Antennas and Free Space Propagation

WIRELESS SHORT COURSE

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Learning Objectives

- Mathematically describe an EM wave:
  - Direction of motion, wavenumber, frequency, polarization, …
- Identify radio spectrum and power levels used in common commercial wireless products
- Perform basic mathematical operations in polar coordinates
  - Conversions to cartesian coordinates, rotations, integrals, averages, …
- Use tools from MATLAB to compute and plot key antenna parameters
  - Directivity, gain, efficiency, …
- Compute received power in an angular region using the radiation density and intensity.
- Compute the free-space path loss using Friis Law
- Derive Friis Law

Outline

- Basics of Electromagnetic Waves
- Basics of Antennas
- Free Space Propagation

Electric and Magnetic Forces

- Two closely related forces:
  - Electric: Forces between charged particles
  - Magnetic: Forces between moving charged particles
- Forces operate at a distance:
  - Enables communication.
  - … and many other phenomena in the universe
- Represented by a vector field
  - Force strength has a direction and magnitude
  - Changes with position $\boldsymbol{r} = (x, y, z)$ and time t
  - E-field: $\boldsymbol{E}(\boldsymbol{r}, t)$ in N/C (force / unit charge)
  - B-field: $\boldsymbol{B}(\boldsymbol{r}, t)$ in N/(Am) (force / unit charge / velocity)

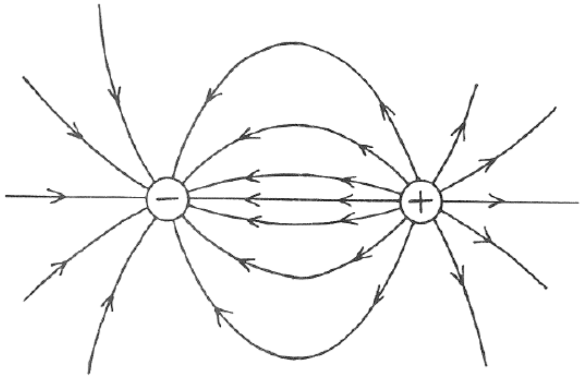
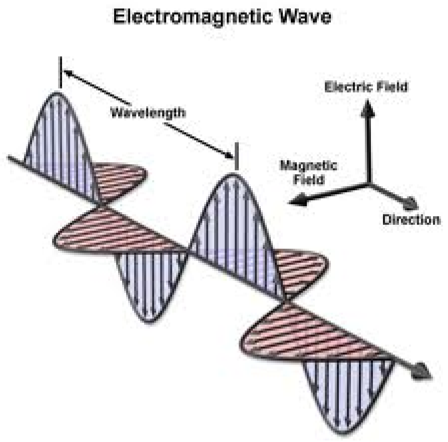


FIGURE 2.4 Electric field lines begin and end on charges.

Plane Waves

- EM field governed by Maxwell's equations
- All solutions can be decomposed into plane waves

- EM plane wave at position $\boldsymbol{r} = (x, y, z)$
  - $\boldsymbol{E}(\boldsymbol{r}, t) = E_0 \boldsymbol{e}_x \cos(2\pi(ft + \lambda^{-1}z) + \phi)$
  - $\boldsymbol{B}(\boldsymbol{r}, t) = B_0 \boldsymbol{e}_y \cos(2\pi(ft + \lambda^{-1}z) + \phi)$
  - $B_0 = (1/c)E_0, \;\; c = \lambda f =$ speed of light
- Sometimes write:
  - $\boldsymbol{E}(\boldsymbol{r}, t) = E_0 \boldsymbol{e}_x \cos(\omega t + kz + \phi)$
  - $\boldsymbol{B}(\boldsymbol{r}, t) = B_0 \boldsymbol{e}_y \cos(\omega t + kz + \phi)$
  - $k = \frac{2\pi}{\lambda} =$ wave number



Plane Waves Illustrated

- EM plane wave
  - $\boldsymbol{E}(\boldsymbol{r},t) = E_0\boldsymbol{e}_x \cos(2\pi(ft - \lambda^{-1}z) + \phi)$
  - $\boldsymbol{B}(\boldsymbol{r},t) = B_0\boldsymbol{e}_y \cos(2\pi(ft - \lambda^{-1}z) + \phi)\,, B_0 = c^{-1}E_0$
- Five key parameters:
  - Amplitude, frequency, direction of motion, phase
  - Polarization (see below)
- Diagrams on board
  - Fixed position, variation in time
  - Fixed time, variation in position.
- Phasor notation: $\boldsymbol{E}(\boldsymbol{r},t) = Real\left[\boldsymbol{E}(\boldsymbol{r})e^{i\omega t}\right]$

Plane Wave Direction of Motion

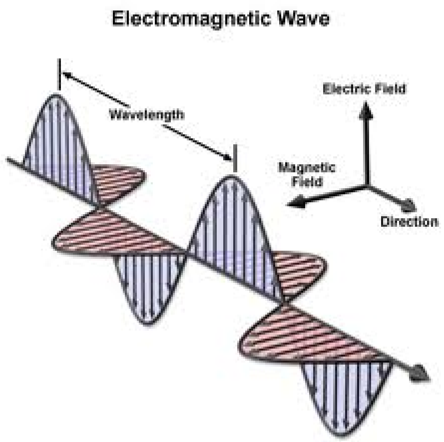
- EM field constant in $x - y$ plane
- Moves along z-direction:

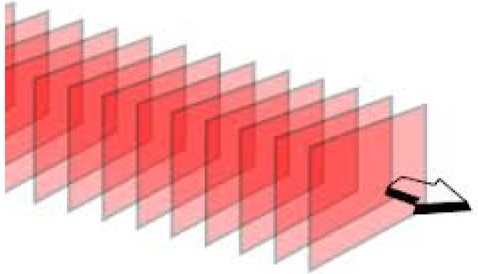
$$E(x, y, z, t + \delta t) = E(x, y, z - c\delta t, t)$$

- "Poynting" vector:

$$\boldsymbol{S} = \frac{1}{\mu_0}\boldsymbol{E} \times \boldsymbol{B} = \frac{|E_0|^2}{c\mu_0}\cos^2(2\pi(ft - \lambda^{-1}z))\boldsymbol{e}_z$$

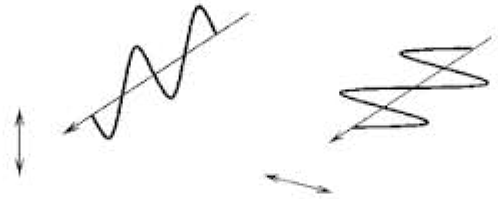
  - Represents "energy flux"
  - Energy consumed = $\nabla \cdot \boldsymbol{S}$
  - Units = W/m^2



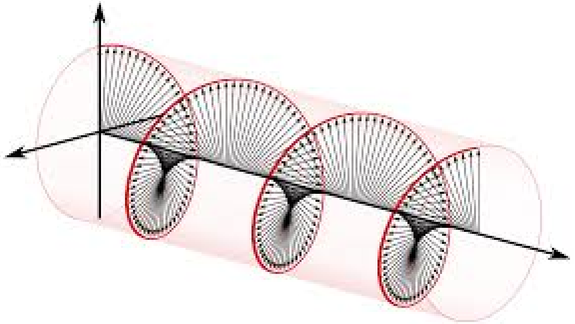


Polarization

- Polarization: Orientation of E-field relative to direction of motion
- Linearly polarized: Constant orientation
  - Vertical: $\boldsymbol{E}(\boldsymbol{r},t) = E_0\boldsymbol{e}_x\cos(\omega t + kz)$
  - Horizontal: $\boldsymbol{E}(\boldsymbol{r},t) = E_0\boldsymbol{e}_y\cos(\omega t + kz)$



