Quantum probability and many worlds

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April 11, 2007

Abstract

We discuss the meaning of probabilities in the many worlds interpretation of quantum mechanics. We start by presenting very briefly the many worlds theory, how the problem of probability arises, and some unsuccessful attempts to solve it in the past. Then we criticize a recent attempt by Deutsch to *derive* the quantum mechanical probabilities from the non-probabilistic parts of quantum mechanics and classical decision theory. We further argue that the Born probability does not make sense even as an additional probability rule in the many worlds theory. Our conclusion is that the many worlds theory fails to account for the probabilistic statements of standard (collapse) quantum mechanics.

Key words: Many worlds, quantum probability, rational decision theory.

1 Introduction

The problem of probability in the many worlds interpretation of quantum mechanics arises because the splitting of worlds is *un*related to the Born probabilities. The theory implies that *any* possible combinatorial sequence of measurement outcomes is realized in some branch of the quantum state regardless of the size of its quantum amplitude (provided it is non-zero). This seems to mean that the theory has no empirical content.

The problem has several vicissitudes in the short history of the many worlds theory. Everett (1957) and DeWitt (1970) argued that the Born probabilities can be *derived* from the theory itself (that is, from unitary quantum mechanics and its relative state interpretation; see below). This is their famous 'self-interpretation' claim which was later refuted by various authors essentially on pain of circularity. Recently, the claim has been revived by Deutsch (1999), followed by Wallace (2003a, 2005), Saunders (2004) and others, who argue that the Born rule can be derived from unitary quantum mechanics together with some

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(non-probabilistic) axioms of rational decision theory.[1] Here, the idea is to apply rational decision theory to unitary quantum mechanics and prove a quantum version of the decision theoretic representation theorem. The upshot would be an Everettian 'Principal Principle' according to which splitting observers in the many worlds universe, if they are rational, are constrained to adopt the quantum mechanical probability as their unique subjective probability for action (see Wallace 2002, sec. 8). Therefore, despite the overall deterministic dynamics of the quantum state, rational observers will believe that the outcomes of quantum measurements are genuinely probabilistic, where the probabilities are given by the Born rule, just as standard quantum mechanics with collapse has it. Or so it seems.

Note two points that will prove to be important later. First, one cannot argue that worlds with non-quantum frequencies have zero, or near zero probability to occur, since such a claim wold be flatly inconsistent with the deterministic dynamics of the quantum state and the splitting picture itself. In other words, in the many worlds theory it makes no sense to identify quantum measure zero with probability zero! Second, it is not at all clear from the outset how one could solve the problem by appealing to rational decision theory, since rational observers (in whatever sense depicted by the principles of pure rationally) would have no reason to believe that the statistical predictions of quantum mechanics are true if they also believed that the many worlds theory is true.

In what follows we argue that Deutsch's decision theoretic approach fails to establish its claim. In particular, we argue that Born's rule *cannot* be derived from the many worlds theory. If the many worlds theory is true, only the frequencies occuring in some subset of all the worlds will match Born's rule. Therefore, only observers that track the worlds in *this* subset will consider both Born's rule (and in fact quantum theory) to be true. Moreover, in the many worlds theory, not only Born's rule but any probability rule is meaningless. The only way to solve the problem, we argue, is by adding to the many worlds theory some stochastic element.

The paper is structured as follows. In Section 2 we give a very brief guide to the many worlds theory. In Section 3 we present the probability problem sketched above and some old attempts to solve it. In Section 4 we review Deutsch's (1999) decision theoretic approach in which he claims to prove that the Born rule follows from the non-probabilistic parts of quantum mechanics and decision theory. We criticize Deutsch's approach in Section 5. Finally, in Section 6, we argue that without adding some stochastic chance element into the many worlds theory, probability statements in the physically relevant sense are meaningless.

[1]See Barrett (1999) for other attempts to solve the problem, e.g. by Lockwood, Saunders, Vaidman; see also Lewis (2004) and Greaves (2004).

