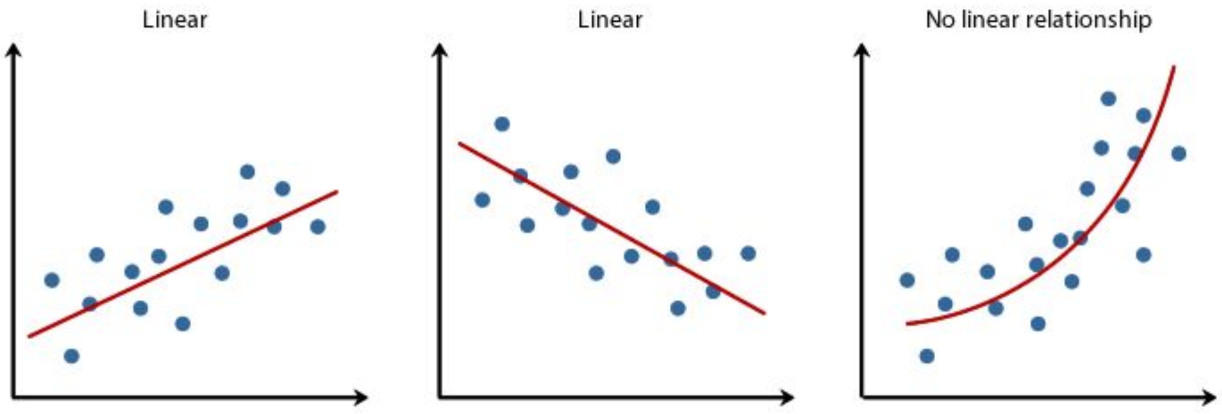
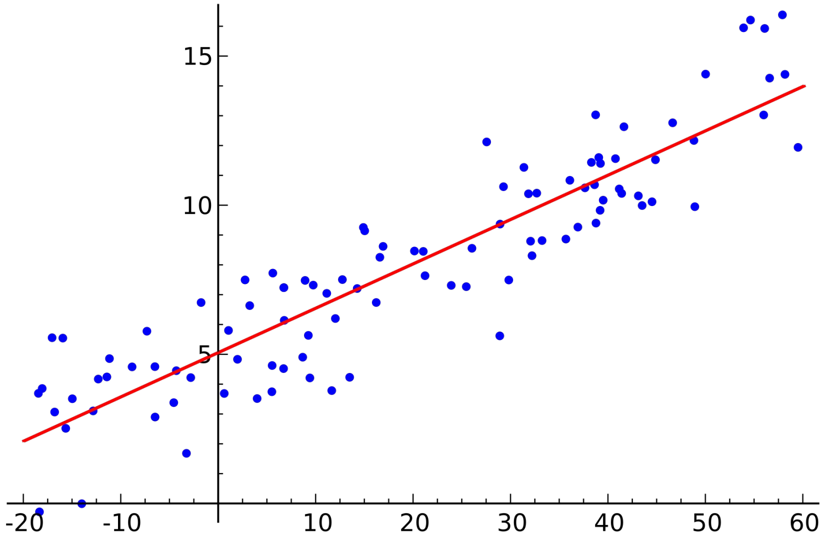
Linear Regression

Line of regression

- A line that can be taken as representative of the ideal variation is called as the line of best fit



