Consider the following equations:

$$2x - y = 0$$
$$\& \quad -x + 2y = 3$$

$\}$ here, $n=2$; 2 unknowns
& 2 equations

There are two ways to go about representing a linear alg. problem
- Row Picture
- Column Picture.

Consider the previous two eqns

$$2x - y = 0$$
$$\& \quad -x + 2y = 3$$

◦ **ROW PICTURE**

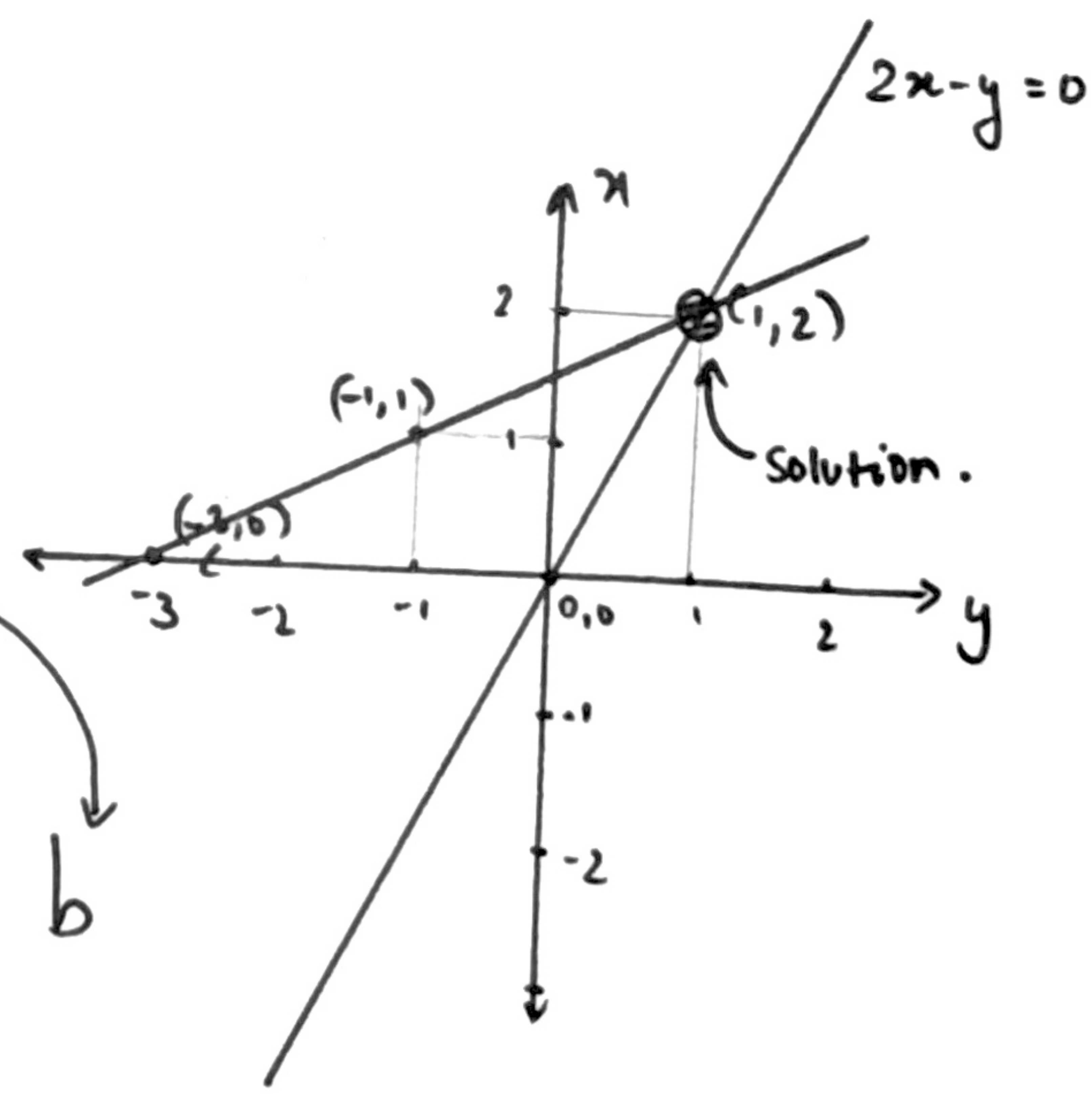
$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix} \leftarrow \text{result (b)}$$