2 The many worlds theory

Everett (1957) and DeWitt (1970) proposed the *many worlds* (or *relative state*) interpretation of quantum mechanics (henceforth many worlds theory) as an explicit solution to the measurement problem in the quantum mechanical theory of measurement.[2] To a large extent, the motivation for the many worlds theory was from the outset the fact that it could be written in a relativistic formulation (at least in the case of spatial relativity). In this respect, it seemed superior to both standard quantum mechanics with collapse and hidden variables theories.[3]

Since the many worlds theory takes unitary quantum mechanics, without collapse, to be a *complete* theory, a radical change of interpretation of the quantum state is required in order to understand what the theory says about the world. This goes as follows. Given the pure quantum state of the universe, any subsystem of it can always be assigned a quantum state of its own. In general this state will not be pure but rather mixed. However, given the total state of the universe at a time t, any state of a subsystem of the universe that has nonzero amplitude uniquely determines a *relative state* of the rest of the universe at the same time t. This is essentially the whole theory.

To see what is meant by 'relative state' consider the following. Let $\mathcal{H}$ be the Hilbert space of the universe, and suppose that the universe has a (pure) quantum state $|\Psi\rangle$. Consider a division of the universe into two subsystems $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$. We can think of $\mathcal{H}_1$as a space associated with of a given physical system, and $\mathcal{H}_2$ with the "rest of the universe". Take a basis $\{|\psi_i\rangle\}$ in $\mathcal{H}_1$ and a basis $\{|\phi_j\rangle\}$ in $\mathcal{H}_2$. The state of the universe $|\Psi\rangle$ can be written in terms of these bases as:

$$|\Psi\rangle = \sum_{ij} \lambda_{ij} |\psi_i\rangle \otimes |\phi_j\rangle \tag{1}$$

with $\sum_{ij} |\lambda_{ij}|^2 = 1$. Since $|\Psi\rangle$ is, in general, not factorizable into the product form $|\psi_i\rangle \otimes |\phi_j\rangle$ the subsystems $\mathcal{H}_1$ and $\mathcal{H}_2$ are not in pure relative states; they can only be assigned mixed relative states. For example, relative to the basis $\{|\psi_i\rangle\}$ in $\mathcal{H}_1$, put

$$\mu_i = \sum_j |\lambda_{ij}|^2 \tag{2}$$

so that and $\mu_i \geq 0$, $\sum_i \mu_i = 1$. Now define

$$|E_i\rangle = \sum_j \frac{\lambda_{ij}}{\sqrt{\mu_i}} |\phi_j\rangle \tag{3}$$

then $|\Psi\rangle$ can be rewritten as:

$$|\Psi\rangle = \sum_i \sqrt{\mu_i}\, |\psi_i\rangle \otimes |E_i\rangle \tag{4}$$

[2]We don't distinguish between Everett's (1957) and DeWitt's (1970) formulations, since they agree on the interpretation of probability in the many worlds theory.

[3]In the GRW dynamical collapse theory some progress with respect to relativity has been recently made. See Tumulka (2006).

where the $|E_i\rangle$ need not be orthogonal nor form a basis, so the expansion (4) is not, in general, a bi-orthogonal decomposition. The relative state of the system 2 is the mixture on $\mathcal{H}_2$ given by $\rho_2 = \sum_i \mu_i |E_i\rangle\langle E_i|$, and each individual $|E_i\rangle$ is the state of the rest of the universe relative to the state $|\psi_i\rangle$ of the system. Everett calls each term $|\psi_i\rangle \otimes |E_i\rangle$ in the global superposition (4) a *branch* (we shall sometimes call it also a *world*).

Given the state of the universe $|\Psi\rangle$ and the orthonormal basis $\{|\psi_i\rangle\}$, the expansion of $|\Psi\rangle$ in (4) is unique. Furthermore, given $|\Psi\rangle$, for each component state $|\psi_i\rangle$, the corresponding relative state $|E_i\rangle$ is unique and determined by $|\psi_i\rangle$ alone: the relative state $|E_i\rangle$ with respect to the state $|\psi_i\rangle$ is independent of the choice of basis for the orthogonal complement of $|\psi_i\rangle$. One may consider the set of $|\psi_i\rangle$'s, or the projections $|\psi_i\rangle\langle\psi_i|$, as a family of alternative and exhaustive events (since $\langle\psi_i|\psi_j\rangle = \delta_{ij}$ and $\sum_i |\psi_i\rangle\langle\psi_i| = I$). The expansion of $|\Psi\rangle$ in (4) can be then taken to describe a one-to-one correlation between the relative states and the events $|\psi_i\rangle$. In a similar way we define the relative states in $\mathcal{H}_1$ corresponding to a choice of basis in $\mathcal{H}_2$.

