API-201 ABC REVIEW SESSION #6

Friday, October 21

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Lecture recap

Sampling distribution

Suppose there is some proportion ***p*** that we want to measure. We usually don't have access to information for all the population, so we estimate ***p*** by computing the proportion ***p̂*** in a random sample.

The sample proportion ***p̂*** is random, as it may produce a different estimate if we apply it to a different sample. The **sampling distribution** is the distribution of the sample proportion ***p̂***. In other words, it is a probability distribution formed by the estimates we obtain from calculating the proportion for different samples from the population of interest.

The sample proportion ***p̂*** is a random variable, so it has an expected value and a standard deviation. The **Central Limit Theorem** states that for a large enough sample (n > 30), the distribution of ***p̂*** is approximately normal:

$$N\left(p, \sqrt{\frac{p(1-p)}{n}}\right)$$

If we were interested in measuring a population mean μ instead of a population proportion ***p***, the distribution of $\hat{\mu}$ will have an approximately normal sampling distribution with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$:

$$N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

The standard deviation of the sampling deviation is called the **standard error**.

Sampling distribution in R

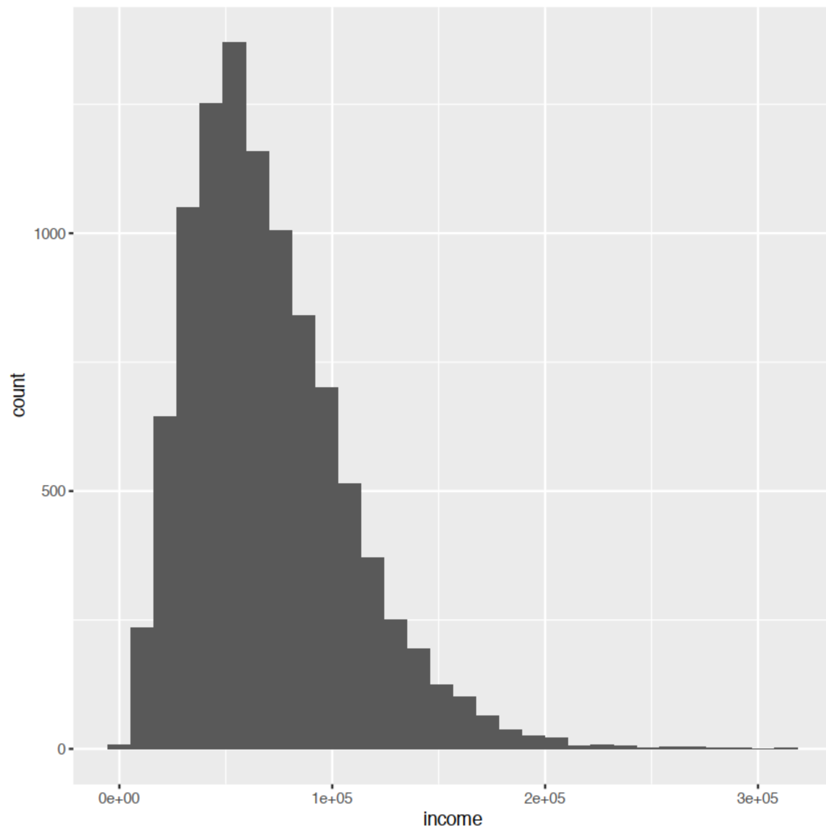
Suppose we had income data for the entire population of a city. Does this look like a normal distribution?

```
library(tidyverse)
```

```
# Simulate population data
set.seed(101)
data <- tibble(income = 1e4 * rchisq(10000, df = 7))

# Plot population distribution
ggplot(data) + geom_histogram(aes(x = income))

# Calculate population mean and std. deviation
c("Population Income Mean μ" = mean(data$income),
  "Population Income Std. Dev σ" = sd(data$income))
```

```
`stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```

Population Income Mean μ: 70069.4567841333 Population Income Std. Dev σ: 37279.4133939986

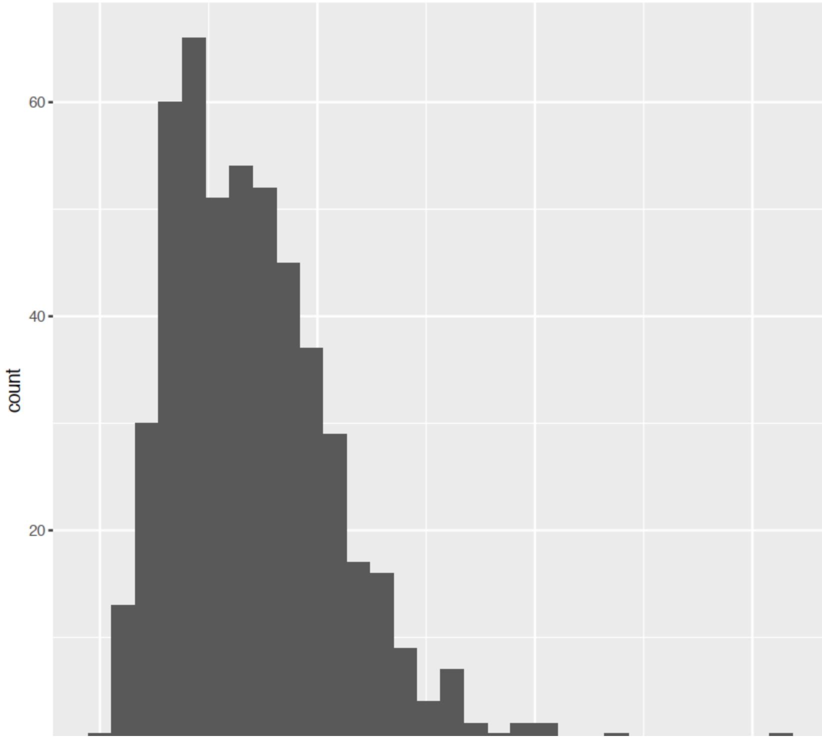


We can sample 500 people from the population using the function `sample`. Does it look like a normal distribution?