$A \cdot X = b$

coefficients (A)

unknowns (X)

(matrix picture)



Using the row pic we plotted the lines for the equations, & their intersection is the solution for the equations

~~form for row picture is A.X = b.~~

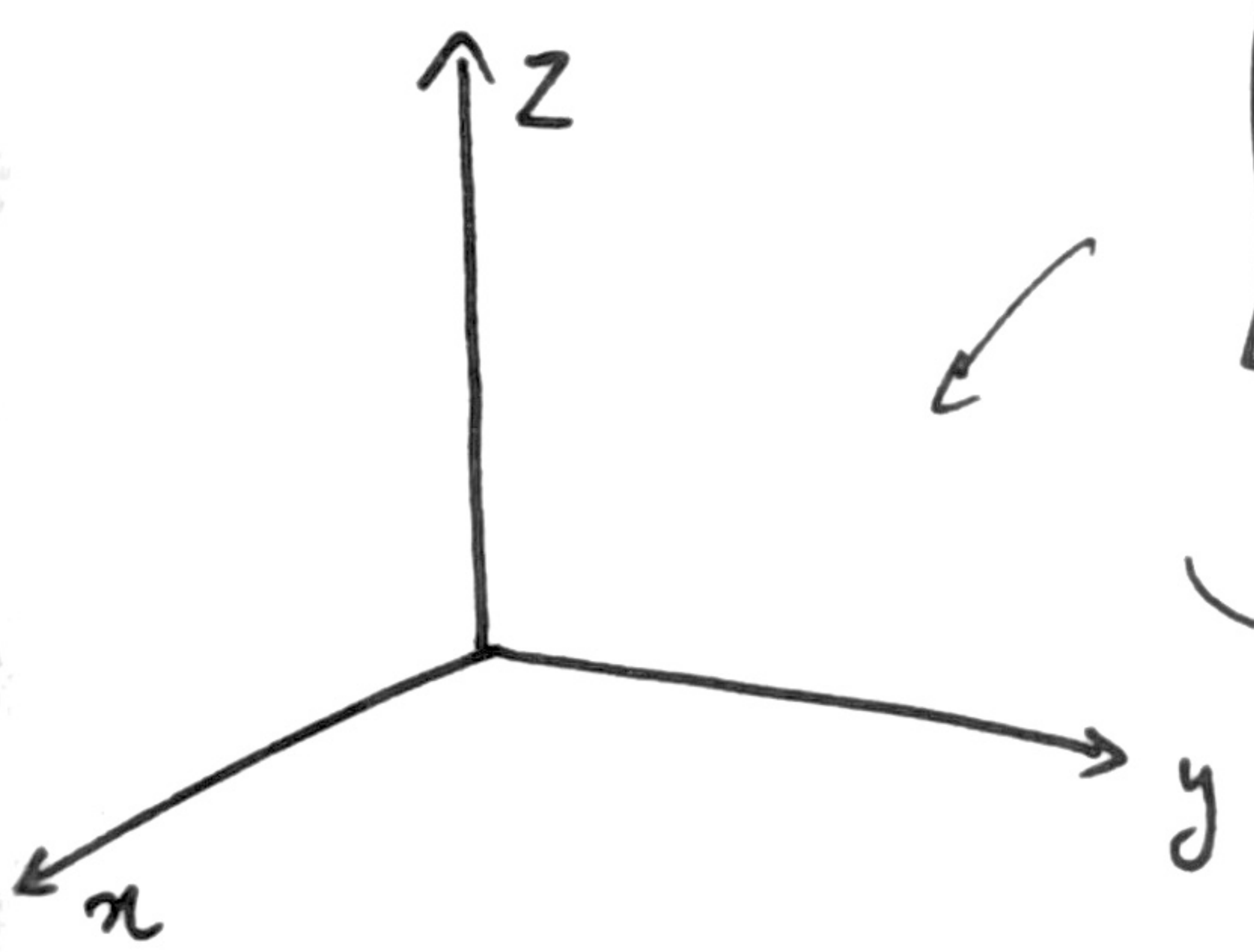
for three dimentional row representation,

