CSC-257
Theory Of Computation
(BSc CSIT, TU)

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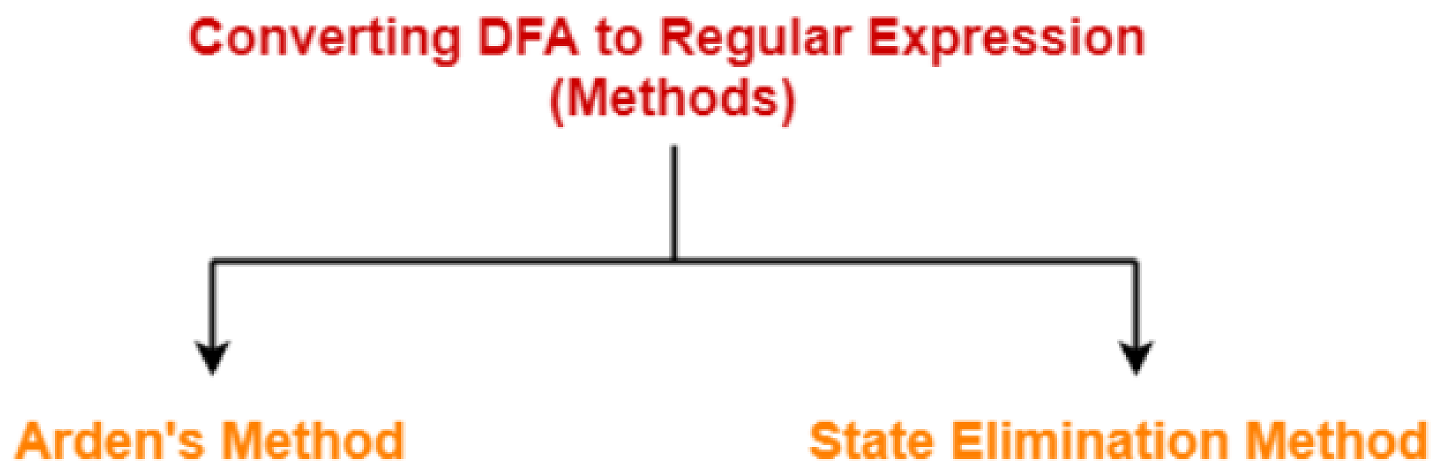
Conversion of DFA to Regular Expression

- A DFA can be converted into RE using two methods.



Conversion of DFA to Regular Expression

- **Arden's Theorem :** Let p and q be two regular expressions over the alphabet Σ, if p does not contain any empty string then **r = q + rp** has a unique solution **r = qp***.
- **Proof :** Here, we have, $r = q + rp$ ……………… (i)
- putting the value of $r = q + rp$ on the right hand side of the relation (i),
- We have, $r = q + (q + rp)p$
- $r = q + qp + rp^2$……………… (ii)
- Again putting value of $r = q + rp$ in relation (ii),
- we get, $r = q + qp + (q +rp)p^2$
- $r = q+ qp + qp^2 + rp^3$

Conversion of DFA to Regular Expression

- Continuing the process in same way, we will get as;
- $r = q + qp + qp^2 + qp^3 + \ldots\ldots\ldots\ldots\ldots$
- $r = q(\epsilon + p + p^2 + p^3 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots$
- Thus, $r = qp^*$ Proved.

Use of Arden's rule to convert a DFA to RE

- To convert the given DFA into a regular expression, here are some of the assumptions regarding the transition system:

1. The transition diagram should not have the Є-transitions
2. There must be only one initial state
3. Form an equation for each state considering the transitions which comes towards that state. The vertices or the states in the DFA are as; $q_1,q_2,\ldots\ldots\ldots\ldots q_n$ (Any q_i is final state).
4. Add '∈' in the equation of initial state
5. Bring final state in the form $r = q + rp$ to get the required regular expression
6. Note : If there exists multiple final states, then :
   - Write a regular expression for each final state separately.
   - Add all the regular expressions to get the final regular expression.