Simple Linear Regression

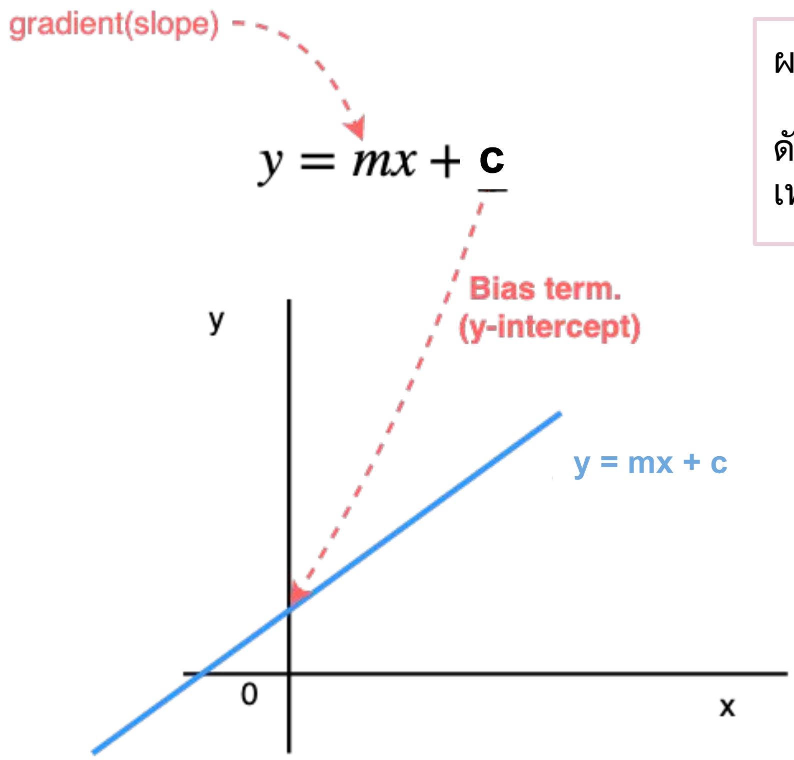


$$Y = mX + c$$

ซึ่ง

Y = ตัวแปรตาม
X = ตัวแปรต้น
m = ความชัน
c = จุดตัดแกน Y

Simple Linear Regression

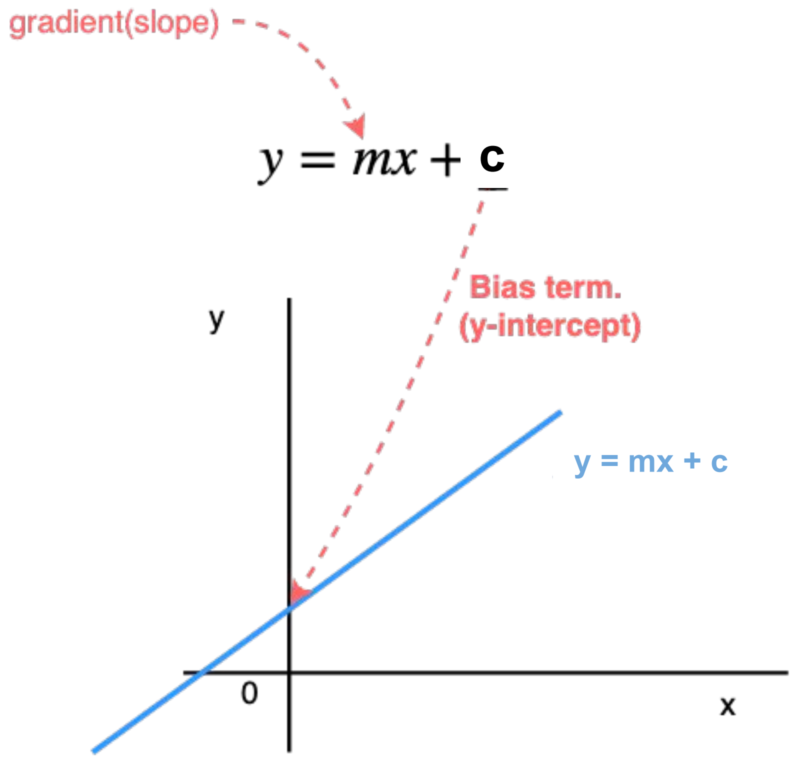


ผลลัพธ์ที่ได้ คือ ผลคูณของค่า **x** กับ **m** และ บวกกับ bias **c**

ดังนั้นเมื่อ **x**=0 (ไม่มีข้อมูลเข้ามาในตัวแปรต้น) output จะมีค่าเท่ากับ bias **c**.

https://www.kdnuggets.com/2019/08/numpy-neural-networks-computational-graphs.html?fbclid=IwAR3hpahD1P1NTx5G4pXnEcXbdfyD3pbeAtbyEVnmiXwNI_bmWIVm96HtSIo