Intersection of two 3 variabled equations is an intersection of 2 planes, which gives a line; & when the third eq^n intersects the line it forms ~~the plane~~ a point which is the solution for the set of eq^{n}s.

$$\left.\begin{aligned} 2x - y &= 0 \\ -x + 2y - z &= -1 \\ -3y + 4z &= 4 \end{aligned}\right\} \text{— Row picture}$$

$\downarrow$

$$\underset{A}{\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -3 & 4 \end{bmatrix}} \cdot \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{b}{\begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}}$$

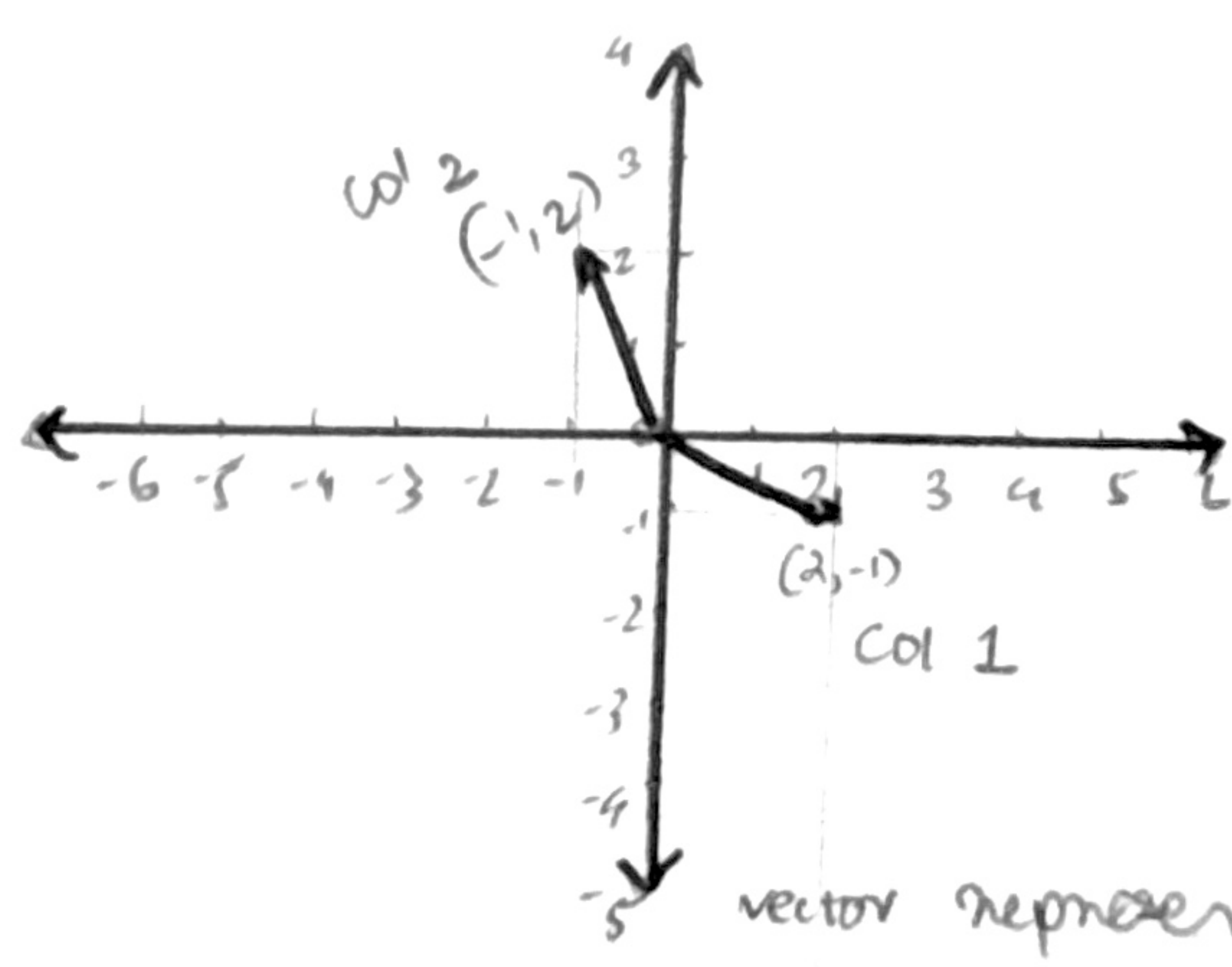


→ plane intersects plane gives line.
line intersects plane gives point.
(Ideally)

COLUMN PICTURE

for the equations :

$$2x - y = 0$$
$$\& \; -x + 2y = 3$$



vector representation.

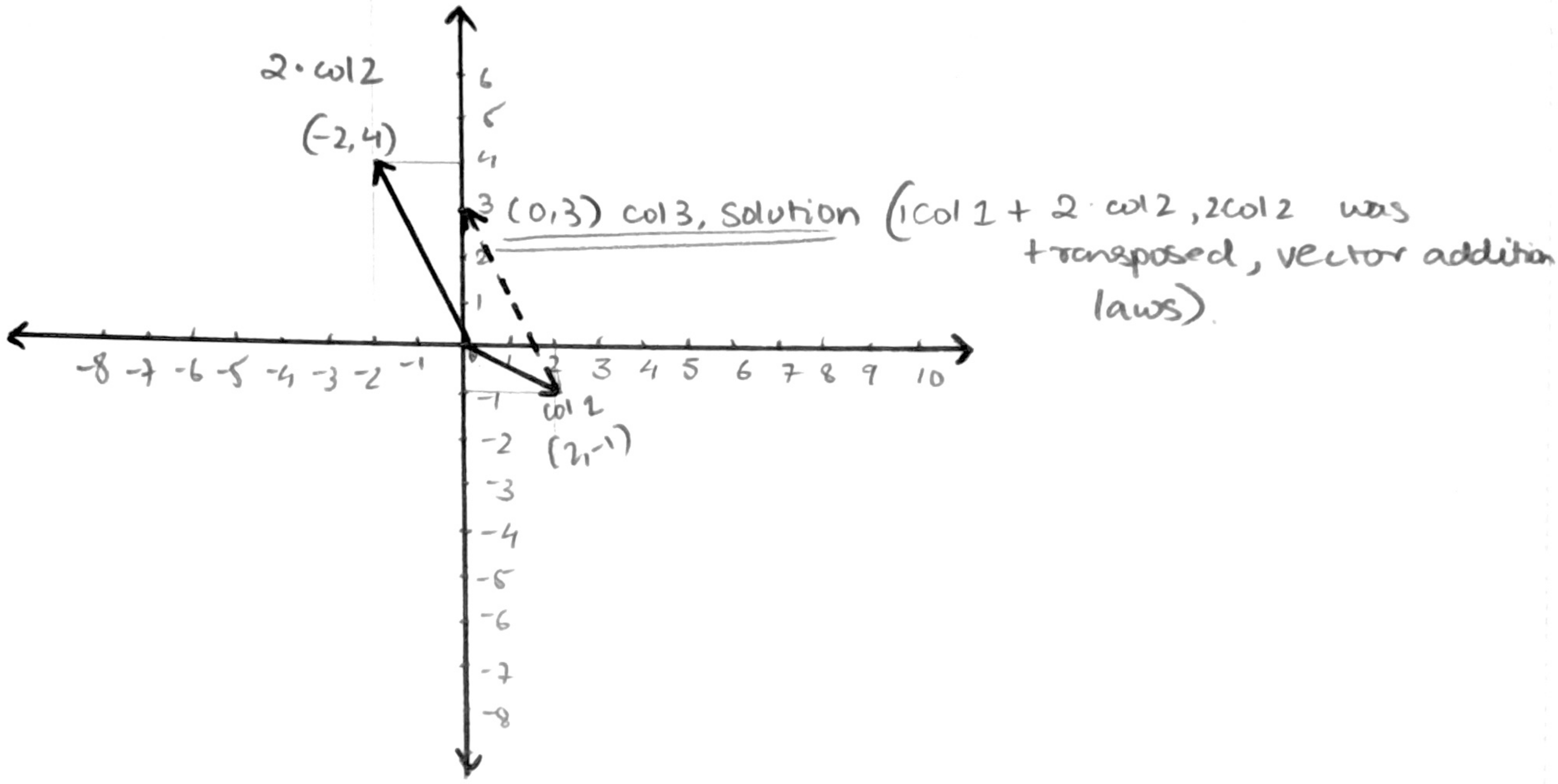
col pic representation :

