CSC-257
Theory Of Computation
(BSc CSIT, TU)

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Derivation Using Grammar Rule

- We apply the production rule of a CFG to infer that certain strings are in the language of a certain variable
- A derivation of a context free grammar is a finite sequence of strings $\beta_0\ \beta_1\ \beta_2 \ldots\ldots\ldots \beta_n$ such that :
  - for $0 \leq i \leq n$, the string $\beta_i \in (V \cup T)^*$
  - $\beta_0 = S$
  - for $0 \leq i \leq n$, there is a production of P that applied to β_i yields β_{i+1}
  - $\beta_n \in T^*$
- There are two possible approaches of derivation
  - Body to head (Bottom Up) approach
  - Head to body (Top Down) approach

Body to head (Bottom Up) approach

- Here, we take strings known to be in the language of each of the variables of the body, concatenate them, in the proper order, with any terminals appearing in the body, and infer that the resulting string is the language of the variables in the head
- Consider a grammar :
  - S → S + S
  - S → S – S
  - S → S * S ……………………………………… Grammar (1)
  - S → S / S
  - S → (S)
  - S → a
- And expression to be derived is a + (a*a) / a – a
- Now using this approach,

Body to head (Bottom Up) approach

- Below table is the process that shows how this approach works

SN	String Inferred	Variable	Production	Strings Used
1	a	S	S → a	
2	a*a	S	S → S * S	String 1
3	(a*a)	S	S → (S)	String 2
4	(a*a) / a	S	S → S / S	String 1 and String 3
5	a + (a*a) / a	S	S → S + S	String 1 and String 4
6	a + (a*a) / a – a	S	S → S - S	String 1 and String 5

- Thus, in this process we start with any terminal appearing in the body and use the available rules from body to head.

Head to Body (Top Down) approach

- Here, we use production from head to body.
- We expand the start symbol using a production, whose head is the start symbol.
- Here we expand the resulting string until all strings of terminal are obtained.
- There are two approaches under this :
  - **Leftmost Derivation** : leftmost symbol (variable) is replaced first
  - **Rightmost Derivation** : rightmost symbol (variable) is replaced first

Head to Body (Top Down) approach

- **Leftmost Derivation :**
- Consider the grammar (1) and deriving string is a + (a*a) / a – a
- Now leftmost derivation for the given string is :
- S → S + S rule S → S + S
- S → a + S rule S → a
- S → a + S - S rule S→ S – S
- S → a + S / S - S rule S → S / S
- S → a + (S) / S - S rule S → (S)
- S → a + (S*S) / S - S rule S → S*S
- S → a + (a*S) / S - S rule S → a
- S → a + (a*a) / S - S rule S → a
- S → a + (a*a) / a - S rule S → a
- S → a + (a*a) / a - a rule S → a

Head to Body (Top Down) approach

- **Rightmost Derivation :**
- Consider the grammar (1) and deriving string is a + (a*a) / a – a
- Now rightmost derivation for the given string is :
- S → S - S rule S → S - S
- S → S - a rule S → a
- S → S + S - a rule S→ S + S
- S → S + S / S - a rule S → S / S
- S → S + S / a - a rule S → a
- S → S + (S) / a - a rule S → (S)
- S → S + (S*S) / a - a rule S → S*S
- S → S + (S*a) / a - a rule S → a
- S → S + (a*a) / a - a rule S → a
- S → a + (a*a) / a - a rule S → a

Language of Context Free Grammar

- Let $G = (V, T, P, S)$ is a context free grammar.
- Then the language of G denoted by L(G) is the set of terminal strings that have derivation from the start symbol in G
  i.e. $L(G) = \{ x \in T^* \mid S \rightarrow^* x\}$
- The language generated by a CFG is called the Context Free Language (CFL)