```
# Take one random sample
one_sample <- tibble(income = sample(data$income, 500))

# Plot sample distribution
ggplot(one_sample) + geom_histogram(aes(x = income))

# Calculate sample mean and std. deviation
c("Sample Income Mean μhat" = mean(one_sample$income),
  "Sample Income Std. Dev σhat" = sd(one_sample$income))
```

```
`stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```

Sample Income Mean μhat: 70780.1807103224 Sample Income Std. Dev σhat: 39195.5567297244

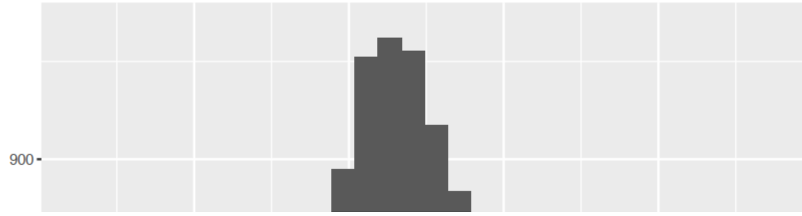


We can use the `replicate` function to sample from the population many times and calculate for each simulation the sample mean $\hat{\mu}$.

```
# Sample many times and calculate many sample means
set.seed(1)
income_means <- tibble(income_mean = replicate(10000, mean(sample(data$income, 500))))

# Plot sampling distribution
ggplot(income_means) + geom_histogram(aes(x = income_mean))

# Calculate mean and standard deviation of sampling distribution
c("Sampling Distribution Mean" = mean(income_means$income_mean),
  "Standard error" = sd(income_means$income_mean))
```

```
`stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```

Sampling Distribution Mean: 70055.5222839946 Standard error: 1624.10868816474



As you can see, even though the population distribution is not normally distributed, the distribution of the sample mean is approximately normally distributed.

Confidence intervals

Going back to the example about sample proportions, we can use the sampling distribution of $\hat{p}$ to tell us how confident we are in our estimate. if the sampling distribution is normal, then we can construct a 95% confidence interval around $\hat{p}$ by using the mean and standard deviation of $\hat{p}$. However, given that we don't know p, we use both the sample proportion and the estimated standard error instead, such that:

$$CI = \hat{p} \pm 2SD(\hat{p})$$

where:

$$SD(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

This implies that 95% of all possible confidence intervals will contain the true value of p.

If we wanted to estimate a mean instead, the 95% confidence interval has the same form as before:

$$CI = \hat{\mu} \pm 2SD(\hat{\mu})$$

Plugging in the standard deviation of the sample mean, we get:

$$CI = \hat{\mu} \pm \frac{2\hat{\sigma}}{\sqrt{n}}$$

Exercise: Project STAR

The Project STAR (for Student-Teacher Achievement Ratio) was designed to determine the effect of smaller class size in the earliest grades on short-term and long-term pupil performance (source). Over 7,000 students in 79 schools across the state of Tennessee were randomly assigned into one of three interventions: small class (13 to 17 students per teacher), regular class (22 to 25 students per teacher), and regular-with-aide class (22 to 25 students with a full-time teacher's aide). Classroom teachers were also randomly assigned to the classes they would teach. The interventions were initiated as the students entered school in kindergarten and continued through third grade.

In this exercise, we are going to use data from the STAR Project to learn about the pupils involved in the project through visualization and measure the association between classroom size and student achievement.

[Download the data using this link.]

Data Dictionary

- `stdntid`: unique student ID
- `gender`: student gender; F - female, M - male
- `race`: student's race; W - White, B - Black, A - Asian, H - Hispanic, O - Other
- `gkschid`: kindergarten school ID
- `gktchid`: kindergarten teacher ID
- `gkclasstype`: kindergarten class type
- `gkclasssize`: kindergarten class size; S - Small, R - Regular/Large
- `gktyears`: teacher experience in years
- `gktreadss`: student's kindergarten reading score
- `gktmathss`: student's kindergarten math score
- `gkpresent`: number of days student present in kindergarten
- `hsactenglish`: student's high school ACT english score
- `hsactmath`: student's high school ACT math score
- `hsactread`: student's high school ACT reading score
- `hsactscience`:student's high school ACT science score

1. Upload the Excel file `STAR_data.xlsx` to Google Colab and use `read_excel` to read its first worksheet as a new table called `star_data`. How many rows and columns does this dataset have? Examine the first 10 rows of the data.

```
library(tidyverse)
library(readxl)

# Your answer here!

# START
star_data <- read_excel(path = "STAR_data.xlsx", sheet = 1)

dim(star_data)

head(star_data, 10)
# END
```

11601 · 15

A tibble: 10 × 15

stdntid	gender	race	gkschid	gktchid	gkclasstype	gkclasssize	gktyears	gktreadss	g
<dbl>	<chr>	<chr>	<dbl>	<dbl>	<chr>	<dbl>	<dbl>	<dbl>	
10000	M	W	NA	NA	NA	NA	NA	NA	
10001	M	W	169229	16922904	R	24	5	NA	
10002	F	B	NA	NA	NA	NA	NA	NA	
10003	M	W	NA	NA	NA	NA	NA	NA	
10004	F	B	NA	NA	NA	NA	NA	NA	
10005	M	B	NA	NA	NA	NA	NA	NA	
10006	F	B	NA	NA	NA	NA	NA	NA	
10007	M	B	NA	NA	NA	NA	NA	NA	

2. The code below counts the number of observations with *any* missing value. Drop these observations using the function `drop_na` and create a new dataset called `star_data_clean`. How many observations does this new dataset have?

```
sum(!complete.cases(star_data))