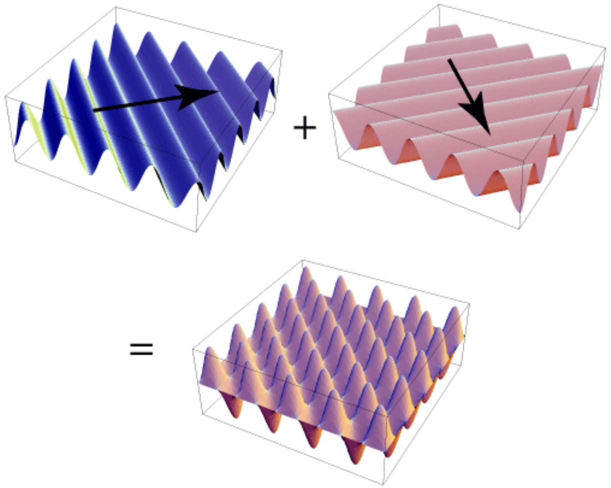
- Also, circularly polarized
  - Sum of V and H that are out of phase
  - $E_0[\boldsymbol{e}_x\cos(\omega t + kz) \pm \boldsymbol{e}_y\sin(\omega t + kz)]$
  - Called left hand and right hand



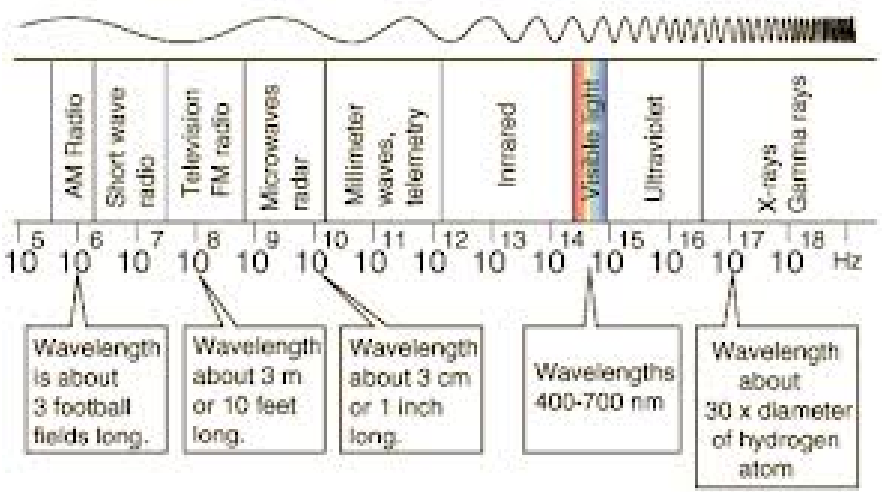
- Two degrees of freedom:
  - Consider any plane wave in some direction
  - Can be decomposed as V + H or LH + RH

Plane Wave Decomposition

- Every electric field is a linear combination of plane waves
- Each plane wave in the decomposition has:
  - Frequency
  - Direction of motion
  - Gain, Phase
  - One of two polarization
- Decomposition can be found from a 4D Fourier transform
  - $\boldsymbol{E}(x,y,z,t) \Rightarrow \hat{E}_V(k_x,k_y,k_z,f)$ and $\hat{E}_H(k_x,k_y,k_z,f)$
  - Converts time + space $\Rightarrow$ wavenumber and frequency
  - Note that there are two polarization components
- This decomposition is used in many EM solvers
  - And your EM class if you take it

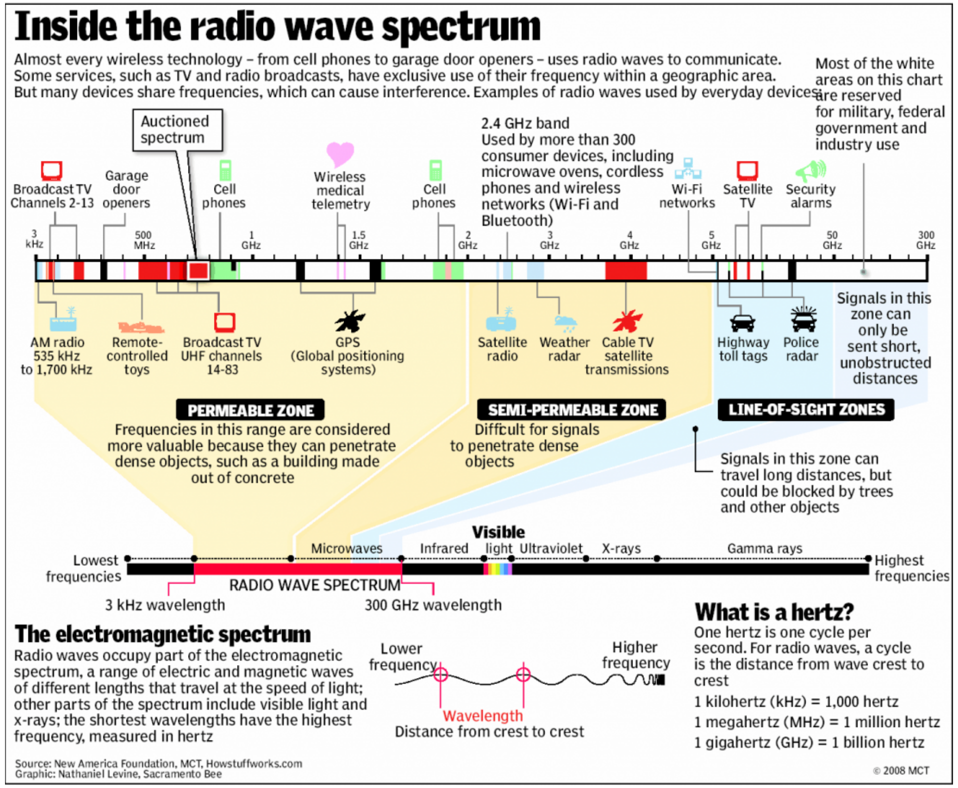


EM Spectrum



- ❑Frequency of EM radiation has wide range
- ❑Encompasses many forms of radiation
- ❑Radio waves are uniquely valuable since they can propagate far

Radio Spectrum



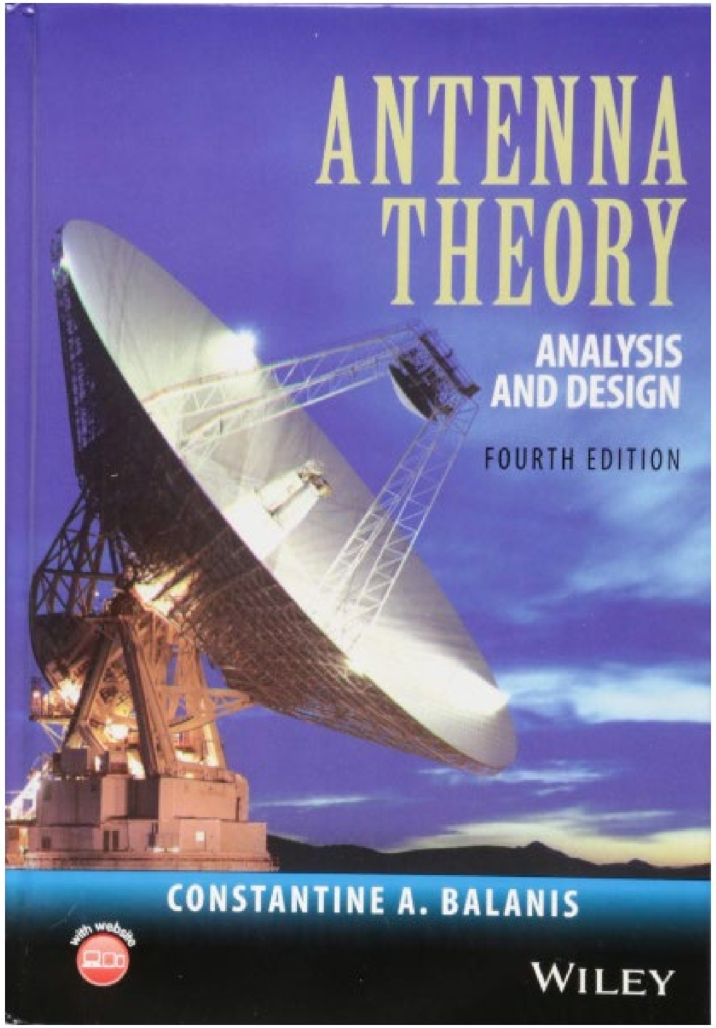
Outline

- ❑ Basics of Electromagnetic Waves
- ❑ Basics of Antennas
- ❑ Free Space Propagation

