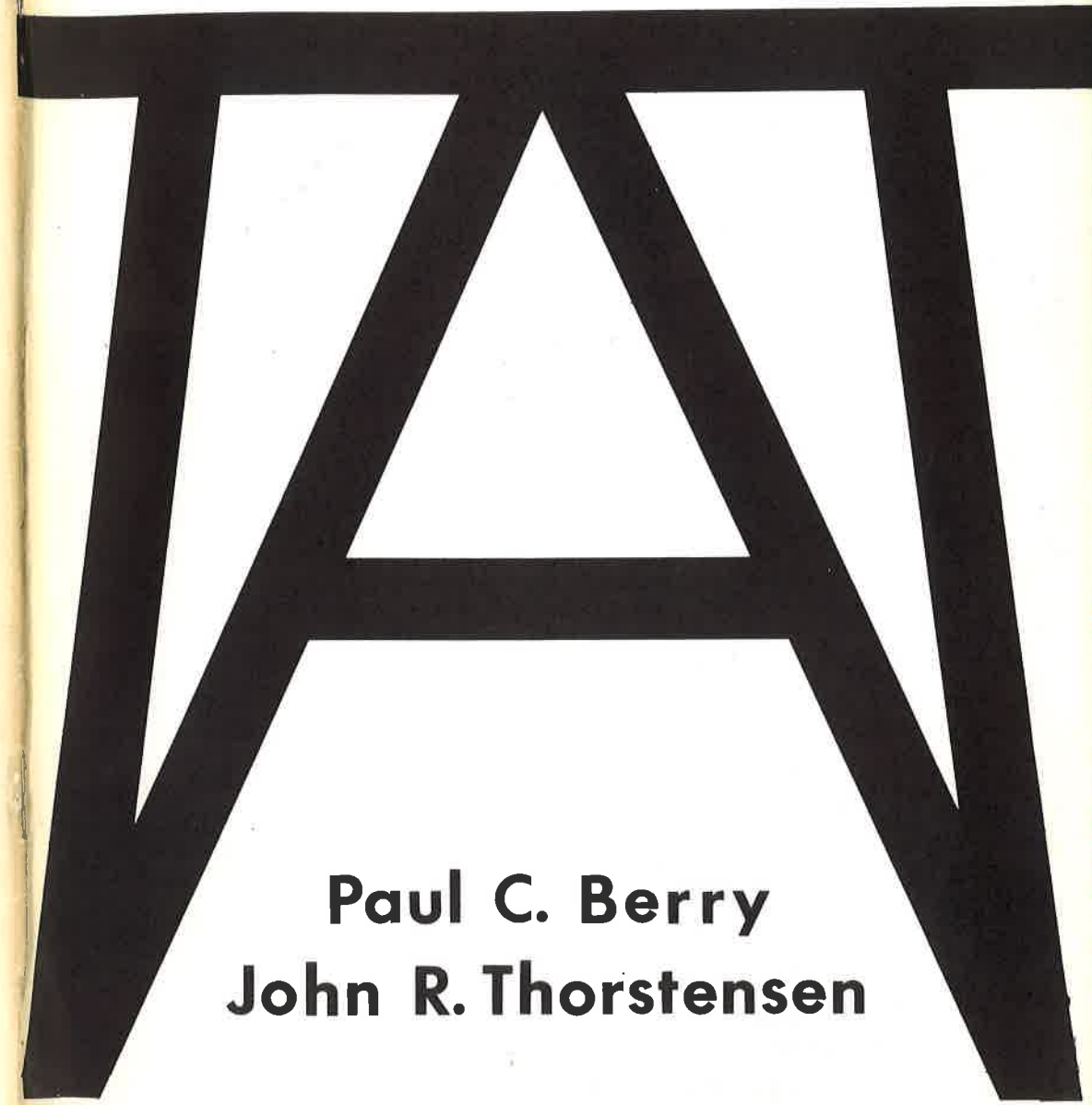


STARMAP



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Introduction

In many fields of science or commerce, it is possible to define a set of functions (i.e. computer programs) in such a way that each corresponds to a term or concept used in that discipline. Such a set of functions in effect constitutes a "user language" for that particular area of application. If that language has a simple and consistent syntax, and if its various functions refer to the data on which they work in a consistent way, it is possible to achieve a programming package that is easy to understand, to revise, or to adapt to new applications. At the same time, such a package constitutes an executable model of some of the concepts of that discipline.

Constructing a particular definition by referring to a set of simpler or more general sub-definitions is the principal technique of what has come to be called modular programming; the program used for a particular job consists of a brief invocation of the concepts or components from which it is made. They in turn are defined by invoking modules at the next level of detail. Reading programs that have been written in that way, the student sees at the outer level a brief summary of the organization of the work; pursuing the definitions further, he or she may obtain explanation to whatever level of detail is desired.

The aim of this paper is to illustrate this style of programming in APL by presenting in detail the definitions used in a particular project. We selected for this purpose the set of programs contained in an APL workspace called STARMAP, which was in use as part of a display on astronomy at the IBM Exhibit Center in New York during 1973 and 1974. That set of programs served to print at a terminal a map showing the positions of the brighter stars, the planets, and the comet Kohoutek, as they would appear above any point on Earth at any time on any date --at least for some number of years on either side of the present.

To generate a map, the user started work by invoking a function called *DISPLAY*. Thereupon he or she was asked to specify the date for which a map was wanted, the local time, and the latitude and longitude. The program then computed the positions of the stars and planets, and drew the map, either at the typewriter terminal (using a special type element for fine plotting) or on a cathode-ray display tube. A sample of the conversation in which a user enters the specifications of a map is shown in Figure 1; the resulting chart (photographically reduced from the original size of about 35 cm. square) is shown in Figure 2.

)LOAD 5 STARMAP SAVED 13,28,39 12/23/73 COPYRIGHT 1973 IBM CORPORATION	The STARMAP workspace is loaded for use.
GUEST APL 6, ANAHEIM, CALIFORNIA,	Copyright notice is printed automatically when the workspace is loaded.
DISPLAY YEAR MONTH DAY 1974 5 15 TIME OF DAY OR NIGHT 10 PM DAYLIGHT LATITUDE 33 46 NORTH LONGITUDE 117 50 WEST	Optionally, the user may indicate the name of a person place, etc., to appear at the bottom of the finished map. Prompts issued by the functions called by ENTER appear at the left and the user's responses to the right.

Fig. 1 Dialog during user's request for a star map

Following this dialog, the keyboard is unlocked, awaiting a carriage return from the user to indicate that the fine plotting element has been inserted and the paper is ready to receive the printed map.

On the opposite page appears the map generated in response to the request shown in Figure 1. The map has been photographically reduced; the actual print-out is about 35 cm. high. The fine plotting type element (number 114) carries fifteen dots and fifteen crosses, one for each position of a 3-by-5 matrix, giving a resolution of about .85 mm (1/30 inch) between adjacent points vertically or horizontally. Four additional star maps are shown beginning on page 28.

VIEW FROM 33 DEGREES 46 MINUTES NORTH, 117 DEGREES 50 MINUTES WEST, ON WEDNESDAY 1974/5/15 AT 10 00 PM DAYLIGHT TIME

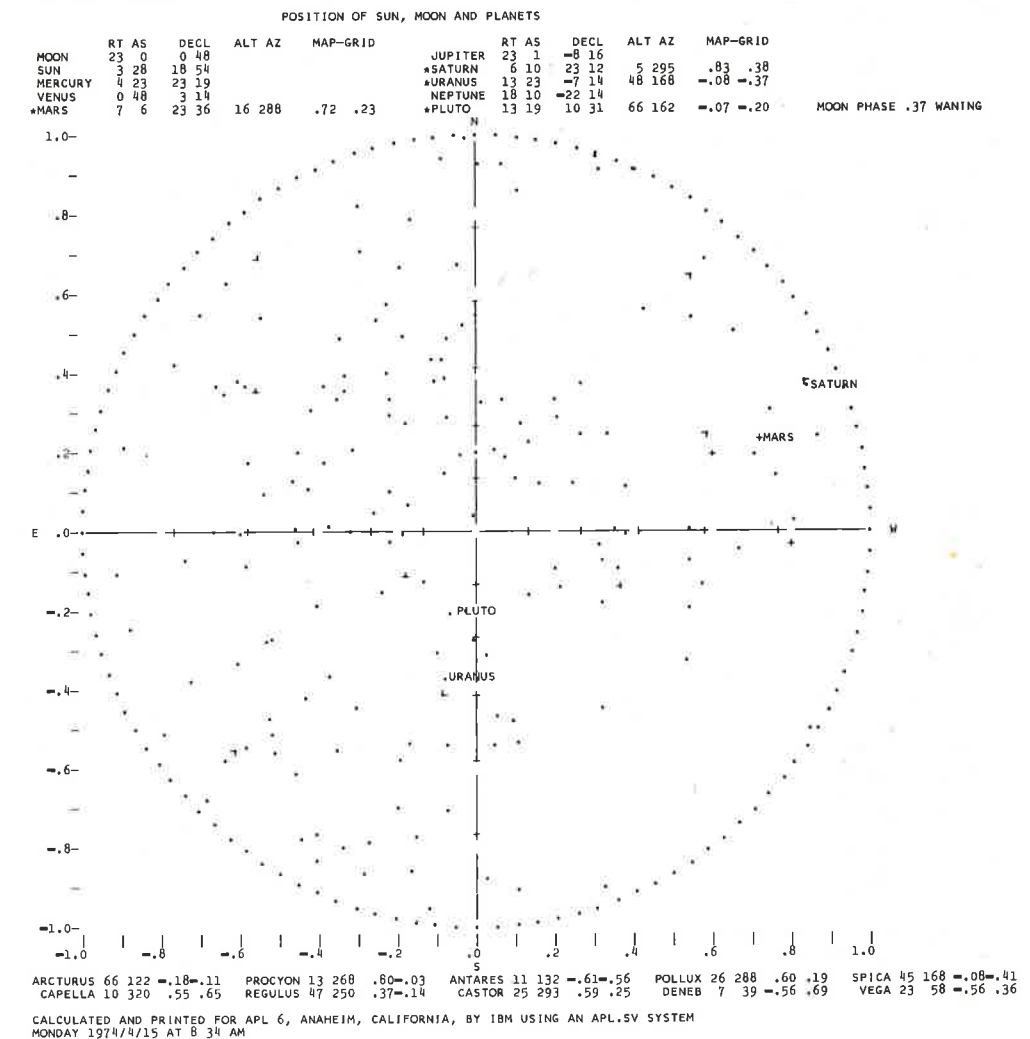


Fig. 2 Sample chart produced by the STARMAP workspace

To generate such a map requires solving the formulas for planetary motion in order to know where in the solar system the planets should be located at the date requested, and then to translate those coordinates to show their apparent positions as seen from the Earth. The coordinates thus obtained, together with those of the fixed stars, could then be rotated for the desired time and location on the surface of the Earth.

The motions of the planets may be described by formulas that were first developed by Kepler early in the seventeenth century. Kepler's function states the time needed for a planet to traverse a given angle through its orbit. To find the position at a given time requires the inverse of that function; a general iterative method, applicable to evaluating that inverse, was worked out by Newton later in the same century. The rest of the task is an exercise in analytic geometry, to translate and rotate the coordinates appropriately. Formulas for doing so were familiar in the seventeenth and eighteenth centuries, but were simplified by the matrix algebra developed in the nineteenth century. Thus the programming task involved here consists mainly of representing in terms executable by the computer a set of classical formulas.

The interest in these programs lies not in the method itself, which has been well known for many years, but in the style in which classical and familiar formulas may be stated in APL. Our aim was to provide APL definitions for a vocabulary of terms which would not only make clear the process by which the work is done, but would permit the student of astronomy to apply them to new problems or applications. Where possible, we used names that correspond to those in general use in astronomy. Occasionally we had to coin names for functions which are not usually explicitly identified, but even here our terms should be recognizable to astronomers. To make our APL definitions correspond more directly to familiar formulas, we also made free use of defined functions such as *SIN*, *COS*, *PI*, *RADIAN*, etc. despite the fact that their effects could be readily obtained from APL primitives.

Clearly, programs to convert the coordinates into a printed map or video display were also necessary to this project, but these are standard packages in widespread use, and we have not chosen to discuss them here.

Organization and Key Parameters

The function *DISPLAY* initiates the entire task. It calls functions corresponding to the various stages of work. A tabular outline of the functions used is shown in Table 1. At the top left of the table the name *DISPLAY* appears, indicating that this is the primary function. Indented five spaces below it appear the names of each of the functions called by *DISPLAY*. Indented below them appear the names of functions they call, and so on. (For simplicity, some 98 references to functions such as *SIN*, *COS*, *RADIAN*, etc. have been omitted from the table.) The rest of this paper provides an explication of the work being done at each of these stages, in top-down order.