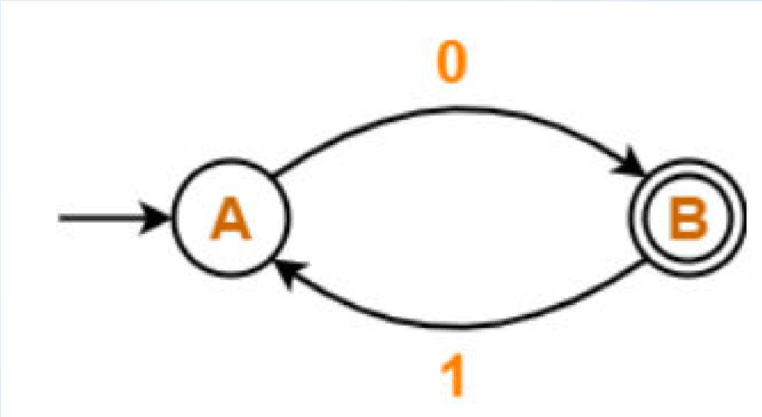
Use of Arden's rule to convert a DFA to RE

7. w_{ij} denotes the regular expressions representing the set of labels of edges from q_i to q_j , We can get the following conditions as :
   - $q_1=q_1w_{11}+q_2w_{21}+q_3w_{31}+\ldots\ldots\ldots\ldots q_nw_{n1} + \epsilon$
   - $q_2=q_1w_{12}+q2w_{22}+q_3w_{32}+\ldots\ldots\ldots\ldots+q_nw_{n2}$
   - $q_3=q_1w_{13}+q_2w_{23}+q_3w_{33}+\ldots\ldots\ldots\ldots+q_nw_{n3}$
   - ………………………………………………………………………
   - ………………………………………………………………………
   - $q_n= q_1w_{1n} + q_2w_{2n} + q_3w_{3n} +\ldots\ldots\ldots\ldots\ldots\ldots+ q_nw_{nn}$

- Hence, solving these equations for q_i in terms of w_{ij} gives RE

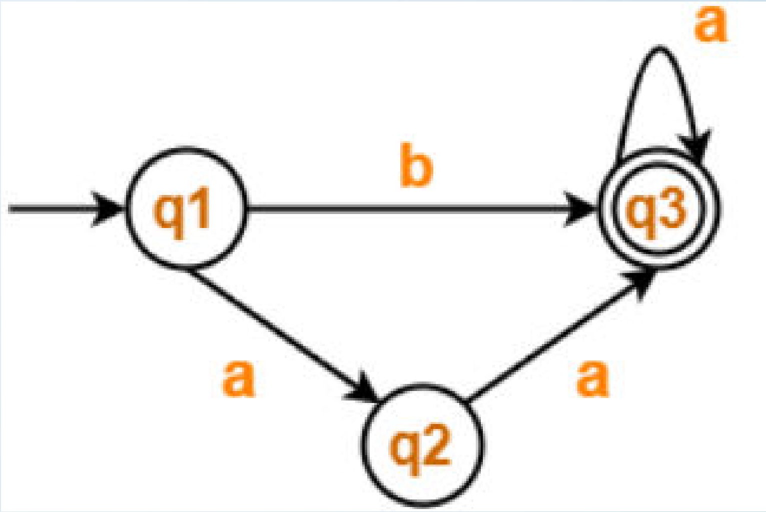
Use of Arden's rule to convert a DFA to RE : Ex

