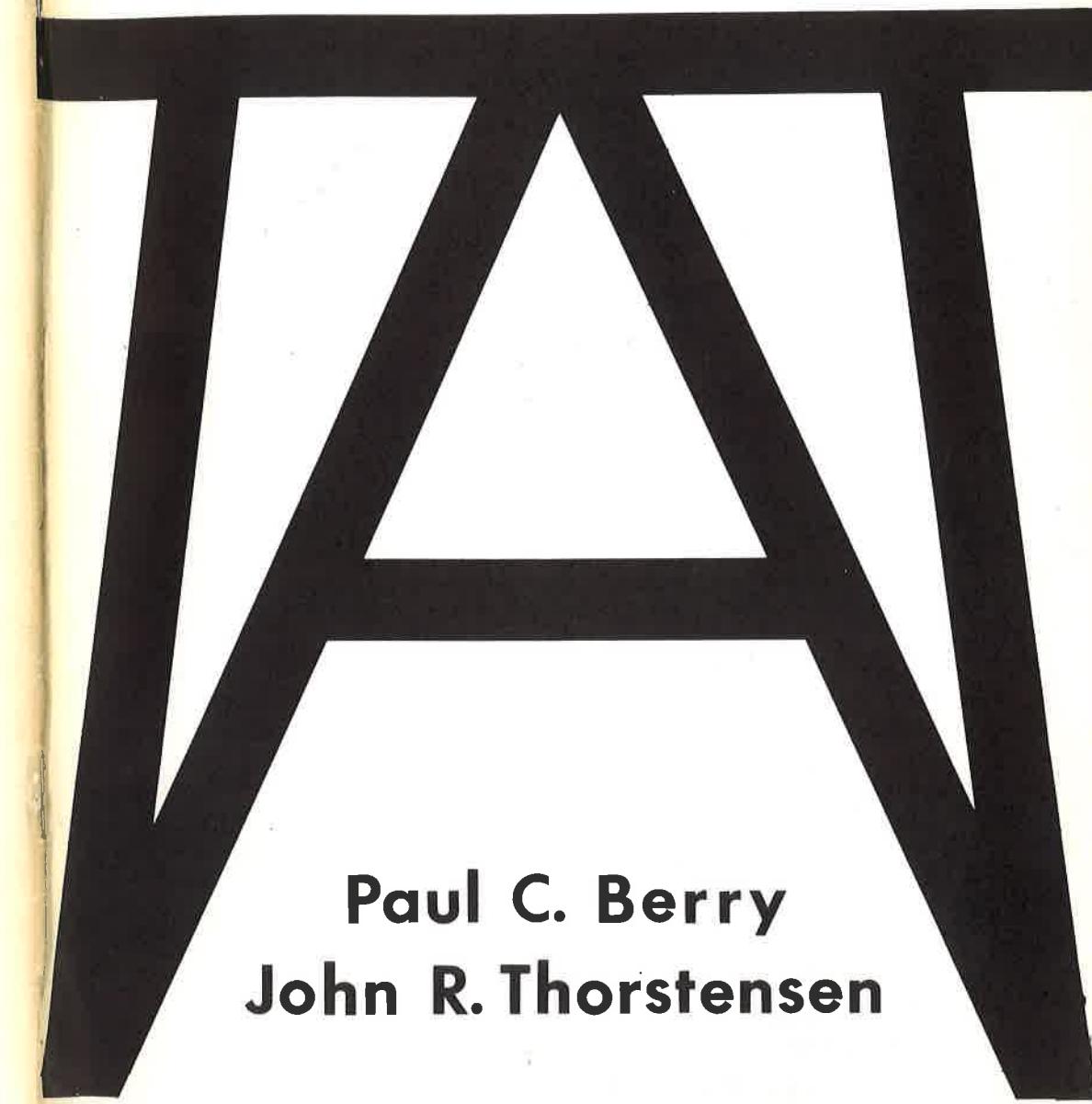


STARMAP



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Introduction

In many fields of science or commerce, it is possible to define a set of functions (i.e. computer programs) in such a way that each corresponds to a term or concept used in that discipline. Such a set of functions in effect constitutes a "user language" for that particular area of application. If that language has a simple and consistent syntax, and if its various functions refer to the data on which they work in a consistent way, it is possible to achieve a programming package that is easy to understand, to revise, or to adapt to new applications. At the same time, such a package constitutes an executable model of some of the concepts of that discipline.

Constructing a particular definition by referring to a set of simpler or more general sub-definitions is the principal technique of what has come to be called modular programming; the program used for a particular job consists of a brief invocation of the concepts or components from which it is made. They in turn are defined by invoking modules at the next level of detail. Reading programs that have been written in that way, the student sees at the outer level a brief summary of the organization of the work; pursuing the definitions further, he or she may obtain explanation to whatever level of detail is desired.

The aim of this paper is to illustrate this style of programming in APL by presenting in detail the definitions used in a particular project. We selected for this purpose the set of programs contained in an APL workspace called STARMAP, which was in use as part of a display on astronomy at the IBM Exhibit Center in New York during 1973 and 1974. That set of programs served to print at a terminal a map showing the positions of the brighter stars, the planets, and the comet Kohoutek, as they would appear above any point on Earth at any time on any date --at least for some number of years on either side of the present.

To generate a map, the user started work by invoking a function called *DISPLAY*. Thereupon he or she was asked to specify the date for which a map was wanted, the local time, and the latitude and longitude. The program then computed the positions of the stars and planets, and drew the map, either at the typewriter terminal (using a special type element for fine plotting) or on a cathode-ray display tube. A sample of the conversation in which a user enters the specifications of a map is shown in Figure 1; the resulting chart (photographically reduced from the original size of about 35 cm. square) is shown in Figure 2.

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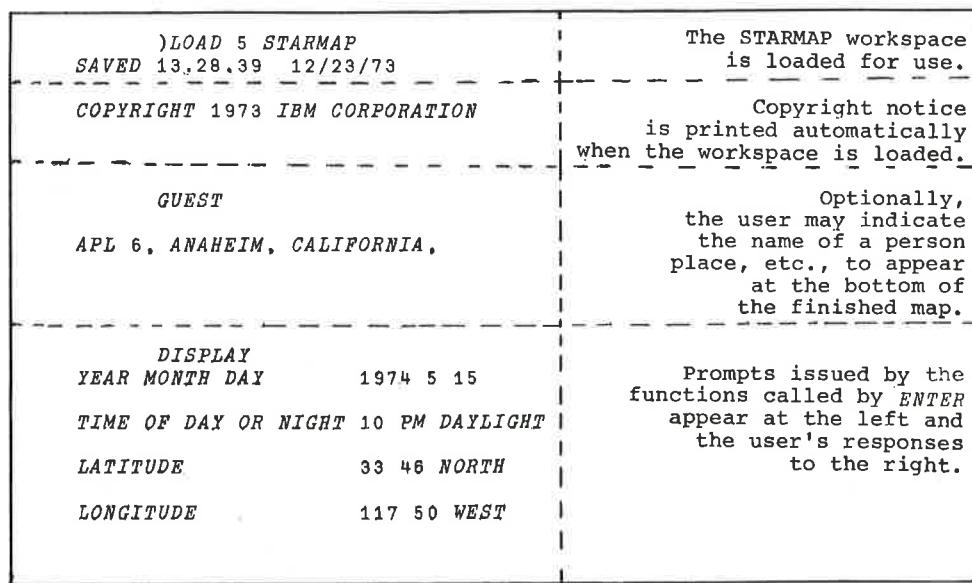


Fig. 1 Dialog during user's request for a star map

Following this dialog, the keyboard is unlocked, awaiting a carriage return from the user to indicate that the fine plotting element has been inserted and the paper is ready to receive the printed map.

On the opposite page appears the map generated in response to the request shown in Figure 1. The map has been photographically reduced; the actual print-out is about 35 cm. high. The fine plotting type element (number 114) carries fifteen dots and fifteen crosses, one for each position of a 3-by-5 matrix, giving a resolution of about .85 mm (1/30 inch) between adjacent points vertically or horizontally. Four additional star maps are shown beginning on page 28.

VIEW FROM 33 DEGREES 46 MINUTES NORTH, 117 DEGREES 50 MINUTES WEST, ON WEDNESDAY 1974/5/15 AT 10 00 PM DAYLIGHT TIME
POSITION OF SUN, MOON AND PLANETS

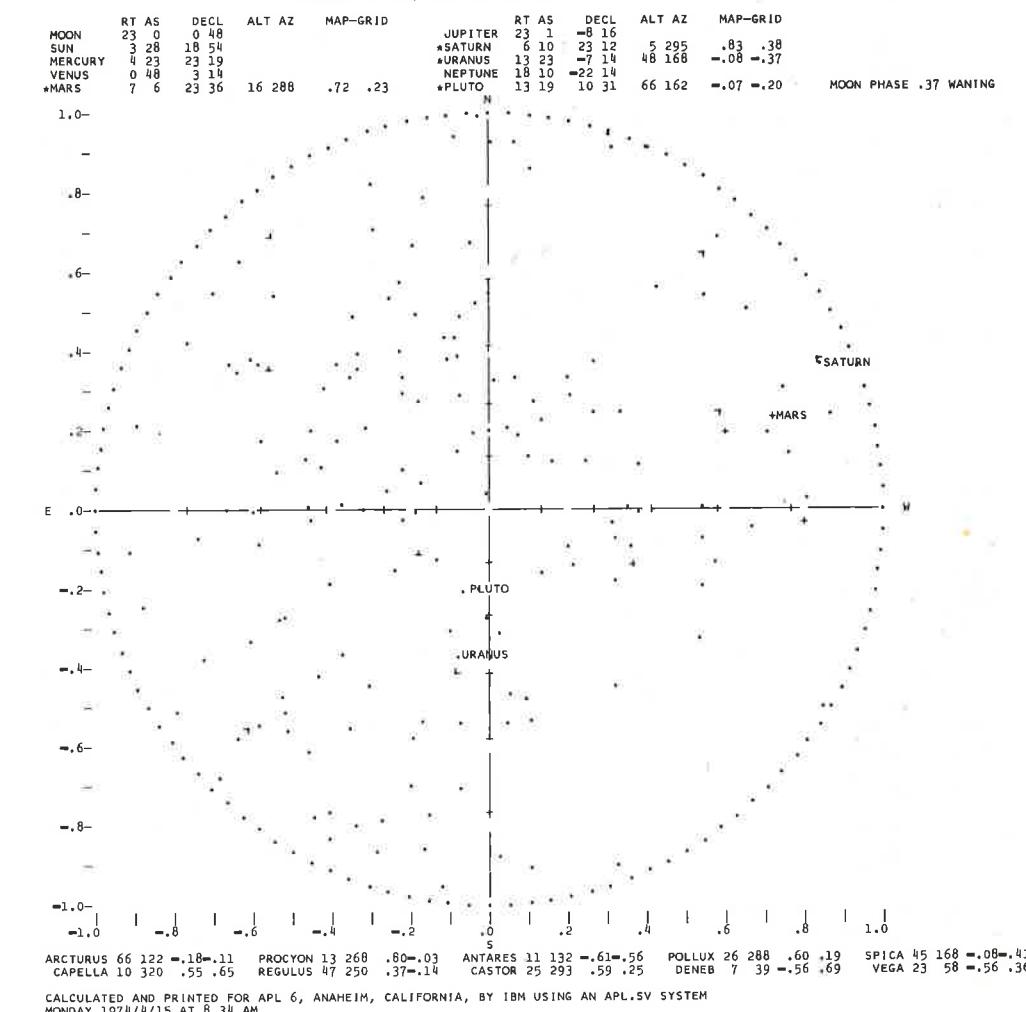


Fig. 2 Sample chart produced by the STARMAP workspace

To generate such a map requires solving the formulas for planetary motion in order to know where in the solar system the planets should be located at the date requested, and then to translate those coordinates to show their apparent positions as seen from the Earth. The coordinates thus obtained, together with those of the fixed stars, could then be rotated for the desired time and location on the surface of the Earth.

The motions of the planets may be described by formulas that were first developed by Kepler early in the seventeenth century. Kepler's function states the time needed for a planet to traverse a given angle through its orbit. To find the position at a given time requires the inverse of that function; a general iterative method, applicable to evaluating that inverse, was worked out by Newton later in the same century. The rest of the task is an exercise in analytic geometry, to translate and rotate the coordinates appropriately. Formulas for doing so were familiar in the seventeenth and eighteenth centuries, but were simplified by the matrix algebra developed in the nineteenth century. Thus the programming task involved here consists mainly of representing in terms executable by the computer a set of classical formulas.

The interest in these programs lies not in the method itself, which has been well known for many years, but in the style in which classical and familiar formulas may be stated in APL. Our aim was to provide APL definitions for a vocabulary of terms which would not only make clear the process by which the work is done, but would permit the student of astronomy to apply them to new problems or applications. Where possible, we used names that correspond to those in general use in astronomy. Occasionally we had to coin names for functions which are not usually explicitly identified, but even here our terms should be recognizable to astronomers. To make our APL definitions correspond more directly to familiar formulas, we also made free use of defined functions such as *SIN*, *COS*, *PI*, *RADIAN*, etc. despite the fact that their effects could be readily obtained from APL primitives.