In the text that follows, the symbol ∇ (del) is used to indicate the header of a function, containing the name of the function, names for its arguments and results, and names for temporary variables used as intermediate steps in its definition, if any. As may be seen below, *DISPLAY* calls *ENTRY* (which requests input for the parameters governing a particular map), then pauses to permit the user to align the paper and insert the fine plotting element (indicated by the input symbol \square), and then calls *WORK*, which does the necessary calculations and prints the map:

```
 $\nabla$  DISPLAY
  ENTRY
   $\square$ 
  WORK
```

The function *ENTRY* establishes the values of the input parameters by calling the functions *GETDATE*, *GETTIME*, *GETLAT* and *GETLONG*, and adjusts the date and time stated by the user to correct for the indicated longitude:

```
 $\nabla$  ENTRY
  STATEDDAYNO←GETDATE
  STATEDTIME←GETTIME
  LAT←GETLAT
  LONG←GETLONG
  TIME←LONG TIMEADJUST STATEDTIME
  DATE←STATEDDAYNO+(TIME÷24)-LONG÷360
```

When execution of *ENTRY* is complete, values have been established for the following variables:

TIME Time is a single number indicating the number of hours since midnight in the exact local time for the indicated longitude. (However, the user enters the time in conventional form as it would appear on a clock in the nearest time zone.)

DATE Although entered in a conventional form, the date is represented internally as the Julian day number; a function *JNU* converts the date to that form. (The Julian day number of 1 January 1974 was 2442049.) The fractional part of the day number indicates how far through the day by universal time the indicated time is. Thus before the major calculations take place, all information on time is contained in the single number *DATE*.

LAT Number of degrees north of the Equator.

LONG Number of degrees east of the prime meridian.

The time and date as entered by the user are preserved as *STATEDTIME* and *STATEDDATE*, and the Julian day number of the stated date as *STATEDDAYNO*; in some circumstances the adjusted value of *DATE* (in universal time) may fall within a day 1 more or less than the stated date.

Stages of Calculation

The task of calculation and printing may be divided into seven stages, each defined by a single function:

▽ **WORK**
CAPTION
CALCULATEPLANETS
REPORTPLANETS
CALCULATESTARS
PLOTSTARS
REPORTSTARS
PRINTED

Table 1
Levels of Function Calls in the STARMAP Workspace

<p>DISPLAY</p> <p>ENTRY</p> <p>GETDATE JNU JULCAL TODAY GREGORIAN JNU</p> <p>GETLAT DECIMALDEG</p> <p>GETLONG DECIMALDEG</p> <p>GETTIME NOW TIMEADJUST</p> <p>WORK</p> <p>CAPTION DATEREPR LATREPR LONGREPR TIMEREPR</p> <p>CALCULATEPLANETS</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>PLANETPOS</p> <p>EPOCHADJUST SECULAR KEPLINVERSE KEPLERFN KEPLERIV ORBROTATE INCLROTATE LONGROTATE INCLINATION ASCENDING PERIANGLE TIMES CARTESIAN RADIUS SEMIMAJOR ECCENTRICITY ECCENTRICITY ANOMALYDATE PERIODER PERIOD ANOMALY ANOMALYDATE TRUEANOMALY</p> </div> <p>MOONPOS</p> <p>PLANETPOS (here repeat calls from block listed previously)</p> <p>INCLROTATE RADECDIST</p> <p>MOONPHASE PARALLAXADJUST</p> <p>COMETPOS</p> <p>COMETSOLVE AREA PERIDIST AREADERIV PERIDIST PERIDIST</p>	<p>ONBROTATE INCLROTATE LONGROTATE INCLINATION ASCENDING PERIANGLE TIMES PARABOLA PERIDIST PERIDIST ANOMALYDATE</p> <p>EARTHVIEW PLANETPOS (here repeat calls from block listed previously)</p> <p>RADECDIST</p> <p>VISIBLE PROJECTION COALITUDE MAPCARTESIAN CARTESIAN JNU</p> <p>SKYPOS</p> <p>CARTRIPLET LATROTATE INCLROTATE LONGROTATE NORM</p> <p>REPORTPLANETS</p> <p>PLANETTABLE DEGMIN HOURMINSEC REPORTPHASE</p> <p>CALCULATESTARS</p> <p>PRECESS CARTRIPLET INCLROTATE LONGROTATE RADECDIST</p> <p>VISIBLE PROJECTION COALITUDE MAPCARTESIAN CARTESIAN JNU</p> <p>SKYPOS</p> <p>CARTRIPLET LATROTATE INCLROTATE LONGROTATE NORM</p> <p>STARPLOT</p> <p>FLOT</p> <p>REPORTSTARS</p> <p>PRINTED</p> <p>DATEREPR GREGORIAN JNU NOW PRINTNAME TIMEREPR TODAY GREGORIAN JNU</p>
---	---

The sequence of segments is designed to overlap output to the terminal (produced by *CAPTION* or *REPORTPLANETS*) with the segments that require substantial calculation.

The function *CAPTION* recapitulates the stated input parameters, and adds the day of the week (directly obtainable from 7|*STATEDDAYNO*). The function *REPORTPLANETS* prints a table showing for each planet (and the moon, sun, and comet) its right ascension and declination. For those that are visible, the altitude and azimuth are included, together with the coordinates on the map grid. The phase of the moon is reported.

The function *PLOTSTARS* calls *FPLLOT*, which is adapted from the fine-plotting function in IBM program 5798-AGL, "Graphs and Histograms in APL." The stars and planets visible above the horizon are plotted, together with a circular frame of dots at 3-degree intervals around the horizon, and cross marks at intervals of 15 degrees of elevation. The standard plotting program was modified to insert a label showing the name of each planet, and to print a special symbol for the sun and moon.

The function *REPORTSTARS* prints a table showing the names of bright stars appearing in the plot, together with the altitude, azimuth and map-grid coordinates of each. The function *PRINTED* permits the finished map to be labelled with the name of the person for whom it was prepared, and reports the date and time at which it was printed. The input and output functions are not described further in this article.

The functions *CALCULATEPLANETS* and *CALCULATESTARS* use the global arguments *DATE*, *TIME*, *LAT*, and *LONG*, as well as the following reference tables:

<i>STARS</i>	A table containing the right ascension and declination of about 300 bright stars.
<i>PLANETS</i>	A table of the elements for the elliptical orbits of the nine planets.
<i>MOON</i>	A similar table for the elements of the moon's elliptical orbit about the Earth.
<i>KOHOUTEK</i>	A table of the elements for the parabolic orbit of the comet.
<i>BRIGHT</i>	A logical vector indicating which members of <i>STARS</i> represent stars of magnitude 1.5 or brighter.

BP

A logical vector indicating which planets are usually of magnitude 1.5 or brighter.

The positions of the stars are taken from the *Yale Catalog of Bright Stars*, and the elements of the planetary orbits from the *American Ephemeris and Nautical Almanac* for 1973. The orbital functions which follow were written after consulting the text by Marion (1965) *Classical Dynamics of Particles and Systems*.

Coordinate Systems Used in Describing the Positions of the Planets

Calculating the appearance of the heavens can be divided into two principal tasks: finding the locations of the planets in the solar system, and then calculating how they appear to an observer. A large part of the work thus involves rotation of coordinate axes, or translation from one system of coordinates to another. It will help to understand the programs that determine the positions of the planets if the various coordinate systems are first described.

Two-dimensional coordinates in the plane of each planet. Each of the objects in orbit around the sun is first considered to be moving along an ellipse (or, in the case of the comet, along a parabola) lying in a plane. Each planet can thus be located by two coordinates. During the initial solution of the orbits, these are polar coordinates; they are then converted to Cartesian coordinates, describing the planet's position by its distance from the solar focus along the major and minor axes of the ellipse. Two-dimensional coordinates appear only within the functions *PLANETPOS*, *MOONPOS*, and *COMETPOS*.

Heliocentric Cartesian coordinates. The two-dimensional Cartesian coordinates that specify each planet's position within the plane of its own orbit are converted to a common three-dimensional coordinate system whose center is in the sun. The first coordinate points from the sun in a direction opposite to the Earth at the moment of the vernal equinox. The second points perpendicularly out of the plane of the ecliptic, on the same side as the north pole. The third points in the plane of the ecliptic, perpendicularly to the other two, so that the three form a right-handed coordinate system. It intersects the celestial sphere at a right ascension of 18 hours (i.e. 270 degrees) and a declination (due to the tilt of the Earth's axis) of -23.45 degrees.

Positions stated in the heliocentric system are given the name *H* in the functions *PLANETPOS*, *MOONPOS*, *COMETPOS*, and in *EARTHVIEW* (which translates from heliocentric to geocentric coordinates).

The function *ORBROTATE* converts the two-dimensional Cartesian coordinates of the planets within their own planes to three-dimensional heliocentric coordinates, taking into account the orientation and tilt of the plane of each orbit, by reference to the elements *PERIANGLE* (angle of perihelion), *INCLINATION*, and *ASCENDING* (the angle of the ascending node); see pp. 14-15.

Geocentric ecliptic coordinates. The axes of this system are parallel to those of the heliocentric system, but have their origin in the center of the Earth rather than in the sun; values in this system are obtained simply by subtracting the heliocentric coordinates of the Earth from those of the object in question. Coordinates stated in this system are given the name *GC*. They appear as intermediate steps in the function *EARTHVIEW*.

Geocentric equatorial coordinates. This is a Cartesian form of the standard astronomical system of right ascension and declination. The first axis points (as before) to the vernal equinox. The second points to the north celestial pole. The third points at a location on the equator at the right ascension of the winter solstice.

Positions in this coordinate system are obtained from those stated in the geocentric ecliptic system by a rotation of 23.45 degrees around the first axis. Variables stated in these coordinates are given the name *GQ*.

Geocentric coordinates. The final transformation is to adjust for the position on Earth of the observer for whom the map is calculated. The first axis points due south. The second points to the zenith (above the observer). The third points due west. Positions in this system are obtained from positions in the geocentric equatorial system by a sequence of rotations in the course of the function *SKYPOS*, whose arguments are the positions of the planets in geocentric equatorial coordinates (*GQ*) and the latitude, date, and time of the viewing point on Earth. The result is in units of altitude and azimuth, and such variables are given the name *AA*.

Calculating the Positions of the Planetary Bodies

The function *CALCULATEPLANETS* finds *PLANETS*, a table of the positions of the sun, moon, and planets at the desired date. When first calculated by the function *PLANETPOS*, these positions are stated in 3-dimensional heliocentric Cartesian coordinates. But the function *EARTHVIEW* converts them to geocentric polar coordinates (right ascension in hours, declination in degrees, and distance in astronomical units), locating the planets with respect to the center of the Earth.

In order to plot the sky above a particular place, the function *SKYPOS* (see p. 29) is used to calculate *AA*, a table of altitude and azimuth with respect to given time and location on the Earth's surface. The function *VISIBLE* is used to select from *P* those members that are above the horizon, and saves them in the table *AAP*. Finally, the *PROJECTION* of these coordinates onto a flat surface is calculated, and translated to the Cartesian form expected by the plotting function; the function *IF* is simply a compression of the left argument by the right, along the first axis, defined by the APL symbol \uparrow .

```

▽ CALCULATEPLANETS; AA; MOON; SUN; KOHOUTEK
  PLANETCOORD←AAP←PLANETS←VP←10
  PLANETS←DATE EARTHVIEW DATE PLANETPOS (3#19)↑PLANETS
  SUN←DATE EARTHVIEW 0 0 0
  K←100≥|DATE-JNU 12 28 1973
  KOHOUTEK←DATE EARTHVIEW (DATE IF K) COMETPOS KOHOUTEK
  MOON←MOONPOS DATE
  PHASE←MOON[1;1] MOONPHASE SUN[1;1]
  PLANETS←MOON,[1] SUN,[1] PLANETS,[1] KOHOUTEK
  MOON←MOON[;3] PARALLAXADJUST (LAT,DATE,TIME) SKYPOS MOON
  AA←MOON,[1] (LAT,DATE,TIME) SKYPOS 1 0↑PLANETS
  PLANETCOORD←MAPCARTESIAN PROJECTION AAP←AA IF VP←VISIBLE AA

```

Execution of *CALCULATEPLANETS* causes new values to be assigned to four global variables. (These are initially set to 10 in the first statement, mainly to draw attention to a list of the global variables which will be reset as a consequence of executing this function.) The four are:

<i>PLANETS</i>	The right ascension and declination of the moon, sun, planets, and Kohoutek.
<i>VP</i>	A logical vector indicating which planets are visible from the place, date, and time requested.
<i>AAP</i>	The altitude and azimuth of the visible planets.
<i>PLANETCOORD</i>	The Cartesian coordinates used to plot the projection of the visible planets.

Orbital Parameters

The functions that locate the positions of the planets in their orbits make reference to a set of parameters usually called the elements of the orbit. The reference set of orbital elements for the planets is stored in the matrix PLANETS. Each row contains the set of elements for a particular planet. For example:

```

V Z←EARTH
  Z←PLANETS[,3:]

```

Each column corresponds to a particular element of the various orbits. Each of the functions that makes use of the orbital elements (PLANETPOS, MOONPOS, or COMETPOS) take as one of its arguments a matrix containing the rows of the table PLANETS that are appropriate: i.e. those corresponding to the particular planets being considered. This sub-table is given the name ORB. Functions are provided corresponding to each orbital element (for example, PERIOD, ECCENTRICITY, INCLINATION, etc.). Those functions select the appropriate column of the table ORB. In that way, terms such as PERIOD, ECCENTRICITY or INCLINATION refer to those elements for the planets currently under consideration, whatever those may be. This is achieved by making ORB, the table from which the values are selected, global with respect to these selection functions, but local to the functions such as PLANETPOS which use the elements, since ORB there appears as the explicit argument.