Thus in the many worlds theory physical systems are assigned definite quantum states not absolutely, but relative to the state of another system in the same branch. This is taken to justify the intuition that each branch $|\psi_i\rangle \otimes |E_i\rangle$ in the global superposition (4) can be considered to have some kind of independent existence, and that all branches should be treated on equal status. The desideratum put explicitly by Everett and DeWitt was that *all* the usual features of standard (collapse) quantum theory and its statistical predictions should be *derivable* within the scheme of relative states from the description given by the universal quantum state together with the unitary quantum mechanical dynamics. In particular, no collapse is assumed and no extra or hidden variables over and above the quantum mechanical state are postulated.

In the many worlds theory environmental decoherence (see e.g. Joos, Zeh & Kiefer 2003) is the key to solving the so-called preferred basis problem (i.e. roughly the question why our experience is associated with some particular (coarse grained) states). It can be shown that decoherence yields coarse grained branches that are defined in terms of the decoherent states and this is just enough to explain our usual quasi-classical experience. The decoherent branches effectively evolve independently of each other and in such branches observable effects of re-interference are negligible and observers cannot be aware of the splitting of the wave function (nor of the splitting of their own states). What is left to be shown in this context is that the neurophysiological systems upon which our phenomenal experience supervenes are typically of the kind described by decoherence theory. Given our best current knowledge this is indeed most plausible.[4]

[4]See on the issue of decoherence and the branching, Zeh (1973, 2003), Saunders (1995), Hemmo (1996), Wallace (2003b). Everett (1957) implicitly assumed that branches specified by the memory states of an observer in which sequences of different measurement results are recorded don't reinterfere.

3 The probability problem

The major problem of the many worlds theory (as construed above) is how to account for the probabilistic content of standard collapse quantum mechanics. The problem arises because the Everett-DeWitt branching is *un*related to the quantum probability. In a finite set of repeated measurements, *any* sequence of outcomes certainly occurs, as long as the sequence has nonzero quantum amplitude, including sequences with relative frequencies that differ radically from the predictions of the standard theory. By contrast, the standard theory makes the link to probability via the collapse postulate and Born's rule. Thus, there is a natural distinction between *typical* branches which are likely to occur, and deviant branches which are *a-typical*, have diminishing probability, and are rarely if ever observed. The problem in the many worlds theory is to account for this feature of the standard theory which explains our experience.

Let us consider the problem and some of its implications in more detail. Consider a finite temporal sequence of N repeated measurements of an observable $\mathcal{O}$ on N identical systems, all prepared in the state $|\varphi\rangle$ (this is formally equivalent to a single measurement of the N copies tensor product $\mathcal{O}\otimes...\otimes\mathcal{O}$ on the state $|\varphi\rangle\otimes...\otimes|\varphi\rangle$). Let $\{|\psi_i\rangle\}$ be the eigenstates of $\mathcal{O}$, and assume for simplicity that $\mathcal{O}$ is a maximal operator, so that all the eigenvalues of $\mathcal{O}$ are distinct. We can represent $|\varphi\rangle=\sum_{i=1}^{K}\mu_i|\psi_i\rangle$, where all coefficients $\mu_i\neq 0$. According to quantum theory, the measurements are independent of each other, and the outcome $|\psi_i\rangle$ is occurring with probability $|\mu_i|^2$. If the initial state at t_0 of the composite system and apparata is represented as the tensor product state

$$|\Psi^N(t_0)\rangle=|\varphi\rangle\otimes...\otimes|\varphi\rangle\otimes|\Phi_0\rangle\otimes...\otimes|\Phi_0\rangle \tag{5}$$

where $|\Phi_0\rangle$ is the ready state of the apparata, then the Hamiltonian evolves the state so that after the N measurements the final state is the superposition

$$|\Psi^N\rangle=\sum_{i_1,i_2...i_N}\mu_{i_1}\mu_{i_2}...\mu_{i_N}|\psi_{i_1}\rangle\otimes...\otimes|\psi_{i_N}\rangle\otimes|\Phi_{i_1}\rangle\otimes...\otimes|\Phi_{i_N}\rangle. \tag{6}$$

Here the $|\psi_{i_k}\rangle$'s are the eigenstates of $\mathcal{O}$ and the $|\Phi_{i_k}\rangle$ are the pointer states (or the memory states) of the apparata which we assume are of the kind described by decoherence theory.