Clearly, programs to convert the coordinates into a printed map or video display were also necessary to this project, but these are standard packages in widespread use, and we have not chosen to discuss them here.

Organization and Key Parameters

The function *DISPLAY* initiates the entire task. It calls functions corresponding to the various stages of work. A tabular outline of the functions used is shown in Table 1. At the top left of the table the name *DISPLAY* appears, indicating that this is the primary function. Indented five spaces below it appear the names of each of the functions called by *DISPLAY*. Indented below them appear the names of functions they call, and so on. (For simplicity, some 98 references to functions such as *SIN*, *COS*, *RADIAN*, etc. have been omitted from the table.) The rest of this paper provides an explication of the work being done at each of these stages, in top-down order.

In the text that follows, the symbol ∇ (del) is used to indicate the header of a function, containing the name of the function, names for its arguments and results, and names for temporary variables used as intermediate steps in its definition, if any. As may be seen below, *DISPLAY* calls *ENTRY* (which requests input for the parameters governing a particular map), then pauses to permit the user to align the paper and insert the fine plotting element (indicated by the input symbol \Box), and then calls *WORK*, which does the necessary calculations and prints the map:

```
 $\nabla$  DISPLAY
  ENTRY
   $\Box$ 
  WORK
```

The function *ENTRY* establishes the values of the input parameters by calling the functions *GETDATE*, *GETTIME*, *GETLAT* and *GETLONG*, and adjusts the date and time stated by the user to correct for the indicated longitude:

```
 $\nabla$  ENTRY
  STATEDDAYNO←GETDATE
  STATEDTIME←GETTIME
  LAT←GETLAT
  LONG←GETLONG
  TIME←LONG TIMEADJUST STATEDTIME
  DATE←STATEDDAYNO+(TIME÷24)-LONG÷360
```

When execution of *ENTRY* is complete, values have been established for the following variables:

TIME Time is a single number indicating the number of hours since midnight in the exact local time for the indicated longitude. (However, the user enters the time in conventional form as it would appear on a clock in the nearest time zone.)

DATE Although entered in a conventional form, the date is represented internally as the Julian day number; a function *JNU* converts the date to that form. (The Julian day number of 1 January 1974 was 2442049.) The fractional part of the day number indicates how far through the day by universal time the indicated time is. Thus before the major calculations take place, all information on time is contained in the single number *DATE*.

LAT Number of degrees north of the Equator.

LONG Number of degrees east of the prime meridian.

The time and date as entered by the user are preserved as *STATEDTIME* and *STATEDDATE*, and the Julian day number of the stated date as *STATEDDAYNO*; in some circumstances the adjusted value of *DATE* (in universal time) may fall within a day 1 more or less than the stated date.

Stages of Calculation

The task of calculation and printing may be divided into seven stages, each defined by a single function:

- ▼ **WORK**
- CAPTION**
- CALCULATEPLANETS**
- REPORTPLANETS**
- CALCULATESTARS**
- PLOTSTARS**
- REPORTSTARS**
- PRINTED**

Table 1
Levels of Function Calls in the STARMAP Workspace

DISPLAY	
ENTRY	<i>ORBROTATE</i> <i>INCLROTATE</i> <i>LONGROTATE</i> <i>INCLINATION</i> <i>ASCENDING</i> <i>PERIANGLE</i> <i>TIMES</i> <i>PARABOLA</i> <i>PERIDIST</i> <i>PRRIDIST</i> <i>ANOMALYDATE</i>
	<i>EARTHVIEW</i> <i>PLANETPOS</i> (here repeat calls from block listed previously) <i>RADECDIST</i>
WORK	<i>CARTOGRAPHS</i> <i>VISIBLE</i> <i>PROJECTION</i> <i>COALTITUDE</i> <i>MAPCARTESIAN</i> <i>CARTESIAN</i> <i>JNU</i> <i>SKYPOS</i> <i>CARTRIPLET</i> <i>LATROTATE</i> <i>INCLROTATE</i> <i>LONGROTATE</i> <i>NORM</i>
	<i>REPORTPLANETS</i> <i>PLANETTABLE</i> <i>DEGMIN</i> <i>HOURMINSEC</i> <i>REPORTPHASE</i>
	<i>CALCULATESTARS</i> <i>PRECESS</i> <i>CARTRIPLET</i> <i>INCLROTATE</i> <i>LONGROTATE</i> <i>RADECDIST</i>
	<i>VISUAL</i> <i>PROJECTION</i> <i>COALTITUDE</i> <i>MAPCARTESIAN</i> <i>CARTESIAN</i> <i>JNU</i> <i>SKYPOS</i> <i>CARTRIPLET</i> <i>LATROTATE</i> <i>INCLROTATE</i> <i>LONGROTATE</i> <i>NORM</i>
	<i>STARPLOT</i> <i>FPLLOT</i>
	<i>REPORTSTARS</i>
	<i>PRINTED</i> <i>DATEREP</i> <i>GREGORIAN</i> <i>JNU</i> <i>NOW</i> <i>PRINTNAME</i> <i>TIMEREP</i> <i>TODAY</i> <i>GREGORIAN</i> <i>JNU</i>
MOONPOS	<i>PLANETPOS</i> (here repeat calls from block listed previously)
	<i>INCLROTATE</i> <i>RADECDIST</i> <i>MOONPHASE</i> <i>PARALLAXADJUST</i>
	<i>COMETPOS</i> <i>COMETSOLVE</i> <i>AREA</i> <i>PERIDIST</i> <i>AREADERIV</i> <i>PERIDIST</i> <i>PERIDIST</i>

The sequence of segments is designed to overlap output to the terminal (produced by *CAPTION* or *REPORTPLANETS*) with the segments that require substantial calculation.

The function *CAPTION* recapitulates the stated input parameters, and adds the day of the week (directly obtainable from *7|STATEDDAYNO*). The function *REPORTPLANETS* prints a table showing for each planet (and the moon, sun, and comet) its right ascension and declination. For those that are visible, the altitude and azimuth are included, together with the coordinates on the map grid. The phase of the moon is reported.

The function *PLOTSTARS* calls *FPLOT*, which is adapted from the fine-plotting function in IBM program 5798-AGL, "Graphs and Histograms in APL." The stars and planets visible above the horizon are plotted, together with a circular frame of dots at 3-degree intervals around the horizon, and cross marks at intervals of 15 degrees of elevation. The standard plotting program was modified to insert a label showing the name of each planet, and to print a special symbol for the sun and moon.

The function *REPORTSTARS* prints a table showing the names of bright stars appearing in the plot, together with the altitude, azimuth and map-grid coordinates of each. The function *PRINTED* permits the finished map to be labelled with the name of the person for whom it was prepared, and reports the date and time at which it was printed. The input and output functions are not described further in this article.

The functions *CALCULATEPLANETS* and *CALCULATESTARS* use the global arguments *DATE*, *TIME*, *LAT*, and *LONG*, as well as the following reference tables:

- | | |
|-----------------|---|
| <i>STARS</i> | A table containing the right ascension and declination of about 300 bright stars. |
| <i>PLANETS</i> | A table of the elements for the elliptical orbits of the nine planets. |
| <i>MOON</i> | A similar table for the elements of the moon's elliptical orbit about the Earth. |
| <i>KOHOUTEK</i> | A table of the elements for the parabolic orbit of the comet. |
| <i>BRIGHT</i> | A logical vector indicating which members of <i>STARS</i> represent stars of magnitude 1.5 or brighter. |

BP A logical vector indicating which planets are usually of magnitude 1.5 or brighter.

The positions of the stars are taken from the Yale Catalog of Bright Stars, and the elements of the planetary orbits from the American Ephemeris and Nautical Almanac for 1973. The orbital functions which follow were written after consulting the text by Marion (1965) Classical Dynamics of Particles and Systems.

Coordinate Systems Used in Describing the Positions of the Planets

Calculating the appearance of the heavens can be divided into two principal tasks: finding the locations of the planets in the solar system, and then calculating how they appear to an observer. A large part of the work thus involves rotation of coordinate axes, or translation from one system of coordinates to another. It will help to understand the programs that determine the positions of the planets if the various coordinate systems are first described.

Two-dimensional coordinates in the plane of each planet. Each of the objects in orbit around the sun is first considered to be moving along an ellipse (or, in the case of the comet, along a parabola) lying in a plane. Each planet can thus be located by two coordinates. During the initial solution of the orbits, these are polar coordinates; they are then converted to Cartesian coordinates, describing the planet's position by its distance from the solar focus along the major and minor axes of the ellipse. Two-dimensional coordinates appear only within the functions *PLANETPOS*, *MOONPOS*, and *COMETPOS*.

Heliocentric Cartesian coordinates. The two-dimensional Cartesian coordinates that specify each planet's position within the plane of its own orbit are converted to a common three-dimensional coordinate system whose center is in the sun. The first coordinate points from the sun in a direction opposite to the Earth at the moment of the vernal equinox. The second points perpendicularly out of the plane of the ecliptic, on the same side as the north pole. The third points in the plane of the ecliptic, perpendicularly to the other two, so that the three form a right-handed coordinate system. It intersects the celestial sphere at a right ascension of 18 hours (i.e. 270 degrees) and a declination (due to the tilt of the Earth's axis) of -23.45 degrees.

Positions stated in the heliocentric system are given the name *H* in the functions *PLANETPOS*, *MOONPOS*, *COMETPOS*, and in *EARTHVIEW* (which translates from heliocentric to geocentric coordinates).

The function *ORBROTATE* converts the two-dimensional Cartesian coordinates of the planets within their own planes to three-dimensional heliocentric coordinates, taking into account the orientation and tilt of the plane of each orbit, by reference to the elements *PERIANGLE* (angle of perihelion), *INCLINATION*, and *ASCENDING* (the angle of the ascending node); see pp. 14-15.

Geocentric ecliptic coordinates. The axes of this system are parallel to those of the heliocentric system, but have their origin in the center of the Earth rather than in the sun; values in this system are obtained simply by subtracting the heliocentric coordinates of the Earth from those of the object in question. Coordinates stated in this system are given the name *GC*. They appear as intermediate steps in the function *EARTHVIEW*.

Geocentric equatorial coordinates. This is a Cartesian form of the standard astronomical system of right ascension and declination. The first axis points (as before) to the vernal equinox. The second points to the north celestial pole. The third points at a location on the equator at the right ascension of the winter solstice.

Positions in this coordinate system are obtained from those stated in the geocentric ecliptic system by a rotation of 23.45 degrees around the first axis. Variables stated in these coordinates are given the name *GQ*.

Egocentric coordinates. The final transformation is to adjust for the position on Earth of the observer for whom the map is calculated. The first axis points due south. The second points to the zenith (above the observer). The third points due west. Positions in this system are obtained from positions in the geocentric equatorial system by a sequence of rotations in the course of the function *SKYPOS*, whose arguments are the positions of the planets in geocentric equatorial coordinates (*GQ*) and the latitude, date, and time of the viewing point on Earth. The result is in units of altitude and azimuth, and such variables are given the name *AA*.

Calculating the Positions of the Planetary Bodies

The function *CALCULATEPLANETS* finds *PLANETS*, a table of the positions of the sun, moon, and planets at the desired date. When first calculated by the function *PLANETPOS*, these positions are stated in 3-dimensional heliocentric Cartesian coordinates. But the function *EARTHVIEW* converts them to geocentric polar coordinates (right ascension in hours, declination in degrees, and distance in astronomical units), locating the planets with respect to the center of the Earth.

In order to plot the sky above a particular place, the function *SKYPOS* (see p. 29) is used to calculate *AA*, a table of altitude and azimuth with respect to given time and location on the Earth's surface. The function *VISIBLE* is used to select from *P* those members that are above the horizon, and saves them in the table *AAP*. Finally, the *PROJECTION* of these coordinates onto a flat surface is calculated, and translated to the Cartesian form expected by the plotting function; the function *IF* is simply a compression of the left argument by the right, along the first axis, defined by the APL symbol */*.

```
▽ CALCULATEPLANETS; AA; MOON; SUN; KOHOUTEK
PLANETCOORD←AAP+PLANETS+VP+10
PLANETS←DATE EARTHVIEW DATE PLANETPOS (3×19)+PLANETS
SUN←DATE EARTHVIEW 0 0 0
K←100≥|DATE-JNU 12 28 1973
KOHOUTEK←DATE EARTHVIEW (DATE IF K) COMETPOS KOHOUTEK
MOON←MOONPOS DATE
PHASE←MOON[1;1] MOONPHASE SUN[1;1]
PLANETS+MOON,[1] SUN,[1] PLANETS,[1] KOHOUTEK
MOON←MOON[;3] PARALLAXADJUST (LAT,DATE,TIME) SKYPOS MOON
AA←MOON,[1] (LAT,DATE,TIME) SKYPOS 1 0+PLANETS
PLANETCOORD←MAPCARTESIAN PROJECTION AAP←AA IF VP+VISIBLE AA
```

Execution of *CALCULATEPLANETS* causes new values to be assigned to four global variables. (These are initially set to 10 in the first statement, mainly to draw attention to a list of the global variables which will be reset as a consequence of executing this function.) The four are:

<i>PLANETS</i>	The right ascension and declination of the moon, sun, planets, and Kohoutek.
<i>VP</i>	A logical vector indicating which planets are visible from the place, date, and time requested.
<i>AAP</i>	The altitude and azimuth of the visible planets.
<i>PLANETCOORD</i>	The Cartesian coordinates used to plot the projection of the visible planets.

