Air University

(End Semester Examination: Spring-2024)

Subject:	**Linear Algebra**	Total Marks:	**100**
Course Code:	**MA-201**	Date:	-06-2024
Class:	**BS-CYS**	Time:	
Semester:	**II**	Duration:	**3 Hours**
Section:	**A (Morning Session)**	FM Name:	**Mr. Umair Habib**

HoD Signature:________ FM Signature:________

IMPORTANT INSTRUCTIONS:

- **Attempt all questions**
- **This examination carries 45% weight toward the final grade**
- **Scientific calculator is allowed**

Q. No. 1 (CLO-3) (PLO-2)		**20 Marks**
a	*Diagonalize* the following matrix (if possible). i.e. **Solve** to find an invertible matrix P and a diagonal matrix D. WHERE $\lambda = 1, -2, -2$. $$A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$$	20
Q. No. 2 (CLO-2) (PLO-1)		**20 Marks**
a	Find the vectors that **describe** the basis for Col A. $$A = \begin{bmatrix} -2 & 4 & -2 & -4 \\ 2 & -6 & -3 & 1 \\ -3 & 8 & 2 & -3 \end{bmatrix}$$	10
b	Find an explicit description of Nul A by listing vectors that span the null space. $$A = \begin{bmatrix} 1 & 5 & -4 & -3 & 1 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$	10
Q. No. 3 (CLO-4) (PLO-2)		**20 Marks**
a	Decipher the following Hill 2-cipher by **Implementing** the knowledge of Gaussian elimination and modular arithmetic if the last four plaintext letters are known to be *ATOM*. ***LNGIHGYBVRENJYQO***	20
Q. No. 4 (CLO-5) (PLO-1)		**20 Marks**
a	**Illustrate** the use of inner products to find an *orthonormal basis* of the subspace spanned by the given set of vectors. $$\left\{\begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix}, \begin{bmatrix} -3 \\ 14 \\ -7 \end{bmatrix}\right\}$$	12
b	Show that the set $\{u_1, u_2, u_3\}$ is an orthogonal basis for R^3. Then express "x" as a linear combination of the u's. $$u_1 = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}, u_2 = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, u_3 = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}, x = \begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}$$	8

Q. No. 5 (CLO-1) (PLO-1)		20 Marks
a	**Identify** the *LDU* factorization of the given matrix A. $$A = \begin{bmatrix} 3 & -6 & 3 \\ 6 & -7 & 2 \\ -1 & 7 & 0 \end{bmatrix}$$	10
b	Use the Invertible Matrix Theorem to **Recognize** whether matrix A is invertible or not. $$A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 1 & -2 \\ -5 & -1 & 9 \end{bmatrix}$$	5
c	Use a cofactor expansion across the second column to **Find** the determinant of A, where $$A = \begin{bmatrix} 5 & -2 & 2 \\ 0 & 3 & -3 \\ 2 & -4 & 7 \end{bmatrix}$$	5

**************Best of Luck**************

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