Standard quantum mechanics implies, in virtue of the *probabilistic* collapse postulate, that the sequence of the outcomes is random with multinomial probability $|\mu_{i_1}...\mu_{i_N}|^2$, which is just the product of the single probabilities. In the standard theory these probabilities are interpreted as corresponding to a genuine *stochastic* process: From all the combinatorialy possible sequences only one is realized. Moreover, as N approaches ∞ the probability that the real sequence exhibits relative frequencies that match the quantum probabilities approaches one. In other words, the fact that a *typical* sequence of outcomes follows Born rule is a simple consequence of the law of large numbers of the classical theory of random variables.

However, in the many worlds theory the above sequence of measurements is completely described by the branching in the state $|\Psi^N\rangle$ (in 6)). This state is a superposition of K^N branches that in fact exhaust all the combinatorially possible arrangements of sequences of length N, *and all of them are real*. In most of the sequences (on a simple count) the relative frequencies are radically different from the quantum probabilities (except for the special case where all the μ_i are equal). This means that the combinatorial majority of observers would find that quantum mechanics contradicts their experience. So, as a general rule one clearly cannot claim that the quantum probabilities might be inferred (say, as an empirical conjecture) from the observed frequencies.[5] Moreover, even if one were to define (by brute force) the squared modulus of a branch as its probability it would still be irrelevant, since all branches are actual. In other words, in the absence of measurement as a genuine stochastic process the combinatorial majority of worlds is still a-typical. We do not argue that the combinatorial counting measure is the right one. Our point is that there is no sense at all in which *any* probability measure is naturally picked out by the many worlds theory (see also Putnam 2005).

So far we have assumed that the number of branches in a measurement is determined by the number of all possible outcomes. This is the way that Everett formulated the theory. However, in more recent accounts based on decoherence it has been pointed out (see Wallace 2005, Greaves 2004) that the number of branches associated with each outcome is indefinite. This means that this number depends sensitively on the exact choice of decoherence basis, the degree of coarse graining, and perhaps other factors. Nevertheless, in what follows we shall keep with the Everettian account, but this does not change our argument in any way. The reason is that our argument only assumes the following two uncontroversial premises. (i) Any finite set of possible outcomes of a sequence of measurements corresponds to at least one real branch (world). (ii) The many worlds theory by itself does not provide a measure over branches. In the absence of such measure, there is no way to tell whether one branch is more typical than another.

The problem is to derive Born's rule from the many worlds theory without assuming directly or indirectly a notion of typicality or a measure over branches. Our aim is to show that this is impossible.

For our later discussion it will be instructive to consider two old attempts at solving this problem. The first attempt associates subsets of worlds with the branches described by states of the type (6). Suppose that the measure of each subset is given by the square of the amplitude of the corresponding branch, with the intended interpretation that a subset with larger weight contains more worlds than a subset with smaller weight. For example, if $|\mu_1| > |\mu_2|$ then the subset of worlds associated with the branch $|\psi_1\rangle|\psi_1\rangle...|\psi_1\rangle$ in the post measurement state (6) contains more worlds than the subset associated with the $|\psi_2\rangle|\psi_1\rangle...|\psi_1\rangle$ branch. Such a theory could be made compatible with the

[5]By contrast to the standard theory in which collapses are genuinely probabilistic, or the many minds theory by Albert and Loewer (1988) in which the minds perform a random walk on the Everett tree with the quantum probabilities indicated on the branches.

predictions of standard quantum mechanics with respect to measurement outcomes. However, as we have argued in our (2003), it would be a hidden variables theory in disguise in which the *individual* worlds would presumably evolve in a stochastic way with contextual transition probability for each individual world. Moreover, in such a theory subsets of worlds have quantitative properties that transcend, and cannot be inferred from quantum theory itself.[6]