# Your answer here!

# START
star_data_clean <- drop_na(star_data)
nrow(star_data_clean)
# END
```

9354
2247

3a. Plot the racial composition of children in STAR by gender using `geom_col`. The proportions have already been calculated for you below. Sort the bars from largest to smallest.

i. Using `arrange` won't sort the bars for you. Instead you need to reorder the race variable by proportions. To do this, specify the x aesthetic as `race = fct_reorder(race, desc(prop))`.

ii. Map gender to the `fill` aesthetic so that proportions for boys and girls appear as different colors.

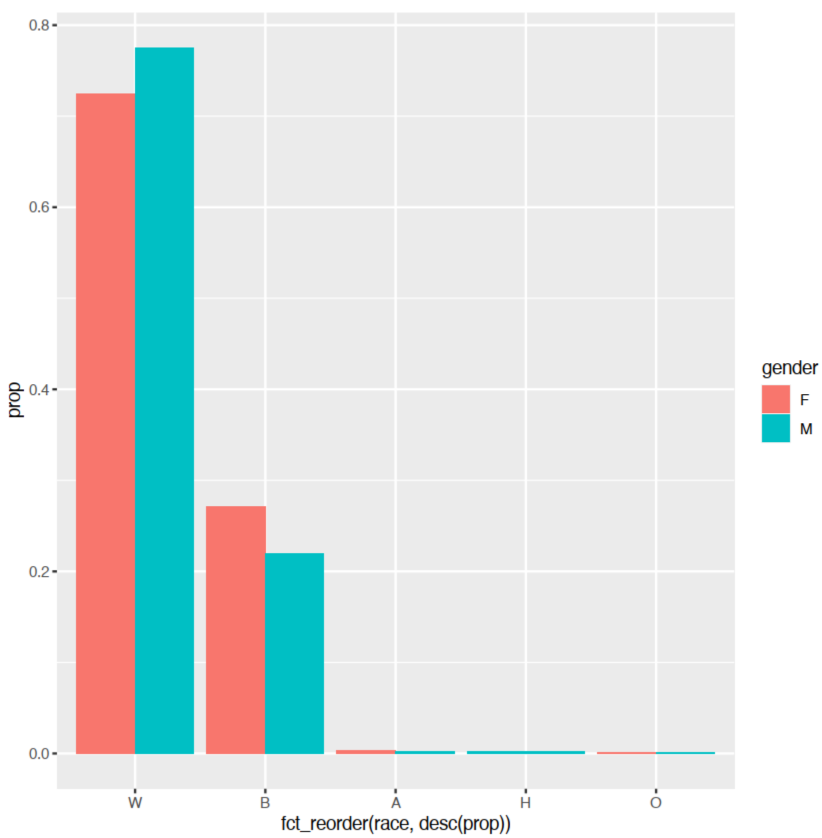
iii. Use the argument `position = "dodge"` of `geom_col` to prevent bars from overlapping.

```
# Calculate proportions
star_race_props <- star_data_clean %>%
    count(race, gender) %>%
    group_by(gender) %>%
    mutate(prop = n / sum(n))