Excellent Text for Antennas

- This section based on classic text
  - Figures are from this text
- Balanis, “Antenna Theory”
- Full EM theory
- Many excellent problems and examples
- Designed for RF engineers

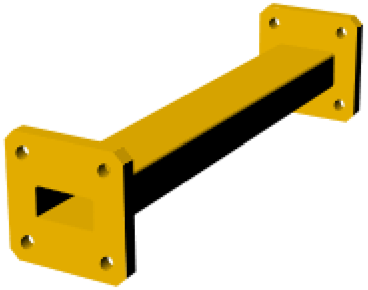
- We will use only a small portion here
- Take an EM class for more!



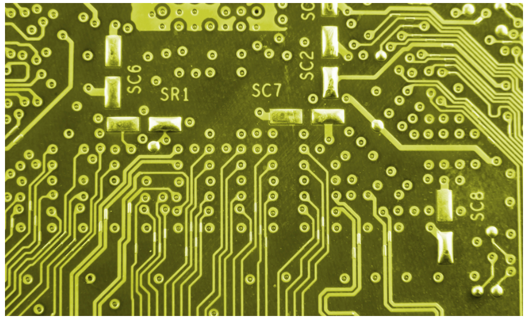
Waveguides and Transmission Lines

- Transmission lines and waveguides: Any structure to guide waves with minimal loss
- Some texts:
  - Transmission lines refer to conductors and waveguides to hollow structures
- Many examples

Coaxial cable



Waveguide



PCB traces
Microstrip: External layer
Stripline: Internal layer

Antenna

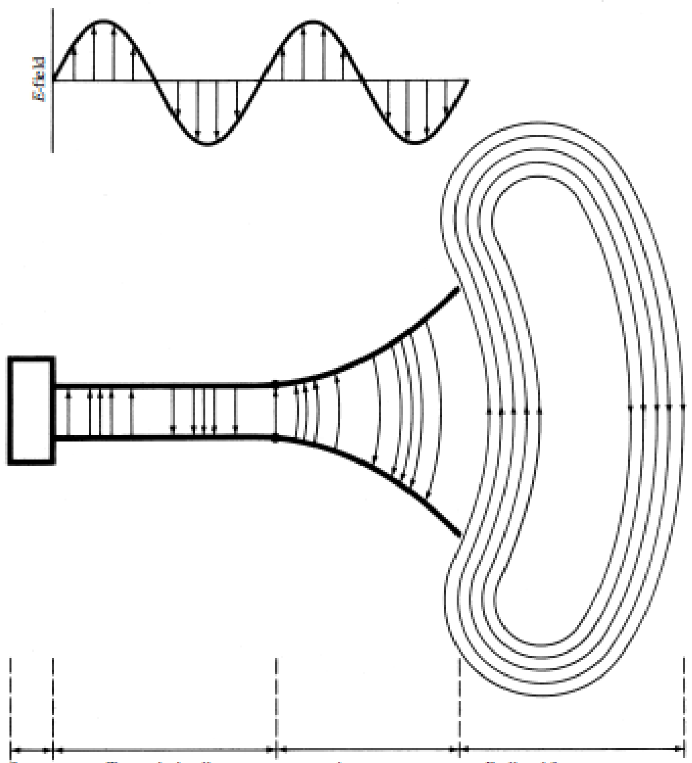


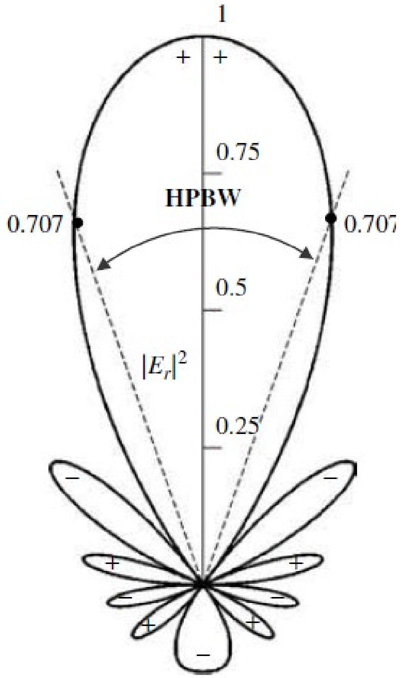
Figure 1.1 Antenna as a transition device.

- Transmit antenna: Radiates electromagnetic waves
- Converts signals:
  - From guided signals in transmission lines to
  - To radiation in free space
- Receive antenna: Collects EM wave

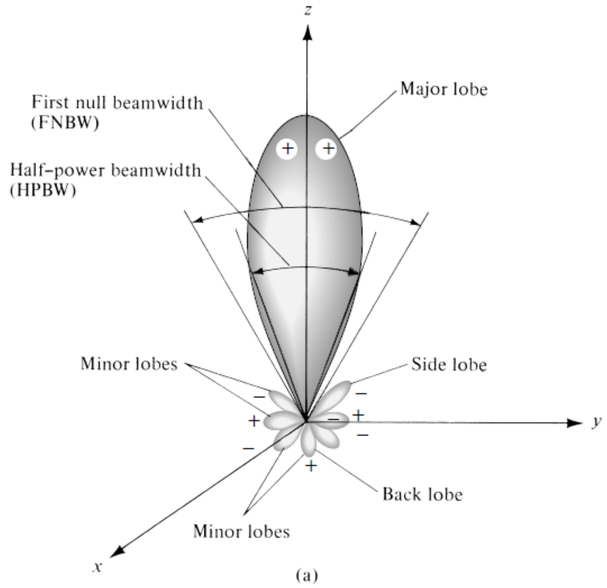
USRP with four vertical antenas

Radiation Patterns

- Aantenna radiation typically shown via a pattern
  - Value of scalar as a function of position
  - Antenna usually at origin
  - Orientation of the antenna is important
- Many possible quantities:
  - Power, electric field, …
  - Normalized or un-normalized
- Can be 2D or 3D



2D

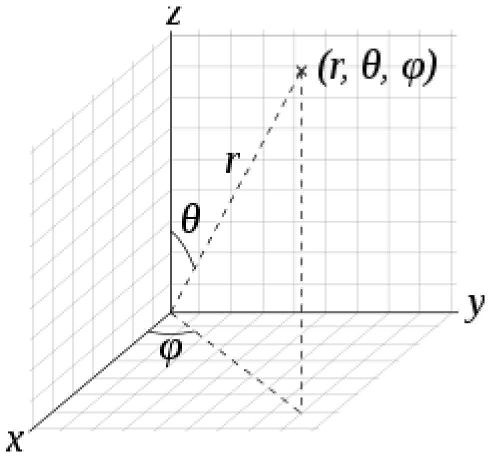


3D

Spherical Coordinates

- Radiation patterns are often given in spherical coordinates
- Spherical coordinates: (φ, θ, r)
  - $\varphi \in [-\pi,\ \pi]$: Azimuth, counter-clockwise angle in xy plane
  - $\theta = \theta_{el} \in [\frac{\pi}{2}\ \frac{\pi}{2}]$: Elevation, angle from xy plane
  - $r \geq 0$: Radius from origin
- Many texts use polar or inclination angle:
  - Use $\theta_{inc} = \frac{\pi}{2} - \theta_{el} \in [0, \pi]$
  - Measures angle from z axis
  - Most antenna and math texts use polar form
  - But, MATLAB antenna toolbox uses elevation form
- Remember right hand rule!