Simple Linear Regression

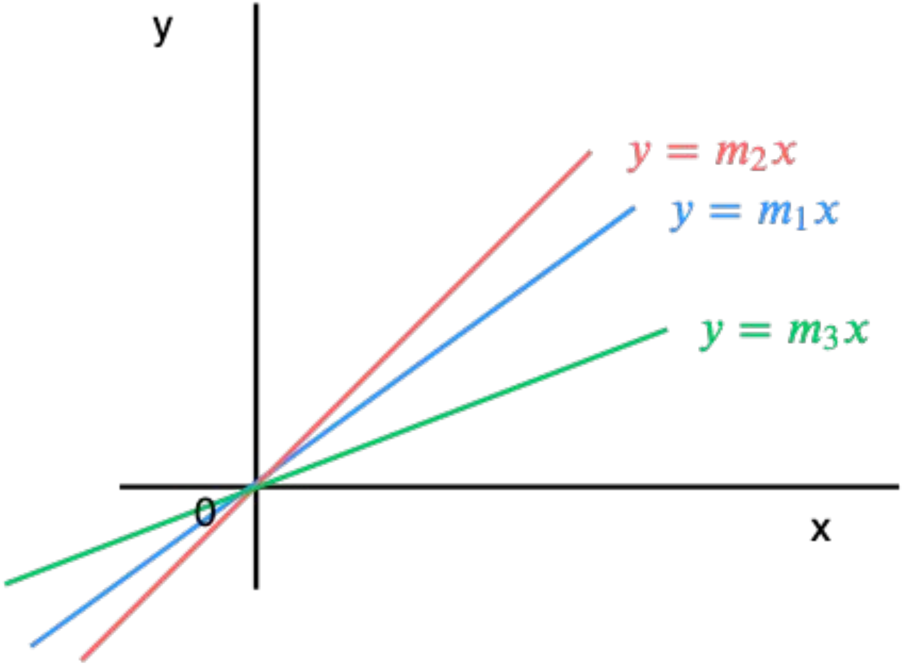


$$Y = mX + c$$

$$h_\theta(x) = \theta_0 + \theta_1 x_1$$

https://www.kdnuggets.com/2019/08/numpy-neural-networks-computational-graphs.html?fbclid=IwAR3hpahD1P1NTx5G4pXnEcXbdfyD3pbeAtbyEVnmiXwNI_bmWIVm96HtSIo

Simple Linear Regression



ถ้าหากไม่มี bias term นั่นหมายความว่า เส้นตรงจะตัดผ่านจุด origin (0,0) เสมอ จะต่างเพียงแค่ความชัน m เท่านั้น

https://www.kdnuggets.com/2019/08/numpy-neural-networks-computational-graphs.html?fbclid=IwAR3hpahD1P1NTx5G4pXnEcXbdfyD3pbeAtbyEVnmiXwNI_bmWIVm96HtSIo

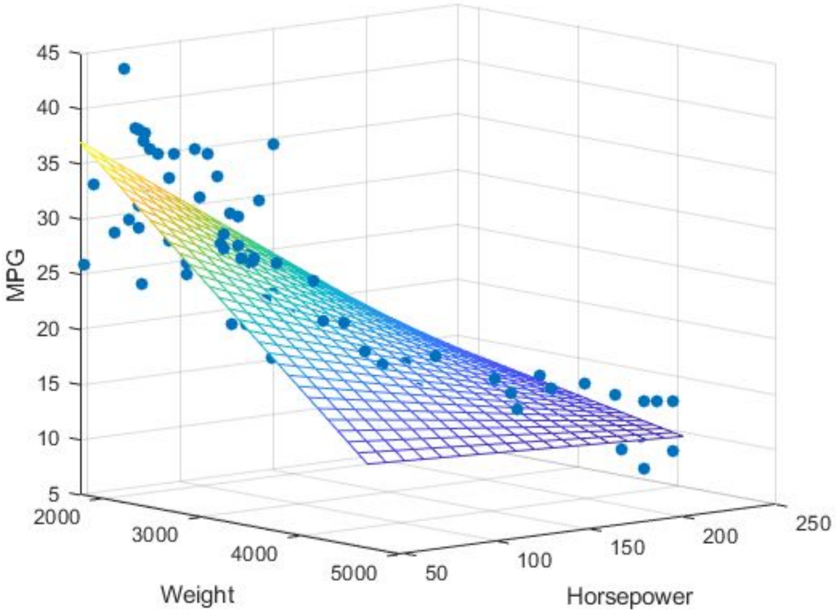
Simple Linear Regression

Years of work	Salary
2.3	7,432 Bath
4.5	14,143 Bath
7.2	22,175 Bath
10.5	32,108 Bath
20.7	? Bath

Can we create a model that predict the amount of salary?

- What is the output ?
- What is the input (feature) ?

Multivariate Linear Regression



$$y = m_1x_1 + m_2x_2 + c$$

https://www.kdnuggets.com/2019/08/numpy-neural-networks-computational-graphs.html?fbclid=IwAR3hpahD1P1NTx5G4pXnEcXbdfyD3pbeAtbyEVnmiXwNl_bmWIVm96HtSIo