The other attempt which goes back to DeWitt (1970) aims at proving the Born rule from the non-probabilistic part of quantum mechanics. DeWitt's approach relies on theorems about the properties of quantum mechanical relative frequency operators (see Hartle 1968, DeWitt 1970, Ochs 1979, Farhi, Goldstone and Gutmann 1989) which say the following. Consider an actual infinite ensemble of systems, each associated with the state $|\phi\rangle$ in the Hilbert space $\mathcal{H}$. The infinite ensemble is then associated with the infinite tensor product $\mathcal{H}^{\infty} = \mathcal{H} \otimes ... \otimes \mathcal{H} \otimes ...$ and the state

$$|\phi^{\infty}\rangle = |\phi\rangle \otimes ... \otimes |\phi\rangle \otimes ... \tag{7}$$

Let $|\psi_k\rangle$ be the eigenstates of the operator $\mathcal{O}$ on $\mathcal{H}$. Define for positive integers N and k the operator $\mathcal{P}_k^N$ on $\mathcal{H}^N = \mathcal{H} \otimes ... \otimes \mathcal{H}$ (N copies) by

$$\mathcal{P}_k^N = \frac{1}{N} \sum_{i_1, i_2, ... i_N} \left(\sum_{j=1}^{N} \delta_{k,i_j} \right) |\psi_{i_1}\rangle \langle\psi_{i_1}| \otimes |\psi_{i_2}\rangle \langle\psi_{i_2}| \otimes ... \otimes |\psi_{i_N}\rangle \langle\psi_{i_N}| \tag{8}$$

The expression $\sum_{j=1}^{N} \delta_{k,i_j}$ gives the number of copies of $|\psi_k\rangle \langle\psi_k|$ in the tensor product $|\psi_{i_1}\rangle \langle\psi_{i_1}| \otimes ... \otimes |\psi_{i_N}\rangle \langle\psi_{i_N}|$. Now, extend $\mathcal{P}_k^N$ to $\mathcal{H}^{\infty}$ by putting $\mathcal{R}_k^N = \mathcal{P}_k^N \otimes \mathcal{I} \otimes \mathcal{I} \otimes ...$, where $\mathcal{I}$ is the unit operator on $\mathcal{H}$. Then the operator $\mathcal{R}_k^{\infty} = \lim_{N \to \infty} \mathcal{R}_k^N$ on $\mathcal{H}^{\infty}$ exists in the strong limit. Hartle (*ibid*) proved that *any* state $|\phi^{\infty}\rangle$ of the type (7) is an eigenstate of $\mathcal{R}_k^{\infty}$ and

$$\mathcal{R}_k^{\infty} |\phi^{\infty}\rangle = |\langle\psi_k|\phi\rangle|^2 |\phi^{\infty}\rangle \tag{9}$$

where the eigenvalue $|\langle\psi_k|\phi\rangle|^2$ is just the quantum mechanical probability of the result $|\psi_i\rangle$ in the state $|\phi\rangle$. Therefore, the limit of relative frequencies of each possible outcome of an $\mathcal{O}$ measurement on identical systems in the state $|\phi\rangle$ is correctly predicted without reference to Born's rule, or so it seems.

DeWitt argued that the interpretation of the quantum weights as probabilities simply follows (by the above theorem) from the non-probabilistic part of quantum mechanics without a collapse postulate. The argument goes as follows: Since the quantum measure of branches with frequencies that do not match the quantum probabilities is zero in the infinite limit, the quantum mechanical branches are in fact typical (have measure one), and the branches with frequencies that differ from the non-quantum probabilities are a-typical, and have zero probability to occur.

[6]See Hemmo and Pitowsky (2003) for the argument. Theories of this kind were proposed by Albert and Loewer (1988) and Deutsch (1985), and recently by Lewis (2004).