Orbital Parameters

The functions that locate the positions of the planets in their orbits make reference to a set of parameters usually called the elements of the orbit. The reference set of orbital elements for the planets is stored in the matrix PLANETS. Each row contains the set of elements for a particular planet. For example:

```
▽ Z←EARTH
Z←PLANETS[,3;]
```

Each column corresponds to a particular element of the various orbits. Each of the functions that makes use of the orbital elements (PLANETPOS, MOONPOS, or COMETPOS) take as one of its arguments a matrix containing the rows of the table PLANETS that are appropriate: i.e. those corresponding to the particular planets being considered. This sub-table is given the name ORB. Functions are provided corresponding to each orbital element (for example, PERIOD, ECCENTRICITY, INCLINATION, etc.). Those functions select the appropriate column of the table ORB. In that way, terms such as PERIOD, ECCENTRICITY or INCLINATION refer to those elements for the planets currently under consideration, whatever those may be. This is achieved by making ORB, the table from which the values are selected, global with respect to these selection functions, but local to the functions such as PLANETPOS which use the elements, since ORB there appears as the explicit argument.

The geometrical meanings of the terms inclination, ascending node, and angle of perihelion are illustrated in Figure 5.

▽ Z←SEMIMAJOR Z←ORB[,1]	▽ Z←ASCENDING Z←ORB[,5]
▽ Z←PERIOD Z←ORB[,2]	▽ Z←PERIANGLE Z←ORB[,6]
▽ Z←ECCENTRICITY Z←ORB[,3]	▽ Z←ANOMALY Z←ORB[,7]
▽ Z←INCLINATION Z←ORB[,4]	▽ Z←ANOMALYDATE Z←ORB[,10]

The date of perihelion is computed from the elements already tabled:

```
▽ Z←PERIDATE
Z←ANOMALYDATE - PERIOD×ANOMALY÷360
```

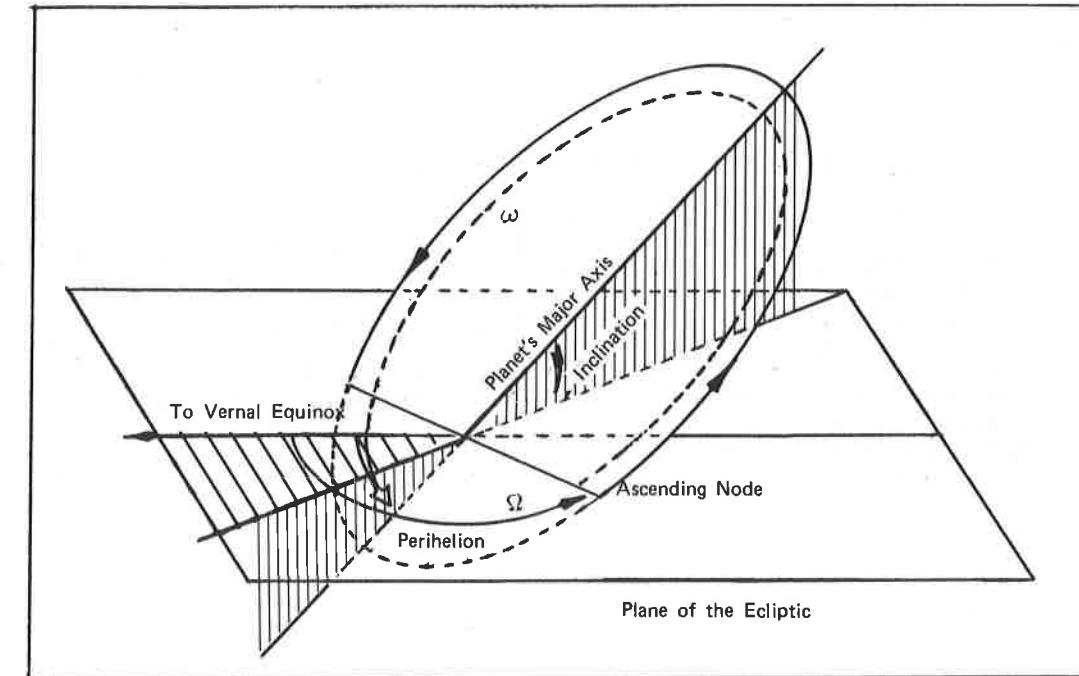


Fig. 3 Elements of an elliptical orbit

The rectangular plane represents the plane of the ecliptic. The focus of the planet's elliptical orbit is the sun. INCLINATION is the angle between the plane of the ellipse and the plane of the ecliptic.

The ASCENDING node is the point at which the planet's orbit passes through the plane of the ecliptic from south to north.

The angle Ω is measured in the plane of the ecliptic, from a line from the sun through the vernal equinox, to a line from the sun to the ascending node.

The angle ω is measured in the plane of the planet's orbit, from a line from the sun to the ascending node, to the major axis on the side of perihelion.

The parameter PERIANGLE used in this article is defined as $\Omega + \omega$.

In finding where a planet is located at a particular date, one must know what portion of its total period has elapsed since its last perihelion. This is provided by the function *PERIODER*:

```
▽ Z←PERIODER DATE
Z←1|(ANOMALY÷360) + (DATE-ANOMALYDATE) ÷ PERIOD×TROPYR
```

Epochal Adjustment of Planetary Elements

The orientations of the major axes of the elliptical orbits of the planets are not fixed, but themselves rotate steadily; the effect is appreciable over long intervals. Allowance for this secular shift requires an adjustment to the elements *ASCENDING* (the angular coordinate of the ascending node) and *PERIANGLE* (the angular coordinate of perihelion). An approximate adjustment is made by the function *EPOCHADJUST*. It revises the values in columns 5 and 6 of *ORB* (i.e. the ascending node and the angle of perihelion) by the size of the secular shift per unit time, multiplied by the interval since the epoch date. The secular effect is here considered to be linear with time:

```
▽ Z←INTERVAL EPOCHADJUST ORB
ORB[; 5 6]←ORB[; 5 6] + SECULAR × INTERVAL
Z←ORB
```

```
▽ Z←SECULAR
Z←ORB[; 8 9]
```

```
▽ Z←EPOCHDATE
Z←ORB[;10 11]
```

Procedure for Locating the Planets

The function *PLANETPOS* finds the positions of any or all the planets as a function of the date and their orbital elements.

```
▽ H←DATE PLANETPOS ORB; E; THETA
ORB←(DATE-EPOCHDATE) EPOCHADJUST ORB
E←ECCENTRICITY
THETA←E TRUEANOMALY E KEPLINVERSE 2×PI×PERIODER DATE
H←ORBROTATE CARTESIAN THETA,[1.5] RADIUS THETA
```

The third statement of *PLANETPOS* finds *THETA*, the angle between each planet's position at perihelion and its position on the indicated date. The function *RADIUS* finds the distance from the sun at which that angle intersects the ellipse:

```
▽ Z←RADIUS THETA; E
E←ECCENTRICITY
Z←SEMITMAJOR×(1-E×2)÷1+E×COS THETA
```

In the last statement of *PLANETPOS*, the polar coordinates *THETA* calculated in the preceding step are converted to Cartesian heliocentric coordinates *H*.

The Inverse of Kepler's Function

The formula for an ellipse permits us to state the distance from the solar focus to a point on the ellipse (that is, the radius at that point) as a function of the angle *THETA* between the major axis and a line through the focus to that point. However, finding the true anomaly *THETA* directly as a function of time is difficult. An easier method is due to Kepler. He discovered that a closely related angle *PSI* could be constructed (see Figure 4) for which the solution is simpler. A quantity proportional to the time is computed by *KEPLERFN* as a function of *PSI* and the eccentricity *E*:

```
▽ TIME←E KEPLERFN PSI
E←Q((`1↑ρPSI),ρE)ρE
TIME←PSI - E×SIN PSI
```

Notice that as *E* goes to zero (meaning that the ellipse approaches a circle) *KEPLERFN PSI* approaches *PSI*.

To find *PSI* as a function of time, *KEPLERFN* must be inverted. Because *KEPLERFN* involves both *PSI* and *SIN PSI*, it is transcendental, and approximations must be used to evaluate its inverse. We used an iterative method. In this procedure, each estimate of *PSI* is adjusted by correcting the previous approximation by an amount inversely proportional to the derivative. That general procedure is known as Newton's method; it was while working on solutions to Kepler's equations that Newton developed the method:

```
▽ PSI←E KEPLINVERSE TIME; ERROR; TOL
TOL←1E-10
TIME←PSI←((ρE),ρTIME)ρTIME
TEST: →END IF ^/,TOL>|ERROR←TIME-E KEPLERFN PSI
PSI←PSI+ERROR÷E KEPDERIV PSI
→TEST
END: PSI←+/PSI×(2ρρE)ρ(1+ρE)1
```

The restructuring appearing in the second statement and the last statement (and also in *KEPDERIV*, below) is introduced to

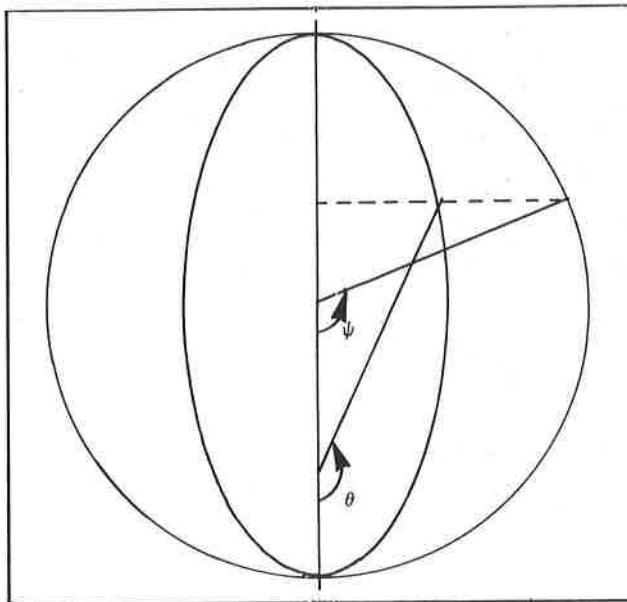


Fig. 4 Angles θ and ψ in the calculation of true anomaly

The angle $THETA$ is measured between the major axis and a line drawn from the focus to the planet's position on the ellipse.

The angle PSI is measured from the major axis to a line drawn from the center of a circle circumscribed about the ellipse, to the point where a line drawn perpendicular to the axis and passing through the planet intersects the Earth.

permit parallel solution for multiple values of E and $TIME$, so that all planets can be treated at once.

The derivative of Kepler's functions is given as follows:

```

V Z←E KEPDERIV PSI
E←Q((‐1+pPSI),pE)pE
Z←1‐E×COS PSI

```

Now that PSI has been found, the more useful true anomaly can be found by analytic geometry:

```

V THETA+E TRUEANOMALY PSI
THETA+(2×PI)|2×ARCTAN (SQRT(1+E)÷1‐E) × TAN PSI÷2

```

The function $RADIUS$ can now be used to find the planet's distance from the sun in astronomical units.

Plotting the Heliocentric Coordinates of the Planets

The aim in preparing this set of functions was to draw maps showing the sky as it appears above a particular place on Earth. To achieve that, the heliocentric coordinates just calculated must be further translated and rotated to allow for the position of the Earth in the solar system and of the observer on the Earth. However, before introducing the functions that carry out that part of the task, we illustrate a use of the heliocentric coordinates. A function $PLANETSPOS$ constructs (iteratively) a table showing for a selected set of dates the positions of selected planets (and also of the comet Kohoutek) for each of an array of dates:

```

V H←DATES PLANETSPOS P; I; D; PL
DATES←,DATES
PL←PLANETS[P;]
H←(0,(1+pP),3)pI+0
TEST: →0 IF (pDATES)<I+I+1
D←DATES[I]
H←H,[1] (D PLANETPOS PL),[1] D COMETPOS KOHOUTEK
→TEST

```

The result is a 3-dimensional array, dates by planets by coordinates. Plotting the first coordinate against the third, we obtain a diagram showing the positions of the planets projected in the plane of the ecliptic (Figure 5).

Positions of the Earth and Moon

In order to find the geocentric coordinates of the other bodies, the heliocentric coordinates of the Earth are required. However, this does not require a special function, since they are directly obtainable from the expression

DATE PLANETPOS EARTH

in which *EARTH* is the function which selects the orbital elements of the Earth.

Since the moon is in an elliptical orbit about the Earth, the position of the moon with respect to the earth can be found by the same procedure used to locate the planets with respect to the sun. In calculating the position of the moon, the positions of the ascending node and the angle of perihelion are subject to linear epochal adjustments that are larger than those for the planets, but they are computed in exactly the same way:

```
▽ GQ←MOONPOS DATE; GC  
GC←DATE PLANETPOS MOON  
GQ←3 RADECDIST GC+.×&INCLROTATE RADIANT AXITILT+23.4428
```

In the case of the moon, the unit of distance is the semimajor axis of the orbit of the moon rather than of the Earth.