Polar coordinates



Spherical (polar form) ⇔ Cartesian

$$r = \sqrt{x^2 + y^2 + z^2}, \qquad x = r \sin\theta \cos\varphi,$$

$$\varphi = \arctan\frac{y}{x}, \qquad y = r \sin\theta \sin\varphi,$$

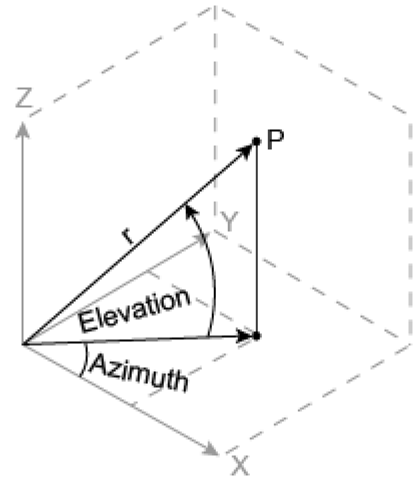
$$\theta = \arccos\frac{z}{\sqrt{x^2 + y^2 + z^2}} \qquad z = r\cos\theta.$$

Spherical Coordinates in MATLAB

- Conversion between spherical and cartesian

```
% Generate four random points in 3D
X = randn(3,4);

% Compute spherical coordinates of a matrix of points
% Note these are in radians!
[az, el, rad] = cart2sph(X(1,:), X(2,:), X(3,:));

% Convert back
[x,y,z] = sph2cart(az,el,rad);
Xhat = [x; y; z];
```

```
x = r .* cos(elevation) .* cos(azimuth)
y = r .* cos(elevation) .* sin(azimuth)
z = r .* sin(elevation)
```



- Conversion to a coordinate system

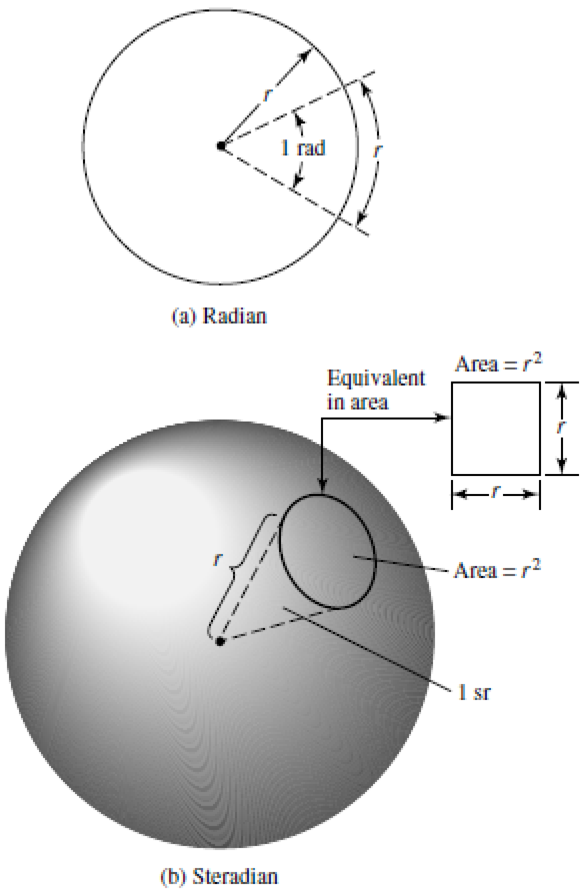
```
%% Conversion to a new frame of reference

% Angles of new frame of reference
% Note these are in degrees!
az1 = 0;
el1 = 45;

% Rotate to the new frame of reference
% This takes row vectors!
X1 = cart2sphvec(X,az1,el1);
```

Radians and Steradians

- Radian:
  - Circle of radius one
  - Angle for unit length on circumference
  - 2π radians in the circle
- Steradian
  - Defined on sphere of radius one
  - Angles corresponding to unit area on surface
  - 4π sr in the sphere
- Infinitesimal area and solid angle:

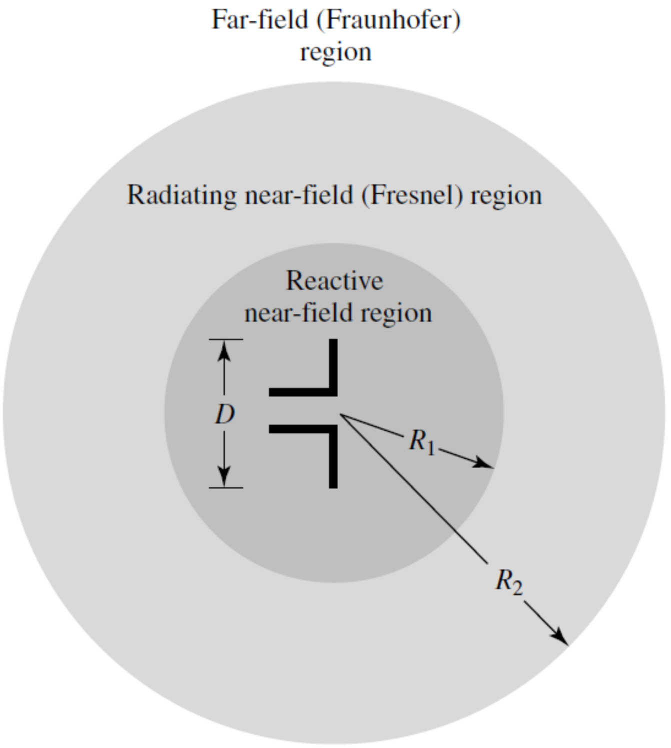
$$dA = r^2 \sin\theta \, d\theta \, d\phi \quad (\mathrm{m}^2) \qquad d\Omega = \frac{dA}{r^2} = \sin\theta \, d\theta \, d\phi \quad (\mathrm{sr})$$

  - Note: θ is the inclination angle not elevation



Field Regions

- ❑Antenna patterns depend on the region
- ❑Reactive near field:
  - Reactive pattern dominates
- ❑Radiating near field or Fresnel region:
  - Angular pattern depends on distance
- ❑Far field or Fraunhofer region:
  - Angular pattern independent of distance
  - Radiation is approximately plane waves
- ❑Can be approximately calculated using:
  - D: Maximum antenna dimension
  - λ: Wavelength