The geometrical meanings of the terms inclination, ascending node, and angle of perihelion are illustrated in Figure 5.

V Z←SEMIMAJOR Z←ORB[;1]	V Z←ASCENDING Z←ORB[;5]
V Z←PERIOD Z←ORB[;2]	V Z←PERIANGLE Z←ORB[;6]
V Z←ECCENTRICITY Z←ORB[;3]	V Z←ANOMALY Z←ORB[;7]
V Z←INCLINATION Z←ORB[;4]	V Z←ANOMALYDATE Z←ORB[;10]

The date of perihelion is computed from the elements already tabled:

```

V Z←PERIDATE
  Z←ANOMALYDATE - PERIOD×ANOMALY÷360

```

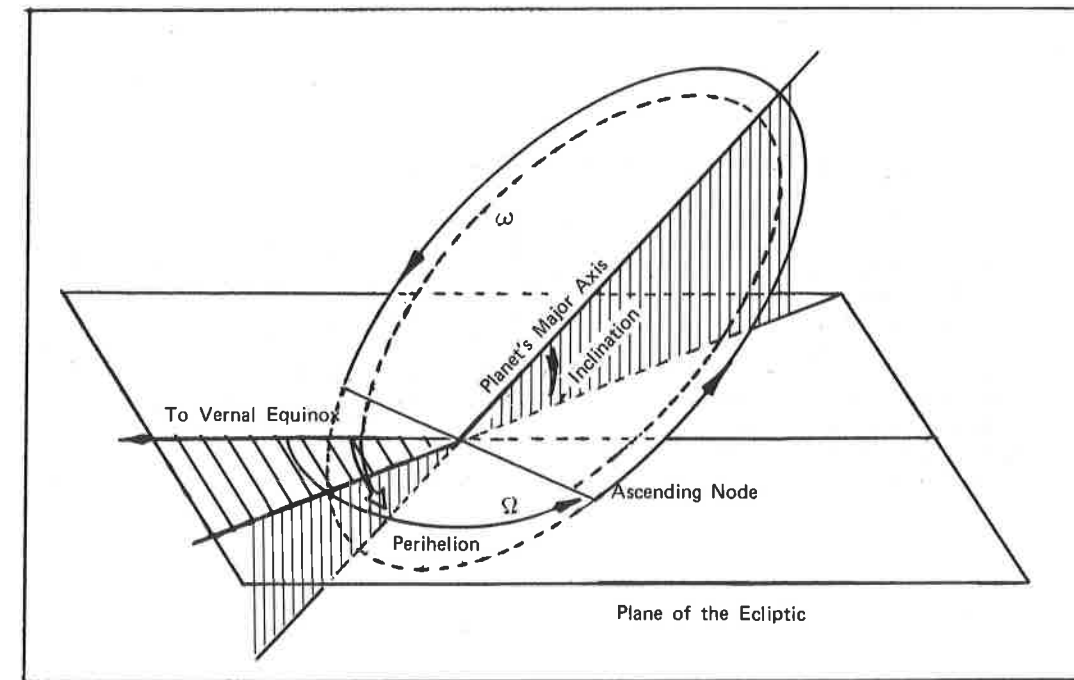


Fig. 3 Elements of an elliptical orbit

The rectangular plane represents the plane of the ecliptic. The focus of the planet's elliptical orbit is the sun. INCLINATION is the angle between the plane of the ellipse and the plane of the ecliptic.

The ASCENDING node is the point at which the planet's orbit passes through the plane of the ecliptic from south to north.

The angle Ω is measured in the plane of the ecliptic, from a line from the sun through the vernal equinox, to a line from the sun to the ascending node.

The angle ω is measured in the plane of the planet's orbit, from a line from the sun to the ascending node, to the major axis on the side of perihelion.

The parameter PERIANGLE used in this article is defined as $\Omega + \omega$.

In finding where a planet is located at a particular date, one must know what portion of its total period has elapsed since its last perihelion. This is provided by the function PERIODER:

```

V Z+PERIODER DATE
Z+1|(ANOMALY+360) + (DATE-ANOMALYDATE) ÷ PERIOD×TROPYR

```

Epochal Adjustment of Planetary Elements

The orientations of the major axes of the elliptical orbits of the planets are not fixed, but themselves rotate steadily; the effect is appreciable over long intervals. Allowance for this secular shift requires an adjustment to the elements ASCENDING (the angular coordinate of the ascending node) and PERIANGLE (the angular coordinate of perihelion). An approximate adjustment is made by the function EPOCHADJUST. It revises the values in columns 5 and 6 of ORB (i.e. the ascending node and the angle of perihelion) by the size of the secular shift per unit time, multiplied by the interval since the epoch date. The secular effect is here considered to be linear with time:

```

V Z+INTERVAL EPOCHADJUST ORB
ORB[; 5 6]+ORB[; 5 6] + SECULAR × INTERVAL
Z+ORB

V Z+SECULAR
Z+ORB[;8 9]

V Z+EPOCHDATE
Z+ORB[;10 11]

```

Procedure for Locating the Planets

The function PLANETPOS finds the positions of any or all the planets as a function of the date and their orbital elements.

```

V H+DATE PLANETPOS ORB; E; THETA
ORB+(DATE-EPOCHDATE) EPOCHADJUST ORB
E+ECCENTRICITY
THETA+E TRUEANOMALY E KEPLINVERSE 2×PI×PERIODER DATE
H+ORBROTATE CARTESIAN THETA,[1.5] RADIUS THETA

```

The third statement of PLANETPOS finds THETA, the angle between each planet's position at perihelion and its position on the indicated date. The function RADIUS finds the distance from the sun at which that angle intersects the ellipse:

```

V Z+RADIUS THETA; E
E+ECCENTRICITY
Z+SEMIMAJOR×(1-E*2)÷1+E×COS THETA

```

In the last statement of PLANETPOS, the polar coordinates THETA calculated in the preceding step are converted to Cartesian heliocentric coordinates H.

The Inverse of Kepler's Function

The formula for an ellipse permits us to state the distance from the solar focus to a point on the ellipse (that is, the radius at that point) as a function of the angle THETA between the major axis and a line through the focus to that point. However, finding the true anomaly THETA directly as a function of time is difficult. An easier method is due to Kepler. He discovered that a closely related angle PSI could be constructed (see Figure 4) for which the solution is simpler. A quantity proportional to the time is computed by KEPLERFN as a function of PSI and the eccentricity E:

```

V TIME+E KEPLERFN PSI
E+Q((-1+ρPSI),ρE)ρE
TIME+PSI - E×SIN PSI

```

Notice that as E goes to zero (meaning that the ellipse approaches a circle) KEPLERFN PSI approaches PSI.

To find PSI as a function of time, KEPLERFN must be inverted. Because KEPLERFN involves both PSI and SIN PSI, it is transcendental, and approximations must be used to evaluate its inverse. We used an iterative method. In this procedure, each estimate of PSI is adjusted by correcting the previous approximation by an amount inversely proportional to the derivative. That general procedure is known as Newton's method; it was while working on solutions to Kepler's equations that Newton developed the method:

```

V PSI+E KEPLINVERSE TIME; ERROR; TOL
TOL+1E-10
TIME+PSI+((ρE),ρTIME)ρTIME
TEST: →END IF ^/,TOL>|ERROR+TIME-E KEPLERFN PSI
PSI+PSI+ERROR÷E KEPDERIV PSI
→TEST
END: PSI+÷/PSI×(2ρρE)ρ(1+ρE)↑1

```

The restructuring appearing in the second statement and the last statement (and also in KEPDERIV, below) is introduced to

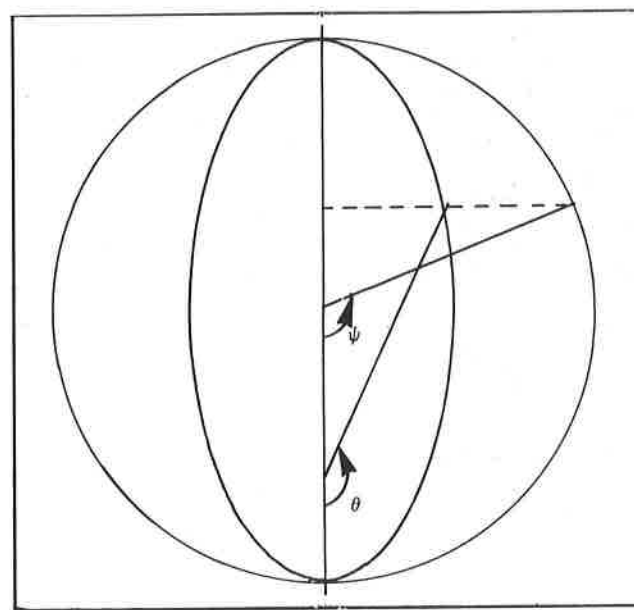


Fig. 4 Angles θ and ψ in the calculation of true anomaly

The angle *THETA* is measured between the major axis and a line drawn from the focus to the planet's position on the ellipse.

The angle *PSI* is measured from the major axis to a line drawn from the center of a circle circumscribed about the ellipse, to the point where a line drawn perpendicular to the axis and passing through the planet intersects the Earth.

permit parallel solution for multiple values of *E* and *TIME*, so that all planets can be treated at once.

The derivative of Kepler's functions is given as follows:

```

V Z←E KEPDERIV PSI
E←Q((-1+ρPSI),ρE)ρE
Z←1-E×COS PSI

```

Now that *PSI* has been found, the more useful true anomaly can be found by analytic geometry:

```

V THETA←E TRUEANOMALY PSI
THETA←(2×PI)|2×ARCTAN (SQRT(1+E)÷1-E) × TAN PSI÷2

```

The function *RADIUS* can now be used to find the planet's distance from the sun in astronomical units.

Plotting the Heliocentric Coordinates of the Planets

The aim in preparing this set of functions was to draw maps showing the sky as it appears above a particular place on Earth. To achieve that, the heliocentric coordinates just calculated must be further translated and rotated to allow for the position of the Earth in the solar system and of the observer on the Earth. However, before introducing the functions that carry out that part of the task, we illustrate a use of the heliocentric coordinates. A function *PLANETSPOS* constructs (iteratively) a table showing for a selected set of dates the positions of selected planets (and also of the comet Kohoutek) for each of an array of dates:

```

V H←DATES PLANETSPOS P; I; D; PL
DATES←,DATES
PL←PLANETS[P;]
H←(0,(1+ρP),3)ρI←0
TEST: →0 IF (ρDATES)<I+1
D←DATES[I]
H←H,[1] (D PLANETPOS PL),[1] D COMETPOS KOHOUTEK
→TEST

```

The result is a 3-dimensional array, dates by planets by coordinates. Plotting the first coordinate against the third, we obtain a diagram showing the positions of the planets projected in the plane of the ecliptic (Figure 5).

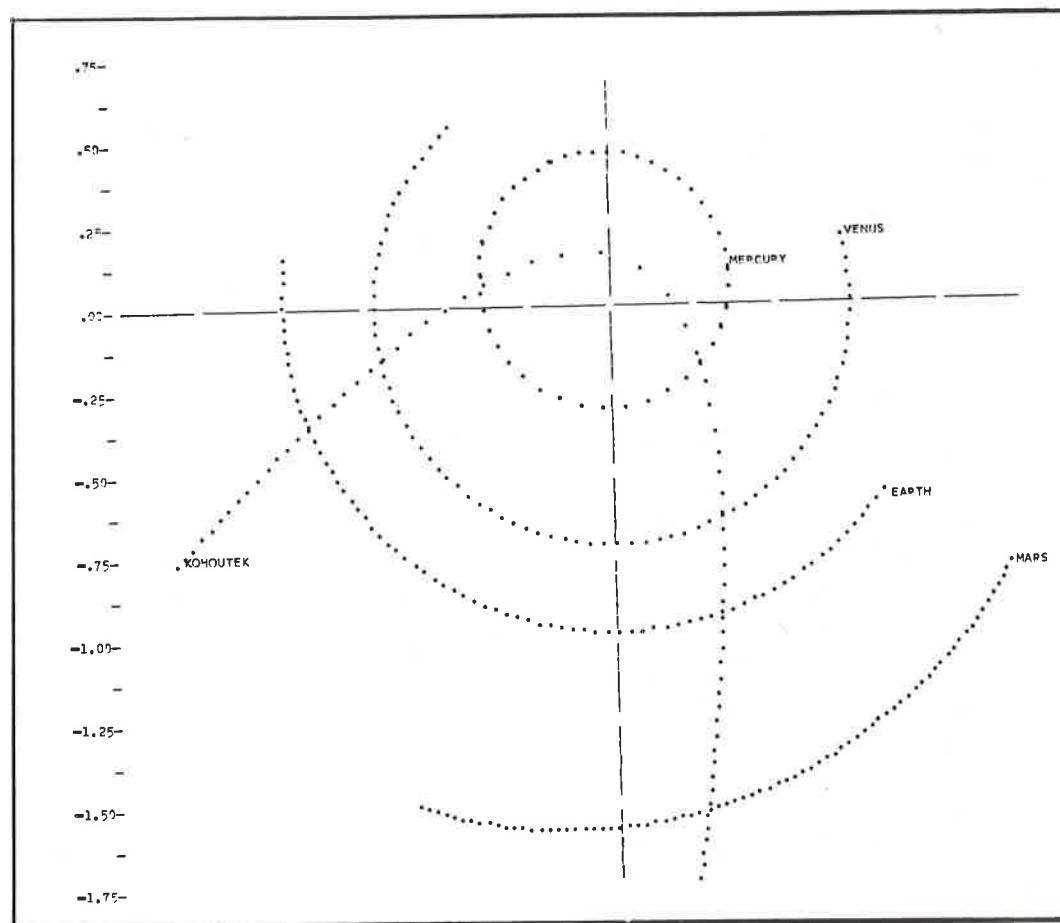


Fig. 5 Sample output of program to plot heliocentric coordinates

The plot shows the orbits of the four inner planets and the comet Kohoutek at 2-day intervals from 20 October 1973 through 30 March 1974.