Multivariate Linear Regression

$$y = m_1x_1 + m_2x_2 + c$$

$$\downarrow$$

$$h_\theta(x) = \theta_0 + \theta_1x_1 + \theta_2x_2 + \theta_3x_3$$

Multivariate Linear Regression

CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	B	LSTAT	PRICE
0.00632	18.0	2.31	0.0	0.538	6.575	65.2	4.0900	1.0	296.0	15.3	396.90	4.98	24.0
0.02731	0.0	7.07	0.0	0.469	6.421	78.9	4.9671	2.0	242.0	17.8	396.90	9.14	21.6
0.02729	0.0	7.07	0.0	0.469	7.185	61.1	4.9671	2.0	242.0	17.8	392.83	4.03	34.7
0.03237	0.0	2.18	0.0	0.458	6.998	45.8	6.0622	3.0	222.0	18.7	394.63	2.94	33.4
0.06905	0.0	2.18	0.0	0.458	7.147	54.2	6.0622	3.0	222.0	18.7	396.90	5.33	36.2

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \ldots$$

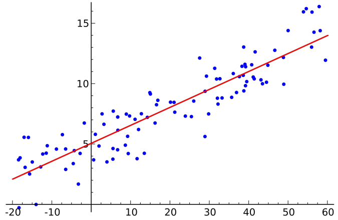
Where θ_s are the parameter of the model

X_s are values in the table

https://github.com/ekapolc/pattern_2019/blob/master/lecture/L02-regression.pdf

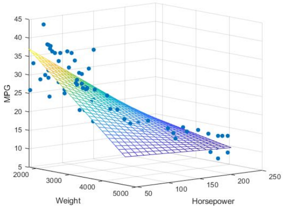
Linear Regression

Simple $$h_\theta(x) = \theta_0 + \theta_1 x_1$$



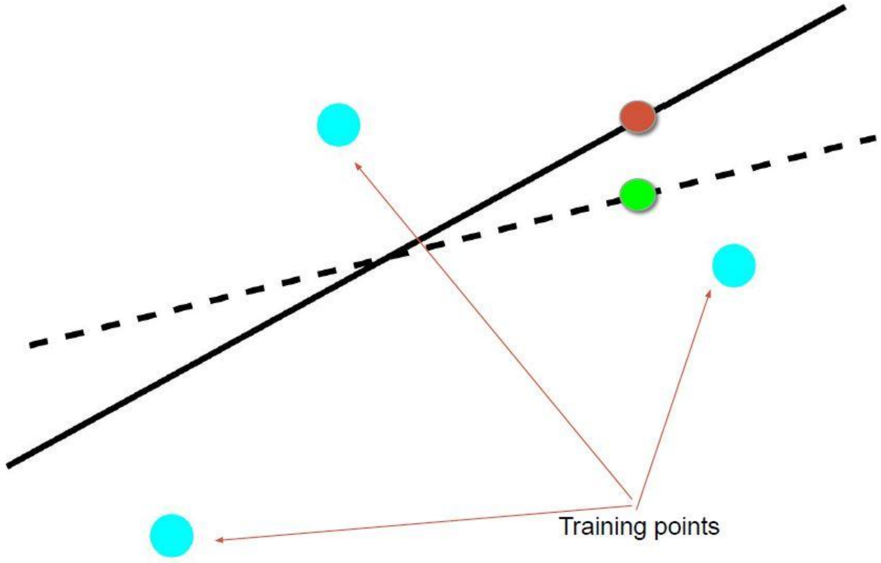
Multivariate $$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \dots + \theta_n x_n$$

$$\begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}^T \begin{bmatrix} 1 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \theta^T x$$



Linear Regression

$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \ldots + \theta_n x_n = \theta^T x$



https://chem.libretexts.org/Textbook_Maps/Analytical_Chemistry_Textbook_Maps/Map%3A_Analytical_Chemistry_2.0_

Cost function (Loss function)

Loss function คือฟังก์ชันที่ทำหน้าที่คำนวนความผิดพลาดระหว่างข้อมูลที่ทำนายออกมา กับค่าความเป็นจริง เพื่อนำไปใช้การปรับปรุงให้โมเดลพยายามที่เรียนปรับค่าที่ทำนาย ออกมาให้เข้าใกล้ค่าจริงมากยิ่งขึ้น

- Let's use the mean square error (MSE)

m is the number of training examples

$$J(\theta) = \frac{1}{m} \Sigma_{i=1}^{m} (y_i - \theta^T \mathbf{x_i})^2$$

We want to pick **θ** that minimize the loss

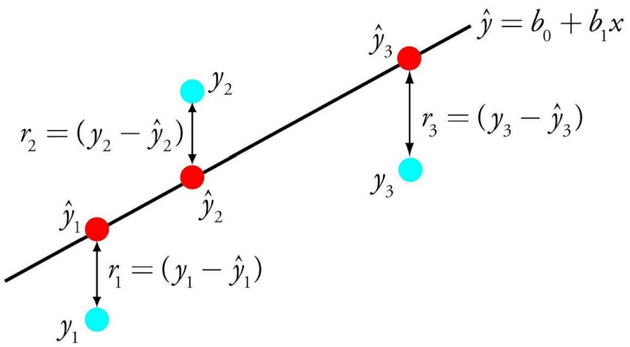
i here is the index of the training example
Note how **x** is bolded