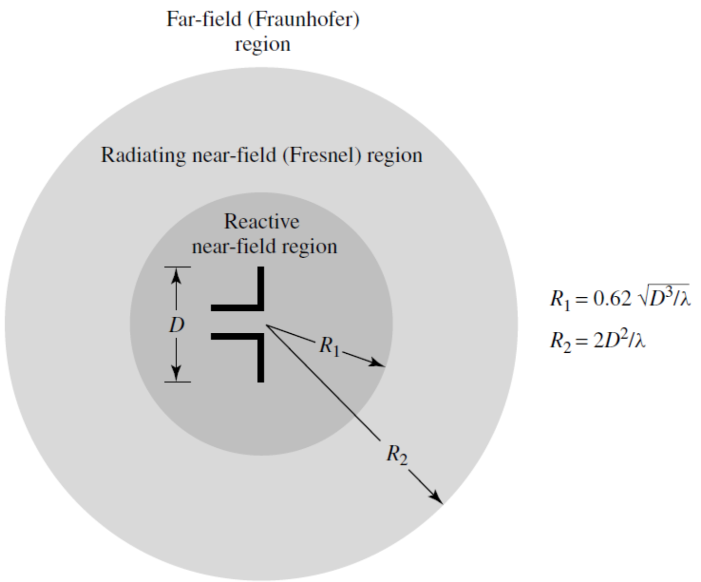


$R_1 = 0.62\sqrt{D^3/\lambda}$

$R_2 = 2D^2/\lambda$

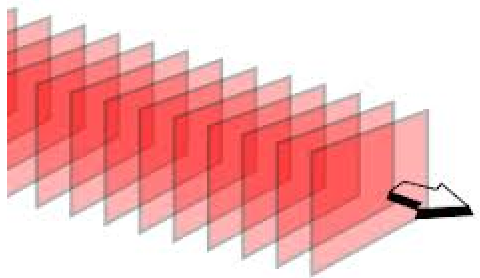
Rayleigh Distance

- Distance R_2 to far-field = Rayleigh distance
- Most cellular / WLAN systems operate in far field
- Ex 1: Half wavelength dipole antenna
  - f_c = 2.3 GHz:
  - $D = \frac{\lambda}{2}$, $R_2 = \frac{2D^2}{\lambda} = \frac{\lambda}{2} = 6.5$ cm
- Ex 2: Large cellular base station
  - $D \approx 7$m, f_c = 2.3 GHz
  - $R_2 = 751$ m
- Ex 3: MmWave wide aperture antenna
  - $D \approx 40$ cm, f_c = 140 GHz
  - $R_2 = 149$ m



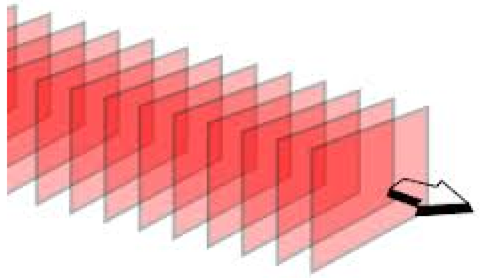
Radiation Density

- Recall instantaneous energy flux for a plane wave: $\boldsymbol{S}(t) = \frac{1}{\mu}\boldsymbol{E}(t) \times \boldsymbol{B}(t) = \frac{1}{\mu}\|\boldsymbol{E}(t)\|^2\boldsymbol{n}$
  - $\boldsymbol{n}$ = normal vector in direction of the plane wave
- Typically consider fields at some frequency $\omega = 2\pi f$: $\boldsymbol{E}(t) = Re\left[\boldsymbol{E}e^{i\omega t}\right]$
- Time average power $\langle \boldsymbol{S}(t) \rangle = \frac{1}{2\mu}\|\boldsymbol{E}\|^2\boldsymbol{n}$
  - Note factor of 2
- Can write $\langle \boldsymbol{S}(t) \rangle = W\,\boldsymbol{n}$, $W = \frac{1}{2\mu}\|\boldsymbol{E}\|^2$
  - Radiation density: $W = W(r,\theta,\phi) = \frac{1}{2\mu}|E(r,\theta,\phi)|^2$ = radiation density
  - Maximum power available if aligned in the direction $\boldsymbol{n}$
  - Units W/m^2
  - This is a function of position $W(r,\theta,\phi)$



Radiation Intensity

- From previous slide: Radiation density: $W = W(r,\theta,\phi) = \frac{1}{2\mu}|\boldsymbol{E}(r,\theta,\phi)|^2$
  - Units $\frac{W}{m^2}$
- Also define radiation intensity: $U = r^2 W = \frac{r^2}{2\mu}|\boldsymbol{E}(r,\theta,\phi)|^2$
  - Watts per solid angle: $\frac{W}{sr}$
- In far field, radiation pattern typically decays as:
  - $\boldsymbol{E}(r,\theta,\phi) \approx \frac{1}{r}\boldsymbol{E}_0(\theta,\phi)$
  - In this case, $U(r,\theta,\phi) = r^2 W(r,\theta,\phi) = \frac{r^2}{2\mu}|\boldsymbol{E}(r,\theta,\phi)|^2 \approx \frac{1}{2\mu}|\boldsymbol{E}_0(\theta,\phi)|^2$
  - Only depends on angular position $U(r,\theta,\phi) = U(\theta,\phi)$
  - Does not depend on distance r



Total Radiated Power

- Total radiated power:

$$P_{rad} = \iint U d\Omega = \int_{-\pi/2}^{\pi/2} \int_{-\pi}^{\pi} U(\theta, \phi) \cos\theta \, d\phi d\theta$$

  - Units is Watts
  - Note $\cos\theta$ term! Angle here is elevation angle not polar angle
- Typically measured in dBm or dBW:
  - $P_{rad}[\mathrm{dBm}] = 10\log_{10}\left[\frac{P_{rad}}{1\,\mathrm{mW}}\right], P_{rad}[\mathrm{dBW}] = 10\log_{10}\left[\frac{P_{rad}}{1\,\mathrm{W}}\right]$
  - Power relative to mW or W
- Review dB calculations if you forgot!
  - Ex: A mobile transmitter transmits 250 mW. What is the power in dBm?
  - Ans: 250 = 1000/4=$\frac{10^3}{2^2}$. In dBm: $3(10) - 2(3) = 24$ dBm

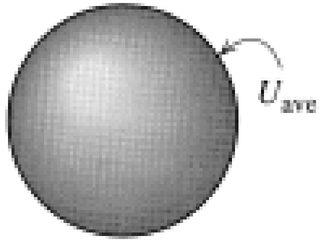
Typical Wireless Power Transmit Levels

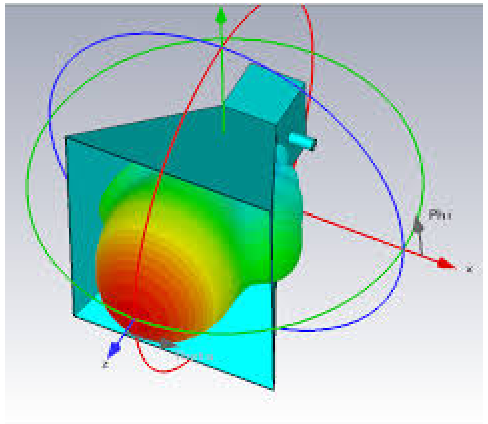
- 100 kW = 80 dBm: Typical FM radio transmission with 50 km radius
- 1 kW = 60 dBm: Microwave oven element (most of this doesn't escape)
- ~300 W = 55 dBm: Geostationary satellite
- 250 mW = 24 dBm: Cellular phone maximum power (class 2)
- 200 mW = 23 dBm: WiFi access point
- 32 mW = 15 dBm: WiFi transmitter in a laptop
- 4 mW = 6 dBm: Bluetooth 10 m range
- 1 mW = 0 dBm: Bluetooth, 1 m range

Antenna Directivity

- Isotropic antenna: Radiates uniformly in all directions
  - Theoretically construct
- Most antennas concentrate power in certain angles
- Antenna directivity:
  - $D(\theta,\phi) = \frac{4\pi U(\theta,\phi)}{P_{rad}}$ [dimensionless]
  - Measures power at an angle relative to average
  - Average in linear domain is one
- Max directivity: $D_{max} = \max D(\theta,\phi)$
  - Directivity in direction with maximum power
- Typically measured in dBi
  - dB relative to isotropic
  - $D(\theta,\phi)\ [dBi] = 10\log\left[\frac{4\pi U(\theta,\phi)}{P_{rad}}\right]$