- Convert the following DFA to RE



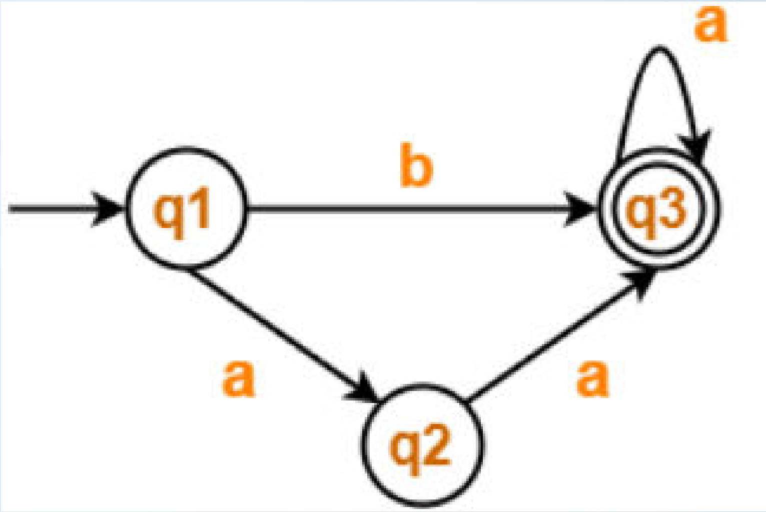
- **Solution :**
- Form equation for each state as :
  - $A = \in + B1$ …… (1)
  - $B = A0$ …… (2)
- Bring final state in the form $r = q + rp$.
- Using (1) in (2), we get :
  - $B = (\in + B1)0$
  - $B = \in 0 + B1$
  - $B = 0 + B(10)$ …… (3)
- Using Arden's Theorem in (3), we get :
  - $B = 0(10)^*$, where $r = B$, $q = 0$ and $p = (10)$
- Thus, Regular Expression for the given DFA = $0(10)^*$ [since, $r = qp^*$]

Use of Arden's rule to convert a DFA to RE : Ex



- Convert the following DFA to RE
- **Solution :**
- Forming equation for each state as :
  - $q_1 = \in$ …………. (1)
  - $q_2 = q_1 a$ …………… (2)
  - $q3 = q_1 b + q_2 a + q_3 a$ ……….. (3)
- Bring final state in the form r = q + rp as :
- Using (1) in (2), we get :
  - $q_2 = \in a$
  - $q2 = a$ ………… (4)

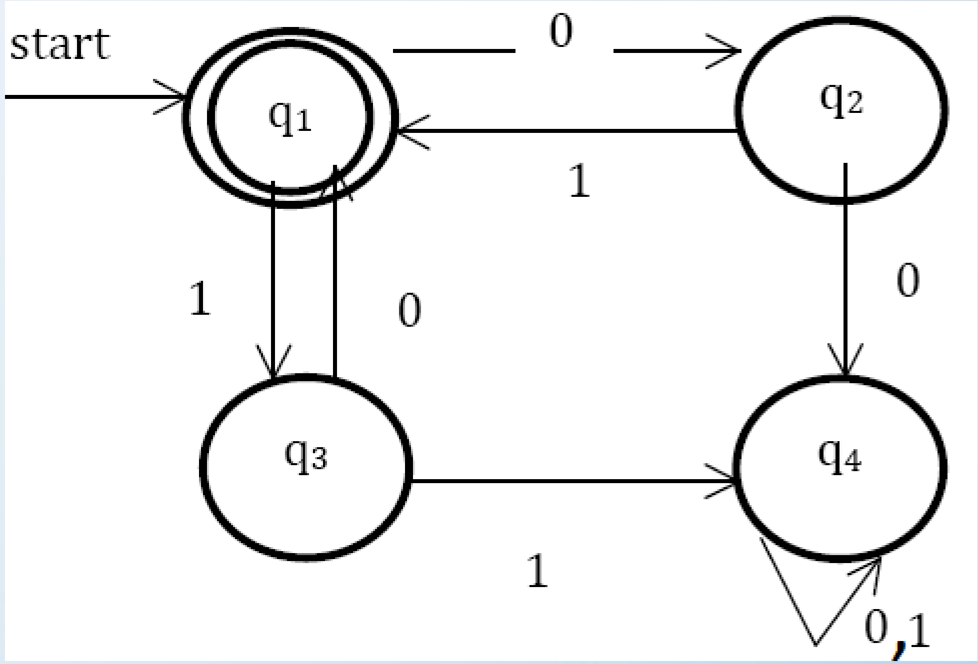
Use of Arden's rule to convert a DFA to RE : Ex



- Using (1) and (4) in (3), we get :
  - $q_3 = q_1b + q_2a + q_3a$
  - $q_3 = \in b + aa + q_3a$
  - $q_3 = (b + aa) + q_3a$ .... (5)
- Using Arden's Theorem in (5), we get :
  - $q_3 = (b + aa)a^*$
- Thus, Regular Expression for the given DFA = $(b + aa)a^*$