However, as noted by many authors (Deutsch 1985, Albert and Loewer 1988, Kent 1990, Butterfield 1996, Barrett 1999, Ch. 6, Hemmo and Pitowsky 2003, and others), DeWitt's argument is circular. The relative frequency theorem entails only that branches with quantum mechanical frequencies have *quantum* measure one. The sequence $\{\mathcal{R}_i^N\}$ converges to $\mathcal{R}_i^\infty$ in the strong limit which is induced by the Hilbert space inner product. Born's rule is thus implicitly assumed when we identify the eigenvalues of $\mathcal{R}_i^\infty$ with the limiting frequency of *typical* sequences.[7] DeWitt's argument is analogous to the attempt to apply the classical strong law of large numbers to define probability as the limit of relative frequencies. In this case too the set of badly behaving infinite sequences has measure zero, and the circularity stems from the identification of measure zero with probability zero. Similarly, in classical statistical mechanics the effort to define probability in terms of limiting relative frequencies suffers the same fate. Here the tool that is supposed to accomplish the task is the pointwise ergodic theorem.

Two additional remarks are relevant here. First, the quantum relative frequency theorem applies only to *actual* infinities, i. e. there is no analogue of the result in (9) for finite sequences. Note also that for infinite systems the state $|\phi^\infty\rangle$ of the type (7) is an eigenstate of the observer reporting the Born frequencies (see Albert 1991). But, again, this is true only in the infinite limit, since for finite sequences all the low weight branches exist. Moreover, for the same reason, there is no sense in which one can rely on the *approach* to the infinite limit without begging the question. Second, as we briefly argued above, postulating a probability measure over the branches seems to be unintelligible in the Everett-DeWitt approach: since all the branches (with nonzero amplitudes) are realized no matter what their probability, what could be the probability about? We believe that in the many worlds framework the problem can be solved only by adding some chance process to the theory. We shall come back to this point in the Conclusion.

4 The Decision-theoretic approach

We now turn to the decision-theoretic approach in the many worlds theory originated by Deutsch (1999) and advanced by Wallace (2003a, 2005), Saunders (2004), Greaves (2004) and others. Deutsch (1999) claims to derive the Born probabilities from what he takes to be the non-probabilistic axioms of quantum theory and the non-probabilistic axioms of classical decision theory.[8] Deutsch's essential idea in deriving the Born rule is that his theorem is supposed to rely only on the quantum mechanical Hilbert space structure or else on *a priori* (and so completely innocuous) principles of rationality. Therefore Deutsch interprets his theorem to mean that *rational* agents (in the sense defined by *the non*-probabilistic axioms of decision theory) who also believe quantum mechanics stripped of the Born rule are constrained to adopt the Born rule as their

[7] In fact, even DeWitt (1970) himself seems to have noticed this circularity.
[8] For classical decision theory, see e. g. Savage 1972, Fishburn 1981, Joyce 1999.

unique probability function for action. This has been taken to mean that if the many worlds theory is true (which is supposed to be just quantum mechanics stripped of collapse and the Born rule), then a rational agent will necessarily make all decisions that depend on predicting the outcomes of quantum measurements *as if* (she believd) the standard collapse postulate were true (even though in fact it is flase, since the branch structure is completely governed by determistic dynamics). Following Deutsch (1999), Wallace (2003a, 2005), Saunders (2004) and others claim that the entire probabilistic interpretation of standard quantum mechanics with collapse is derivable in the many worlds theory, and consequently that the probability problem is solved.

Deutsch's approach belongs to the subjectivist tradition in the philosophy of probability developed by Ramsey (1931), Savage (1972), de Finetti (1974) and others who tried to define probabilities on the basis of preferences ordering of rational agents. However, his theorem is much stronger than the familiar results of classical decision theory which do not lead to a *unique* probability function over a given set of preferences. Obviously, if Deutsch's approach to the quantum probabilies were successful, it would have given strong support to the subjective approaches to probability in general. However, we do not think that Deutsch's theorem solves the probability problem in the *many worlds* theory. To see why, let us backtrack by sketching very briefly Deutsch's proof (for details and variations on the proof, see Wallace 2002, 2003a). We postpone criticism to the next section.

The set up is a decision theoretic one in a quantum mechanical context. We consider *quantum* games in which an agent receives a payoff depending on the outcome of a quantum mechanical measurement of an observable $\mathcal{O}$ on a system in some arbitrary state $|\phi\rangle$. The *non*-probabilistic axioms of quantum theory are given by the usual Hilbert space structure of a quantum system. The only link to experiment is made by assuming, as usual, that a measurement of an observable on a system in one of its eigenstates yields with certainty (probability one) the corresponding eigenvalue.