Positions of the Earth and Moon

In order to find the geocentric coordinates of the other bodies, the heliocentric coordinates of the Earth are required. However, this does not require a special function, since they are directly obtainable from the expression

`DATE PLANETPOS EARTH`

in which `EARTH` is the function which selects the orbital elements of the Earth.

Since the moon is in an elliptical orbit about the Earth, the position of the moon with respect to the earth can be found by the same procedure used to locate the planets with respect to the sun. In calculating the position of the moon, the positions of the ascending node and the angle of perihelion are subject to linear epochal adjustments that are larger than those for the planets, but they are computed in exactly the same way:

```
V GQ+MOONPOS DATE; GC
GC+DATE PLANETPOS MOON
GQ+3 RADECDIST GC+.XQINCLROTATE RADIANT AXITILT+23.4428
```

In the case of the moon, the unit of distance is the semimajor axis of the orbit of the moon rather than of the Earth.

The rotation functions will be discussed below (see pp. 26-27); the function `RADECDIST` calculates polar coordinates in units of right ascension, declination, and distance; the left argument 3 indicates that in this case all three are to be retained.

Since `MOONPOS` finds the moon's position with respect to the Earth, the result is stated with respect to the Earth, and there is no need for subsequent translation from heliocentric to geocentric coordinates. (In the definition of `CALCULATEPLANETS`, p. 13, the expressions for `PLANETS`, `SUN` and `KOHOUTEK` require the application of the function `EARTHVIEW`, whereas the expression for `MOON` does not.) However, the moon is sufficiently close to the Earth that in calculating its apparent position allowance must be made for the parallax introduced by the fact that the observer's position on the surface of the Earth may depart significantly from a line between the center of the Earth and the center of the moon. Such a correction to the moon's altitude is used in `CALCULATEPLANETS`:

```

V Z←DIST PARALLAXADJUST AA; ALT
  ALT←AA[;1]
  Z←AA
  Z[;1]←ALT - (COS RADIAN ALT)×MOONRATIO÷DIST

```

in which *MOONRATIO* is the ratio of the semimajor axis of the moon's orbit to the radius of the Earth, expressed in radians; the value is about 0.95.

The phase of the moon depends upon the difference between the right ascensions of the sun and moon:

```

V Z←MOON MOONPHASE SUN
  Z←1|(MOON-SUN)÷24

```

The moon is full when their right ascensions differ by 12 hours, and new when they are equal. When both the right ascension and the declination of the moon are equal to those of the sun, there is an eclipse of the sun; when their right ascensions differ by 12 hours and their declinations are equal but of opposite sign, there is an eclipse of the moon.

Position of the Comet

The position of Kohoutek is calculated only for dates within 100 days of its perihelion, 28 December 1973. The logical variable *K* (set in *CALCULATEPLANETS*) has the value 1 when Kohoutek is within range, 0 otherwise. The expression *K/DATE* thus makes the date empty when the position of the comet is not needed.

```

V H←DATE COMETPOS ORB; X
  H← 0 3p0
  →0 IF 0=p,DATE
  X←COMETSOLVE (PI×SQRT 2×PERIDIST)×(DATE-ANOMALYDATE)÷TROPYR
  H←ORBROTATE (PARABOLA X), -X

```

The method used to locate the comet is similar to that used for the planets. However, for several reasons the polar coordinates used in the initial two-dimensional solution for the planets are here replaced with Cartesian coordinates. The approximations for planets (whose orbits are nearly circular) do not converge easily when applied to the comet, whose orbit is almost exactly parabolic. The usual polar expression in the function *RADIUS* is singular when *E* is 1 (parabola) and *THETA* is *PI*. Moreover, the Cartesian expression for a parabola is simple to integrate; hence Kepler's equal-areas equal-times law is easily applied.

The time required to reach a point on the parabolic path of the comet as a function of the distance from the axis of the parabola is given by the function *AREA*:

```

V Z←AREA X
  Z←(PERIDIST×X÷2) + (X×3)÷24×PERIDIST

```

in which the orbital element *PERIDIST* is the distance from the sun at perihelion, in astronomical units:

```

V Z←PERIDIST
  Z←ORB[;1]

```

The function *COMETSOLVE* provides an iterative definition for the inverse of *AREA*, giving the perpendicular distance from the axis of the parabola as a function of the time interval from perihelion:

```

V X←COMETSOLVE TIME; ERROR
  X←2×TIME÷PERIDIST
TEST: →0 IF 1E-8>|ERROR+TIME-AREA X
  X←X+ERROR÷AREADERIV X
  →TEST

```

Here again the inverse is found by Newton's method; convergence is speeded by the use of the derivative of the area function with respect to the abscissa:

```

V Z←AREADERIV X
  Z←(PERIDIST÷2) + (X×2)÷8×PERIDIST

```

The second coordinate of the comet's position (within the plane of its orbit) is measured in the direction of the axis of the parabola. It is obtained from the first coordinate by the function *PARABOLA*:

```

V Z←PARABOLA X
  Z←PERIDIST - (X×2)÷4×PERIDIST

```

Rotation of the Stars

The positions of the stars are represented by a table of their right ascensions and declinations, as of 1 January 2000, contained in the matrix *STARS*. There is no provision for the proper motions of the stars, nor for the effects of parallax between different positions on the Earth's orbit, since both these effects are small compared to the precision of the rest of the calculation or to the resolution of the plotting program. The calculation thus reduces to the correction for

the observer's position at a given latitude, date, and time, and the long-run variation introduced by precession.

```

V CALCULATESTARS; STARS
VS←BRIGHT←STARCOORD←AAS←10
STARS←(LAT,DATE,TIME) SKYPOS DATE PRECESS STARS
BRIGHT←BRIGHT IF VS←VISIBLE STARS
AAS←STARS IF BRIGHT←VS
STARCOORD←MAPCARTESIAN PROJECTION STARS IF VS

```

The global results of this function (initially set to 10 in the first statement) are as follows:

STARCOORD	Cartesian coordinates on the map for the stars visible from the indicated time, date, and location.
BRIGHT	A logical vector indicating which of the visible stars are of magnitude 1.5 or brighter.
VS	A logical vector indicating which stars are visible.
AAS	A matrix containing the altitude and azimuth of the visible bright stars.

Correction for Precession

The effect of precession is to alter the direction in which the Earth's axis is tilted. A line drawn from the north pole to the zenith (which today points approximately to the star Polaris) in the course of 25800 years describes a complete circle, with radius 23.45 degrees. What changes with precession is the direction in which the Earth's north pole departs from a point perpendicular to the plane of the Earth's orbit. However, since the direction of the equinox enters into the definition of one of the axes of both the heliocentric and the geocentric ecliptic coordinates, the effect appears as a systematic rotation of the entire star table. The function PRECESS makes this adjustment by first removing the Earth's axial tilt, then rotating about the second axis through an angle that would amount to a complete rotation in 25800 years, and then restoring the axial tilt.

```

V Z←INTERVAL PRECESS X; PRECESSION; ROT; TILT; DETILT; RETILT
X←CARTRIPLET X
RETILT←INCLROTATE TILT←RADIAN AXITILT
DETILT←INCLROTATE -TILT
PRECESSION←LONGROTATE INTERVAL × 2×PI÷25800×YRLENGTH
ROT←RETILT+.×PRECESSION+.×DETILT
Z←2 RADECDIST X+.×ROT

```

Projection of the Visible Sky

Once the altitude and azimuth of moon, sun, planets, and comet have been calculated, it remains only to select those that are visible, and calculate a suitable projection for the map. Objects are considered to be visible if they are on or above the horizon, i.e. if they have non-negative altitude:

```

V Z←VISIBLE X; ALT
ALT←X[;1]
Z←ALT≥0

```

To preserve the apparent shapes of constellations when projected onto a flat surface, the altitudes near the zenith are condensed and those near the horizon expanded by the function PROJECTION which makes the distance from the center of the map proportional to the tangent of one half the coaltitude:

```

V Z←PROJECTION X
Z←(TAN 0.5×COALTITUDE X[;1]),[1.5] RADIAN X[;2]

```

in which coaltitude is defined thus:

```

V Z←COALTITUDE X
Z←RADIAN 90-X

```

Since the plotting routine expects its data to be stated in Cartesian coordinates, the projected polar coordinates are converted back to that form. The function MAPCARTESIAN makes allowance for the fact that altitude and azimuth are conventionally grouped in the opposite order from right ascension and declination:

```

V Z←MAPCARTESIAN X
Z←ΦCARTESIANΦX

```

Functions for Rotation and Translation of Coordinates

The function ORBROTATE converts the two-dimensional Cartesian coordinates of the planets within their own planes to three-dimensional heliocentric coordinates, taking into account the orientation and tilt of the plane of each orbit:


```

V H+ORBROTATE X; INCL; I; OMEGA; O; OMEGA; Q
X+((ρX),1)ρX+ 1 0 1 \X
OMEGA+RADIAN PERIANGLE-ASCENDING
OMEGA+RADIAN ASCENDING
INCL+RADIAN INCLINATION
I+INCLROTATE INCL
O+LONGROTATE OMEGA
Q+LONGROTATE OMEGA
H+(Q TIMES I TIMES O) TIMES X
H+((1+ρH),×/1+ρH)ρH

```

The rotations are achieved by a series of matrix products. The functions *INCLROTATE* and *LONGROTATE* generate the appropriate matrices of sines and cosines, stacking them in a three-dimensional array since several sets of coordinates are to be rotated at once. The function *TIMES* (not shown) calculates the ordinary matrix product of the corresponding pairs of matrices in a three-dimensional stack.

The functions *INCLROTATE* and *LONGROTATE* generate stacks of matrices containing the appropriate sines and cosines of the angles through which rotation is to occur (see Figure 6):

```

V Z+INCLROTATE INCL; RHO
RHO+ρINCL
Z+((ρ,INCL), 3 3)ρ9+1
Z[;2;2]+Z[;3;3]+COS INCL
Z[;2;3]+-Z[;3;2]+SIN INCL
→(0<ρRHO)/0
Z+ 3 3 ρZ

```

```

V Z+LONGROTATE OMEGA; RHO
RHO+ρOMEGA
Z+((ρ,OMEGA), 3 3)ρ 0 0 0 0 1 0 0 0 0
Z[;1;1]+Z[;3;3]+COS OMEGA
Z[;3;1]+-Z[;1;3]+SIN OMEGA
→(0<ρRHO)/0
Z+ 3 3 ρZ

```

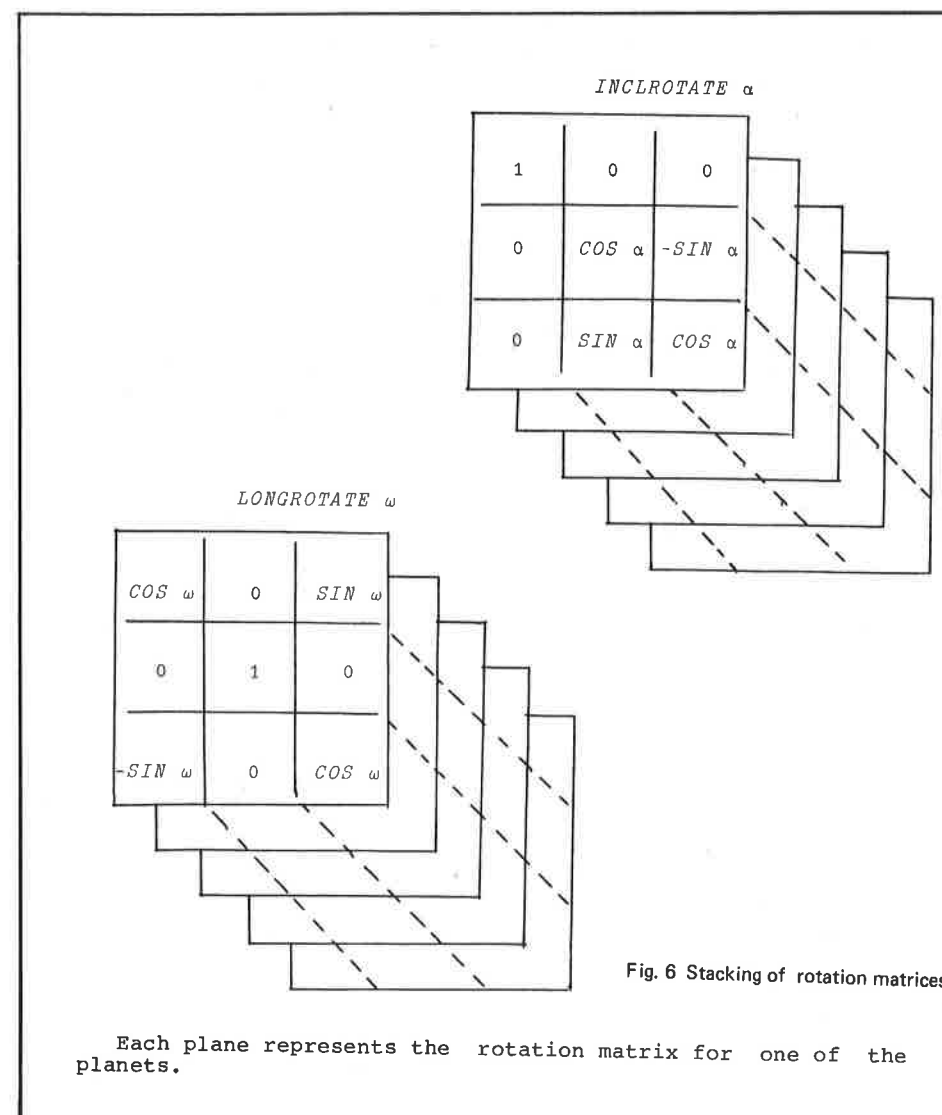
These functions are used in translating the heliocentric coordinates of the planets to geocentric equatorial coordinates (i.e. the view from the center of the Earth):

```

V GQ+DATE EARTHVIEW H; GC
GC+H-(ρH)ρDATE PLANETPOS EARTH
GQ+3 RADECDIST GC+.*INCLROTATE -RADIAN AXITILT

```

in which *AXITILT* is the angle between the axis of the Earth and the plane of the ecliptic.



