Convex Optimization - First Call 30/01/2023

Name: _____ ID: _____

1. [3pt] Consider the following set: $\{(x, y) \in \mathbb{R}^2_{++} | \frac{x}{y} \leq 1\}$ This set is:

   (a) convex but nonlinear
   (b) nonconvex but linear
   (c) convex and linear
   (d) nonconvex and nonlinear

2. [2pt] Consider an unconstrained minimization problem, with $f(x)$ strongly convex on S, where S is the sublevel set defined by the initial point, with S closed. Can we bound the optimality gap of a given point $x \in S$?

3. [2pt] Is the Lagrangian dual function always a concave function?

4. [2pt] Does strong duality always hold for convex optimization problems?

5. [4pt] Consider the following linear program:

$$\begin{cases} \min x_1 - 2x_2 \\ 2x_1 + 3x_3 = 1 \\ 3x_1 + 2x_2 - x_3 = 5 \\ x \geq 0 \end{cases}$$

   and let a basis be $B = \{x_1, x_2\}$. The basis B is:

   (a) primal and dual feasible
   (b) primal feasible and dual infeasible
   (c) primal infeasible and dual feasible
   (d) primal and dual infeasible

6. [2pt] Consider a convex equality-constrained optimization problem. Can the Newton method be started only from a feasible point?

7. [3pt] Let a B&B tree for a minimization be given with four open nodes $N_1, \ldots, N_4$. The optimal objective z_i of the linear programming relaxation of each node N_i is known and we have $(z_1, \ldots, z_4) = (101, 100, 103, 101)$. The current incumbent has value $\overline{z} = 105$. What is the absolute optimality gap after branching on node N_2, if we assume that the LP relaxations of its child nodes N_5 and N_6 have value 102 and 106, respectively?

8. [4pt] Write the dual of the following linear program:

$$\begin{cases} \min x_2 - 3x_3 + 5x_4 \\ x_1 - x_2 + 2x_4 \geq 2 \\ 9x_2 + x_3 = -4 \\ x_1 + x_3 - 2x_4 \leq 1 \\ x_1 \geq 0 \\ x_2 \leq 0 \\ x_3, x_4 \text{ free} \end{cases}$$

9. [9pt] Write an integer linear program that models a binary knapsack problem with the following additional constraints:

   (a) the average weight of the selected items does not exceed a given threshold T;

   (b) items 1 and 2 are mutually exclusive;

   (c) the number of selected items of odd index is equal to the number of selected items of even index.