The rotation functions will be discussed below (see pp. 26-27); the function *RADECDIST* calculates polar coordinates in units of right ascension, declination, and distance; the left argument 3 indicates that in this case all three are to be retained.

Since *MOONPOS* finds the moon's position with respect to the Earth, the result is stated with respect to the Earth, and there is no need for subsequent translation from heliocentric to geocentric coordinates. (In the definition of *CALCULATEPLANETS*, p. 13, the expressions for *PLANETS*, *SUN* and *KOHOUTEK* require the application of the function *EARTHVIEW*, whereas the expression for *MOON* does not.) However, the moon is sufficiently close to the Earth that in calculating its apparent position allowance must be made for the parallax introduced by the fact that the observer's position on the surface of the Earth may depart significantly from a line between the center of the Earth and the center of the moon. Such a correction to the moon's altitude is used in *CALCULATEPLANETS*:

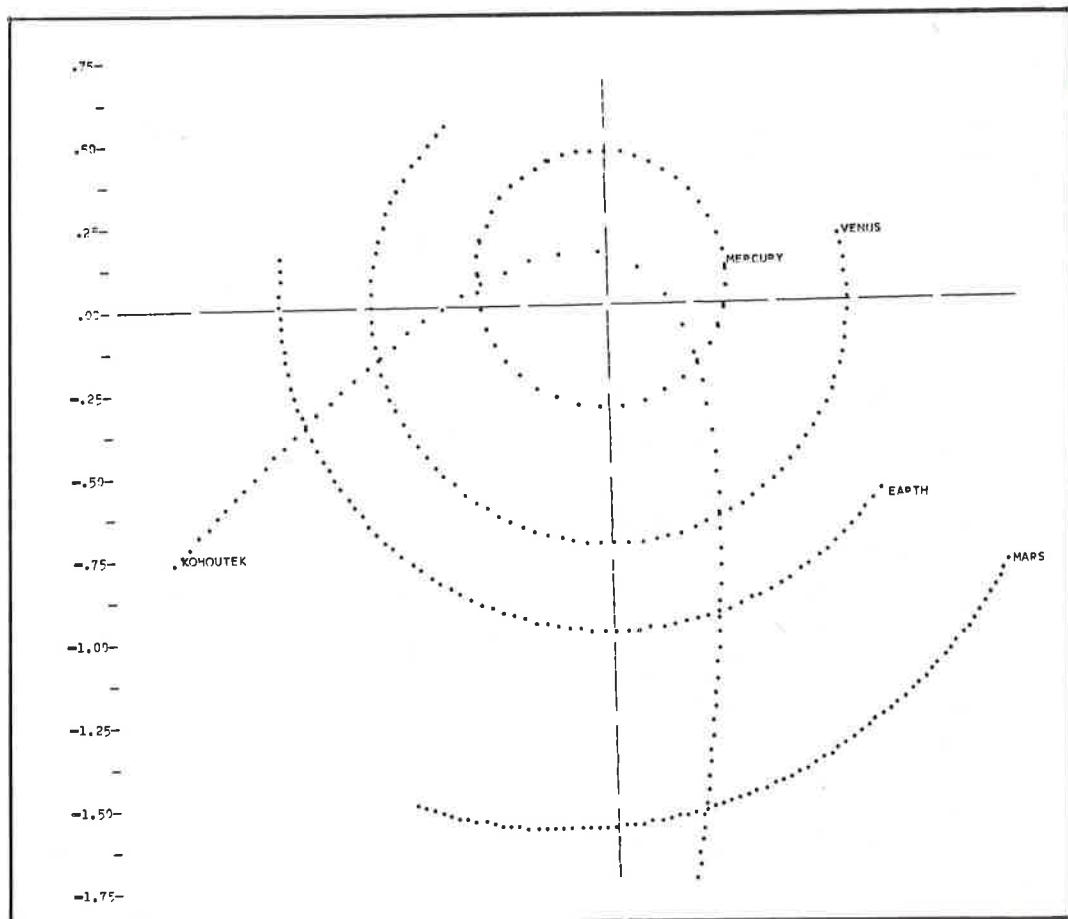


Fig. 5 Sample output of program to plot heliocentric coordinates

The plot shows the orbits of the four inner planets and the comet Kohoutek at 2-day intervals from 20 October 1973 through 30 March 1974.

```

    V Z←DIST PARALLAXADJUST AA; ALT
    ALT←AA[;1]
    Z←AA
    Z[;1]←ALT - (COS RADIANT ALT)×MOONRATIO÷DIST

```

in which *MOONRATIO* is the ratio of the semimajor axis of the moon's orbit to the radius of the Earth, expressed in radians; the value is about 0.95.

The phase of the moon depends upon the difference between the right ascensions of the sun and moon:

```

    V Z←MOON MOONPHASE SUN
    Z←1|(MOON-SUN)÷24

```

The moon is full when their right ascensions differ by 12 hours, and new when they are equal. When both the right ascension and the declination of the moon are equal to those of the sun, there is an eclipse of the sun; when their right ascensions differ by 12 hours and their declinations are equal but of opposite sign, there is an eclipse of the moon.

Position of the Comet

The position of Kohoutek is calculated only for dates within 100 days of its perihelion, 28 December 1973. The logical variable *K* (set in *CALCULATEPLANETS*) has the value 1 when Kohoutek is within range, 0 otherwise. The expression *K/DATE* thus makes the date empty when the position of the comet is not needed.

```

    V H←DATE COMETPOS ORB; X
    H← 0 3p0
    →0 IF 0=p,DATE
    X←COMETSOLVE (PI×SQRT 2×PERIDIST)×(DATE-ANOMALYDATE)÷TROPYR
    H←ORBROTATE (PARABOLA X), -X

```

The method used to locate the comet is similar to that used for the planets. However, for several reasons the polar coordinates used in the initial two-dimensional solution for the planets are here replaced with Cartesian coordinates. The approximations for planets (whose orbits are nearly circular) do not converge easily when applied to the comet, whose orbit is almost exactly parabolic. The usual polar expression in the function *RADIUS* is singular when *E* is 1 (parabola) and *THETA* is *PI*. Moreover, the Cartesian expression for a parabola is simple to integrate; hence Kepler's equal-areas equal-times law is easily applied.

The time required to reach a point on the parabolic path of the comet as a function of the distance from the axis of the parabola is given by the function *AREA*:

```

    V Z←AREA X
    Z←(PERIDIST×X÷2) + (X*3)÷24×PERIDIST

```

in which the orbital element *PERIDIST* is the distance from the sun at perihelion, in astronomical units:

```

    V Z←PERIDIST
    Z←ORB[;1]

```

The function *COMETSOLVE* provides an iterative definition for the inverse of *AREA*, giving the perpendicular distance from the axis of the parabola as a function of the time interval from perihelion:

```

    V X←COMETSOLVE TIME; ERROR
    X←2×TIME÷PERIDIST
    TEST: →0 IF 1E-8>|ERROR-TIME-AREA X
    X←X+ERROR÷AREADERIV X
    →TEST

```

Here again the inverse is found by Newton's method; convergence is speeded by the use of the derivative of the area function with respect to the abscissa:

```

    V Z←AREADERIV X
    Z←(PERIDIST÷2) + (X*2)÷8×PERIDIST

```

The second coordinate of the comet's position (within the plane of its orbit) is measured in the direction of the axis of the parabola. It is obtained from the first coordinate by the function *PARABOLA*:

```

    V Z←PARABOLA X
    Z←PERIDIST - (X*2)÷4×PERIDIST

```

Rotation of the Stars

The positions of the stars are represented by a table of their right ascensions and declinations, as of 1 January 2000, contained in the matrix *STARS*. There is no provision for the proper motions of the stars, nor for the effects of parallax between different positions on the Earth's orbit, since both these effects are small compared to the precision of the rest of the calculation or to the resolution of the plotting program. The calculation thus reduces to the correction for

the observer's position at a given latitude, date, and time, and the long-run variation introduced by precession.

```

▽ CALCULATESTARS; STARS
VS←BRIGHT←STARCOORD←AAS+10
STARS←(LAT,DATE,TIME) SKYPOS DATE PRECESS STARS
BRIGHT←BRIGHT IF VS←VISIBLE STARS
AAS←STARS IF BRIGHT^VS
STARCOORD←MAPCARTESIAN PROJECTION STARS IF VS

```

The global results of this function (initially set to 10 in the first statement) are as follows:

STARCOORD	Cartesian coordinates on the map for the stars visible from the indicated time, date, and location.
BRIGHT	A logical vector indicating which of the visible stars are of magnitude 1.5 or brighter.
VS	A logical vector indicating which stars are visible.
AAS	A matrix containing the altitude and azimuth of the visible bright stars.

Correction for Precession

The effect of precession is to alter the direction in which the Earth's axis is tilted. A line drawn from the north pole to the zenith (which today points approximately to the star Polaris) in the course of 25800 years describes a complete circle, with radius 23.45 degrees. What changes with precession is the direction in which the Earth's north pole departs from a point perpendicular to the plane of the Earth's orbit. However, since the direction of the equinox enters into the definition of one of the axes of both the heliocentric and the geocentric ecliptic coordinates, the effect appears as a systematic rotation of the entire star table. The function PRECESS makes this adjustment by first removing the Earth's axial tilt, then rotating about the second axis through an angle that would amount to a complete rotation in 25800 years, and then restoring the axial tilt.

```

▽ Z←INTERVAL PRECESS X; PRECESSION; ROT; TILT; DETILT; RETILT
X←CARTRIPLET X
RETILT←INCLROTATE TILT←RADIAN AXITILT
DETILT←INCLROTATE -TILT
PRECESSION←LONGROTATE INTERVAL × 2×PI÷25800×YRLENGTH
ROT←RETILT+.×PRECESSION+.×DETILT
Z←2 RADECDIST X+.×QROT

```

Projection of the Visible Sky

Once the altitude and azimuth of moon, sun, planets, and comet have been calculated, it remains only to select those that are visible, and calculate a suitable projection for the map. Objects are considered to be visible if they are on or above the horizon, i.e. if they have non-negative altitude:

```

▽ Z←VISIBLE X; ALT
ALT←X[;1]
Z←ALT≥0

```

To preserve the apparent shapes of constellations when projected onto a flat surface, the altitudes near the zenith are condensed and those near the horizon expanded by the function PROJECTION which makes the distance from the center of the map proportional to the tangent of one half the coaltitude:

```

▽ Z←PROJECTION X
Z←(TAN 0.5×COALTITUDE X[;1]),[1.5] RADIAN X[;2]

```

in which coaltitude is defined thus:

```

▽ Z←COALTITUDE X
Z←RADIAN 90-X

```

Since the plotting routine expects its data to be stated in Cartesian coordinates, the projected polar coordinates are converted back to that form. The function MAPCARTESIAN makes allowance for the fact that altitude and azimuth are conventionally grouped in the opposite order from right ascension and declination:

```

▽ Z←MAPCARTESIAN X
Z←ΦCARTESIANΦX

```

Functions for Rotation and Translation of Coordinates

The function ORBROTADE converts the two-dimensional Cartesian coordinates of the planets within their own planes to three-dimensional heliocentric coordinates, taking into account the orientation and tilt of the plane of each orbit:

```

▽ H←ORBROTATE X; INCL; I; OMEGA; O; QMEGA; Q
X←((ρX),1)ρX← 1 0 1 \X
OMEGA←RADIAN PERIANGLE-ASCENDING
OMEGA←RADIAN ASCENDING
INCL←RADIAN INCLINATION
I←INCLROTATE INCL
O←LONGROTATE OMEGA
Q←LONGROTATE QMEGA
H←(Q TIMES I TIMES O) TIMES X
H←((1↑ρH),×/1↓ρH)ρH

```

The rotations are achieved by a series of matrix products. The functions *INCLROTATE* and *LONGROTATE* generate the appropriate matrices of sines and cosines, stacking them in a three-dimensional array since several sets of coordinates are to be rotated at once. The function *TIMES* (not shown) calculates the ordinary matrix product of the corresponding pairs of matrices in a three-dimensional stack.

The functions *INCLROTATE* and *LONGROTATE* generate stacks of matrices containing the appropriate sines and cosines of the angles through which rotation is to occur (see Figure 6):

```

▽ Z←INCLROTATE INCL; RHO
RHO←ρINCL
Z←((ρ,INCL), 3 3)ρ9↑1
Z[ ;2;2]←Z[ ;3;3]←COS INCL
Z[ ;2;3]←-Z[ ;3;2]←SIN INCL
→(0<ρRHO)/0
Z← 3 3 ρZ

▽ Z←LONGROTATE OMEGA; RHO
RHO←ρOMEGA
Z←((ρ,OMEGA), 3 3)ρ 0 0 0 0 1 0 0 0 0
Z[ ;1;1]←Z[ ;3;3]←COS OMEGA
Z[ ;3;1]←-Z[ ;1;3]←SIN OMEGA
→(0<ρRHO)/0
Z← 3 3 ρZ

```

These functions are used in translating the heliocentric coordinates of the planets to geocentric equatorial coordinates (i.e. the view from the center of the Earth):

```

▽ GQ←DATE EARTHVIEW H; GC
GC←H-(ρH)ρDATE PLANETPOS EARTH
GQ+3 RADECDIST GC+.×INCLROTATE -RADIAN AXITILT

```

in which *AXITILT* is the angle between the axis of the Earth and the plane of the ecliptic.