Cost function (Loss function)

- Let's use the mean square error (MSE)

$$J(\theta) = \frac{1}{m}\Sigma_{i=1}^{m}(y_i - \theta^T \mathbf{x_i})^2$$

We want to pick **θ** that minimize the loss



https://github.com/ekapolc/pattern_2019/blob/master/lecture/L02-regression.pdf

Cost function (Loss function)

$$J(\theta) = \frac{1}{m}\Sigma_{i=1}^{m}(y_i - \theta^T \mathbf{x_i})^2$$

X_1	X_2	θ_0	θ_1	θ_2	y_pred	y_real	loss
3	-7	-10	7	3	-10	-8	4
-2	4	-10	7	3	-12	-2	100
13	-8	-10	7	3	57	43	196
0	9	-10	7	3	17	12	25
-1	2	-10	7	3	3	5	4

SUM = 329

Average = 329/5 = 65.8

https://github.com/ekapolc/pattern_2019/blob/master/lecture/L02-regression.pdf

Cost function (Loss function)

- Let's use the mean square error (MSE)

$$J(\theta) = \frac{1}{m}\Sigma_{i=1}^{m}(y_i - \theta^T \mathbf{x_i})^2$$

We want to pick **θ** that minimize the loss

$$\frac{m}{2}J(\theta) = \boxed{\frac{1}{2}\Sigma_{i=1}^{m}(y_i - \theta^T \mathbf{x_i})^2}$$

https://github.com/ekapolc/pattern_2019/blob/master/lecture/L02-regression.pdf

Picking θ

- Random until you get the best performance?
  - Can we do better than random chance?

- How to quantify best performance?

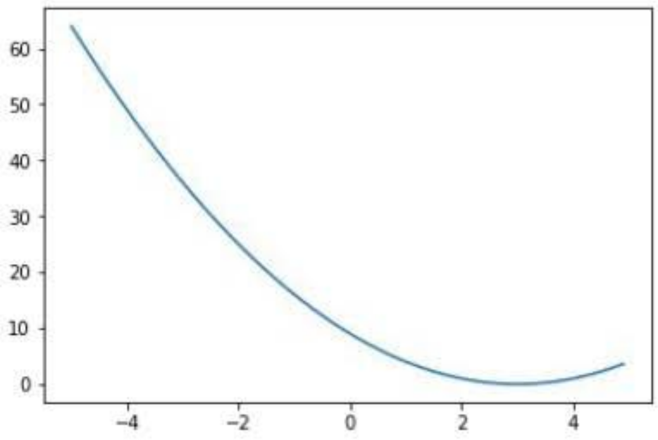
$$\frac{m}{2}J(\theta) = \frac{1}{2}\Sigma_{i=1}^{m}(y_i - \theta^T \mathbf{x_i})^2$$

Minimizing a function

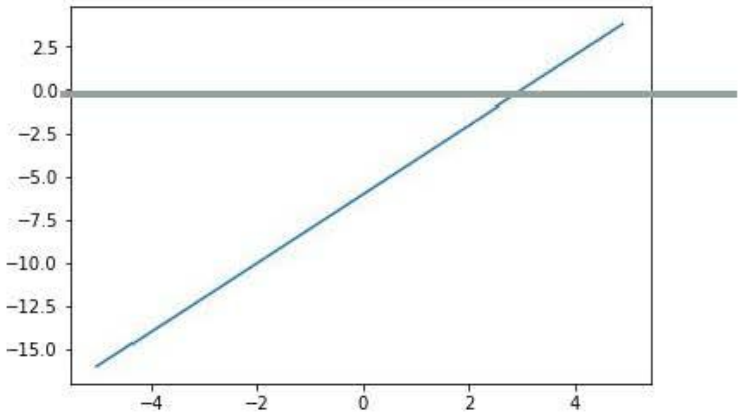
- You have a function
  - $y = (x - a)^2$
- You want to minimize Y with respect to x
  - $dy/dx = 2x - 2a$
  - Take the derivative and set the derivative to 0
    - (And maybe check if it's a minima, maxima or saddle point)

