Air University
(Mid-Term Examination: Spring 2022)
Department of Cyber Security

AIR UNIVERSITY

Lashid 16/5/2022

Subject: Discrete Structures
Course Code: MA-216
Class: BS-CYS (II) MA216
Section: (A, B)

Total Marks: 50 (Weightage 25%)
Date: 06/04/2022
Duration: 2 Hours
FM Name: Noor Zeb Khan

Q. No	Questions	Marks	CLO
(01)	**A).** Construct the Truth Table for $(((p \to q) \to r) \to s)$. **B).** Show that $\neg(p \lor (\neg p \land q))$ and $\neg p \land \neg q$ are logically equivalent by logical laws also mention the names of the laws.	(06) (06)	CLO 1 C2
(02)	**A).** Show that the premises "It is not sunny this afternoon and it is colder than yesterday", "We will go swimming only if it is sunny", "If we don't go swimming, then we will take a canoe trip", 'If we take a canoe trip, then we will be home by sunset", Lead to the conclusion, "We will be home by sunset", By Rules of Inference also give reason with each step. **B).** Prove that if n is an integer and $3n + 2$ is odd, then n is odd. By (i). Contraposition proof (ii). Contradiction.	(08) (08)	CLO 2 C3
(03)	**A).** Suppose that a person deposit $10,000 in a saving account at a bank yielding 11% per year with interest compounded annually. How much will be in the account after 30 years. **B).** Is the sequence $a_n = n4^n$ is the solution of Recurrence relation $a_n = 8a_{n-1} - 16a_{n-2}$.	(06) (06)	CLO 3 C3
(04)	**A).** Find $\sum_{k=50}^{100} K^2$ where $\sum_{K=1}^{n} K^2 = \frac{n(n+1)(2n+1)}{6}$. **B).** Write the linear search algorithm that searches for an element in a list of elements.	(05) (05)	CLO 4 C4

***************** End of Examination Paper *********************

sp-22 (01) Mid Paper BSCYS-II A,B

(03)

Q#01 : (a) Construct the truth table for

$((p \to q) \to r) \to s$.

p	q	r	s	$p \to q$	$((p \to q) \to r)$	$((p \to q) \to r) \to s$
T	T	T	T	T	T	T
T	T	T	F	T	T	F
T	T	F	T	T	F	T
T	T	F	F	T	F	T
T	F	T	T	F	T	T
T	F	T	F	F	T	F
T	F	F	T	F	T	T
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	T	F	T	T	F
F	T	F	T	T	F	T
F	T	F	F	T	F	T
F	F	T	T	T	T	T
F	F	T	F	T	T	F
F	F	F	T	T	F	T
F	F	F	F	T	F	T

(03)

(b) Show that ~~that~~ the $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically Equivalent by logicall laws also mentioned ~~by logical laws~~. the names of laws.

$\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg(\neg p \wedge q)$ second Demorgan law

$\equiv \neg p \wedge (\neg(\neg p) \vee \neg q)$. 1st De Morgan law

$\equiv \neg p \wedge (p \vee \neg q)$ Double negation.

$\equiv (\neg p \wedge p) \vee (\neg p \vee \neg q)$ 2nd Distributive law

$\equiv F \vee (\neg p \vee \neg q)$ $\neg p \wedge p \equiv F$

$\equiv (\neg p \vee \neg q) \vee F$ commutative law

$\equiv \neg p \vee \neg q \equiv$ R.H.S.

Q# 02: (a) show that the premises "It is not sunny this after noon and it is colder than yesterday."
~~"It is colder than yesterday~~
"We will go swimming only if it is sunny".
"If we donot go swimming, then we will take a canoe trip." and "If we take a canoe trip, then we will be home by sunset." lead to the conclusion "We will be home by sunset.
by ~~Use~~ Rules of Inference. also give Reason with each step.

Sol:

p = It is sunny this after noon.
q = It is colder then yesterday
r = We will go swimming
s = We will take a canoe trip.
t = We will be home by sunset.

$P_1 = \neg p \wedge q$, $P_2 = r \to p$, $P_3 = \neg r \to s$, $P_4 = s \to t$

Concln = t

Steps	Reason
1) $\neg p \wedge q$	Premise
2) $\neg p$	Simplification using (1)
3) $r \to p$	Premise
4) $\neg r$	Modus tollens using (2) & (3)
5) $\neg r \to s$	Premise
6) s	Modus ponens using (4) & (5)
7) $s \to t$	Premise
8) t	Modus Ponens using (6) & (7)

#(b): Prove that if n is an integer and $3n+2$ is odd, then n is odd.

i): By Contraposition

We assum that the conclusion of the conditional statement, " If "$3n+2$" is odd $\to$ (i) then "n" is odd" is false.

p q

So; Assume that n is even. ($\sim q$)

$\Rightarrow n = 2k$; $k \in \mathbb{Z}$ $\to$ (ii) $\because$ By def of Even Number

Substitute (ii) in (i) ; We got:

$3n+2 = 3(2k)+2$; $k \in \mathbb{Z}$

$\Rightarrow 3n+2 = 6k+2$.