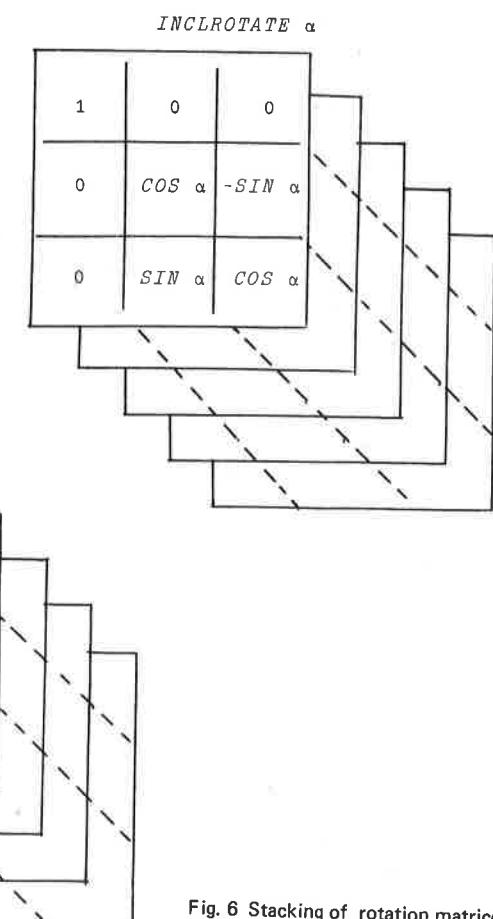


Fig. 6 Stacking of rotation matrices

Each plane represents the rotation matrix for one of the planets.

The next transformation adjusts for the location on the Earth of the observer for whom the map is calculated. The coordinates with respect to the observer are described in a system in which the three coordinates point respectively south, overhead, and west. These are calculated by *SKYPOS* as a function of the geocentric equatorial coordinates *GC*, and the observer's latitude and true local time:

```

▽ AA←EARTH SKYPOS GQ; SUN; ROT; LAT; DATE; TIME;
    ALT; AZ; NEG; S
LAT←EARTH[ 1 ]
DATE←EARTH[ 2 ]
TIME←EARTH[ 3 ]
SUN←(24+YRLENGTH)×YRLENGTH | DATE-EQUINOX
ROT←PI×(SUN+TIME-12)÷12
LAT←RADIAN 90-LAT
GQ←GQ÷Q(Φ, GQ)ρNORM GQ←CARTRIPLET GQ
GQ←GQ+.×Q(LATROTATE LAT)+.×LONGROTATE-ROT
ALT+DEGREES ARCSIN GQ[ ;2]
NEG←-S+.×GQ[ ;3]
AZ←(360×S≥0)+NEG×DEGREES ARCCOS-GQ[ ;1]÷NORM GQ[ ; 1 3]
AA←ALT,[ 1.5 ] AZ

```

The variable *TROPYR* is the length of the tropical year in days; *EQUINOX* is the Julian date of a vernal equinox (in this case, for 1973).

The function *LATROTATE* prepares a matrix of sines and cosines, exploiting the relation between rotation of latitude and rotation of inclination:

```

▽ Z←LATROTATE LAT
Z←ΦΦΦINCLROTATE LAT

```

Conversion of Units

The positions of objects in the sky are described in spherical polar coordinates, usually as right ascension, declination, and distance. The first two are stated as angles in hours or degrees, and the last in astronomical units. The function *RAECDIST* converts from Cartesian to polar coordinates in which right ascension is stated in hours and declination in degrees. Since the distance of celestial objects is not apparent from the Earth, only the right ascension and declination are required for some calculations; by using a left argument of 2, only the first two coordinates are retained, and distance is dropped where it is no longer appropriate:

```

▽ Z+COL RAECDIST GQ; DIST
Z←ARCCOS GQ[ ;1]÷(GQ[ ; 1 3]+.*2)*0.5
Z←(12÷PI)×Z+(GQ[ ;3]>0)×2×PI-Z
DIST←(GQ+.×2)*0.5
Z+Z,[ 1.5 ] (180÷PI)×ARCSIN GQ[ ;2]:DIST
→0 IF COL<3
Z←Z,DIST

```

The norm is defined as the square root of the sum of the squares:

```

▽ Z←NORM X
Z←(X+.×2)*0.5

```

Conversion to Cartesian from polar coordinates is provided by the function *CARTESIAN*:

```

▽ Z←CARTESIAN POLAR; RHO; THETA
THETA←POLAR[ ;1]
RHO←POLAR[ ;2]
Z←(RHO×COS THETA),[ 1.5 ] -RHO×SIN THETA

```

Conversion to non-normalized three-dimensional Cartesian coordinates from spherical polar coordinates is provided by the function *CARTRIPLET*:

```

▽ Z←CARTRIPLET RADEC; Z1; Z2; Z3
Z1←COS PI×RADEC[ ;1]:12
Z2←TAN RADIAN RADEC[ ;2]
Z3←-SIN PI×RADEC[ ;1]:12
Z←Z1, Z2,[ 1.5 ] Z3

```

Sample Star Maps

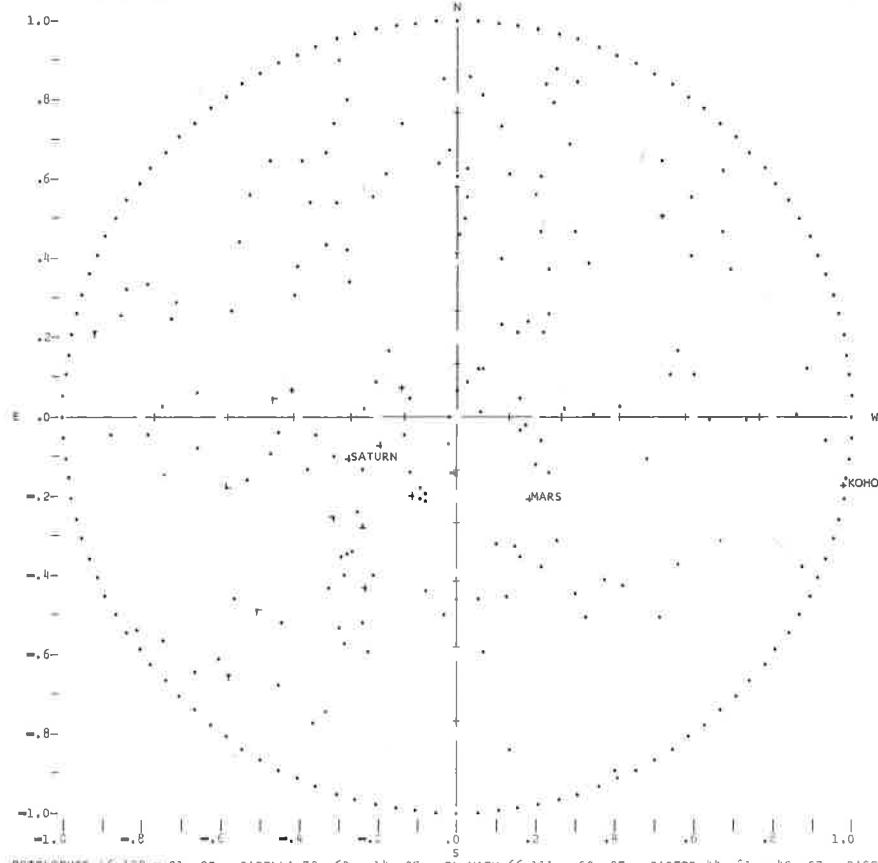
On the following pages maps generated by these programs are reproduced, showing the views from Philadelphia on 14 January 1974 (when Kohoutek was visible), and from the Arctic circle at midnight on 21 June 1974. On two further charts, showing the views from the north and south poles at the vernal equinox, lines linking stars in the same constellation have been drawn in by hand.

VIEW FROM 39 DEGREES 50 MINUTES NORTH, 75 DEGREES 10 MINUTES WEST, ON MONDAY 1974/1/14 AT 8 00 PM STANDARD TIME

POSITION OF SUN, MOON AND PLANETS

	RT AS	DECL	ALT AZ	MAP-GRID		RT AS	DECL	ALT AZ	MAP-GRID
MOON	13 8	-12 21			*SATURN	5 56	23 7	57 111	-.27 -.10
SUN	19 43	-21 17			URANUS	13 35	-8 27		
MERCURY	20 1	-22 38			NEPTUNE	18 5	-22 18		
VENUS	20 36	-13 42			PLUTO	13 27	9 18		
*MARS	2 20	14 58	59 222	.18 =.20	*KOHOUTEK	22 10	-7 27	0 260	.98 =.17
JUPITER	21 20	-17 1							

MOON PHASE .55 WANING



BETELGEUSE 46 120 =-31 =.25 CAPELLA 72 62 =-14 =.07 EL NATH 66 111 =-.20 =.07 CASTOR 44 61 =-.42 =.07 RIGEL 38 151 =.24 =.43
BELLATRIX 50 139 =-24 =.29 PROCYON 27 107 =-.55 =.19 SIRIUS 20 134 =-.51 =.49 POLLUX 40 64 =-.47 =.05
ALDEBARAN 64 151 =-.11 =.20 REGULUS 3 77 =-.92 =.21 ADHARA 0 139 =-.50 =.65 DENEBO 18 314 =.52 =.51

CALCULATED AND PRINTED BY IBM USING AN APL.SV SYSTEM
MONDAY 1974/1/15 AT 5 23 AM

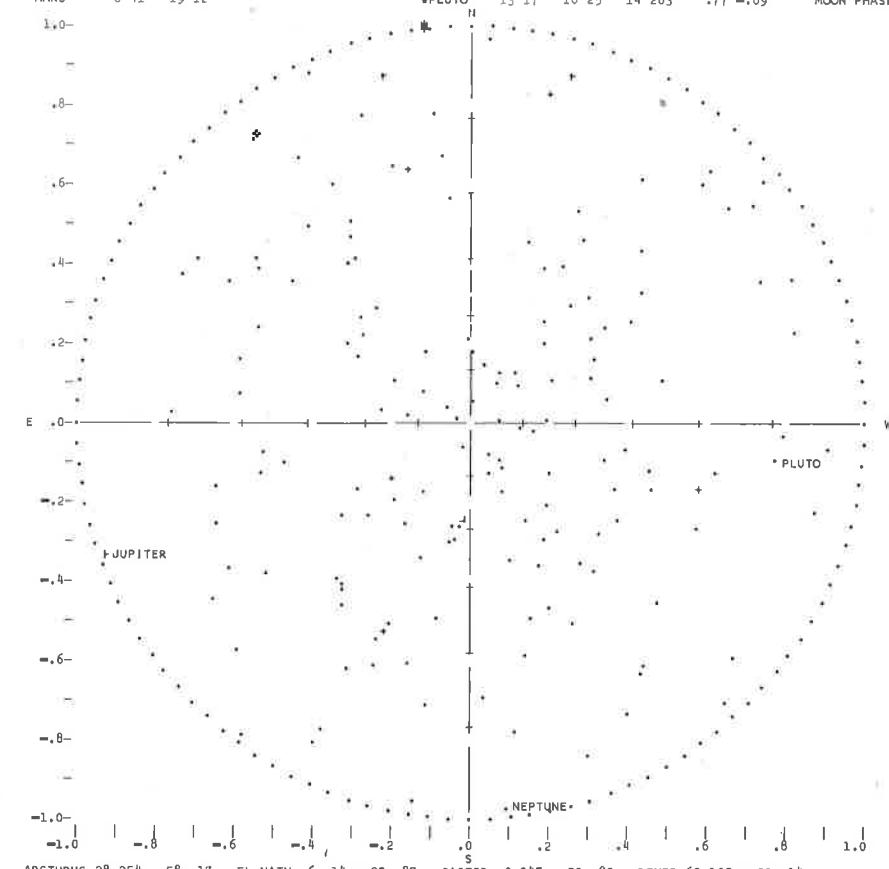
Fig. 7 Star map at Philadelphia on January 14, 1974

VIEW FROM 66 DEGREES 30 MINUTES NORTH, 144 DEGREES 30 MINUTES WEST, ON FRIDAY 1974/6/21 AT MIDNIGHT STANDARD TIME

POSITION OF SUN, MOON AND PLANETS

	RT AS	DECL	ALT AZ	MAP-GRID		RT AS	DECL	ALT AZ	MAP-GRID
*SUN	8 6	16 45			*JUPITER	23 16	=7 6	1 110	-.93 =.33
MERCURY	6 52	23 26	0 7	-.12 .99	SATURN	6 30	23 4		
VENUS	3 34	17 18			URANUS	13 20	6 59		
MARS	8 41	19 12			NEPTUNE	18 6	-22 15	1 186	.10 =.98
					PLUTO	13 17	10 25	14 263	.77 =.09

SUN SHOWN AS # MOON PHASE .17 WAXING



ARCTURUS 28 254 =.58 =.17 EL NATH 6 14 =-.23 =.87 CASTOR 9 347 =.20 =.83 DENEBO 63 125 =-.20 =.14
CAPELLA 23 14 =-.16 =.64 ALTAIR 31 157 =-.22 =.52 POLLUX 6 344 =.25 =.87 VEGA 62 178 =.01 =.25