The next transformation adjusts for the location on the Earth of the observer for whom the map is calculated. The coordinates with respect to the observer are described in a system in which the three coordinates point respectively south, overhead, and west. These are calculated by *SKYPOS* as a function of the geocentric equatorial coordinates *GC*, and the observer's latitude and true local time:

```

V AA←EARTH SKYPOS GQ; SUN; ROT; LAT; DATE; TIME;
  ALT; AZ; NEG; S
LAT←EARTH[1]
DATE←EARTH[2]
TIME←EARTH[3]
SUN←(24+YRLENGTH)×YRLENGTH|DATE-EQUINOX
ROT←PI×(SUN+TIME-12)÷12
LAT←RADIAN 90-LAT
GQ←GQ÷Q(ΦGQ)ρNORM GQ←CARTRIPLET GQ
GQ←GQ+.×Q(LATROTATE LAT)+.×LONGROTATE-ROT
ALT←DEGREES ARCSIN GQ[;2]
NEG←-S×GQ[;3]
AZ←(360×S≥0)+NEG×DEGREES ARCCOS-GQ[;1]÷NORM GQ[; 1 3]
AA←ALT,[1.5] AZ

```

The variable *TROPYR* is the length of the tropical year in days; *EQUINOX* is the Julian date of a vernal equinox (in this case, for 1973).

The function *LATROTATE* prepares a matrix of sines and cosines, exploiting the relation between rotation of latitude and rotation of inclination:

```

V Z←LATROTATE LAT
Z←ΦΦINCLROTATE LAT

```

Conversion of Units

The positions of objects in the sky are described in spherical polar coordinates, usually as right ascension, declination, and distance. The first two are stated as angles in hours or degrees, and the last in astronomical units. The function *RADECDIST* converts from Cartesian to polar coordinates in which right ascension is stated in hours and declination in degrees. Since the distance of celestial objects is not apparent from the Earth, only the right ascension and declination are required for some calculations; by using a left argument of 2, only the first two coordinates are retained, and distance is dropped where it is no longer appropriate:

```

V Z←COL RADECDIST GQ; DIST
Z←ARCCOS GQ[;1]÷(GQ[; 1 3]+.×2)*0.5
Z←(12÷PI)×Z+(GQ[;3]>0)×2×PI-Z
DIST←(GQ+.×2)*0.5
Z←Z,[1.5] (180÷PI)×ARCSIN GQ[;2]÷DIST
→0 IF COL<3
Z←Z,DIST

```

The norm is defined as the square root of the sum of the squares:

```

V Z←NORM X
Z←(X+.×2)*0.5

```

Conversion to Cartesian from polar coordinates is provided by the function *CARTESIAN*:

```

V Z←CARTESIAN POLAR; RHO; THETA
THETA←POLAR[;1]
RHO←POLAR[;2]
Z←(RHO×COS THETA),[1.5] -RHO×SIN THETA

```

Conversion to non-normalized three-dimensional Cartesian coordinates from spherical polar coordinates is provided by the function *CARTRIPLET*:

```

V Z←CARTRIPLET RADEC; Z1; Z2; Z3
Z1←COS PI×RADEC[;1]÷12
Z2←TAN RADIAN RADEC[;2]
Z3←-SIN PI×RADEC[;1]÷12
Z←Z1, Z2,[1.5] Z3

```

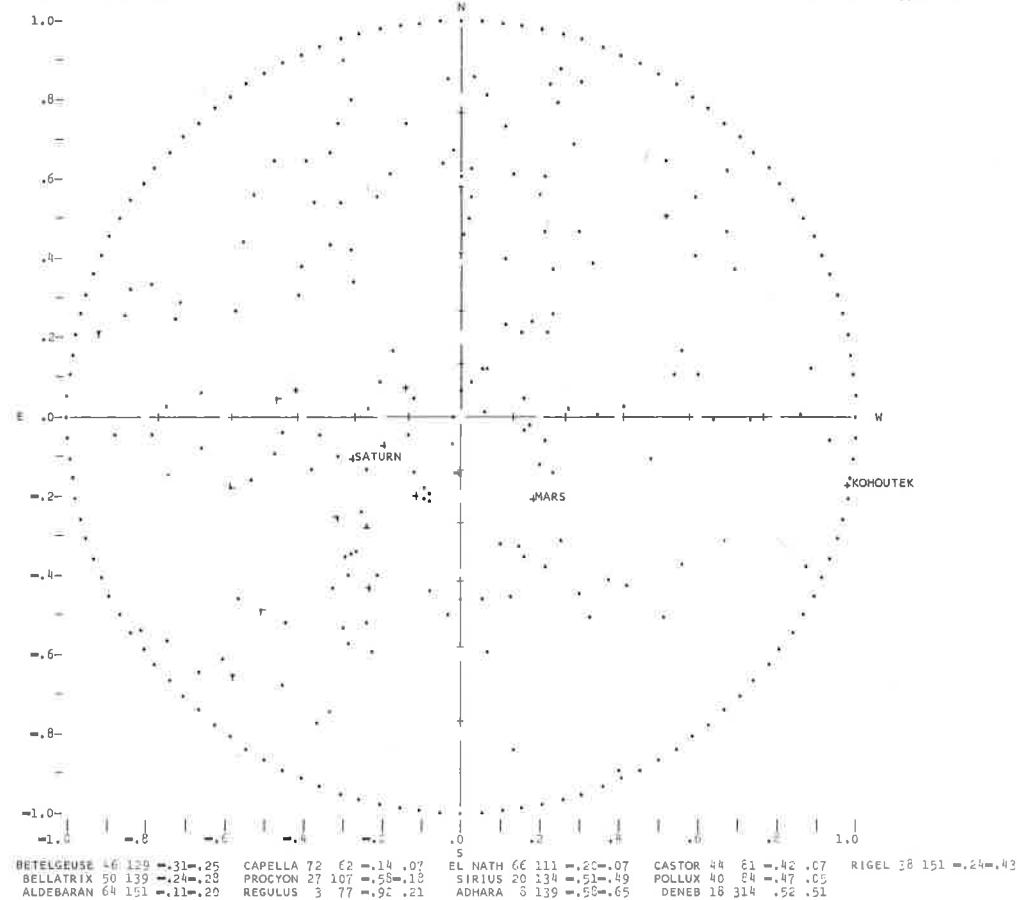
Sample Star Maps

On the following pages maps generated by these programs are reproduced, showing the views from Philadelphia on 14 January 1974 (when Kohoutek was visible), and from the Arctic circle at midnight on 21 June 1974. On two further charts, showing the views from the north and south poles at the vernal equinox, lines linking stars in the same constellation have been drawn in by hand.

VIEW FROM 39 DEGREES 50 MINUTES NORTH, 75 DEGREES 10 MINUTES WEST, ON MONDAY 1974/1/14 AT 8 00 PM STANDARD TIME

POSITION OF SUN, MOON AND PLANETS													
	RT	AS	DECL	ALT	AZ	MAP-GRID		RT	AS	DECL	ALT	AZ	MAP-GRID
MOON	13	8	=12 21				*SATURN	5	56	23 7	57	111	-.27 -.10
SUN	19	43	=21 17				URANUS	13	35	=8 27			
MERCURY	20	1	=22 38				NEPTUNE	18	5	=22 18			
VENUS	20	36	=13 42				PLUTO	13	27	9 18			
*MARS	2	20	14 58	59	222	.18 =.20	*KOHOUTEK	22	10	=7 27	0	260	.98 =.17
JUPITER	21	20	=17 1										
													MOON PHASE .55 WANING

MOON PHASE .55 WANING



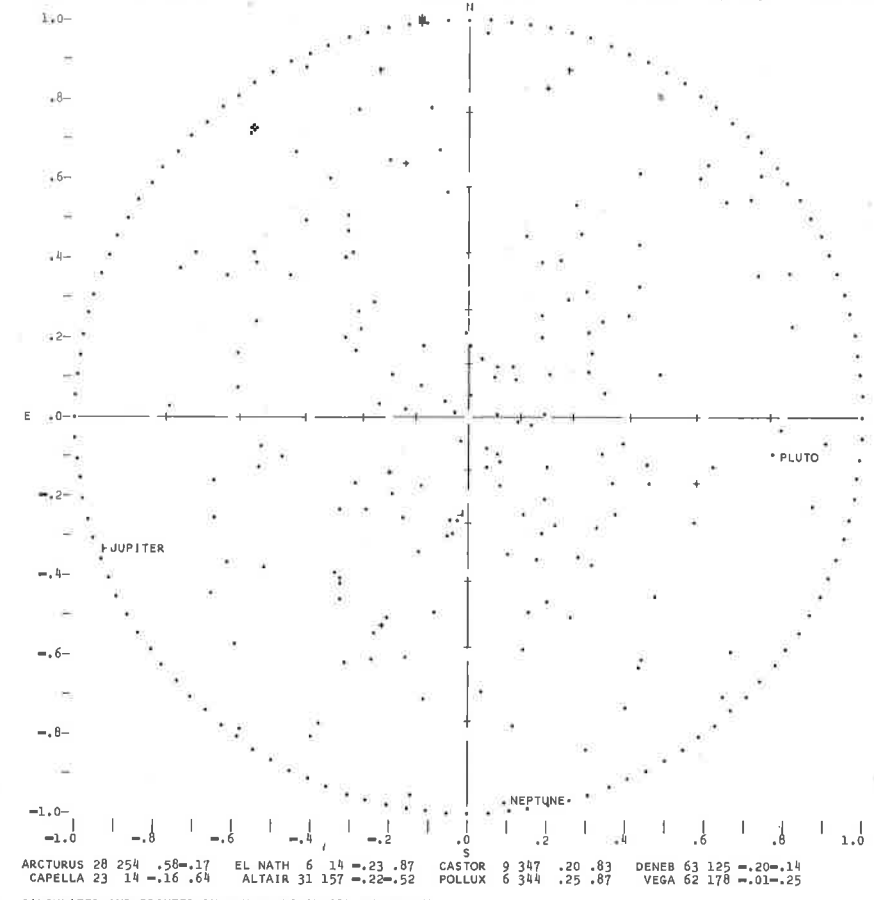
CALCULATED AND PRINTED BY IBM USING AN APL.SV SYSTEM
 MONDAY 1974/1/15 AT 5 23 AM

Fig. 7 Star map at Philadelphia on January 14, 1974

VIEW FROM 66 DEGREES 30 MINUTES NORTH, 144 DEGREES 30 MINUTES WEST, ON FRIDAY 1974/6/21 AT MIDNIGHT STANDARD TIME

POSITION OF SUN, MOON AND PLANETS															
	RT	AS	DECL		ALT	AZ	MAP-GRID		RT	AS	DECL		ALT	AZ	MAP-GRID
MOON	8	6	16	45				*JUPITER	23	16	-7	6			
*SUN	6	0	23	26	0	7	-.12 .99	SATURN	6	30	23	4	1	110	-.93 -.33
MERCURY	6	52	20	18				URANUS	13	20	-6	59			
VENUS	3	34	17	18				*NEPTUNE	18	6	-22	15	1	186	.10 -.98
MARS	8	41	19	12				*PLUTO	13	17	10	25	14	263	.77 -.09
														SUN SHOWN AS ☼	
														MOON PHASE .17 WAXING	