star_race_props
```

```r
# Your answer here!

# START
ggplot(star_race_props) +
    geom_col(aes(x = fct_reorder(race, desc(prop)), y = prop, fill = gender),
             position = "dodge")
# END
```

A grouped_df: 9 × 4

race	gender	n	prop
<chr>	<chr>	<int>	<dbl>
A	F	4	0.003014318
A	M	2	0.002173913
B	F	360	0.271288621
B	M	202	0.219565217
H	M	2	0.002173913
O	F	2	0.001507159
O	M	1	0.001086957
W	F	961	0.724189902
W	M	713	0.775000000



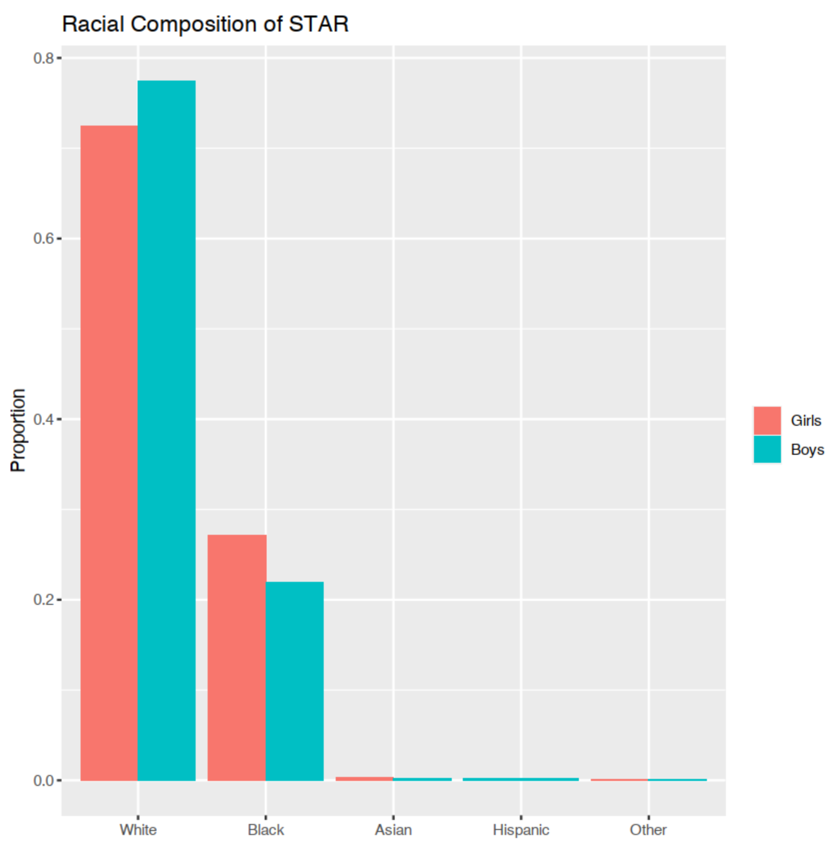
3b. Label each race, label boys and girls, title the plot, and label the axes.

i. To label the races use `scale_x_discrete(labels = c("White", "Black", "Asian", "Hispanic", "Other"))`.

ii. To label boys and girls use `scale_fill_discrete(name = NULL, labels = c("Girls", "Boys"))`

iii. To title and label the axes use `labs(title = "Racial Composition of STAR", x = NULL, y = "Proportion")`.