CALCULATED AND PRINTED BY IBM USING AN APL.SV SYSTEM

MONDAY 1974/6/24 AT 9 04 AM

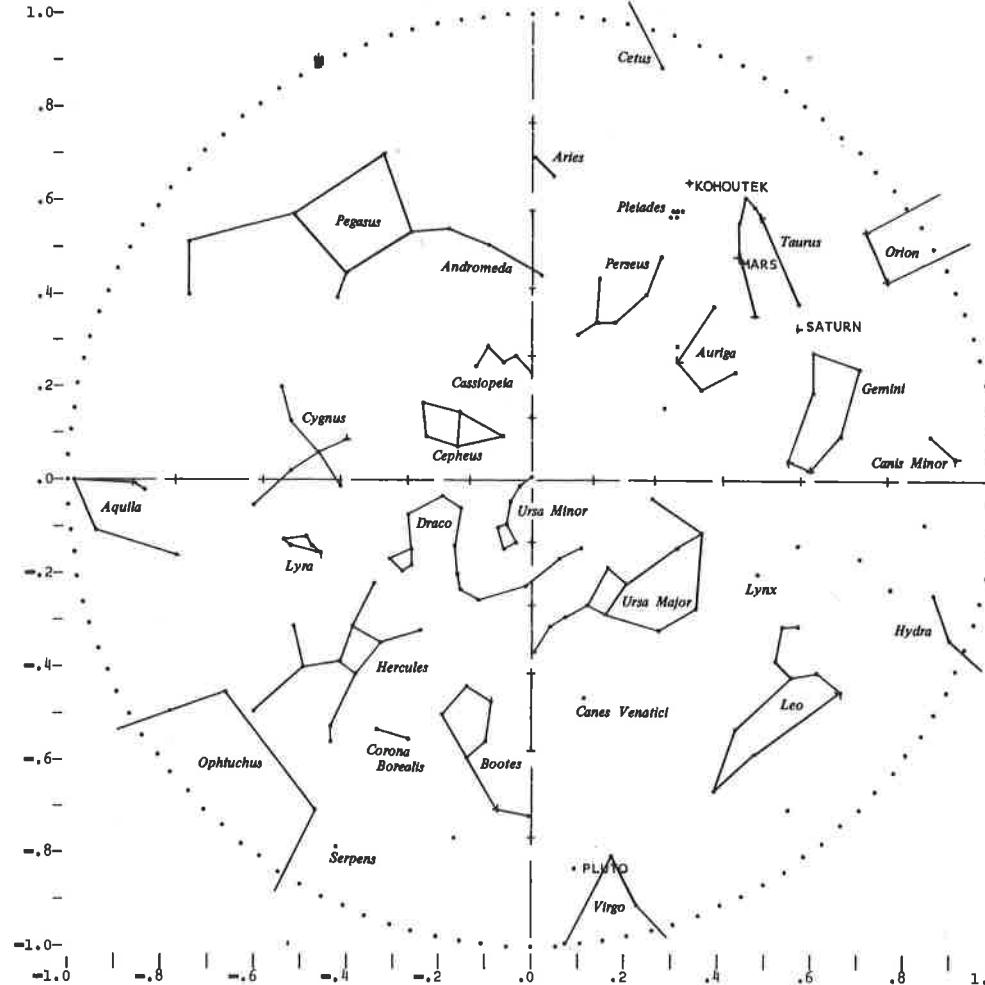
Fig. 8 Star map at Fort Yukon on June 31, 1974

VIEW FROM THE NORTH POLE, 0 DEGREES ON TUESDAY 1974/3/19 AT 1 58 AM

POSITION OF SUN, MOON AND PLANETS

	RT.AS.	DECL.	MAP GRID		RT.AS.	DECL.	MAP GRID
MOON	20 18	-16 15	-.46 .89	*SATURN	5 50	23 10	.57 .33
*SUN	0 0	0 0		URANUS	13 31	-18 3	
MERCURY	22 18	-11 25		NEPTUNE	18 11	-22 15	
VENUS	21 4	-13 48		*PLUTO	13 25	10 5	.09 -.83
*MARS	4 41	23 20	.45 .48	KOHOUTEK	3 42	18 20	.34 -.64
JUPITER	22 20	-12 0					

SUN SHOWN AS ☀
MOON PHASE 0.307 WANING



CALCULATED AND PRINTED FOR DISPLAY BY IBM USING AN APL.SV SYSTEM
FRIDAY 1973/12/7 AT 3 02 PM

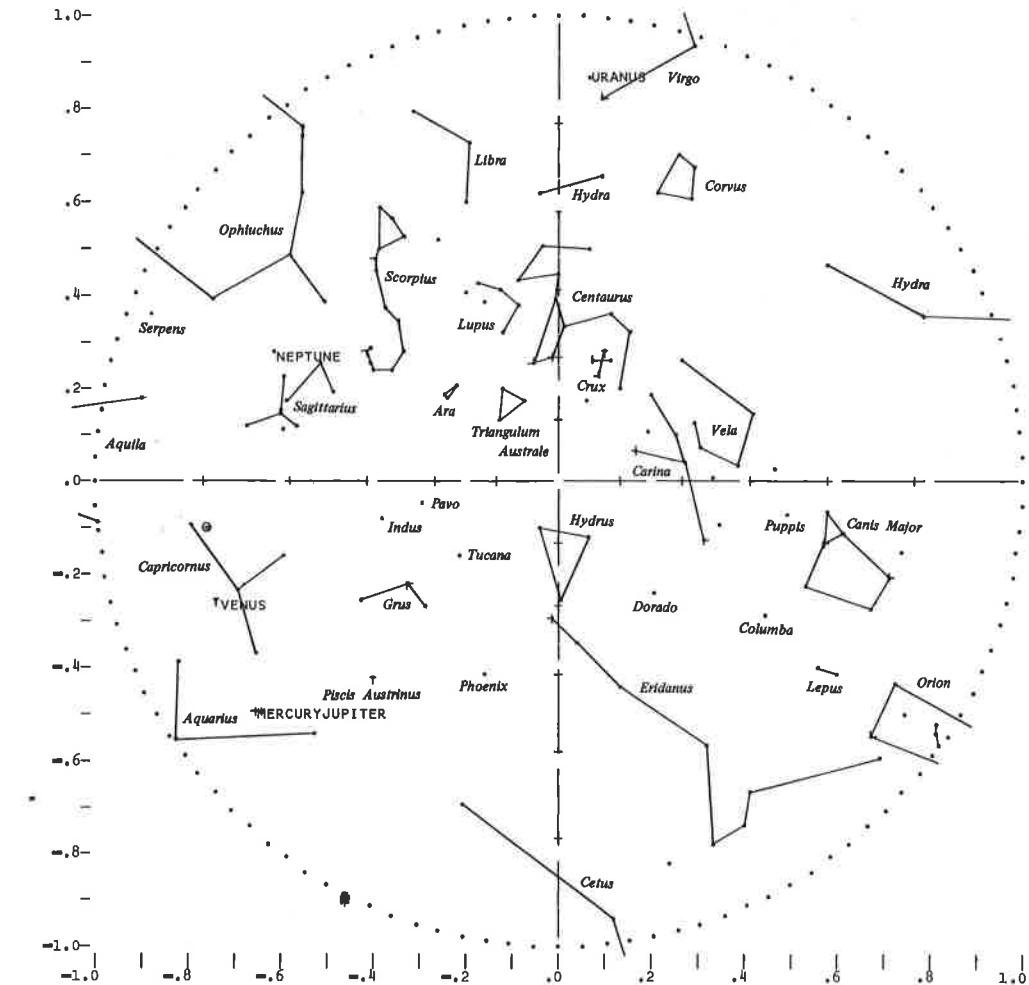
Fig. 9 Star map at North Pole on March 19, 1974

VIEW FROM THE SOUTH POLE, 0 DEGREES ON TUESDAY 1974/3/19 AT 1 57 AM

POSITION OF SUN, MOON AND PLANETS

	RT.AS.	DECL.	MAP GRID		RT.AS.	DECL.	MAP GRID
*MOON	20 18	-16 15	-.76 -.10	SATURN	5 50	23 10	.07 .87
*SUN	23 59	0 0	-.46 -.89	URANUS	13 31	-8 3	
MERCURY	22 18	-11 25	-.65 -.50	NEPTUNE	18 11	-22 15	-.61 .28
VENUS	21 4	-13 48	-.74 -.25	PLUTO	13 25	10 5	
MARS	4 41	23 20		KOHOUTEK	3 42	18 20	
JUPITER	22 20	-12 0	.64 -.50				

SUN SHOWN AS ☀
MOON SHOWN AS O
MOON PHASE 0.307 WANING



CALCULATED AND PRINTED FOR TYCHO BRAHE BY IBM USING AN APL.SV SYSTEM
FRIDAY 1973/12/7 AT 2 50 PM

Fig. 10 Star map at South Pole on March 19, 1974

REFERENCES

- Marion, Jerry B., Classical Dynamics of Particles and Systems, New York: Academic Press, 1965.
- American Ephemeris and Nautical Almanac, Explanatory Supplement, U.S. Naval Observatory, 1961.
- Hoffleit, Dorrit, Catalog of Bright Stars, New Haven: Yale University, 1964.

APPENDIX

TABLES

The tables in this appendix were prepared by Mr. Per Gjerlov of IBM Denmark, using data from the Yale Catalogue of Bright Stars.

On the first page, values are given for the orbital elements of the nine planets, of the moon, and of the comet Kohoutek. These are the values used to produce the sample charts shown in this report.

Following that, there appear the coordinates of 332 stars. The stars included are roughly the first 300 in visual magnitude, plus a handful of others chosen because they help complete the outline of certain constellations. The table shows the popular name (where there is one), the Bayer designation, and the number in the Yale catalogue. The coordinates are shown as right ascension in hours, minutes, and seconds, and declination, in degrees and minutes, epoch 1 January 2000. The last two columns show the visual magnitude, and the annual parallax in seconds. Where there is a bright double star, only one star is listed.

The stars in Pleiades are here named PLE, although they are commonly referred to the constellation Taurus. To improve visual display, they are shown with positions slightly different from the correct ones.

Mean orbital elements for the planets (columns 1-7 of PLANETS)

	SEMI	MAJOR	PERIOD	ECCENT'Y	INCLINAT'N	ASCENDING	PERIANGLE	ANOMALY
MERCURY	0.387	0.24085	0.20563	7.004330	48.07347	77.11704	289.6550	
VENUS	0.723	0.61521	0.00678	3.394420	76.48402	131.26501	150.2801	
EARTH	1	1.00004	0.01672	0	0	102.56635	34.4957	
MARS	1.524	1.88089	0.09338	1.849810	49.38973	335.65866	271.1460	
JUPITER	5.202	11.86223	0.04794	1.305540	100.21550	14.10850	283.9167	
SATURN	9.578	29.45772	0.05759	2.486680	113.49100	94.40310	348.2963	
URANUS	19.178	84.01529	0.04808	0.771410	74.00020	168.86530	25.4394	
NEPTUNE	29.965	164.78829	0.01119	1.772070	131.54740	59.57130	224.9265	
PLUTO	39.543	248.43020	0.24934	17.137130	109.88680	223.14830	335.6904	

Mean orbital elements for the planets (columns 8-11 of PLANETS)

	SECULAR	ASCENDING	SECULAR	PERIANGLE	DATE (ASC)	DATE (PERI)
MERCURY	0.000	032	444	198	0.000 042 559 243	2443600.5
VENUS	0.000	024	641	163	0.000 038 505 620	2443600.5
EARTH	0.000	000	000	000	0.000 047 000 737	2443600.5
MARS	0.000	021	188	358	0.000 050 392 700	2443600.5
JUPITER	0.000	027	683	282	0.000 044 110 724	2443600.5
SATURN	0.000	023	880	633	0.000 053 617 346	2443600.5
URANUS	0.000	013	689	535	0.000 044 110 724	2443600.5
NEPTUNE	0.000	030	040	924	0.000 018 252 713	2443600.5
PLUTO	0.000	038	026	486	0.000 038 026 486	2443600.5

Orbital elements for the moon (with respect to Earth)

	SEMI	MAJOR	PERIOD	ECCENTRICITY	INCLINATION	ASCENDING	PERIANGLE
MOON	1	0.07544	0.05490	5.143412	260.38369	331.80423	
	SECULAR	ASCENDING	SECULAR	PERIANGLE	DATE (ASC)	DATE (PERI)	
MOON	-0.005	295	392	200	0.011 140 408 030	2414997.831	2414997.831

