Computing the Length of Constraint Longest Common Subsequence

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Outline

Brief Introduction to the Problem

Definition (CLCS)

The `Constraint Longest Common Subsequence (CLCS)` problem is to find the longest subsequence common to all the given strings with respect to the constraint sequence. Thus, given two strings X, Y, and a contraint string P, find the longest common subsequence Z of X and Y such that P is a subsequence of Z.

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Example

For example, given input $X = "abc123"$, $Y = "123abc"$, and $P = "ab"$, the LCS of X and Y is $"abc"$ and $"123"$, while the CLCS with respect to P is $"abc"$.

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- Technique
  Dynamic Programming

Algorithm by Yin-Te Tsai

Let $L(x, y, x', y')$ denotes the length of LCS of strings $X[x..x']$ and $Y[y..y']$.
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Recursive Relation.

For $1 \le i \le n$, $1 \le j \le m$ and $1 \le k \le r$, $L_1(i, j) = L(1, 1, i-1, j-1) + 1$ if $X[i] = Y[j] = P[1]$, and $-\infty$ otherwise.
For $2 \le k \le r$, $1 \le i \le n$, and $1 \le j \le m$,

$$L_k(i,j) = \begin{cases} \max\{L_{k-1}(x,y), L(x+1, y+1, i-1, j-1)+1\} & \text{if} X[i] = Y[j] = P[k]; \\ -\infty & \text{otherwise} \end{cases} \quad (1)$$

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After obtaining $L_k(\cdot, \cdot)$, we compute the length of CLCS with the following formula:

$$|Z| = \max_{1 \le i \le n, 1 \le j \le m} \{L_r(i, j) + L(i + 1, j + 1, n, m)\} \quad (2)$$

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Example: $X = $ "aba", $Y = $ "bab", $P = $ "a".

$M_{1,1}$		b	a	b
	0	0	0	0
a	0	0	1	1
b	0	1	1	2
a	0	1	2	2

$M_{1,2}$		a	b
	0	0	0
a	0	1	1
b	0	1	2
a	0	1	2

$M_{1,3}$		b
	0	0
a	0	0
b	0	1
a	0	1

$M_{2,1}$		b	a	b
	0	0	0	0
b	0	1	1	1
a	0	1	2	2

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Finally, we compute the length of CLCS:

$$|Z| = \max_{1 \leq i \leq 3, 1 \leq j \leq 3} \{L_1(i,j) + L(i+1, j+1, 3, 3)\} = 2$$

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Given two strings X, Y of length n, m respectively, and a contraint string P of length r, we define $L(i, j, k)$ as the CLCS length of $X[1..i]$, $Y[1..j]$ w.r.t $P[1..k]$.

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Recursive Relation.

For any $0 \leq i \leq n$, $0 \leq j \leq m$ and $0 \leq k \leq r$:

$$L(i,j,k) = \begin{cases} 1 + L(i-1, j-1, k-1) & \text{if } i,j,k > 0 \text{ and } X[i] = Y[j] = P[k] \\ 1 + L(i-1, j-1, k) & \text{if } i,j > 0, X[i] = Y[j] \text{ and } (k = 0 \text{ or } X[i] \neq P[k]) \\ \max(L(i-1,j,k), L(i,j-1,k)) & \text{if } i,j > 0 \text{ and } X[i] \neq Y[j] \end{cases} \quad (3)$$

with boundary conditions, $L(i,0,0) = L(0,j,0) = 0$ and $L(0,j,k) = L(i,0,k) = -\infty$ for any $0 \leq i \leq n$, $0 \leq j \leq m$ and $0 < k \leq r$. □

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The length of CLCS of X, Y with respect to P is given by $L(n,m,r)$.

Algorithm by Francis Chin et al.

Let $m' = n - a + 1$ and $n' = m - b + 1$, the pseudocode for the algorithm is as follows (Algorithm 1). It can be easily seen that the time complexity is $O(m'n')$.

Algorithm 1: LCS-LENGTH(x, y, m', n')

initialize $L_{a,b}[i, 0] \leftarrow 0$ for $1 \leq i \leq m'$ initialize $L_{a,b}[0, j] \leftarrow 0$ for $1 \leq j \leq n'$

```
for i = 1 → m' do
    for j = 1 → n' do
        if x[i] = y[j] then
            L_{a,b}[i,j] ← L_{a,b}[i−1, j−1] + 1
        else
            L_{a,b}[i,j] ← max{L_{a,b}[i−1, j], L_{a,b}[i, j−1]}
        end
    end
end
return L_{a,b}
```

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From analysis of the computation path, we know that it takes at most $O(n+m+r)$ steps.And we can also say that it takes $O(n*m*r)$ time and space to build and calculate the constraint longest common sequence.