```
# Your answer here!

# START
ggplot(star_race_props) +
    geom_col(aes(x = fct_reorder(race, desc(prop)), y = prop, fill = gender),
             position = "dodge") +
    scale_x_discrete(labels = c("White", "Black", "Asian", "Hispanic", "Other")) +
    scale_fill_discrete(name = NULL, labels = c("Girls", "Boys")) +
    labs(title = "Racial Composition of STAR", x = NULL, y = "Proportion")
# END
```



3c. Resize the racial category labels, title, and axis titles using the `theme()` function. Center the title.

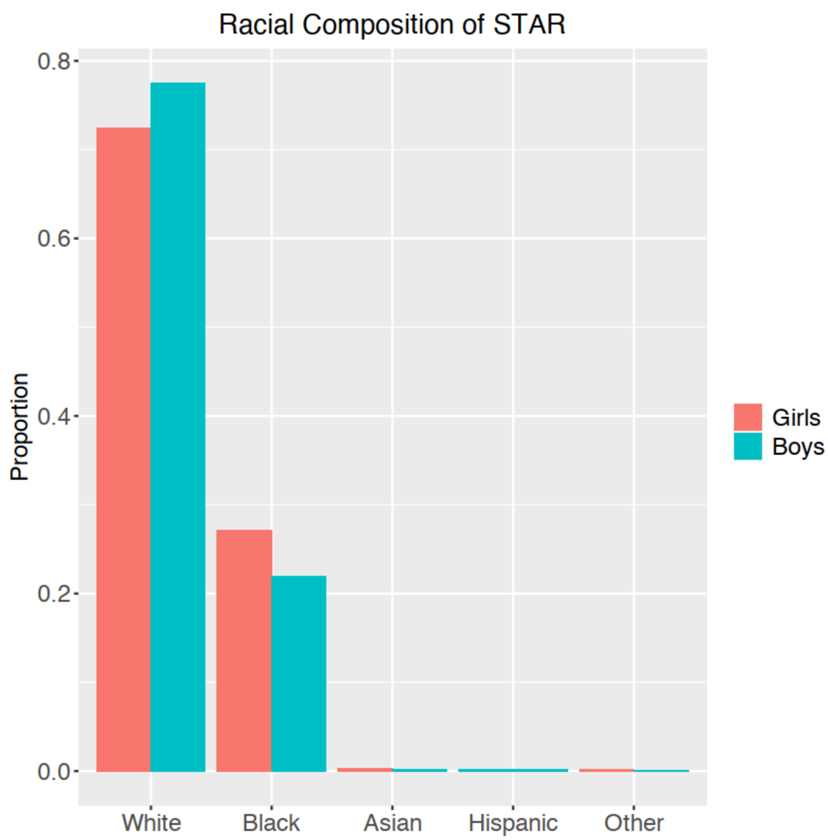
i. To resize and center the title use the argument: `plot.title = element_text(size = 16, hjust = 0.5)`.

ii. To resize the axis *values* use the argument: `axis.text = element_text(size = 14)`.

iii. To resize the axis *titles* use the argument: `axis.title = element_text(size = 14))`

iv. To resize the legend values, use the argument: `legend.text = element_text(size = 14)`.