Orbital elements for the comet Kohoutek

	PERIDIST	INCLINATION	ASCENDING	PERIANGLE	PERIDATE
KOHOUTEK	0.142	14.2969	257.7153	295.5891	2442046.463

Popular Name	Bayer No.	Right Asc.			Decl.		Mag.	Prlx Secs
		Hr	Min	Sec	Deg	Min		
ALpheratz	1 α AND	15	0	8 23	29	5	2.00	.024
Mirach	2 β AND	337	1	9 44	35	37	2.00	.043
Almach	3 γ AND	603	2	3 53	42	20	2.30	.005
	4 δ AND	165	0	39 20	30	52	3.20	.024
Altair	5 α AQL	7557	19	50 47	8	52	.77	.198
	6 β AQL	7602	19	55 19	6	24	3.71	.070
	7 γ AQL	7525	19	46 15	10	37	2.60	.006
	8 δ AQL	7377	19	25 29	3	7	3.36	.062
	9 ε AQL	7235	19	5 25	13	52	3.00	.036
	10 η AQL	7570	19	52 29	1	0	3.50	.005
	11 θ AQL	7710	20	11 18	0	49	3.24	.008
	12 λ AQL	7236	19	6 15	-4	53	3.40	.025
	13 α AQR	8414	22	5 47	0	19	2.93	.003
	14 β AQR	8232	21	31 34	-5	35	2.89	.000
	15 δ AQR	8709	22	54 39	-15	49	3.30	.039
	16 β ARA	6461	17	25 18	-55	32	2.80	.026
	17 γ ARA	6462	17	25 24	-56	23	3.30	.000
	18 ζ ARA	6285	16	58 38	-55	59	3.10	.036
	Hamal	19 α ARI	617	2 7	10	23 27	2.00	.043
	Sheratan	20 β ARI	553	1 54	39	20 48	2.65	.063
	Capella	21 α AUR	1708	5 16	41	46 0	.09	.073
	Menikalinan	22 β AUR	2088	5 59	32	44 57	1.90	.037
		23 δ AUR	2077	5 59	32	54 17	3.70	.020
		24 ε AUR	1605	5 1	58	43 50	3.00	.004
		25 θ AUR	2095	5 59	43	37 12	2.70	.018
	Arcturus	26 ι AUR	1577	4 57	0	33 9	2.70	.015
		27 α BOO	5340	14 15	40	19 11	.06	.090
		28 β BOO	5602	15 1	57	40 23	3.50	.022
		29 γ BOO	5435	14 32	5	38 19	3.00	.016
		30 δ BOO	5681	15 15	30	33 19	3.50	.028
		31 ε BOO	5506	14 44	59	27 5	2.70	.013
		32 ζ BOO	5477	14 41	8	13 43	4.10	.007
		33 η BOO	5235	13 54	41	18 24	2.70	.102
		34 ρ BOO	5429	14 31	50	30 23	3.60	.025
		35 α CAP	7754	20 18	3	-12 32	3.60	.033
		36 β CAP	7776	20 21	1	-14 47	3.10	.005
		37 δ CAP	8322	21 47	2	-16 8	2.80	.065
		38 θ CAP	8075	21 5	57	-17 14	4.10	.010
		39 ω CAP	7980	20 51	49	-26 56	4.10	.000
	Canopus	40 α CAR	2326	6 23	57	-52 41	.73	.018
	MIAPLACIDUS	41 β CAR	3685	9 13	12	-69 43	1.70	.038
		42 ε CAR	3307	8 22	31	-59 30	1.85	.000
		43 η CAR	4210	10 45	4	-59 42	2.00	.000
		44 θ CAR	4199	10 42	57	-64 23	2.76	.000
		45 ι CAR	3699	9 17	6	-59 16	2.24	.011
		46 ν CAR	3890	9 47	6	-65 4	3.00	.020
		47 χ CAR	3117	7 56	47	-52 59	3.50	.000
		48 ω CAR	4037	10 13	45	-70 2	3.31	.000
		49 CAR	4050	10 17	5	-61 20	3.44	.018
	Schedir	50 α CAS	168	0 40	31	56 32	2.20	.009

Popular Name	Bayer	Yale	Right Asc.	Decl.	Mag.	Prlx	Popular Name	Bayer	Yale	Right Asc.	Decl.	Mag.	Prlx
	No.		Hr Min Sec	Deg Min		Secs		No.		Hr Min Sec	Deg Min		Secs
CAPH	51	β CAS	21 0 9 10	59 9	2.30	.072		101	β CRV	4786 12 34 23	-23 -24	2.70	.027
	52	γ CAS	264 0 56 42	60 43	2.65	.034		102	γ CRV	4662 12 15 49	-17 -32	2.60	.023
RUCHBAH	53	δ CAS	403 1 25 49	60 14	2.70	.029		103	δ CRV	4757 12 29 51	-16 -31	3.00	.018
	54	ϵ CAS	542 1 54 24	63 41	3.40	.007		104	ϵ CRV	4630 12 10 8	-22 -37	3.00	.020
RIGIL KENTAUROS	55	α CEN	5459 14 39 36	-60 -50	.10	.751		105	α CVN	4914 12 56 1	38 19	2.80	.023
	56	β CEN	5267 14 3 50	-60 -22	.06	.016	COR CAROLI	106	α CYG	7924 20 41 26	45 16	1.26	.000
	57	γ CEN	4819 12 41 31	-48 -58	2.00	.006	DENEBO	107	β CYG	7417 19 30 43	27 58	3.24	.000
	58	δ CEN	4621 12 8 21	-50 -43	2.88	.020	ALBIREO	108	γ CYG	7796 20 22 13	40 15	2.24	.000
	59	ϵ CEN	5132 13 39 53	-53 -28	2.30	.000		109	δ CYG	7528 19 44 58	45 8	2.92	.021
	60	ζ CEN	5231 13 55 32	-47 -18	2.50	.000		110	ϵ CYG	7949 20 46 13	33 58	2.45	.044
	61	η CEN	5440 14 35 30	-42 -9	2.35	.000		111	ζ CYG	8115 21 12 56	30 14	3.20	.021
	62	θ CEN	5288 14 6 41	-36 -23	2.00	.059		112	η CYG	7615 19 59 1	35 5	3.90	.009
	63	ι CEN	5028 13 20 35	-36 -43	2.76	.046		113	τ CYG	8130 21 14 48	38 3	3.69	.047
	64	λ CEN	4467 11 35 47	-63 -1	3.12	.000		114	α DEL	7906 20 39 39	15 55	3.77	.002
	65	ν CEN	5190 13 49 30	-41 -41	3.40	.000		115	β DEL	7882 20 37 33	14 36	3.78	.026
ALDERAMIN	66	α CEP	8162 21 18 35	62 35	2.40	.063		116	γ DEL	7947 20 46 38	16 8	3.00	.196
	67	β CEP	8238 21 28 39	70 33	3.20	.005		117	ϵ DEL	7852 20 33 13	11 18	3.98	.016
	68	γ CEP	8974 23 39 20	77 37	3.20	.064		118	α DOR	1465 4 34 0	-55 3	3.26	.011
	69	ζ CEP	8465 22 10 51	58 12	3.40	.019	THUBAN	119	β DOR	1922 5 33 37	-62 -29	3.40	.007
	70	ι CEP	8694 22 49 41	66 12	3.60	.036		120	α DRA	5291 14 4 24	64 22	3.60	.011
MENKAR	71	α CET	911 3 2 17	4 6	2.50	.003		121	β DRA	6536 17 30 26	52 19	2.90	.009
DENEBO-KAITOS	72	β CET	188 0 43 35	-17 -59	2.00	.057	ETAMIN	122	γ DRA	6705 17 56 36	51 29	2.20	.017
	73	γ CET	804 2 43 18	3 14	3.47	.048		123	δ DRA	7310 19 12 33	67 40	3.10	.028
	74	δ CET	779 2 39 29	0 20	4.07	.001		124	ϵ DRA	7582 19 48 10	70 16	3.88	.001
	75	ζ CET	539 1 51 27	-10 -20	3.72	.038		125	ζ DRA	6396 17 8 48	65 43	3.20	.017
	76	η CET	334 1 8 36	-10 -11	3.44	.032		126	η DRA	6132 16 23 59	61 30	2.80	.043
	77	θ CET	402 1 24 1	-8 -11	3.61	.034		127	θ DRA	5986 16 1 54	58 34	4.00	.046
	78	ι CET	74 0 19 26	-8 -50	3.56	.010		128	ι DRA	5744 15 24 56	58 58	3.26	.032
SIRIUS	79	\circ CET	681 2 19 21	-2 -59	2.00	.013		129	κ DRA	4787 12 33 29	69 47	3.80	.010
	80	τ CET	509 1 44 4	-15 -56	3.50	.275		130	λ DRA	4434 11 31 24	69 20	3.80	.024
	81	α CMA	2491 6 45 9	-16 -43	-1.47	.375		131	ν DRA	6554 17 32 10	55 11	4.90	.000
	82	β CMA	2294 6 22 42	-17 -57	2.00	.014		132	ξ DRA	6688 17 53 32	56 52	3.80	.031
	83	γ CMA	2657 7 3 45	-15 -38	4.10	.000		133	\circ DRA	7125 18 51 13	59 23	4.70	.003
	84	δ CMA	2693 7 8 24	-26 -24	1.80	.000	ACHERNAR	134	\times DRA	6927 18 21 4	-72 -44	3.58	.120
ADHARA	85	ϵ CMA	2618 6 58 38	-28 -58	1.50	.001		135	α ERI	472 1 37 42	-57 -15	.47	.023
	86	ζ CMA	2282 6 20 18	-30 -4	3.02	.003		136	β ERI	1666 5 7 51	-5 5	2.80	.042
	87	η CMA	2827 7 24 5	-29 -18	2.40	.000		137	γ ERI	1231 3 58 2	-13 -31	3.00	.003
PROCYON	88	α CMI	2943 7 39 18	5 14	.34	.288		138	δ ERI	1136 3 43 14	-9 -46	3.55	.109
	89	β CMI	2845 7 27 9	8 17	2.80	.020		139	ϵ ERI	1084 3 23 56	-9 -28	3.73	.303
	90	α CNC	3572 8 58 29	11 52	4.25	.018		140	η ERI	874 2 56 25	-8 -54	3.89	.027
	91	β CNC	3249 8 16 31	9 12	3.52	.014	ACAMAR	141	θ ERI	897 2 58 15	-40 -18	3.42	.028
	92	δ CNC	3461 8 44 41	18 9	4.00	.000		142	\circ ERI	1325 4 15 6	-7 -40	3.00	.200
PHACT	93	α COL	1956 5 39 39	-34 -5	2.63	.005		143	τ ERI	1003 3 19 31	-21 -45	3.67	.017
	94	β COL	2040 5 50 58	-35 -46	3.11	.023		144	ϕ ERI	674 2 16 30	-51 -31	3.55	.000
	95	α CRB	5793 15 34 41	26 43	2.20	.043	CASTOR	145	α GEM	2890 7 34 36	31 53	1.50	.072
ALPHECCA	96	β CRB	5958 15 59 30	25 55	2.00	.000	POLLUX	146	β GEM	2990 7 45 19	28 1	1.15	.093
	97	α CRU	4730 12 26 36	-63 -6	1.00	.008		147	γ GEM	2421 6 37 43	16 24	1.93	.031
	98	β CRU	4853 12 47 44	-59 -42	1.24	.000		148	δ GEM	2777 7 20 7	21 59	3.50	.059
	99	γ CRU	4763 12 31 10	-57 -7	1.60	.000		149	ϵ GEM	2473 6 43 56	25 8	3.10	.009
	100	δ CRU	4656 12 15 9	-58 -45	2.80	.000		150	η GEM	2216 6 14 42	22 30	3.20	.013