- We can also go with an iterative approach

Gradient descent



y

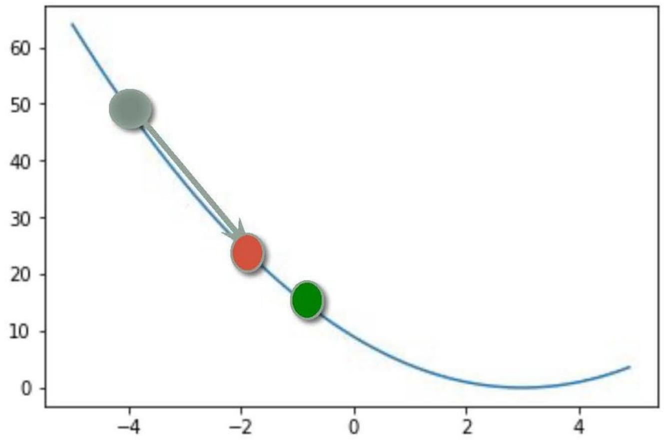


dy/dx

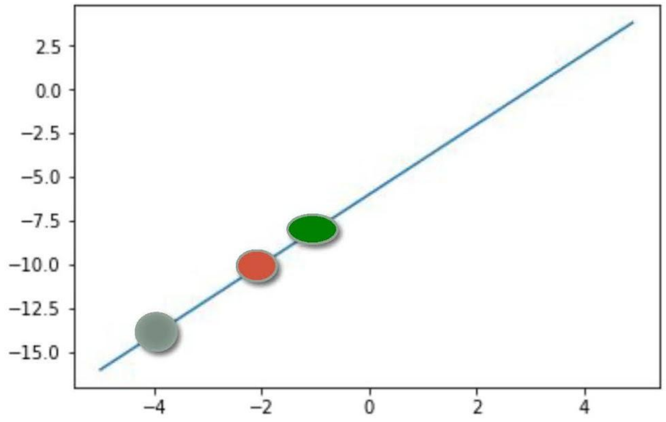
First what does dy/dx means?

https://github.com/ekapolc/pattern_2019/blob/master/lecture/L02-regression.pdf

Gradient descent



y

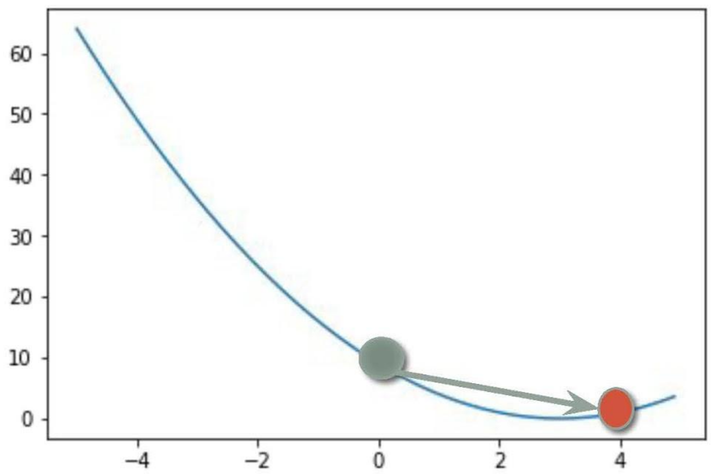


dy/dx

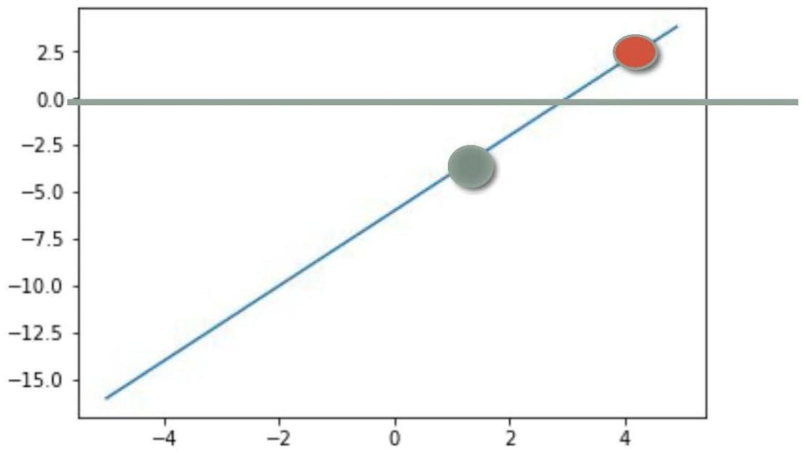
Move along the negative direction of the gradient
The bigger the gradient the bigger step you move