$$\underbrace{x\underset{\text{col 1}}{\begin{bmatrix} 2 \\ -1 \end{bmatrix}} + y\underset{\text{col 2}}{\begin{bmatrix} -1 \\ 2 \end{bmatrix}}}_{\text{linear combination of the columns.}} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

for $x = 1$ & $y = 2$ (the solution) the equation becomes:

$$1\begin{bmatrix} 2 \\ -1 \end{bmatrix} + 2\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$\therefore$ to get the solution add col 1 vector to $2 \cdot$ col2 vector. the resulting vector is the solution.



for the equations

$$\left.\begin{aligned} 2x - y &= 0 \\ -x + 2y - z &= -1 \\ -3y + 4z &= 4 \end{aligned}\right\}$$

column representation:

$$x\begin{bmatrix}2\\-1\\0\end{bmatrix} + y\begin{bmatrix}-1\\2\\-3\end{bmatrix} + z\begin{bmatrix}0\\-1\\4\end{bmatrix} = \begin{bmatrix}0\\-1\\4\end{bmatrix}$$

By inspection, the solution to this eq^n would be $x=0; y=0;$ & $z=1$.

∴ The solution for this set of eq^ns ~~are~~ is the vector 'z' = {0, -1, 4}

∴ Row picture, geometrically is intersection of the equations in a co-ordinate system

& The column picture leverages matrix addition.

Can $AX = b$ be true, solved for every b?
↳ do the linear ~~equations~~ ~~fill up~~ combinations of the columns fill up the ~~entire~~ 3D space?

↳ Yes, for an ~~___~~ invertible matrix,

No, for cases when say 3 column vectors lie on the same plane in a 3 variable eq^n set.

Imagine vectors with 9 components, AKA 9 eq^{n}s, 9 unknowns represented by 9 columns. In cases where two columns are not independent, they contribute nothing new and there will be scenarios where the solution (b) would be impossible to solve.

~~When all columns~~ In the best case senario the ~~__~~ 9 eq^{n}s will fill out ~~all~~ the whole 9 dimentional space.

If 2 colums are the same (assume) The solution would be an 8th dimentional ~~space~~ plane in a 9 ~~__~~ dimentional space.

MATRIX MULTIPLICATION

$AX = b$

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1\begin{bmatrix} 2 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 10 \\ 6 \end{bmatrix} = \begin{bmatrix} 12 \\ 7 \end{bmatrix}$$

column method.

or $\begin{bmatrix} 2\times1 + 5\times2 \\ 1\times1 + 3\times2 \end{bmatrix}$ ← standard row method

$= \begin{bmatrix} 12 \\ 7 \end{bmatrix}$