Popular Name	Bayer	Yale	Right Asc.			Decl.		Mag.	Prlx	
			No.	Hr	Min	Sec	Deg			
<i>AL NAIR</i>	151	λ <i>GEM</i>	2763	7	18	6	16	32	3.58	.041
	152	α <i>GRU</i>	8425	22	8	14	-46	-58	1.73	.051
	153	β <i>GRU</i>	8636	22	42	40	-46	-53	2.20	.003
	154	γ <i>GRU</i>	8353	21	53	56	-37	-22	3.00	.008
	155	α <i>HER</i>	6406	17	14	39	14	23	2.90	.000
	156	β <i>HER</i>	6148	16	30	13	21	29	2.80	.017
	157	γ <i>HER</i>	6095	16	21	56	19	9	3.74	.015
	158	δ <i>HER</i>	6410	17	15	2	24	50	3.10	.034
	159	ϵ <i>HER</i>	6324	17	0	18	30	55	3.90	.022
	160	ζ <i>HER</i>	6212	16	41	17	31	36	2.80	.110
	161	η <i>HER</i>	6220	16	42	54	38	56	3.50	.053
	162	ι <i>HER</i>	6588	17	39	27	46	1	3.80	.002
	163	μ <i>HER</i>	6623	17	46	28	27	44	3.35	.108
	164	π <i>HER</i>	6418	17	15	3	36	48	3.20	.020
<i>ALPHARD</i>	165	τ <i>HER</i>	6092	16	19	44	46	19	3.90	.027
	166	α <i>HYA</i>	3748	9	27	35	-8	-40	1.99	.017
	167	γ <i>HYA</i>	5020	13	18	55	-23	-11	3.02	.021
	168	ζ <i>HYA</i>	3547	8	55	24	5	57	3.10	.029
	169	θ <i>HYA</i>	3665	9	14	22	2	19	3.88	.019
	170	ν <i>HYA</i>	4232	10	49	37	-16	-11	3.12	.022
	171	π <i>HYA</i>	5287	14	6	23	-26	-41	3.25	.039
	172	α <i>HYI</i>	591	1	58	46	-61	-34	2.90	.041
	173	β <i>HYI</i>	98	0	25	45	-77	-15	2.79	.153
	174	γ <i>HYI</i>	1208	3	47	14	-74	-15	3.24	.001
	175	α <i>IND</i>	7869	20	37	34	-47	-17	3.10	.039
<i>REGULUS</i> <i>DENEBOLA</i>	176	α <i>LEO</i>	3982	10	8	22	11	58	1.36	.039
	177	β <i>LEO</i>	4534	11	49	4	14	34	2.10	.076
	178	γ <i>LEO</i>	4057	10	19	59	19	51	2.60	.019
	179	δ <i>LEO</i>	4357	11	14	6	20	31	2.60	.040
	180	ϵ <i>LEO</i>	3873	9	45	51	23	46	3.00	.002
	181	ζ <i>LEO</i>	4031	10	16	42	23	25	3.40	.009
	182	η <i>LEO</i>	3975	10	7	20	16	46	3.50	.000
	183	θ <i>LEO</i>	4359	11	14	15	15	26	3.30	.019
	184	μ <i>LEO</i>	3905	9	52	46	26	1	3.90	.022
	185	ρ <i>LEO</i>	4133	10	32	49	9	18	3.85	.005
<i>ARNEB</i>	186	α <i>LEP</i>	1865	5	32	44	-17	-50	2.60	.002
	187	β <i>LEP</i>	1829	5	28	15	-20	-45	2.84	.014
	188	ϵ <i>LEP</i>	1654	5	5	28	-22	-22	3.18	.006
	189	μ <i>LEP</i>	1702	5	12	56	-16	-12	3.28	.018
	190	α <i>LIB</i>	5530	14	50	53	-16	-3	2.75	.049
	191	β <i>LIB</i>	5685	15	17	0	-9	-23	2.61	.012
	192	γ <i>LIB</i>	5787	15	35	32	-14	-47	3.90	.033
	193	σ <i>LIB</i>	5603	15	4	4	-25	-17	3.30	.056
	194	τ <i>LIB</i>	5812	15	38	40	-29	-47	3.70	.000
	195	α <i>LUP</i>	5469	14	41	56	-47	-24	2.30	.000
	196	β <i>LUP</i>	5571	14	58	32	-43	-8	2.67	.000
	197	γ <i>LUP</i>	5776	15	35	9	-41	-10	2.80	.008
	198	δ <i>LUP</i>	5695	15	21	22	-40	-39	3.20	.000
	199	ϵ <i>LUP</i>	5708	15	22	40	-44	-42	3.40	.009
	200	ζ <i>LUP</i>	5649	15	12	17	-52	-6	3.40	.036

Popular Name	Bayer	Yale	Right Asc.			Decl.		Mag.	Prlx	
			No.	Hr	Min	Sec	Deg			
<i>VEGA</i>	201	α <i>LYN</i>	3705	9	21	3	34	24	3.10	.021
	202	α <i>LYR</i>	7001	18	36	56	38	47	.04	.123
	203	β <i>LYR</i>	7106	18	50	4	33	22	3.40	.000
	204	γ <i>LYR</i>	7178	18	58	56	32	41	3.25	.011
	205	δ <i>LYR</i>	7141	18	54	30	36	54	3.90	.000
	206	ζ <i>LYR</i>	7056	18	44	47	37	36	4.00	.025
	207	α <i>MUS</i>	4798	12	37	11	-69	-8	2.70	.000
	208	α <i>OPH</i>	6556	17	34	56	12	34	2.08	.056
	209	β <i>OPH</i>	6603	17	43	28	4	34	2.80	.023
	210	δ <i>OPH</i>	6056	16	14	20	-3	-41	2.70	.029
	211	ϵ <i>OPH</i>	6075	16	18	19	-4	-42	3.24	.036
	212	ζ <i>OPH</i>	6175	16	37	9	-10	-34	2.60	.000
<i>RASALAGUE</i>	213	η <i>OPH</i>	6378	17	10	23	-15	-43	2.44	.047
	214	θ <i>OPH</i>	6453	17	22	0	-25	0	3.28	.000
	215	κ <i>OPH</i>	6299	16	57	44	9	23	3.31	.026
	216	ν <i>OPH</i>	6698	17	59	1	-9	-47	3.30	.015
	217	α <i>ORI</i>	2061	5	55	10	7	24	.80	.005
	218	β <i>ORI</i>	1713	5	14	32	-8	-12	.08	.000
	219	γ <i>ORI</i>	1790	5	25	8	6	21	1.60	.026
	220	δ <i>ORI</i>	1852	5	32	1	0	-18	2.20	.000
	221	ϵ <i>ORI</i>	1903	5	36	12	-1	-12	1.70	.000
	222	ζ <i>ORI</i>	1948	5	40	46	-1	-57	2.00	.022
	223	ι <i>ORI</i>	1899	5	35	26	-5	-55	2.80	.021
	224	κ <i>ORI</i>	2004	5	47	46	-9	-40	2.00	.009
	225	λ <i>ORI</i>	1879	5	35	8	9	56	3.00	.006
	226	α <i>PAV</i>	7790	20	25	38	-56	-44	1.90	.000
<i>MARKAB</i>	227	α <i>PEG</i>	8781	23	4	46	15	12	2.50	.030
	228	β <i>PEG</i>	8775	23	3	47	28	5	2.56	.015
	229	γ <i>PEG</i>	39	0	13	14	15	11	2.80	.000
	230									

Popular Name	Bayer	Yale	Right Asc.			Decl.		Mag.	Prlx	
			No.	Hr	Min	Sec	Deg			
	251	α <i>PSC</i>	595	2	2	2	-2	46	3.00	.000
	252	ζ <i>PUP</i>	3165	8	3	35	-40	0	2.30	.000
	253	ν <i>PUP</i>	2451	6	37	46	-43	-11	3.17	.000
	254	ξ <i>PUP</i>	3045	7	49	17	-24	-52	3.34	.003
	255	π <i>PUP</i>	2773	7	17	9	-37	-6	2.70	.023
	256	ρ <i>PUP</i>	3185	8	7	33	-24	-18	2.88	.031
	257	σ <i>PUP</i>	2878	7	29	14	-43	-18	3.24	.013
ANTARES	258	τ <i>PUP</i>	2553	6	49	56	-50	-37	2.90	.000
	259	α <i>SCO</i>	6134	16	29	25	-26	-26	1.10	.019
DSCHUBBA	260	β <i>SCO</i>	5984	16	5	26	-19	-48	2.40	.004
	261	δ <i>SCO</i>	5953	16	0	20	-22	-37	2.32	.000
	262	ϵ <i>SCO</i>	6241	16	50	10	-34	-18	2.28	.049
	263	η <i>SCO</i>	6380	17	12	9	-43	-14	3.30	.063
	264	θ <i>SCO</i>	6553	17	37	19	-43	0	1.90	.020
SHAULA	265	ι <i>SCO</i>	6615	17	47	35	-40	-7	3.00	.013
	266	κ <i>SCO</i>	6580	17	42	29	-39	-2	2.40	.000
	267	λ <i>SCO</i>	6527	17	33	36	-37	-6	1.60	.000
	268	μ <i>SCO</i>	6247	16	51	52	-38	-3	3.10	.000
	269	ν <i>SCO</i>	6508	17	30	46	-37	-18	2.70	.000
	270	π <i>SCO</i>	5944	15	58	51	-26	-7	2.90	.005
	271	σ <i>SCO</i>	6084	16	21	12	-25	-35	3.00	.000
UNUK	272	τ <i>SCO</i>	6165	16	35	53	-28	-13	2.80	.014
	273	α <i>SER</i>	5845	15	44	17	-6	-25	2.70	.046
	274	η <i>SER</i>	6869	18	21	18	-2	-53	3.30	.054
KAUS-AUSTRALIS	275	γ <i>SGR</i>	6746	18	5	48	-30	-26	3.00	.018
	276	δ <i>SGR</i>	6859	18	21	0	-29	-49	2.70	.039
	277	ϵ <i>SGR</i>	6879	18	24	10	-34	-23	1.80	.015
	278	ζ <i>SGR</i>	7194	19	2	37	-29	-52	2.60	.020
	279	η <i>SGR</i>	6832	18	17	38	-36	-46	3.12	.038
	280	λ <i>SGR</i>	6913	18	27	58	-25	-26	2.80	.046
NUNKI	281	π <i>SGR</i>	7264	19	9	46	-21	-1	2.90	.016
	282	σ <i>SGR</i>	7121	18	55	16	-26	-18	2.10	.000
	283	τ <i>SGR</i>	7234	19	6	56	-27	-40	3.30	.038
ALDEBARAN	284	ϕ <i>SGR</i>	7039	18	45	40	-27	0	3.18	.000
EL NATH	285	α <i>TAU</i>	1457	4	35	55	16	30	.86	.048
	286	β <i>TAU</i>	1791	5	26	17	28	36	1.70	.018
	287	γ <i>TAU</i>	1346	4	19	48	15	37	3.70	.000
	288	δ <i>TAU</i>	1373	4	22	56	17	32	3.76	.016
	289	ϵ <i>TAU</i>	1409	4	28	37	19	11	3.54	.018
	290	ζ <i>TAU</i>	1910	5	37	39	21	9	3.00	.000
	291	θ <i>TAU</i>	1411	4	28	35	15	57	3.90	.033
	292	τ <i>TAU</i>	1497	4	42	15	22	58	4.30	.008
	293	α <i>TRA</i>	6217	16	48	40	-69	-2	1.91	.024
	294	β <i>TRA</i>	5897	15	55	9	-63	-26	2.84	.078
	295	γ <i>TRA</i>	5671	15	18	55	-68	-41	2.90	.005
	296	α <i>TRI</i>	544	1	53	5	29	35	3.53	.050
	297	β <i>TRI</i>	622	2	9	32	34	59	3.00	.012
	298	γ <i>TRI</i>	664	2	17	19	-33	-51	4.08	.036
DUBHE	299	α <i>TUC</i>	8502	22	18	30	-60	-15	2.90	.019
	300	α <i>UMA</i>	4301	11	3	44	61	45	1.80	.031

Popular Name	Bayer	Yale	Right Asc.			Decl.		Mag.	Prlx	
			No.	Hr	Min	Sec	Deg			
	301	β <i>UMA</i>	4295	11	1	51	56	23	2.40	.042
	302	γ <i>UMA</i>	4554	11	53	49	53	42	2.90	.020
	303	δ <i>UMA</i>	4660	12	15	26	57	2	3.30	.052
MIZAR (+ALCOR)	304	ϵ <i>UMA</i>	4905	12	54	2	55	57	1.80	.008
	305	ζ <i>UMA</i>	5054	13	23	56	54	56	2.40	.037
	306	η <i>UMA</i>	5191	13	47	32	49	19	1.86	.000
	307	θ <i>UMA</i>	3775	9	32	51	51	41	3.18	.052
	308	ι <i>UMA</i>	3569	8	59	13	48	2	3.14	.066
	309	μ <i>UMA</i>	4069	10	22	19	41	30	3.00	.031
	310	ν <i>UMA</i>	3888	9	50	59	59	3	3.77	.036
	311	\circ <i>UMA</i>	3323	8	30	16	60	43	3.36	.004
	312	χ <i>UMA</i>	4518	11	46	3	47	47	3.69	.014
	313	ψ <i>UMA</i>	4335	11	9	40	44	29	3.00	.000
	314	<i>UMA</i>	3757	9	31	32	63	4	3.65	.034
POLARIS	315	α <i>UMI</i>	424	2	31	13	89	15	2.50	.003
KOCHAB	316	β <i>UMI</i>	5563	14	50	43	74	9	2.00	.031
	317	γ <i>UMI</i>	5735	15	20	44	71	50	3.00	.000
	318	δ <i>UMI</i>	6789	17	32	11	86	35	4.40	.000
	319	ϵ <i>UMI</i>	6322	16	45	58	82	2	4.30	.014
	320	ζ <i>UMI</i>	5909	15	44	3	77	48	4.30	.011
AL SUHAIL	321	η <i>UMI</i>	6116	16	17	30	75	45	5.00	.038
	322	γ <i>VEL</i>	3207	8	9	32	-47	-21	1.80	.000
	323	δ <i>VEL</i>	3485	8	44	43	-54	-43	2.00	.043
	324	κ <i>VEL</i>	3734	9	22	7	-55	-1	2.49	.007
	325	λ <i>VEL</i>	3634	9	8	0	-42	-26	2.30	.015
	326	μ <i>VEL</i>	4216	10	46	46	-49	-26	2.70	.022
SPICA	327	\circ <i>VEL</i>	3447	8	40	18	-52	-55	3.61	.000
	328	α <i>VIR</i>	5056	13	25	11	-11	-9	.10	.021
	329	γ <i>VIR</i>	4825	12	41	40	-1	-27	3.60	.101
	330	δ <i>VIR</i>	4910	12	55	36	3	23	3.40	.017
	331	ϵ <i>VIR</i>	4932	13	2	11	10	58	2.80	.036
	332	ζ <i>VIR</i>	5107	13	34	42	0	-36	3.40	.035