Convex Optimization - Second Call 10/02/2023

Name: ______________________________ ID: ____________________

1. [3pt] Let $f(x)$ be a convex function. $g(x) = af(x) + b$ is a convex function if:

   (a) always
   (b) $a \geq 0$
   (c) never
   (d) $b \geq 0$

2. [2pt] What is the largest value of the stepsize in a backtracking line search descent algorithm?

3. [2pt] Assuming all involved functions are differentiable, is the KKT system always a sufficient condition for a primal-dual pair to be optimal given a convex optimization problem?

4. [2pt] Does Slater's condition hold for this problem?

$$\begin{cases} \min x_1^2 + x_2^2 \\ (x_1 - 1)^2 + (x_2 - 1)^2 \leq 1 \\ (x_1 - 1)^2 + (x_2 + 1)^2 \leq 1 \end{cases}$$

5. [4pt] Consider the following linear program:

$$\begin{cases} \max 3x_1 + 5x_2 \\ 7x_1 - 3x_2 \leq 2 \\ -x_1 + x_2 \leq 1 \\ x_1 - 3x_2 \leq 3 \\ x \geq 0 \end{cases}$$

   Bring the program into standard form and consider as initial basis the the set of all slack variables. How many variables are candidates for entering the basis?

6. [2pt] Consider an unconstrained optimization problem. Does the Newton method always find the optimal solution in at most one iteration if the function to minimize is strongly convex?

7. [3pt] Let a mixed integer linear program be given with 5 variables. The domain of the problem is $\{0,1\}^2 \times \mathbb{R}^2 \times \mathbb{Z}$. At the root node, the optimal solution of the LP relaxation is $x^* = (1, 0.5, 0.3, 1.5, 3)$. Assuming a branching rule that branches on fractional variables only, how many variables are candidates for branching?

8. [4pt] Write the dual of the following linear program:

$$\begin{cases} \max x_1 - x_2 + x_4 \\ x_1 + x_2 - x_3 \geq 3 \\ x_2 + x_3 \leq 2 \\ x_1 + x_4 = 6 \\ x_1 \geq 0 \\ x_2 \leq 0 \\ x_3, x_4 \text{ free} \end{cases}$$

9. [9pt] Let $G = (V, E)$ be an undirected graph and $w : E \rightarrow \mathbb{R}^+$ be an associated weight function. A *matching* M in G is a subset of the edges such that not two edges in M share an endpoint. The *maximum weighted matching* problem consists in finding a matching M^* in G with the highest total weight. Write an integer linear programming formulation for such optimization problem. Consider also the following additional constraint: the edges of the graph are partitioned into red edges and black edges and we want the weight of the selected black edges to be at least twice the weight of the selected red edges.