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Algorithm by Francis Chin

Example: X = "*aba*", Y = "*bab*", P = "*a*".

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Finally, $|Z| = L(3,3,1) = 2$.

Algorithm by Francis Chin

```
class CLCSSolver
{
public:
  CLCSSolver(string x, string y, string p);
  ~CLCSSolver();
  void solv();
  int getMaxLength();
  string getCLCS();
protected:
  void solv(string x, string y, string p);
private:
  string _x, _y, _p;
  CLCSState *_states;
  CLCSState *_direct;
  enum {D_NNN, D_NNZ, D_NZZ, D_ZNZ};
};
```

Listing 1: class CLCSSolver

Experimental Study

We took two steps to perform this study.

1. **Correctness**
   First we implemented the algorithms and tested the correctness of implementation with manually selected test cases and program-generated test cases.

Experimental Study

We took two steps to perform this study.

1. **Correctness**
   First we implemented the algorithms and tested the correctness of implementation with manually selected test cases and program-generated test cases.
2. **Performance**
   Then we ran the programs with large program-generated test cases to assess the performance.

Correctness

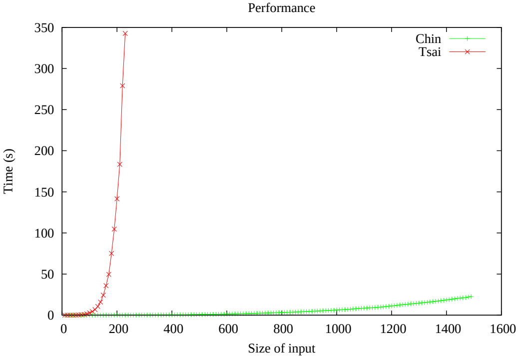
To test whether the algorithms were correctly implemented, we chose a few boundary cases to do white-box test, and generated a series of test cases for black-box test.

The program-generated test cases consist of all the possible combinations of the strings of length less than four. We have $39^3 = 59319$ different test cases.

As the test cases are regularly generated, it is easy to manually find out the answers to the cases.

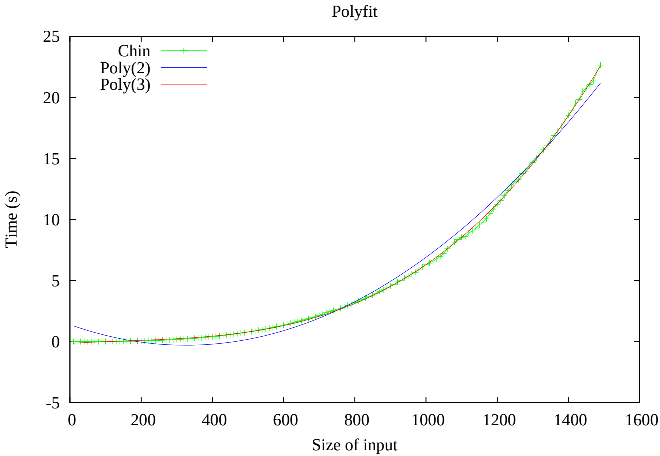
Performance

Randomly generated a series of 3-tuples as test cases. Each tuple consists of two strings and a constraint string, which are defined on the alphabet $\Sigma = \{A, T, G, C\}$. For the k-th tuple (X, Y, P), the lengths of the elements satisfy the condition $|X| = |Y| = 2|P| = 10k$.



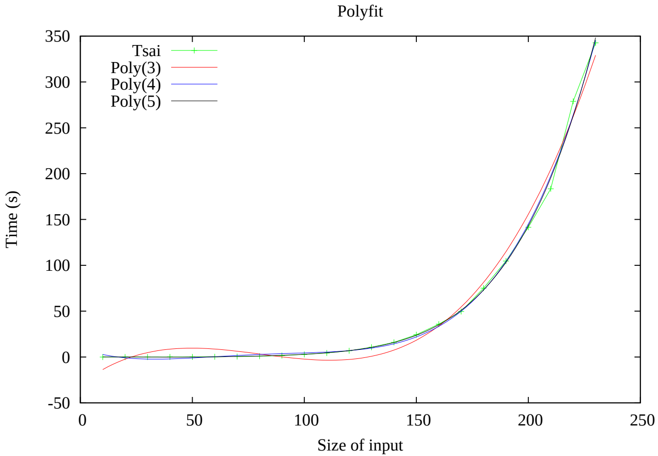
Fitting Data with Polynomials

Chin's algorithm



Fitting Data with Polynomials

Tsai's algorithm



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