}
  - However, following is not a reflexive relation: R1={(1,1), (2,2), (1,2), (1,3)}

Types of Relation

- **Symmetric Relation**
  - In symmetric relation, the instance of relation has a mirror image.
  - It means that if (1,3) is an instance, then (3,1) is also an instance in the relation.
  - Clearly, an ordered pair of element with itself like (1,1) or (2,2) is themselves their mirror images.
  - Consider some of the examples of the symmetric relation, R1={(1,2),(2,1),(1,3),(3,1)} R2={(1,2),(1,3),(2,1),(3,1),(3,3)}
  - Condition of symmetric relation can be stated as
  - Iff$(x,y)\in R \Rightarrow (y,x)\in R$ for all $x,y\in A$
  - The symbol “Iff” means “If and only if”.
  - Here one directional arrow means “implies”.
  - Alternatively, the condition of symmetric relation can be stated as: $xRy \Rightarrow yRx$ for all $x,y\in A$

Types of Relation

- **Transitive Relation**
  - If "R" be the relation on set A, then we state the condition of transitive relation as: Iff$(x,y)\in R$ and $(y,z)\in R \Rightarrow (x,z)\in R$ for all $a,b,c\in A$
  - Alternatively, xRy and $yRz \Rightarrow xRz$ for all $x,y,z\in A$
  - Eg. Let relation $R = \{ (1,1), (2,2), (3,3), (1,2), (2,3), (1,3) \}$
  - Here, $(1,2) \in R$ and $(2,3) \in R$ and $(1,3) \in R$,
  - That's why this relation R is transitive relation

Types of Relation

- **Equivalence Relation**
  - A relation is equivalence relation if it is reflexive, symmetric and transitive at the same time.
  - In order to check whether a relation is equivalent or not, we need to check all three characterizations.
  - Eg. Let A = { 1, 2, 3 } and R = { (1,1), (2,2), (3,3), (1,3), (3,1) }
  - Relation R is equivalence relation because it is reflexive, symmetric and transitive at the same time.

Functions

- A function is a correspondence between two sets (called the domain and the range) such that to each element of the domain, there is assigned exactly one element of the range.
- Determine the domain and range of the given function: $y = -\sqrt{-2x + 3}$
- The domain is all values that x can take on. The only problem with this function is that square root cannot have a negative inside it.
- So, content inside square root should be greater than or equal to zero.
- $-2x + 3 > 0$
- $-2x > -3$
- $2x < 3$
- $x < 3/2 = 1.5$
- Then the domain is "all $x < 3/2$"

Alphabets

- The symbols are generally letters and digits.
- Alphabets are defined as a finite set of symbols.
- It is denoted by 'Σ' symbol.
- E.g.: An alphabet of set of decimal numbers is given by $\Sigma = \{0, 1, \ldots\ldots, 9\}$.
- The alphabet for binary number is $\Sigma = \{0, 1\}$.

Strings

- A string or word is a finite sequence of symbols selected from some alphabets.
- E.g. if $\Sigma=\{a, b\}$ then ‘abab’ is a string over Σ.
- A string is generally denoted by ‘w’.
- The empty string is the string with 0 (zero) occurrence of symbols.
- This string is represented by ε (epsilon)

Closure of an Alphabet

- Closure of an alphabet is defined as the set of all strings over an alphabet Σ including empty string and is denoted by Σ^*
- e.g.
- Let $\Sigma = \{0, 1\}$ then
- $\Sigma^* = \{\varepsilon, 0, 1, 00, 10, 01, 11, \ldots.\}$
- $\Sigma 1 = \{0, 1\}$
- $\Sigma 2 = \{00, 01, 10, 11\}$
- $\Sigma 3 = \{000, 001, 010, 011, 100, 101, 110, 111\}$
- $\Sigma + = \Sigma 1 \cup \Sigma 2 \cup \Sigma 3 \cup \ldots.$
- Therefore, $\Sigma^* = \Sigma + \cup \{\varepsilon\}$

Concatenation of Strings

- Let w1 and w2 be two strings, then w1w2 denotes the concatenation of w1 and w2.
- e.g.
- if w1 = abc, w2 = xyz, then
- w1w2 = abcxyz

Languages

- A set of strings all of which are chosen from Σ^*, where Σ is particular alphabet, is called a language
- Let $\Sigma = \{0, 1\}$ then,
- $L = \{$all strings over Σ with equal number of 0's and 1's$\}$
- $= \{01, 10, 1010, 0101, 0011, 110011 \ldots\ldots\}$
- $L \subseteq \Sigma^*$