$\Rightarrow 3n+2 = 2(3k+1)$

$\Rightarrow 3n+2 = 2m$; $m \in \mathbb{Z}$; $m = 3k+1$

By definition of Even we conclude that

$\therefore$ $3n+2$ is even $\to \sim p$

Therefore; "$3n+2$" is not odd.

Sine $\sim q \to \sim p$ is true.

which is equvalnt to $p \to q$.

$\sim q \to p$ Thus; If "$3n+2$" is odd, then n is odd. $\square$

ii)- By Contraduction:

Let p is "$3n+2$" is odd" and q is "n is odd":

Let us Assume that "$3n+2$ $\to$ (i) is odd ~~and then~~ and n is even"

By def: of even: $n = 2k \to$ (ii) $k \in \mathbb{Z}$.

put (ii) in (i)

$3n+2 = 3(2k)+2$.

$\Rightarrow$ " $= 6k+2$.

$\Rightarrow$ " $= 2(3k+1)$

$\Rightarrow$ " $= 2m$, $m \in \mathbb{Z}$, $m = 3k+1$.

It gives us that $3n+2$ which is contradiction to our supposition that n is even & $3n+2$ is odd. Thus; "If $3n+2$ is odd then n is odd". $\square$

(04) (b) Find $\sum_{k=50}^{100} k^2$

Sol:

$$\sum_{k=1}^{100} k^2 = \sum_{k=1}^{49} k^2 + \sum_{k=50}^{100} k^2$$

$$\sum_{k=50}^{100} k^2 = \sum_{k=1}^{100} k^2 - \sum_{k=1}^{49} k^2$$

$$'' = \frac{100(100+1)(2(100)+1)}{6} - \frac{49(49+1)(2(49)+1)}{6}$$

$$'' = \frac{100 \cdot 101 \cdot 201}{6} - \frac{49 \cdot 50 \cdot 99}{6}$$

$$'' = 338350 - 40425$$

$$\boxed{'' = 297,925}$$

(04) (c) Is the sequence $\{a_n\}$ a solution of Recurrence relation $a_n = 8a_{n-1} - 16a_{n-2}$ if $a_n = n4^n$.

Sol:- Put $a_n = n4^n$ in R.R. $a_n = 8a_{n-1} - 16a_{n-2}$

$\Rightarrow$ $a_n = 8a_{n-1} - 16a_{n-2}$.

$$a_n = 8((n-1)4^{n-1}) - 16((n-2)4^{n-2}).$$

$8 = 2 \cdot 4$

$16 = 4^2$

$32 = 2 \cdot 16 = 2 \cdot 4^2$

$$a_n = 8n \cdot 4^{n-1} - 8 \cdot 4^{n-1} - 16n4^{n-2} + 32 4^{n-2}$$

$$= 2n4^{n-1+1} - 2 \cdot 4^{n-1+1} - n4^{n-2+2} + 2 \cdot 4^{n-2+2}$$

$$= 2n4^n - 2 \cdot 4^n - n \cdot 4^n + 2 \cdot 4^n.$$

$$= 2n4^n - n \cdot 4^n - \cancel{2 4^n} + \cancel{2 4^n}$$

$$= n4^n(2-1)$$

$$\boxed{a_n = n4^n}$$

Thus a_n is the solution of R.R $a_n = ---$

Q#(05): Suppose that a person deposites $ 10,000 in a saving account at a bank yielding 11% per year with interest compounded anually. How much will be in the account after 30 years.

Sol: Let P_0 = initial investment (payment) = 10,000 $.

Initial + intrest after 1 year $\quad P_1 = P_0 + 0.11 P_0 = 1.11 P_0$.

Initial + intrest after 2-years $\quad P_2 = P_1 + 0.11 P_1 = 1.11 P_1 = (1.11)(1.11) P_0$

$$P_2 = (1.11)^2 P_0.$$

Initial + intrest after 3-years $\quad P_3 = P_2 + 0.11 P_2 = 1.11 P_2 = (1.11).(1.11)^2 P_0.$

$$P_3 = (1.11)^3 P_0.$$

Initial + intrest after 4-years $\quad P_4 = (1.11)^4 P_0$

⋮

After n-years $\quad P_n = (1.11)^n . P_0.$

Put $\quad P_0 = 10,000$

$$P_n = (1.11)^n (10,000).$$

$$P_n = (1.11)^n . (10,000).$$

After 30-years: Put $n = 30$.

$$P_{30} = (1.11)^{30} . (10,000).$$

$$P_{30} = (22.89229).(10,000)$$

$$\boxed{P_{30} = 228922.97 \$}$$

Scanned by CamScanner

Q 4 # (a): Write ~~an~~ ~~algorithm~~ The linear search Algorithm.

Sol:

Procedure : linear search (x : integer, $a_1, a_2, \ldots, a_n$: distinct integers)

$i := 1$

while ($i \leq n$ and $x \neq a_i$)

$i := i + 1$

if $i \leq n$ the location := i

else location := 0.

return location { location is the subscript of the term that equal x, or is 0 if x is not found }.