Theoretical isotropic antenna





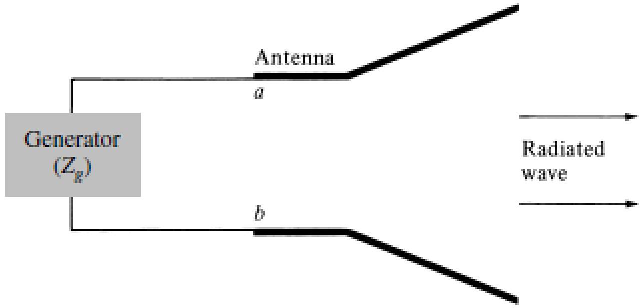
Horn antenna with directivity

Antenna Gain and Efficiency

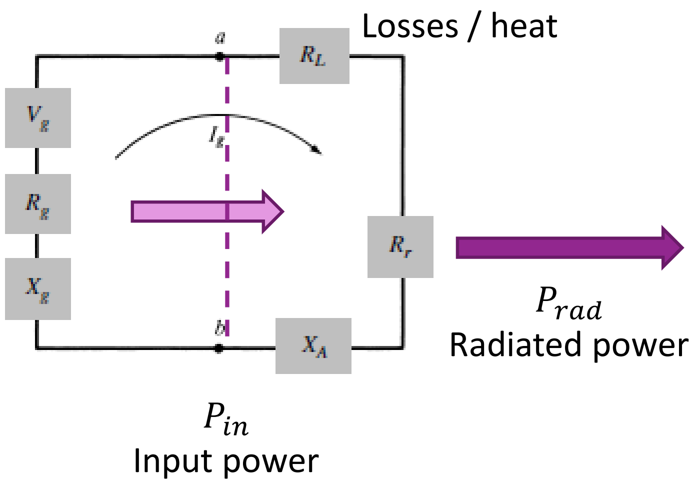
- Most antennas have losses
- Define efficiency:

$$\epsilon = \frac{P_{rad}}{P_{in}} \in [0,1]$$

  - Radiated to input power in TX mode
  - Remaining power is lost in heat in the antenna
  - Note: Some text include losses in the generator
- Lossless antenna: $\epsilon = 1$
- Antenna gain:
  - $G(\theta,\phi) = \epsilon D(\theta,\phi) = \frac{4\pi U(\theta,\phi)}{P_{in}}$
  - Radiation intensity per unit input power
  - For losses antennas, gain = directivity



(a) Antenna in transmitting mode



Antenna Toolbox in MATLAB

- Powerful routines for:
  - Design and analysis of antennas
  - Radiation patterns
- Supports many antennas
- Accurate EM modeling
- Free to NYU students
  - Just download it with your MATLAB

Antenna Toolbox

Design, analyze, and visualize antenna elements and antenna arrays

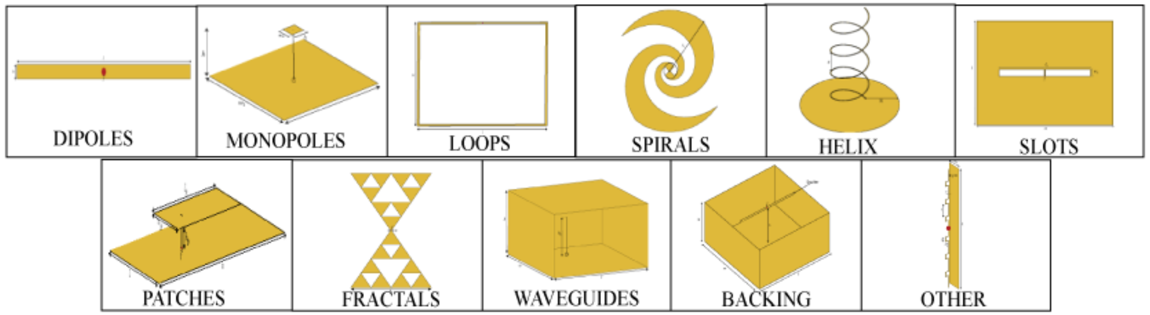
Antenna Toolbox™ provides functions and apps for the design, analysis, and vis
antennas using either predefined elements with parameterized geometry or arbi

Antenna Toolbox uses the method of moments (MoM) to compute port propertie
such as the near-field and far-field radiation pattern. You can visualize antenna g

You can integrate antennas and arrays into wireless systems and use impedanc
beam forming and beam steering algorithms. Gerber files can be generated fron
large platforms such as cars or airplanes and analyze the effects of the structure
using a variety of propagation models.

Get Started

Learn the basics of Antenna Toolbox

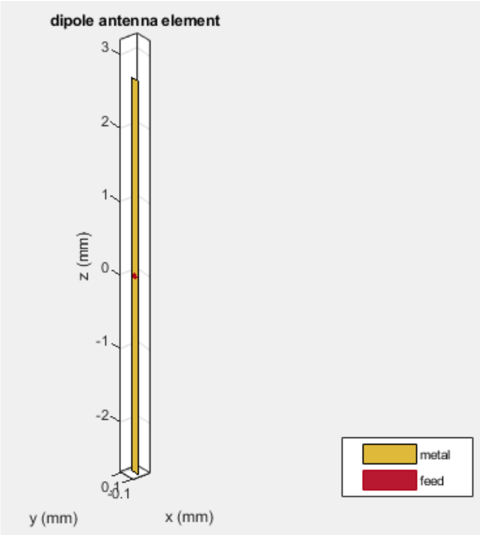


Patterns in MATLAB: Dipole Example

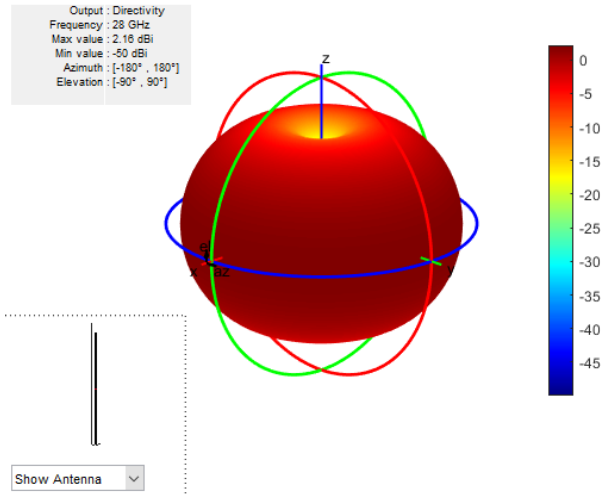
- ❑MATLAB has powerful tools for calculating antenna patterns

```
%% Simulation constants
fc = 28e9;
vp = physconst('lightspeed');
lambda = vp/fc;
```

```
%% Dipole antenna
% Construct the antenna object
ant = dipole(...
    'Length', lambda/2,...
    'Width', 0.01*lambda );
```




```
ant.show();
```