SUN SHOWN AS *
 MOON PHASE .17 WAXING



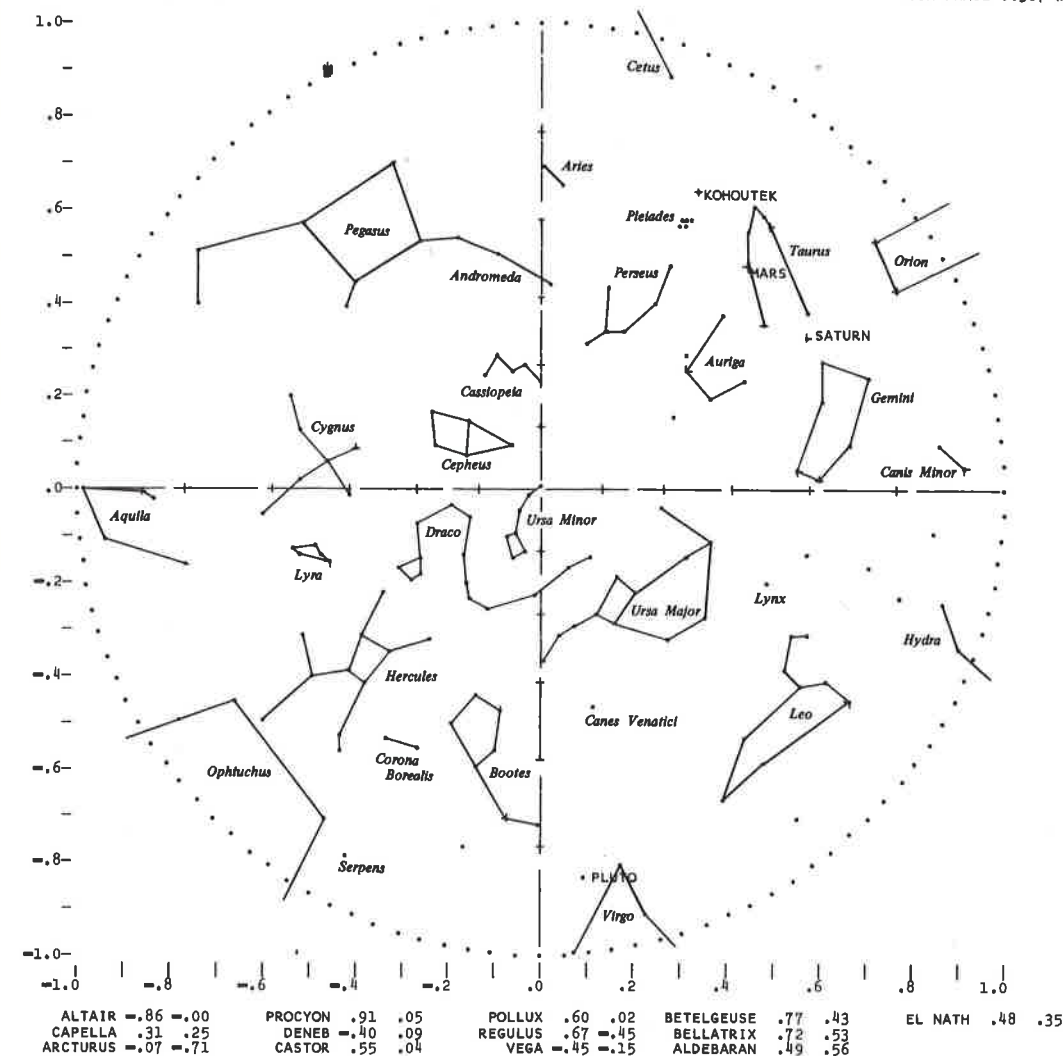
CALCULATED AND PRINTED BY IBM USING AN APL.SV SYSTEM
 MONDAY 1974/4/15 AT 9 04 AM

Fig. 8 Star map at Fort Yukon on June 31, 1974

VIEW FROM THE NORTH POLE, 0 DEGREES ON TUESDAY 1974/3/19 AT 1 58 AM

POSITION OF SUN, MOON AND PLANETS					
	RT.AS.	DECL.	MAP GRID		
*MOON	20 18	-16 15	.46 .89	*SATURN	5 50 23 10
*SUN	0 0	0 0		*URANUS	13 31 -8 3
*MERCURY	22 18	-11 25		*NEPTUNE	18 11 -22 15
*VENUS	21 4	-13 48		*PLUTO	13 25 10 5
*MARS	4 41	23 20	.45 .48	*KOHOUTEK	3 42 18 20
*JUPITER	22 20	-12 0			

SUN SHOWN AS *
MOON PHASE 0.307 WANING



ALTAIR .86 -.00
CAPELLA .31 .25
ARCTURUS .07 -.71

PROCYON .91 .05
DENEK .40 .09
CASTOR .55 .04

POLLUX .60 .02
REGULUS .67 -.45
VEGA -.45 -.15

BETELGEUSE .77 .43
BELLATRIX .72 .53
ALDEBARAN .49 .56

EL NATH .48 .35

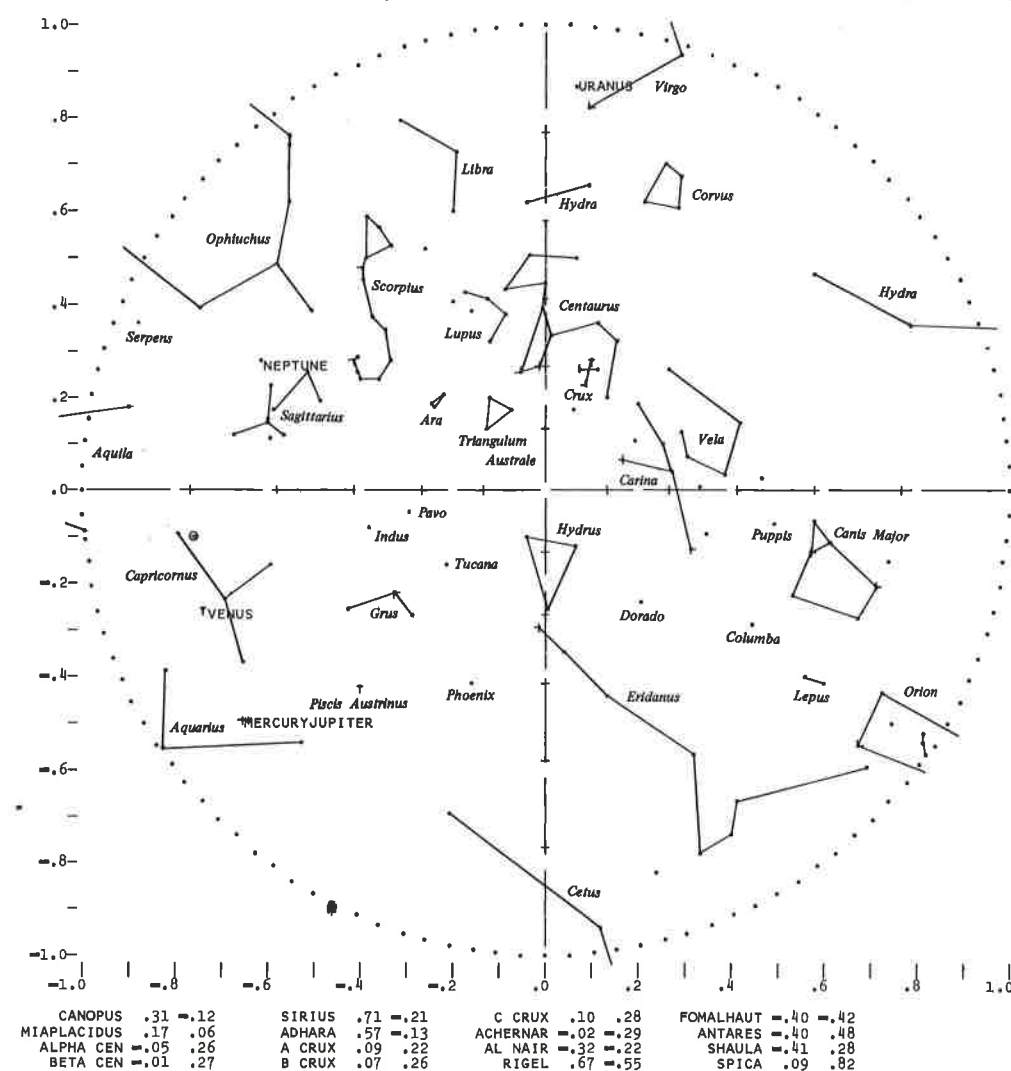
CALCULATED AND PRINTED FOR DISPLAY BY IBM USING AN APL.SV SYSTEM
FRIDAY 1973/12/7 AT 3 02 PM

Fig. 9 Star map at North Pole on March 19, 1974

VIEW FROM THE SOUTH POLE, 0 DEGREES ON TUESDAY 1974/3/19 AT 1 57 AM

POSITION OF SUN, MOON AND PLANETS					
	RT.AS.	DECL.	MAP GRID		
*MOON	20 18	-16 15	.76 -.10	SATURN	5 50 23 10
*SUN	23 59	0 0	.46 -.89	*URANUS	13 31 -8 3
*MERCURY	22 18	-11 25	.65 -.50	*NEPTUNE	18 11 -22 15
*VENUS	21 4	-13 48	.74 -.25	*PLUTO	13 25 10 5
*MARS	4 41	23 20		*KOHOUTEK	3 42 18 20
*JUPITER	22 20	-12 0	.64 -.50		

SUN SHOWN AS *
MOON SHOWN AS O
MOON PHASE 0.307 WANING



CANOPUS .31 -.12
MIAPLACIDUS .17 .06
ALPHA CEN .05 .26
BETA CEN .01 .27

SIRIUS .71 -.21
ADHARA .57 -.13
A CRUX .09 .22
B CRUX .07 .26

C CRUX .10 .28
ACHERNAR .02 .29
AL NAIR .32 .22
RIGEL .67 .55

FOMALHAUT .40 -.42
ANTARES .40 .48
SHAULA .41 .28
SPICA .09 .82

CALCULATED AND PRINTED FOR TYCHO BRAHE BY IBM USING AN APL.SV SYSTEM
FRIDAY 1973/12/7 AT 2 50 PM

Fig. 10 Star map at South Pole on March 19, 1974

REFERENCES

Marion, Jerry B., Classical Dynamics of Particles and Systems, New York: Academic Press, 1965.

American Ephemeris and Nautical Almanac, Explanatory Supplement, U.S. Naval Observatory, 1961.

Hoffleit, Dorrit, Catalog of Bright Stars, New Haven: Yale University, 1964.

APPENDIX

TABLES

The tables in this appendix were prepared by Mr. Per Gjerlov of IBM Denmark, using data from the Yale Catalogue of Bright Stars.

On the first page, values are given for the orbital elements of the nine planets, of the moon, and of the comet Kohoutek. These are the values used to produce the sample charts shown in this report.

Following that, there appear the coordinates of 332 stars. The stars included are roughly the first 300 in visual magnitude, plus a handful of others chosen because they help complete the outline of certain constellations. The table shows the popular name (where there is one), the Bayer designation, and the number in the Yale catalogue. The coordinates are shown as right ascension in hours, minutes, and seconds, and declination, in degrees and minutes, epoch 1 January 2000. The last two columns show the visual magnitude, and the annual parallax in seconds. Where there is a bright double star, only one star is listed.

The stars in Pleiades are here named PLE, although they are commonly referred to the constellation Taurus. To improve visual display, they are shown with positions slightly different from the correct ones.

Mean orbital elements for the planets (columns 1-7 of *PLANETS*)

	SEMIMAJOR	PERIOD	ECCENT'Y	INCLINAT'N	ASCENDING	PERIANGLE	ANOMALY
MERCURY	0.387	0.24085	0.20563	7.004330	48.07347	77.11704	289.6550
VENUS	0.723	0.61521	0.00678	3.394420	76.48402	131.26501	150.2801
EARTH	1	1.00004	0.01672	0	0	102.56635	34.4957
MARS	1.524	1.88089	0.09338	1.849810	49.38973	335.65866	271.1460
JUPITER	5.202	11.86223	0.04794	1.305540	100.21550	14.10850	283.9167
SATURN	9.578	29.45772	0.05759	2.486680	113.49100	94.40310	348.2963
URANUS	19.178	84.01529	0.04808	0.771410	74.00020	168.86530	25.4394
NEPTUNE	29.965	164.78829	0.01119	1.772070	131.54740	59.57130	224.9265
PLUTO	39.543	248.43020	0.24934	17.137130	109.88680	223.14830	335.6904

Mean orbital elements for the planets (columns 8-11 of *PLANETS*)

	SECULAR ASCENDING	SECULAR PERIANGLE	DATE (ASC)	DATE (PERI)
MERCURY	0.000 032 444 198	0.000 042 559 243	2443600.5	2443600.5
VENUS	0.000 024 641 163	0.000 038 505 620	2443600.5	2443600.5
EARTH	0.000 000 000 000	0.000 047 000 737	2443600.5	2443600.5
MARS	0.000 021 188 358	0.000 050 392 700	2443600.5	2443600.5
JUPITER	0.000 027 683 282	0.000 044 110 724	2443600.5	2443600.5
SATURN	0.000 023 880 633	0.000 053 617 346	2443600.5	2443600.5
URANUS	0.000 013 689 535	0.000 044 110 724	2443600.5	2443600.5
NEPTUNE	0.000 030 040 924	0.000 018 252 713	2443600.5	2443600.5
PLUTO	0.000 038 026 486	0.000 038 026 486	2443600.5	2443600.5

Orbital elements for the moon (with respect to Earth)

