Convex Optimization - Second Call 05/02/2025

Name: ______________________________ ID: ____________________

1. [2pt] Is the positive semidefinite cone a convex set?
2. [2pt] Does stochastic gradient descent improves the cost per iteration over standard gradient descent (when it can be applied)?
3. [2pt] Does coordinate descent improves the iteration complexity over standard gradient descent in general?
4. [2pt] Does strong duality always hold for convex optimization problems?
5. [2pt] Assuming all involved functions are differentiable, is the KKT system always a sufficient condition for a primal-dual pair to be optimal?
6. [2pt] Is projected gradient descent a reasonable option when the feasible set is non-convex?
7. [2pt] What is the largest value of the stepsize in a backtracking line search descent algorithm?
8. [4pt] Consider the following linear program:

$$\begin{cases} \min x_1 - 2x_2 \\ 2x_1 + 3x_3 = 1 \\ 3x_1 + 2x_2 - x_3 = 5 \\ x \geq 0 \end{cases}$$

   and let a basis be $B = \{x_1, x_2\}$. The basis B is:

   (a) primal and dual feasible
   (b) primal feasible and dual infeasible
   (c) primal infeasible and dual feasible
   (d) primal and dual infeasible

9. [3pt] Let a B&B tree for a minimization be given with four open nodes $N_1, \ldots, N_4$. The optimal objective z_i of the linear programming relaxation of each node N_i is known and we have $(z_1, \ldots, z_4) = (101, 102, 99, 101)$. The current incumbent has value $\overline{z} = 105$. What is the value of the global dual bound?
10. [4pt] Write the dual of the following linear program:

$$\begin{cases} \min x_2 - 3x_3 \\ x_1 - 3x_2 + 2x_4 \leq 2 \\ 2x_2 + x_3 = -4 \\ x_1 + x_3 - 5x_4 \geq 1 \\ x_1 \leq 0 \;\; x_2 \geq 0 \\ x_3, x_4 \text{ free} \end{cases}$$

11. [7pt] Let X be a discrete random variable with finitely many values $\{a_1, \ldots, a_n\}$. Note that the distribution of such a X is determined by the probability mass function (pmf) $P[X = a_i] = p_i$, for $i = 1, \ldots, n$. We want to compute a pmf for X such that:

- $E[X] = 0.1$
- $E[X^2] = 0.2$;
- $P[X \leq 0] \geq 0.3$;
- the pmf cannot assign a positive probability to more than 20% of the values a_i.

Among all feasible pmf, we want the one that minimizes the probability $P[X \in B]$ for a given subset $B \subset \{a_1, \ldots, a_n\}$.