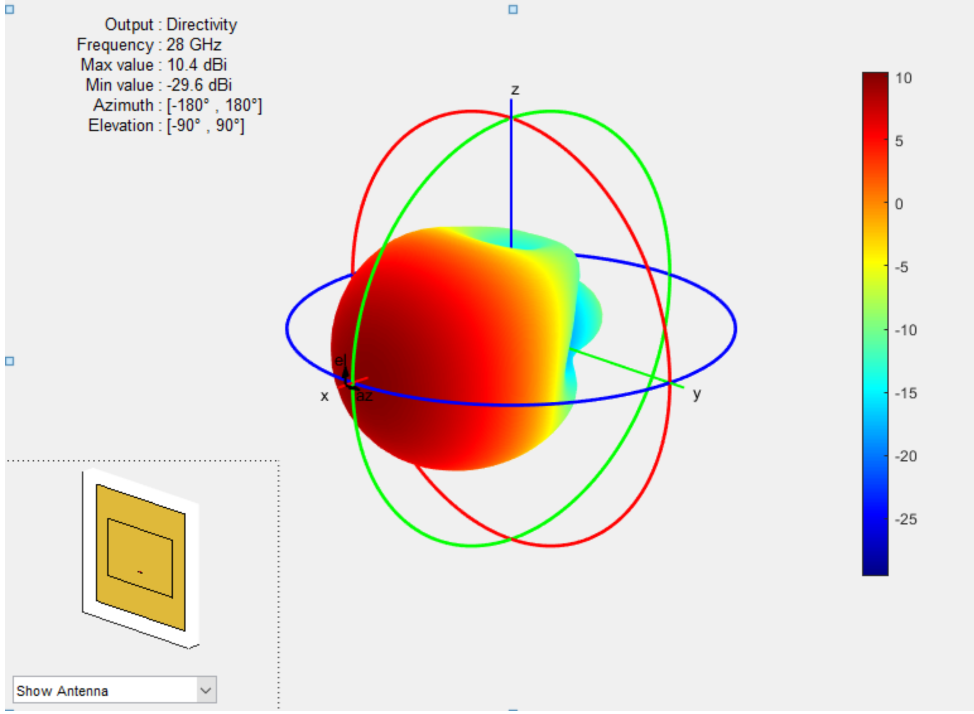

```
ant.pattern(fc)
```

Microstrip Patch Example

- A more complex antenna
- Many other parameters
  - Substrate selection (e.g. FR4, Rogers)
  - Shapes, notches, …

```
%% Create a patch element
len = 0.49*lambda;
groundPlaneLen = lambda;
ant2 = patchMicrostrip(...
    'Length', len, 'Width', 1.5*len, ...
    'GroundPlaneLength', groundPlaneLen, ...
    'GroundPlaneWidth', groundPlaneLen, ...
    'Height', 0.01*lambda, ...
    'FeedOffset', [0.25*len 0]);
```

```
%%
% Tilt the element so that the maximum energy is in the x-axis
ant2.Tilt = 90;
ant2.TiltAxis = [0 1 0];

% Display the antenna pattern after rotation
ant2.pattern(fc);
```



Outline

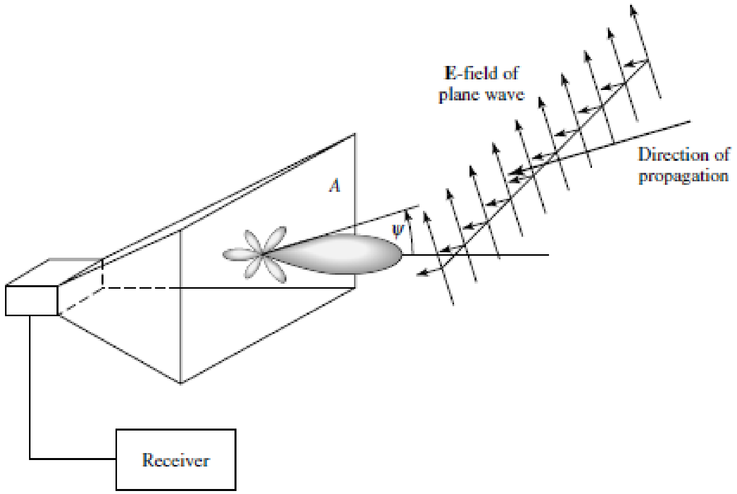
- Basics of Electromagnetic Waves
- Basics of Antennas
- Free Space Propagation

Antenna Effective Aperture

- Suppose RX antenna sees incident plane wave
  - Assume polarization aligned to the antenna
- The effective antenna aperture (or area):

$$A_e(\theta,\phi) = \frac{W(\theta,\phi)}{P_L} \quad [m^2]$$

  - W = Power density of incident wave [W / m^2]
  - P_L = Power delivered to load at the receiver [W]
- The effective area that the antenna collects
  - We will see this is different than the physical aperture
- A_e will depend on the direction of arrival

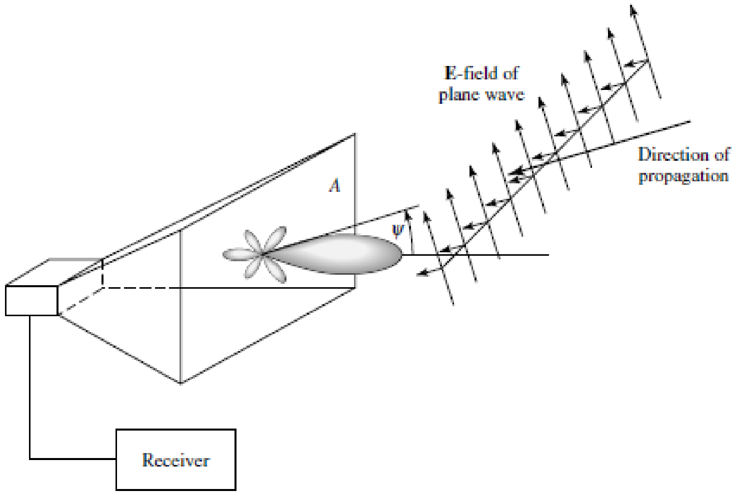


Aperture and Directivity

- From previous slide, effective aperture is: $A_e(\theta,\phi) = \frac{W(\theta,\phi)}{P_L}$ $[m^2]$
  - Ratio of received power to incident radiation density
- Aperture-directivity relation:

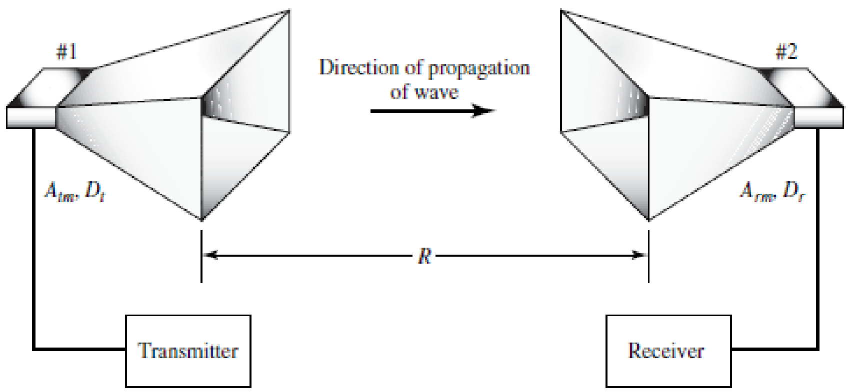
$$\boxed{A_e(\theta,\phi) = D(\theta,\phi)\,\frac{\lambda^2}{4\pi}}$$

  - True for all lossless antennas
  - Proof: next slide
- Consequence: Average aperture is always $\frac{\lambda^2}{4\pi}$
  - Why? $\frac{1}{4\pi}\iint A_e(\theta,\phi)\cos\theta\, d\theta d\phi = \frac{\lambda^2}{(4\pi)^2}\iint D(\theta,\phi)\cos\theta\, d\theta d\phi = \frac{\lambda^2}{4\pi}$
  - Independent of the physical size of the antenna