SEMIMAJOR		PERIOD	ECCENTRICITY	INCLINATION	ASCENDING	PERIANGLE
MOON	1	0.07544	0.05490	5.143412	260.38369	331.80423
SECULAR ASCENDING		SECULAR PERIANGLE		DATE (ASC)	DATE (PERI)	
MOON	-0.005 295 392 200	0.011 140 408 030		2414997.831	2414997.831	

Orbital elements for the comet Kohoutek

	PERIDIST	INCLINATION	ASCENDING	PERIANGLE	PERIDATE
KOHOOTEK	0.142	14.2969	257.7153	295.5891	2442046.463

Popular Name	Bayer	Yale	Right Asc.	Decl.	Mag.	Prlx
	No.		Hr Min Sec	Deg Min		Secs
ALPHERATZ	1	α AND 15	0 8 23	29 5	2.00	.024
MIRACH	2	β AND 337	1 9 44	35 37	2.00	.043
ALMACH	3	γ AND 603	2 3 53	42 20	2.30	.005
	4	δ AND 165	0 39 20	30 52	3.20	.024
ALTAIR	5	α AQL 7557	19 50 47	8 52	.77	.198
	6	β AQL 7602	19 55 19	6 24	3.71	.070
	7	γ AQL 7525	19 46 15	10 37	2.60	.006
	8	δ AQL 7377	19 25 29	3 7	3.36	.062
	9	ζ AQL 7235	19 5 25	13 52	3.00	.036
	10	η AQL 7570	19 52 29	1 0	3.50	.005
	11	θ AQL 7710	20 11 18	0 49	3.24	.008
	12	λ AQL 7236	19 6 15	4 53	3.40	.025
	13	α AQR 8414	22 5 47	0 19	2.93	.003
	14	β AQR 8232	21 31 34	5 35	2.89	.000
	15	δ AQR 8709	22 54 39	15 49	3.30	.039
	16	β ARA 6461	17 25 18	55 32	2.80	.026
	17	γ ARA 6462	17 25 24	56 23	3.30	.000
	18	ζ ARA 6285	16 58 38	55 59	3.10	.036
HAMAL	19	α ARI 617	2 7 10	23 27	2.00	.043
SHERATAN	20	β ARI 553	1 54 39	20 48	2.65	.063
CAPELLA	21	α AUR 1708	5 16 41	46 0	.09	.073
MENIKALINAN	22	β AUR 2088	5 59 32	44 57	1.90	.037
	23	δ AUR 2077	5 59 32	54 17	3.70	.020
	24	ε AUR 1605	5 1 58	43 50	3.00	.004
	25	θ AUR 2095	5 59 43	37 12	2.70	.018
	26	ι AUR 1577	4 57 0	33 9	2.70	.015
ARCTURUS	27	α BOO 5340	14 15 40	19 11	.06	.090
	28	β BOO 5602	15 1 57	40 23	3.50	.022
	29	γ BOO 5435	14 32 5	38 19	3.00	.016
	30	δ BOO 5681	15 15 30	33 19	3.50	.028
	31	ε BOO 5506	14 44 59	27 5	2.70	.013
	32	ζ BOO 5477	14 41 8	13 43	4.10	.007
	33	η BOO 5235	13 54 41	18 24	2.70	.102
	34	ρ BOO 5429	14 31 50	30 23	3.60	.025
	35	α CAP 7754	20 18 3	12 32	3.60	.033
	36	β CAP 7776	20 21 1	14 47	3.10	.005
	37	δ CAP 8322	21 47 2	16 8	2.80	.065
	38	θ CAP 8075	21 5 57	17 14	4.10	.010
	39	ω CAP 7980	20 51 49	26 56	4.10	.000
CANOPUS	40	α CAR 2326	6 23 57	52 41	.73	.018
MIAPLACIDUS	41	β CAR 3685	9 13 12	69 43	1.70	.038
	42	ε CAR 3307	8 22 31	59 30	1.85	.000
	43	η CAR 4210	10 45 4	59 42	2.00	.000
	44	θ CAR 4199	10 42 57	64 23	2.76	.000
	45	ι CAR 3699	9 17 6	59 16	2.24	.011
	46	ν CAR 3890	9 47 6	65 4	3.00	.020
	47	χ CAR 3117	7 56 47	52 59	3.50	.000
	48	ω CAR 4037	10 13 45	70 2	3.31	.000
	49	CAR 4050	10 17 5	61 20	3.44	.018
SCHEDIR	50	α CAS 168	0 40 31	56 32	2.20	.009

Popular Name	Bayer	Yale	Right Asc.			Decl.		Mag.	Prlx
	No.	No.	Hr	Min	Sec	Deg	Min		Secs
CAPH	51	β CAS 21	0	9	10	59	9	2.30	.072
	52	γ CAS 264	0	56	42	60	43	2.65	.034
RUCHBAH	53	δ CAS 403	1	25	49	60	14	2.70	.029
	54	ε CAS 542	1	54	24	63	41	3.40	.007
RIGIL KENTAURUS	55	α CEN 5459	14	39	36	-60	-50	.10	.751
	56	β CEN 5267	14	3	50	-60	-22	.06	.016
	57	γ CEN 4819	12	41	31	-48	-58	2.00	.006
	58	δ CEN 4621	12	8	21	-50	-43	2.88	.020
	59	ε CEN 5132	13	39	53	-53	-28	2.30	.000
	60	ζ CEN 5231	13	55	32	-47	-18	2.50	.000
	61	η CEN 5440	14	35	30	-42	-9	2.35	.000
	62	θ CEN 5288	14	6	41	-36	-23	2.00	.059
	63	ι CEN 5028	13	20	35	-36	-43	2.76	.046
	64	λ CEN 4467	11	35	47	-63	-1	3.12	.000
	65	ν CEN 5190	13	49	30	-41	-41	3.40	.000
ALDERAMIN	66	α CEP 8162	21	18	35	62	35	2.40	.063
	67	β CEP 8238	21	28	39	70	33	3.20	.005
	68	γ CEP 8974	23	39	20	77	37	3.20	.064
	69	ζ CEP 8465	22	10	51	58	12	3.40	.019
	70	ι CEP 8694	22	49	41	66	12	3.60	.036
MENKAR	71	α CET 911	3	2	17	4	6	2.50	.003
DENEK-KAITOS	72	β CET 188	0	43	35	-17	-59	2.00	.057
	73	γ CET 804	2	43	18	3	14	3.47	.048
	74	δ CET 779	2	39	29	0	20	4.07	.001
	75	ζ CET 539	1	51	27	-10	-20	3.72	.038
	76	η CET 334	1	8	36	-10	-11	3.44	.032
	77	θ CET 402	1	24	1	-8	-11	3.61	.034
	78	ι CET 74	0	19	26	-8	-50	3.56	.010
	79	ο CET 681	2	19	21	-2	-59	2.00	.013
	80	τ CET 509	1	44	4	-15	-56	3.50	.275
SIRIUS	81	α CMA 2491	6	45	9	-16	-43	1.47	.375
	82	β CMA 2294	6	22	42	-17	-57	2.00	.014
	83	γ CMA 2657	7	3	45	-15	-38	4.10	.000
	84	δ CMA 2693	7	8	24	-26	-24	1.80	.000
ADHARA	85	ε CMA 2618	6	58	38	-28	-58	1.50	.001
	86	ζ CMA 2282	6	20	18	-30	-4	3.02	.003
	87	η CMA 2827	7	24	5	-29	-18	2.40	.000
PROCYON	88	α CMI 2943	7	39	18	5	14	.34	.288
	89	β CMI 2845	7	27	9	8	17	2.80	.020
	90	α CNC 3572	8	58	29	11	52	4.25	.018
	91	β CNC 3249	8	16	31	9	12	3.52	.014
	92	δ CNC 3461	8	44	41	18	9	4.00	.000
PHACT	93	α COL 1956	5	39	39	-34	-5	2.63	.005
	94	β COL 2040	5	50	58	-35	-46	3.11	.023
ALPHECCA	95	α CRB 5793	15	34	41	26	43	2.20	.043
	96	CRB 5958	15	59	30	25	55	2.00	.000
	97	α CRU 4730	12	26	36	-63	-6	1.00	.008
	98	β CRU 4853	12	47	44	-59	-42	1.24	.000
	99	γ CRU 4763	12	31	10	-57	-7	1.60	.000
	100	δ CRU 4656	12	15	9	-58	-45	2.80	.000

Popular Name	Bayer	Yale	Right Asc.			Decl.		Mag.	Prlx
	No.	No.	Hr	Min	Sec	Deg	Min		Secs
	101	β CRV 4786	12	34	23	-23	-24	2.70	.027
	102	γ CRV 4662	12	15	49	-17	-32	2.60	.023
	103	δ CRV 4757	12	29	51	-16	-31	3.00	.018
	104	ε CRV 4630	12	10	8	-22	-37	3.00	.020
COR CAROLI	105	α CVN 4914	12	56	1	38	19	2.80	.023
DENEK	106	α CYG 7924	20	41	26	45	16	1.26	.000
ALBIREO	107	β CYG 7417	19	30	43	27	58	3.24	.000
	108	γ CYG 7796	20	22	13	40	15	2.24	.000
	109	δ CYG 7528	19	44	58	45	8	2.92	.021
	110	ε CYG 7949	20	46	13	33	58	2.45	.044
	111	ζ CYG 8115	21	12	56	30	14	3.20	.021
	112	η CYG 7615	19	59	1	35	5	3.90	.009
	113	τ CYG 8130	21	14	48	38	3	3.69	.047
	114	α DEL 7906	20	39	39	15	55	3.77	.002
	115	β DEL 7882	20	37	33	14	36	3.78	.026
	116	γ DEL 7947	20	46	38	16	8	3.00	.196
	117	ε DEL 7852	20	33	13	11	18	3.98	.016
	118	α DOR 1465	4	34	0	-55	-3	3.26	.011
THUBAN	119	β DOR 1922	5	33	37	-62	-29	3.40	.007
	120	α DRA 5291	14	4	24	64	22	3.60	.011
ETAMIN	121	β DRA 6536	17	30	26	52	19	2.90	.009
	122	γ DRA 6705	17	56	36	51	29	2.20	.017
	123	δ DRA 7310	19	12	33	67	40	3.10	.028
	124	ε DRA 7582	19	48	10	70	16	3.88	.001
	125	ζ DRA 6396	17	8	48	65	43	3.20	.017
	126	η DRA 6132	16	23	59	61	30	2.80	.043
	127	θ DRA 5986	16	1	54	58	34	4.00	.046
	128	ι DRA 5744	15	24	56	58	58	3.26	.032
	129	κ DRA 4787	12	33	29	69	47	3.80	.010
	130	λ DRA 4434	11	31	24	69	20	3.80	.024
	131	ν DRA 6554	17	32	10	55	11	4.90	.000
	132	ξ DRA 6688	17	53	32	56	52	3.80	.031
	133	ο DRA 7125	18	51	13	59	23	4.70	.003
	134	χ DRA 6927	18	21	4	72	44	3.58	.120
ACHERNAR	135	α ERI 472	1	37	42	-57	-15	.47	.023
	136	β ERI 1666	5	7	51	-5	-5	2.80	.042
	137	γ ERI 1231	3	58	2	-13	-31	3.00	.003
	138	δ ERI 1136	3	43	14	-9	-46	3.55	.109
	139	ε ERI 1084	3	23	56	-9	-28	3.73	.303
	140	η ERI 874	2	56	25	-8	-54	3.89	.027
ACAMAR	141	θ ERI 897	2	58	15	-40	-18	3.42	.028
	142	ο ERI 1325	4	15	6	-7	-40	3.00	.200
	143	τ ERI 1003	3	19	31	-21	-45	3.67	.017
	144	φ ERI 674	2	16	30	-51	-31	3.55	.000
CASIOR	145	α GEM 2890	7	34	36	31	53	1.50	.072
POLLUX	146	β GEM 2990	7	45	19	28	1	1.15	.093
	147	γ GEM 2421	6	37	43	16	24	1.93	.031
	148	δ GEM 2777	7	20	7	21	59	3.50	.059
	149	ε GEM 2473	6	43	56	25	8	3.10	.009
	150	η GEM 2216	6	14	42	22	30	3.20	.013