https://github.com/ekapolc/pattern_2019/blob/master/lecture/L02-regression.pdf

Gradient descent



y

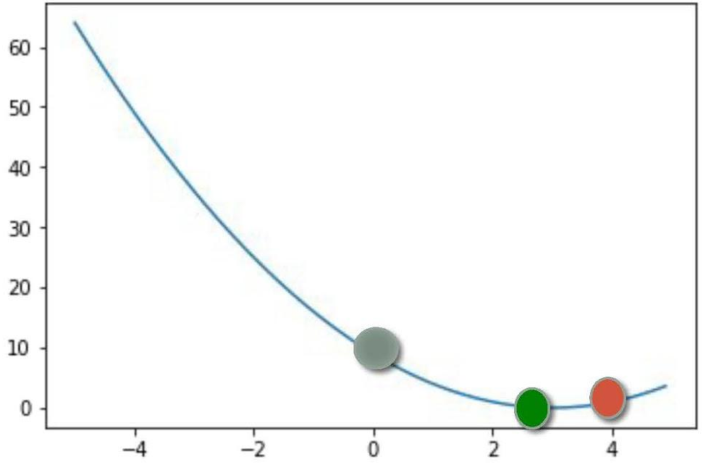


dy/dx

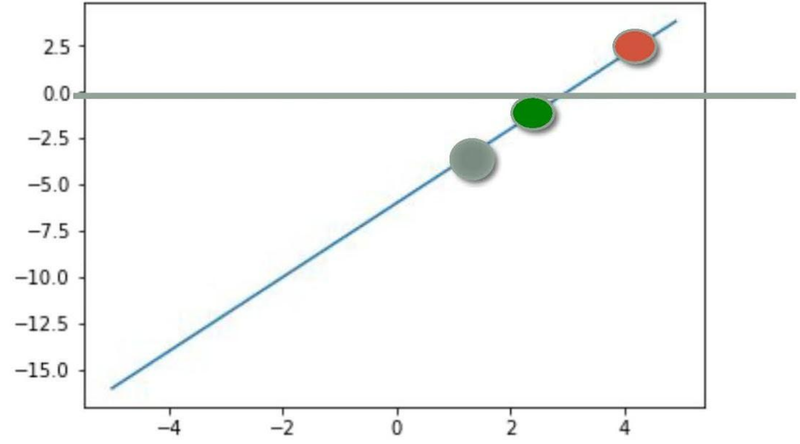
What happens when you overstep?

https://github.com/ekapolc/pattern_2019/blob/master/lecture/L02-regression.pdf

Gradient descent



y



dy/dx

If you over step you can move back

https://github.com/ekapolc/pattern_2019/blob/master/lecture/L02-regression.pdf

Backpropagation

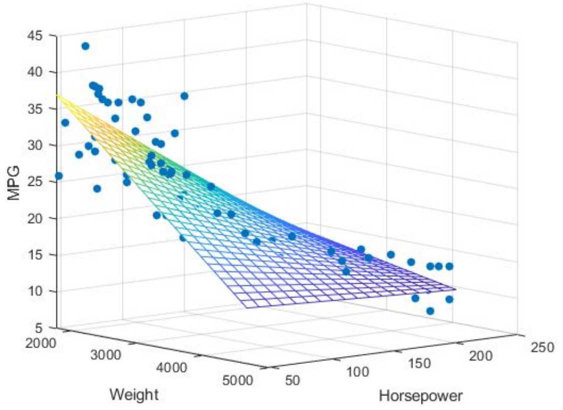
Simple Linear Regression

$$\frac{dLoss}{d\theta_0} = \boldsymbol{\Sigma}_i \; h(\theta_0, \theta_1, x_{i,1}) - y_i$$

$$\frac{dLoss}{d\theta_1} = \boldsymbol{\Sigma}_i \; (h(\theta_0, \theta_1, x_{i,1}) - y_i)*x_{i,1}$$

Derivatives

Multivariate Linear Regression



$$\frac{dLoss}{d\theta_0} = \Sigma_i\, h(\theta_0, \theta_1, \theta_2, \theta_3, x_{i,1}, x_{i,2}, x_{i,3}) - y_i$$

$$\frac{dLoss}{d\theta_1} = \Sigma_i\, (h(\theta_0, \theta_1, \theta_2, \theta_3, x_{i,1}, x_{i,2}, x_{i,3}) - y_i) * x_{i,1}$$

$$\frac{dLoss}{d\theta_2} = \Sigma_i\, (h(\theta_0, \theta_1, \theta_2, \theta_3, x_{i,1}, x_{i,2}, x_{i,3}) - y_i) * x_{i,2}$$

$$\frac{dLoss}{d\theta_3} = \Sigma_i\, (h(\theta_0, \theta_1, \theta_2, \theta_3, x_{i,1}, x_{i,2}, x_{i,3}) - y_i) * x_{i,3}$$