Proof of the Aperture-Directivity Relation

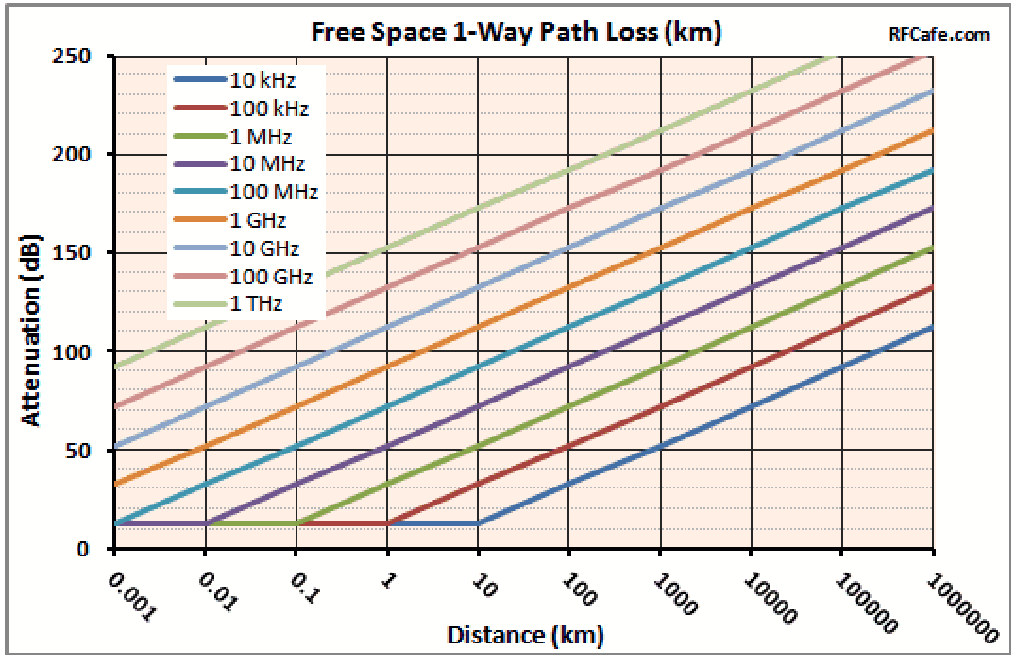
- Suppose Ant 1 transmits power P_t
- Radiation density is: $W = \frac{D_1 P_t}{4\pi R^2}$
- Received power at Ant 2: $P_r = A_2 W = \frac{A_2 D_1 P_t}{4\pi R^2} \Rightarrow \frac{P_r}{P_t} = \frac{A_2 D_1}{4\pi R^2}$
- TX from Ant 2, the gain must be the same: $\frac{P_r}{P_t} = \frac{A_1 D_2}{4\pi R^2}$
  - This is a consequence of reciprocity
- Hence, for any two antennas: $\frac{D_1}{A_1} = \frac{D_2}{A_2}$
- From simple antenna calculations for a short dipole:
  - $D_2 = \frac{3}{2},\ A_2 = \frac{3\lambda^2}{8\pi} \Rightarrow \frac{D_2}{A_2} = \frac{4\pi}{\lambda^2}$ (Needs basic EM theory)



Friis' Law

- Consider two lossless antennas in free space
- From previous slide: $\frac{P_r}{P_t} = \frac{A_1 D_2}{4\pi R^2}$
- From aperture-directivity relation: $A_1 = D_1 \frac{\lambda^2}{4\pi}$
- This leads to Friis' Law (for lossless antennas):

$$\boxed{\frac{P_r}{P_t} = D_1 D_2 \left(\frac{\lambda}{4\pi R}\right)^2}$$



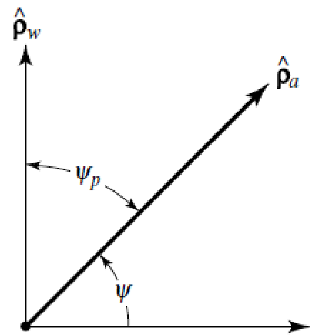
  - Path loss is proportional to R^2
  - Path loss Inversely proportional to $\lambda^2 \Rightarrow$ proportional to f_c^2

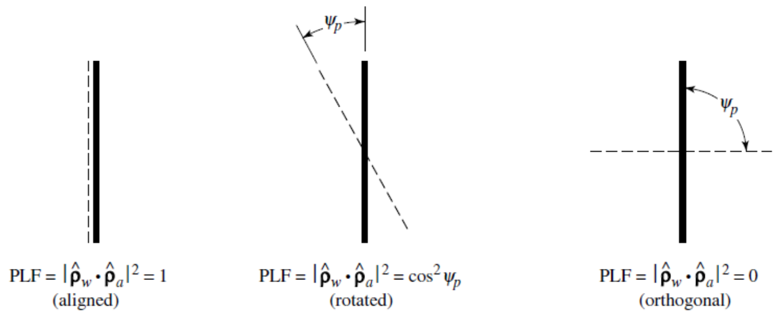
Polarization Loss

- Friis' Law assumes incident wave is aligned in polarization
- In general, need to consider polarization loss
- Recall: polarization vector for a plane wave:
  - Direction of the E-field in phasor notation
  - A complex vector in 3-dim
- Polarization loss factor:

$$PLF = |\boldsymbol{\rho}_a \cdot \boldsymbol{\rho}_w|^2 = \cos^2 \psi_p$$

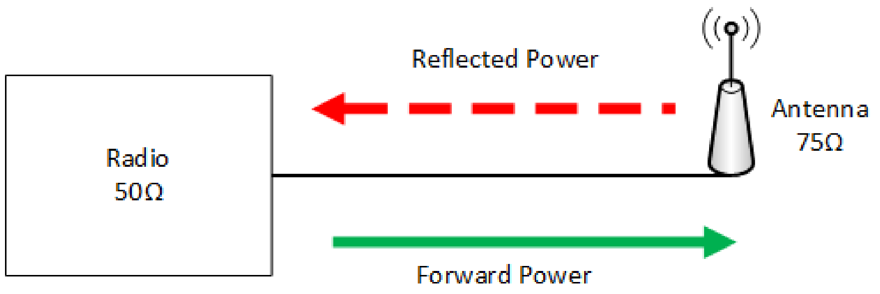
  - $\boldsymbol{\rho}_a$: Polarization vector of the TX wave from antenna
  - $\boldsymbol{\rho}_w$: Polarization vector of the RX incident wave
  - ψ_p: Angle between them

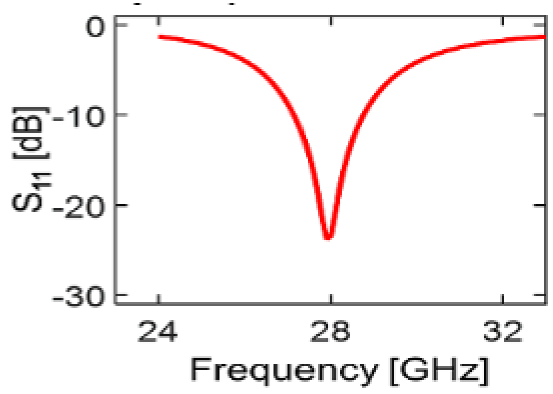




Antenna Impedance and Matching

- Not all power from radio may be delivered to antenna
- Some is reflected back
- Described by reflection coefficient Γ
  - Also referred to as S_{11}
  - Complex ratio of forward to reverse wave
- Also described by impedance mismatch:
  - $\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$
- Fraction of power transferred: $1 - |\Gamma|^2$
- Also given as voltage standing wave ratio (VSWR) $= \frac{1+|\Gamma|}{1-|\Gamma|}$

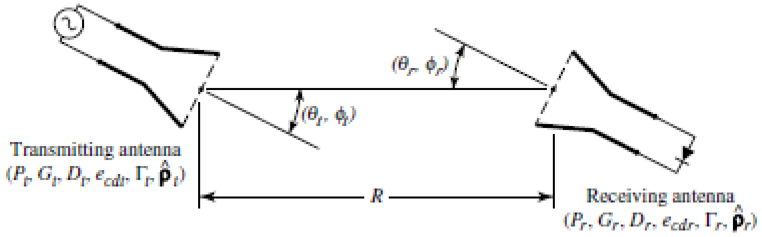




Ali et al, Small Form Factor PIFA Antenna Design at 28 GHz for 5G Applications, 2019

Friis’ Law with Losses

- Three losses in practice:
  - Polarization loss
  - conductive / dielectric loss
  - Impedance mismatch



- Friis’ Law with lossy antennas:

$$\frac{P_r}{P_t} = \epsilon_1 \epsilon_2 (1 - |\Gamma_1|^2)(1 - |\Gamma_2|^2) D_1 D_2 \left(\frac{\lambda}{4\pi R}\right)^2 \cos^2 \theta_{POL}$$

  - ϵ_i: Efficiency of antenna
  - θ_{POL}: Angle between the polarization vectors
  - Note that gain is: $G_i = \epsilon_i D_i$