Popular Name	Bayer	Yale	Right Asc.			Decl.		Mag.	Prlx
	No.		Hr	Min	Sec	Deg	Min		Secs
AL NAIR	151	λ GEM 2763	7	18	6	16	32	3.58	.041
	152	α GRU 8425	22	8	14	-46	-58	1.73	.051
	153	β GRU 8636	22	42	40	-46	-53	2.20	.003
	154	γ GRU 8353	21	53	56	-37	-22	3.00	.008
	155	α HER 6406	17	14	39	14	23	2.90	.000
	156	β HER 6148	16	30	13	21	29	2.80	.017
	157	γ HER 6095	16	21	56	19	9	3.74	.015
	158	δ HER 6410	17	15	2	24	50	3.10	.034
	159	ϵ HER 6324	17	0	18	30	55	3.90	.022
	160	ζ HER 6212	16	41	17	31	36	2.80	.110
ALPHARD	161	η HER 6220	16	42	54	38	56	3.50	.053
	162	ι HER 6588	17	39	27	46	1	3.80	.002
	163	μ HER 6623	17	46	28	27	44	3.35	.108
	164	π HER 6418	17	15	3	36	48	3.20	.020
	165	τ HER 6092	16	19	44	46	19	3.90	.027
	166	α HYA 3748	9	27	35	-8	-40	1.99	.017
	167	γ HYA 5020	13	18	55	-23	-11	3.02	.021
	168	ζ HYA 3547	8	55	24	5	57	3.10	.029
	169	θ HYA 3665	9	14	22	2	19	3.88	.019
	170	ν HYA 4232	10	49	37	-16	-11	3.12	.022
REGULUS DENEbola	171	π HYA 5287	14	6	23	-26	-41	3.25	.039
	172	α HYI 591	1	58	46	-61	-34	2.90	.041
	173	β HYI 98	0	25	45	-77	-15	2.79	.153
	174	γ HYI 1208	3	47	14	-74	-15	3.24	.001
	175	α IND 7869	20	37	34	-47	-17	3.10	.039
	176	α LEO 3982	10	8	22	11	58	1.36	.039
	177	β LEO 4534	11	49	4	14	34	2.10	.076
	178	γ LEO 4057	10	19	59	19	51	2.60	.019
	179	δ LEO 4357	11	14	6	20	31	2.60	.040
	180	ϵ LEO 3873	9	45	51	23	46	3.00	.002
ARNEB	181	ζ LEO 4031	10	16	42	23	25	3.40	.009
	182	η LEO 3975	10	7	20	16	46	3.50	.000
	183	θ LEO 4359	11	14	15	15	26	3.30	.019
	184	μ LEO 3905	9	52	46	26	1	3.90	.022
	185	ρ LEO 4133	10	32	49	9	18	3.85	.005
	186	α LEP 1865	5	32	44	-17	-50	2.60	.002
	187	β LEP 1829	5	28	15	-20	-45	2.84	.014
	188	ϵ LEP 1654	5	5	28	-22	-22	3.18	.006
	189	μ LEP 1702	5	12	56	-16	-12	3.28	.018
	190	α LIB 5530	14	50	53	-16	-3	2.75	.049
	191	β LIB 5685	15	17	0	-9	-23	2.61	.012
	192	γ LIB 5787	15	35	32	-14	-47	3.90	.033
	193	σ LIB 5603	15	4	4	-25	-17	3.30	.056
	194	τ LIB 5812	15	38	40	-29	-47	3.70	.000
	195	α LUP 5469	14	41	56	-47	-24	2.30	.000
	196	β LUP 5571	14	58	32	-43	-8	2.67	.000
	197	γ LUP 5776	15	35	9	-41	-10	2.80	.008
	198	δ LUP 5695	15	21	22	-40	-39	3.20	.000
	199	ϵ LUP 5708	15	22	40	-44	-42	3.40	.009
	200	ζ LUP 5649	15	12	17	-52	-6	3.40	.036

Popular Name	Bayer	Yale	Right Asc.			Decl.		Mag.	Prlx
	No.		Hr	Min	Sec	Deg	Min		Secs
VEGA	201	α LYN 3705	9	21	3	34	24	3.10	.021
	202	α LYR 7001	18	36	56	38	47	.04	.123
	203	β LYR 7106	18	50	4	33	22	3.40	.000
	204	γ LYR 7178	18	58	56	32	41	3.25	.011
	205	δ LYR 7141	18	54	30	36	54	3.90	.000
	206	ζ LYR 7056	18	44	47	37	36	4.00	.025
	207	α MUS 4798	12	37	11	-69	-8	2.70	.000
	208	α OPH 6556	17	34	56	12	34	2.08	.056
	209	β OPH 6603	17	43	28	4	34	2.80	.023
	210	δ OPH 6056	16	14	20	-3	-41	2.70	.029
RASALAGUE	211	ϵ OPH 6075	16	18	19	-4	-42	3.24	.036
	212	ζ OPH 6175	16	37	9	-10	-34	2.60	.000
	213	η OPH 6378	17	10	23	-15	-43	2.44	.047
	214	θ OPH 6453	17	22	0	-25	0	3.28	.000
	215	κ OPH 6299	16	57	44	9	23	3.31	.026
	216	ν OPH 6698	17	59	1	-9	-47	3.30	.015
	217	α ORI 2061	5	55	10	7	24	.80	.005
	218	β ORI 1713	5	14	32	-8	-12	.08	.000
	219	γ ORI 1790	5	25	8	6	21	1.60	.026
	220	δ ORI 1852	5	32	1	0	-18	2.20	.000
SABIK	221	ϵ ORI 1903	5	36	12	-1	-12	1.70	.000
	222	ζ ORI 1948	5	40	46	-1	-57	2.00	.022
	223	ι ORI 1899	5	35	26	-5	-55	2.80	.021
	224	κ ORI 2004	5	47	46	-9	-40	2.00	.009
	225	λ ORI 1879	5	35	8	9	56	3.00	.006
	226	α PAV 7790	20	25	38	-56	-44	1.90	.000
	227	α PEG 8781	23	4	46	15	12	2.50	.030
	228	β PEG 8775	23	3	47	28	5	2.56	.015
	229	γ PEG 39	0	13	14	15	11	2.80	.000
	230	ϵ PEG 8308	21	44	11	9	53	2.40	.000
BETELGEUX RIGEL BELLATRIX	231	ζ PEG 8634	22	41	27	10	50	3.47	.060
	232	η PEG 8650	22	43	0	30	13	3.00	.000
	233	θ PEG 8450	22	10	12	6	12	3.52	.042
	234	μ PEG 8684	22	50	1	24	36	3.50	.032
	235	α PER 1017	3	24	20	49	51	1.79	.029
	236	β PER 936	3	8	11	40	57	2.20	.031
	237	γ PER 915	3	4	48	53	30	2.90	.011
	238	δ PER 1122	3	42	55	47	47	3.00	.007
	239	ϵ PER 1220	3	57	51	40	0	2.90	.000
	240	ζ PER 1203	3	54	8	31	53	2.83	.007
ALNILAM	241	τ PER 854	2	54	16	52	46	3.09	.012
	242	α PHE 99	0	26	17	-42	-18	2.40	.035
	243	β PHE 322	1	6	5	-46	-43	3.30	.017
	244	γ PHE 429	1	28	21	-43	-19	3.40	.003
	245	PLE 1165	3	47	29	24	7	2.86	.005
	246	PLE 1156	3	46	19	23	30	4.16	.000
	247	PLE 1142	3	44	52	24	7	3.69	.019
	248	PLE 1149	3	49	48	23	36	3.86	.000
	249	PLE 1178	3	49	10	24	3	3.62	.028
	250	α PSA 8728	22	57	39	-29	-37	1.16	.144
MARKAB SCHEAT ALGENIB ENIF									
MARFAK ALGOL									
FOMALHAUT									

Popular Name	Bayer	Yale	Right Asc.			Decl.		Mag.	Prlx
	No.	No.	Hr	Min	Sec	Deg	Min		Secs
	251	α PSC 595	2	2	2	2	46	3.00	.000
	252	ζ PUP 3165	8	3	35	-40	0	2.30	.000
	253	ν PUP 2451	6	37	46	-43	-11	3.17	.000
	254	ξ PUP 3045	7	49	17	-24	-52	3.34	.003
	255	π PUP 2773	7	17	9	-37	-6	2.70	.023
	256	ρ PUP 3185	8	7	33	-24	-18	2.88	.031
	257	σ PUP 2878	7	29	14	-43	-18	3.24	.013
	258	τ PUP 2553	6	49	56	-50	-37	2.90	.000
ANTARES	259	α SCO 6134	16	29	25	-26	-26	1.10	.019
	260	β SCO 5984	16	5	26	-19	-48	2.40	.004
DSCHUBBA	261	δ SCO 5953	16	0	20	-22	-37	2.32	.000
	262	ϵ SCO 6241	16	50	10	-34	-18	2.28	.049
	263	η SCO 6380	17	12	9	-43	-14	3.30	.063
	264	θ SCO 6553	17	37	19	-43	0	1.90	.020
	265	ι SCO 6615	17	47	35	-40	-7	3.00	.013
	266	κ SCO 6580	17	42	29	-39	-2	2.40	.000
SHAULA	267	λ SCO 6527	17	33	36	-37	-6	1.60	.000
	268	μ SCO 6247	16	51	52	-38	-3	3.10	.000
	269	ν SCO 6508	17	30	46	-37	-18	2.70	.000
	270	π SCO 5944	15	58	51	-26	-7	2.90	.005
	271	σ SCO 6084	16	21	12	-25	-35	3.00	.000
	272	τ SCO 6165	16	35	53	-28	-13	2.80	.014
UNUK	273	α SER 5845	15	44	17	-6	25	2.70	.046
	274	η SER 6869	18	21	18	-2	-53	3.30	.054
	275	γ SGR 6746	18	5	48	-30	-26	3.00	.018
	276	δ SGR 6859	18	21	0	-29	-49	2.70	.039
KAUS-AUSTRALIS	277	ϵ SGR 6879	18	24	10	-34	-23	1.80	.015
	278	ζ SGR 7194	19	2	37	-29	-52	2.60	.020
	279	η SGR 6832	18	17	38	-36	-46	3.12	.038
	280	λ SGR 6913	18	27	58	-25	-26	2.80	.046
	281	π SGR 7264	19	9	46	-21	-1	2.90	.016
NUNKI	282	σ SGR 7121	18	55	16	-26	-18	2.10	.000
	283	τ SGR 7234	19	6	56	-27	-40	3.30	.038
	284	ϕ SGR 7039	18	45	40	-27	0	3.18	.000
ALDEBARAN	285	α TAU 1457	4	35	55	16	30	.86	.048
EL NATH	286	β TAU 1791	5	26	17	28	36	1.70	.018
	287	γ TAU 1346	4	19	48	15	37	3.70	.000
	288	δ TAU 1373	4	22	56	17	32	3.76	.016
	289	ϵ TAU 1409	4	28	37	19	11	3.54	.018
	290	ζ TAU 1910	5	37	39	21	9	3.00	.000
	291	θ TAU 1411	4	28	35	15	57	3.90	.033
	292	τ TAU 1497	4	42	15	22	58	4.30	.008
	293	α TRA 6217	16	48	40	-69	-2	1.91	.024
	294	β TRA 5897	15	55	9	-63	-26	2.84	.078
	295	γ TRA 5671	15	18	55	-68	-41	2.90	.005
	296	α TRI 544	1	53	5	29	35	3.53	.050
	297	β TRI 622	2	9	32	34	59	3.00	.012
	298	γ TRI 664	2	17	19	33	51	4.08	.036
	299	α TUC 8502	22	18	30	-60	-15	2.90	.019
DUBHE	300	α UMA 4301	11	3	44	61	45	1.80	.031

Popular Name	Bayer	Yale	Right Asc.			Decl.		Mag.	Prlx
	No.	No.	Hr	Min	Sec	Deg	Min		Secs
	301	β UMA 4295	11	1	51	56	23	2.40	.042
MERAK	302	γ UMA 4554	11	53	49	53	42	2.90	.020
PHECDA	303	δ UMA 4660	12	15	26	57	2	3.30	.052
MEGREZ	304	ϵ UMA 4905	12	54	2	55	57	1.80	.008
ALIOOTH	305	ζ UMA 5054	13	23	56	54	56	2.40	.037
MIZAR (+ALCOR)	306	η UMA 5191	13	47	32	49	19	1.86	.000
ALKAID	307	θ UMA 3775	9	32	51	51	41	3.18	.052
	308	ι UMA 3569	8	59	13	48	2	3.14	.066
	309	μ UMA 4069	10	22	19	41	30	3.00	.031
	310	ν UMA 3888	9	50	59	59	3	3.77	.036
	311	ϕ UMA 3323	8	30	16	60	43	3.36	.004
	312	χ UMA 4518	11	46	3	47	47	3.69	.014
	313	ψ UMA 4335	11	9	40	44	29	3.00	.000
	314	ω UMA 3757	9	31	32	63	4	3.65	.034
POLARIS	315	α UMI 424	2	31	13	89	15	2.50	.003
KOCHAB	316	β UMI 5563	14	50	43	74	9	2.00	.031
	317	γ UMI 5735	15	20	44	71	50	3.00	.000
	318	δ UMI 6789	17	32	11	86	35	4.40	.000
	319	ϵ UMI 6322	16	45	58	82	2	4.30	.014
	320	ζ UMI 5909	15	44	3	77	48	4.30	.011
	321	η UMI 6116	16	17	30	75	45	5.00	.038
AL SUHAIL	322	γ VEL 3207	8	9	32	-47	-21	1.80	.000
	323	δ VEL 3485	8	44	43	-54	-43	2.00	.043
	324	κ VEL 3734	9	22	7	-55	-1	2.49	.007
	325	λ VEL 3634	9	8	0	-42	-26	2.30	.015
	326	μ VEL 4216	10	46	46	-49	-26	2.70	.022
	327	ϕ VEL 3447	8	40	18	-52	-55	3.61	.000
SPICA	328	α VIR 5056	13	25	11	-11	-9	.10	.021
	329	γ VIR 4825	12	41	40	-1	-27	3.60	.101
	330	δ VIR 4910	12	55	36	3	23	3.40	.017
	331	ϵ VIR 4932	13	2	11	10	58	2.80	.036
	332	ζ VIR 5107	13	34	42	0	-36	3.40	.035