X_1	X_2	θ_0	θ_1	θ_2	y_pred	y_real	loss
3	-7	-10	7	3	-10	-8	4
-2	4	-10	7	3	-12	-2	100
13	-8	-10	7	3	57	43	196
0	9	-10	7	3	17	12	25
-1	2	-10	7	3	3	5	4

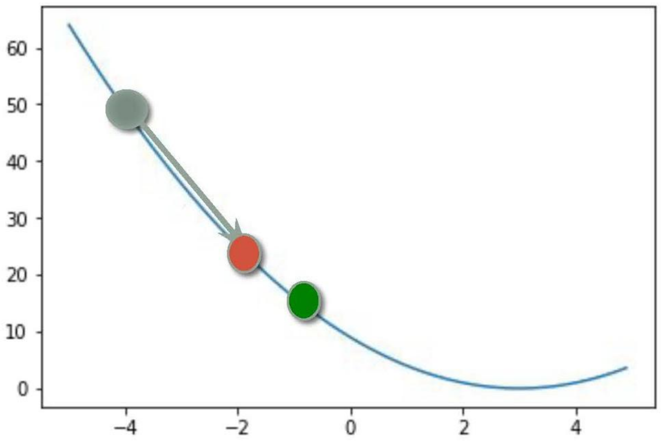
$X_{i,\,j}$

i คือ แถวหรือเลขของจุดข้อมูล

j คือ คอลัมน์หรือมิติในแต่ละจุดข้อมูล

Update weight

$$\theta_0 \leftarrow \theta_0 - \alpha \frac{dLoss}{d\theta_0}$$



y

θ_0 คือ ค่า weight ที่ต้องการปรับ

α (แอลฟ่า) คือ learning rate

Learning rate

The steps which are taken to reach optimal point decides the rate of gradient descent. It is often referred to as ‘Learning rate’

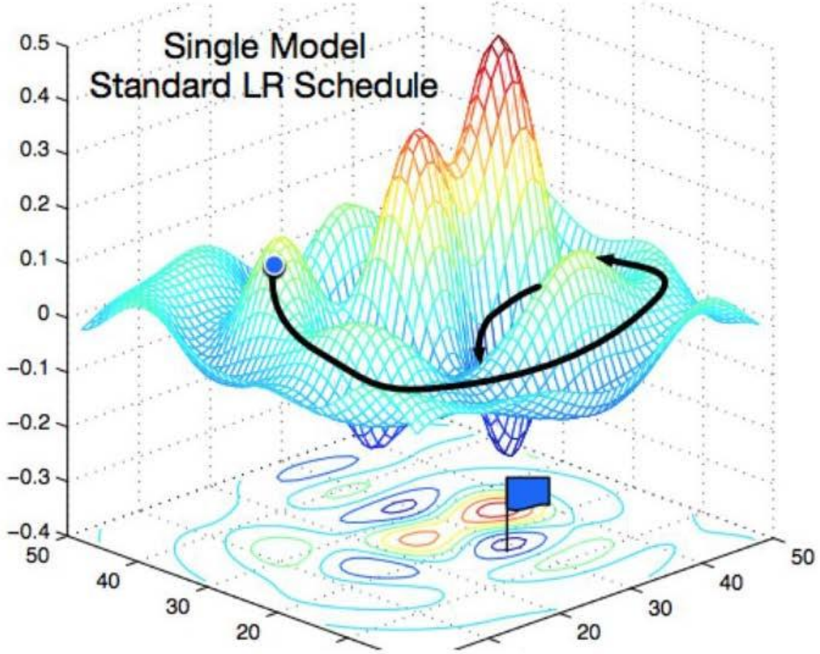
- **Too big**
  bounce between the convex function and may not reach the local minimum

- **Too small**
  gradient descent will eventually reach the local minimum but it will take too much time for that

- **Just right**
  gradient descent will eventually reach the local minimum but it take too much time for that

https://www.slideshare.net/SurajParmar12/linear-regression-with-gradient-descent-115694486

Gradient descent in 3D



https://openreview.net/pdf?id=BJYwwY9ll

Training Step

1. Choose hypothesis function.
2. Create loss function.
3. Calculate loss from hypothesis and ground truth.
4. Compute gradients from loss value.
5. Update weight.