Use of Arden's rule to convert a DFA to RE : Ex

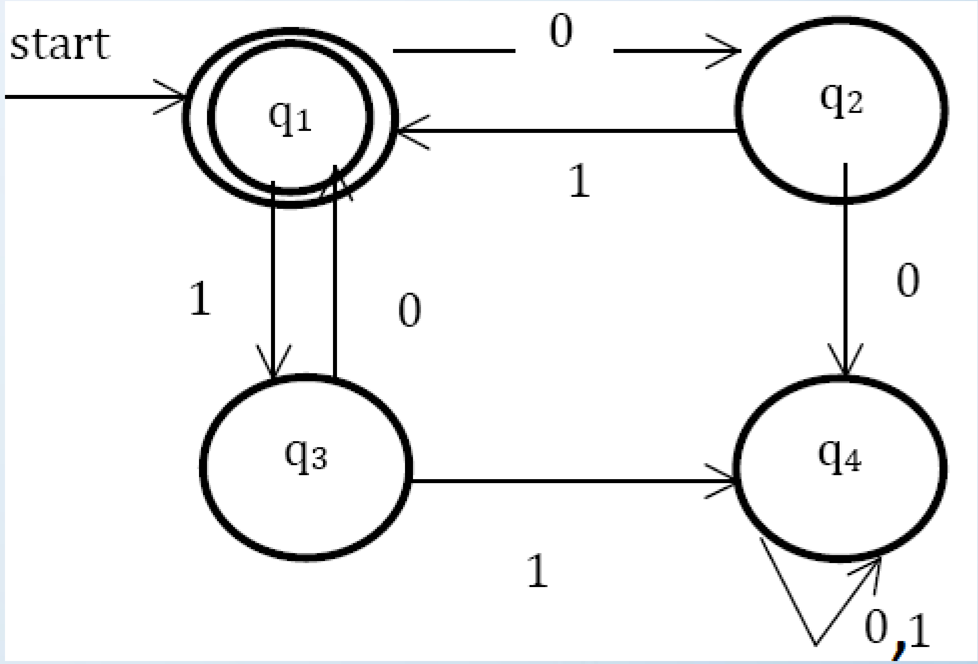
- Convert the following DFA to RE



- **Solution :**
- Here, the equations are
- $q_1 = q_2 1 + q_3 0 + \varepsilon$……………………….(i)
- $q_2 = q_1 0$………………………………. (ii)
- $q_3 = q_1 1$………………………………. (iii)
- $q_4 = q_2 0 + q_3 1 + q_4 0 + q_4 1$……………. (iv)
- Now putting q_2 and q_3 in equation (i), we get
- $q_1 = q_1 01 + q_1 10 + \varepsilon$
- $q_1 = \varepsilon + q_1(01+10)$

Use of Arden's rule to convert a DFA to RE : Ex

- Let,
  - $q = \varepsilon$,
  - $r = q_1$, and
  - $p = (01 + 10)$
- Therefore, according to Arden's rule,
- $q1 = \varepsilon (01 + 10)^*$ [since, $r = qp^*$]
- since, q1 is the final state,
- so, RE= $\varepsilon (01 + 10)^* = (01 + 10)^*$ is the required RE for the given DFA



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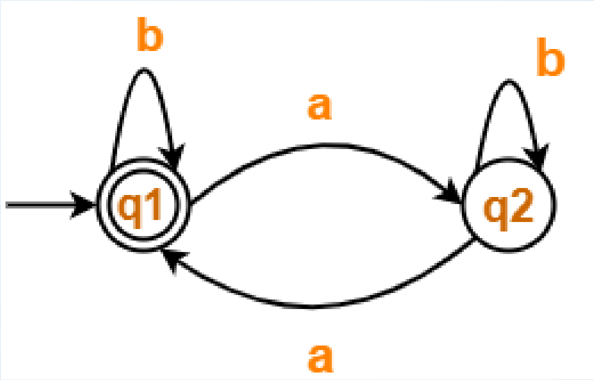

- Convert the following DFA to RE