```
# Your answer here!

# START
ggplot(star_race_props) +
    geom_col(aes(x = fct_reorder(race, desc(prop)), y = prop, fill = gender),
             position = "dodge") +
    scale_x_discrete(labels = c("White", "Black", "Asian", "Hispanic", "Other")) +
    scale_fill_discrete(name = NULL, labels = c("Girls", "Boys")) +
    labs(title = "Racial Composition of STAR", x = NULL, y = "Proportion") +
    theme(plot.title = element_text(size = 16, hjust = 0.5),
          axis.text = element_text(size = 14),
          axis.title = element_text(size = 14),
          legend.text = element_text(size = 14))
# END
```

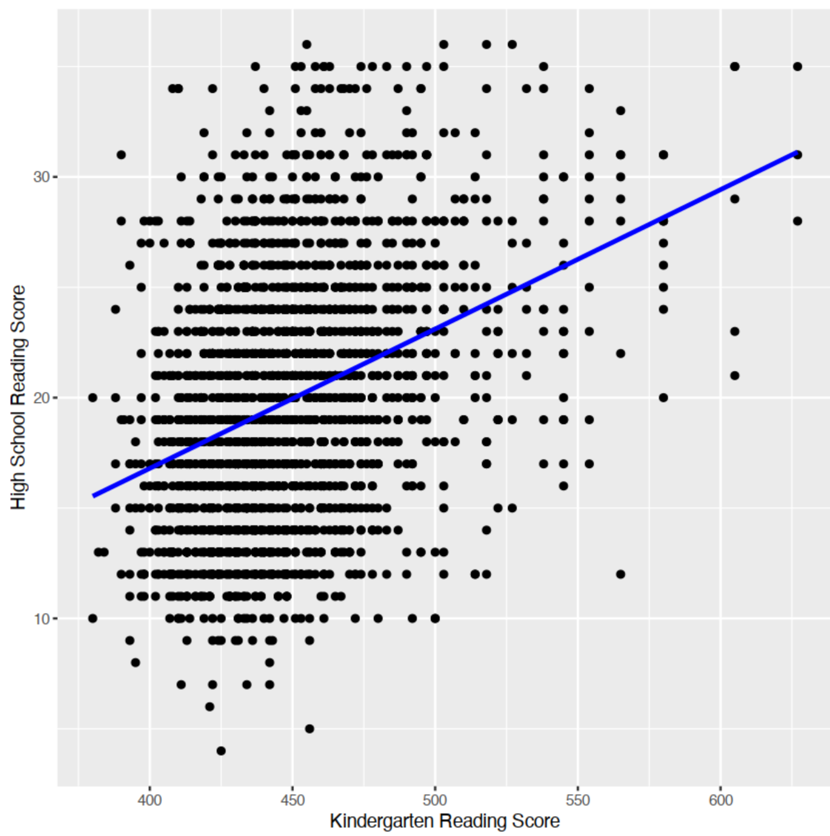


4. Do student test scores in kindergarten predict high school achievement? Plot past and future reading test scores.

```
# Your answer here!

# START

# Calculate correlation between kindergarten and high school reading scores
cor(star_data_clean$gktreadss, star_data_clean$hsactread)

# Plot scores
ggplot(data = star_data_clean) +
  geom_point(aes(x = gktreadss, y = hsactread)) +
  geom_smooth(aes(x = gktreadss, y = hsactread),
```

```
            method = "lm", se = FALSE, color = "blue") +
  labs(x = "Kindergarten Reading Score", y = "High School Reading Score")

# END
```

0.370327523243503
`geom_smooth()` using formula 'y ~ x'



5. Use `summarize` to calculate the sample size and mean and standard deviation of reading scores by classroom size (small vs. large). Then calculate the bounds of the 95% confidence interval for each class size by creating a variable `reading_score_lb` for the lower bound and `reading_score_ub` for the upper bound.

Hint: Recall you can calculate a 95% confidence interval for the mean using $\hat{\mu} \pm 2\frac{\hat{\sigma}}{\sqrt{n}}$.

```
# Your answer here!

