Convex Optimization - First Call 24/01/2025

Name: ______________________________ ID: ____________________

1. [2pt] Is the composition of two convex functions always a convex function?
2. [2pt] Does stochastic gradient descent improves the iteration complexity over standard gradient descent for convex functions?
3. [2pt] Is stochastic gradient descent a monotonically decreasing method for minimizing convex functions?
4. [2pt] Is the Lagrangian dual function always a concave function?
5. [2pt] Is complementary slackness a necessary condition for optimality of a primal-dual pair when strong duality holds and is attained?
6. [2pt] Is projected gradient descent slower than standard gradient descent?
7. [2pt] Consider the Langrange dual of a linear problem: can it be optimized with the gradient method?
8. [3pt] Consider the following linear program:

$$\begin{cases} \max 3x_1 + 5x_2 \\ 7x_1 - 3x_2 \leq 2 \\ -x_1 + x_2 \leq 1 \\ x_1 - 3x_2 \leq 3 \\ x \geq 0 \end{cases}$$

   Bring the program into standard form and consider as initial basis the the set of all slack variables. How many variables are candidates for entering the basis?

9. [2pt] Is a subgradient always a descent direction?
10. [2pt] Let a mixed integer linear program be given with 5 variables. The domain of the problem is $\{0,1\}^2 \times \mathbb{R}^2 \times \mathbb{Z}$. At the root node, the optimal solution of the LP relaxation is $x^* = (1, 0, 0.3, 1.5, 3)$. Assuming a branching rule that branches on fractional variables only, how many variables are candidates for branching?
11. [4pt] Write the dual of the following linear program:

$$\begin{cases} \max x_2 + 3x_3 + 7x_4 \\ x_1 - x_2 + 2x_4 \leq 2 \\ 5x_2 + x_3 = 4 \\ x_1 - x_3 + 2x_4 \geq 1 \\ x_1, x_2 \leq 0 \\ x_3, x_4 \text{ free} \end{cases}$$

12. [6pt] Let $G = (V, E)$ be an undirected graph. A vertex cover of G is a set of vertices that includes at least one endpoint of every edge of the G. Write an integer linear program that models the optimization problem of finding the minimum cardinality vertex cover of G. Consider also the following additional constraint: the vertices of the graph are partitioned into red vertices and black vertices and we want the number of the selected black vertices to be at least twice the number of the selected red vertices.