# START
star_by_size <- star_data_clean %>%
    group_by(gkclasstype) %>%
    summarize(reading_score_mean = mean(gktreadss),
              reading_score_sd = sd(gktreadss),
              n = n()) %>%
    mutate(reading_score_lb = reading_score_mean - 2 * reading_score_sd / sqrt(n),
           reading_score_ub = reading_score_mean + 2 * reading_score_sd / sqrt(n))

star_by_size
```

```
# END
```

A tibble: 2 × 6

gkclasstype	reading_score_mean	reading_score_sd	n	reading_score_lb	reading_score_
<chr>	<dbl>	<dbl>	<int>	<dbl>	<dl
R	448.2054	33.05628	1563	446.5331	449.8
S	453.3860	32.86478	684	450.8727	455.8

6. Do students from small classrooms report higher reading scores in kindergarten? Plot your estimates and add error bars for the 95% confidence interval.

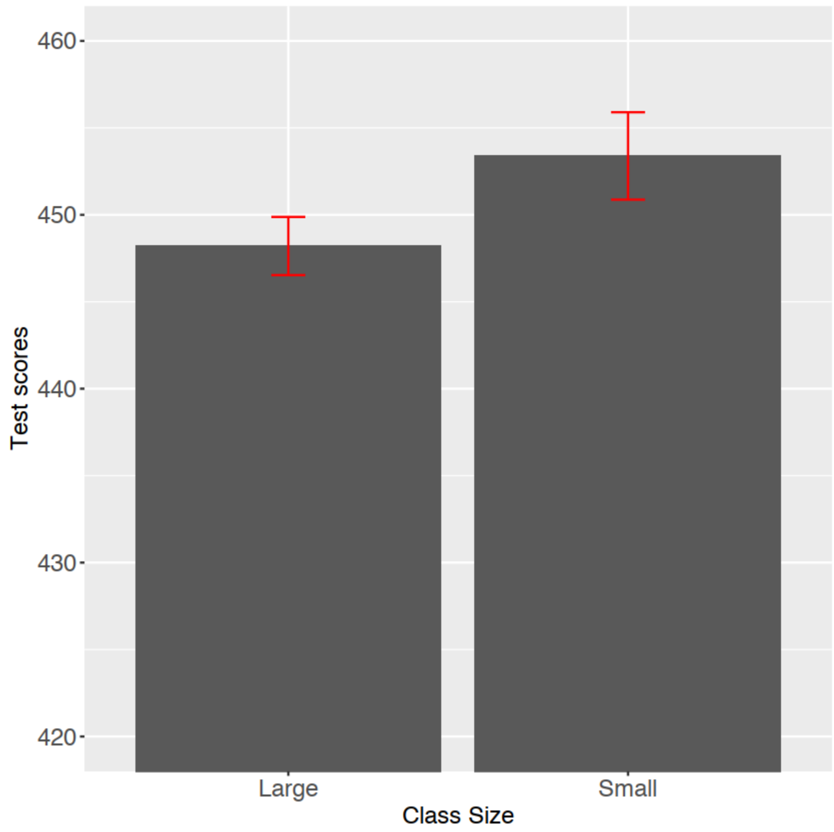
i. Use `geom_col()` to plot the sample mean.

ii. Use `geom_errorbar()` to plot the confidence interval. In addition to `x` it needs a `ymin` and `ymax` aesthetic. These should correspond to the bounds of your confidence interval.

iii. Use `scale_x_discrete(labels = c("Large", "Small"))` to label the x values.

iv. Use `coord_cartesian(ylim = c(420, 460))` to plot only the range of y values between 420 and 460.

```
# Your answer here!

# START
ggplot(star_by_size) +
  geom_col(aes(x = gkclasstype, y = reading_score_mean)) +
  geom_errorbar(aes(x = gkclasstype, ymin = reading_score_lb, ymax = reading_score_ub),
                color = "red", width = .1) +
  scale_x_discrete(labels = c("Large", "Small")) +
  labs(x = "Class Size", y = "Test scores") +
  coord_cartesian(ylim = c(420, 460)) +
  theme(axis.text = element_text(size = 14),
        axis.title = element_text(size = 14))

# END
```


460
450
440
430
420
Test scores
Large
Small
Class Size