A quantum game is defined as an ordered triple $G =:< |\phi\rangle, \mathcal{O}, \Pi >$ where $|\phi\rangle$ is the pre-measurement (pure) state of the system, $\mathcal{O}$ is the observable measured, and Π is some payoff function from the set of measurement outcomes to the reals. Let $V(< |\phi\rangle, \mathcal{O}, \Pi >)$ be the agent's expected payoff, which is a function from the set of games to the reals. The intended interpretation of V is that a rational agent prefers game G_1 to game G_2 just in case $V_1(G_1) > V(G_2)$ (so that completeness and transitivity of preferences is assumed). Assuming that V satisfies some further (arguably, non-probabilistic) decision theoretic axioms (weak-additivity, substitutivity and the zero sum-rule; see Deutsch 1999, Wallace 2002, 2003a), Deutsch claims to derive the following analogue of the Born rule:

$$V(|\phi\rangle, \mathcal{O}, \Pi) = \sum_i |\langle\psi_i|\phi\rangle|^2 \, x_i \tag{10}$$

where the $|\psi_i\rangle$'s are the eigenstates of $\mathcal{O}$, and x_i is the payoffs the agent receives in case the outcome of the measurement $\mathcal{O}$ is $|\psi_i\rangle$. That is, the agent's probability function is obtained by calculating the expectation values over a set of outcomes (branches) whose probabilities are given uniquely by the Born rule. As phrased by Deutsch (1999), "[A] rational decision maker behaves as if he believed that each possible outcome x_i had a probability, given by the conventional formula $|\langle\psi_i|\phi\rangle|^2$, and as if he were maximizing the probabilistic expectation value of the payoff."

We shall only indicate the central steps in Deutsch's proof of (10) which will be relevant to our discussion. Consider first a two component superposition with *equal* amplitudes of the first two eigenstates of $\mathcal{O}$.

$$|\phi\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle). \tag{11}$$

Suppose that our agent's value function V is additive. This means that the agent is indifferent between receiving two separate payoffs x_1 and x_2 or a single payoff of $x_1 + x_2$. Deutsch then proceeds to argue that for the state $|\phi\rangle$ in (11)

$$V(|\phi\rangle, \mathcal{O}, \Pi) = \frac{1}{2}x_1 + \frac{1}{2}x_2 \tag{12}$$

which is just a special case of (10). This is then generalized to equal-amplitude superpositions of n eigenstates of $\mathcal{O}$, for an arbitrary integer n.

The argument for (10) in case of *unequal* amplitudes is reduced to the case of equal amplitudes as follows. Consider, for example, a game that is instantiated by measuring $\mathcal{O}$ on a system in state

$$|\phi\rangle = \frac{1}{\sqrt{3}}|\psi_1\rangle + \sqrt{\frac{2}{3}}|\psi_2\rangle. \tag{13}$$

Suppose that our system is coupled to another, second system so their combined state is given by

$$|\Phi\rangle = \frac{1}{\sqrt{3}}\left(|\psi_1\rangle|\chi_1\rangle + |\psi_2\rangle|\chi_2\rangle + |\psi_2\rangle|\chi_3\rangle\right) \tag{14}$$

where the $|\chi_j\rangle$'s are three eigenstates of some observable $\mathcal{P}$ of the second system corresponding to three distinct eigenvalues. Then we can indirectly measure $\mathcal{O}$ by measuring $\mathcal{P}$ on the second system. The outcome of the $\mathcal{O}$-measurement will be $|\psi_1\rangle$ iff the $\mathcal{P}$-measurement yields $|\chi_1\rangle$, otherwise the outcome is $|\psi_2\rangle$. Now, let y_j be the payoffs that the agent gets when $\mathcal{I} \otimes \mathcal{P}$ is measured on the second system and the outcome of the $\mathcal{P}$-measurement is $|\chi_j\rangle$. Suppose that the payoffs are chosen in such a way that $y_1 = 0$ and $y_2 + y_3 = 0$. Then the agent is indifferent between the following two games

I. The original game in which $\mathcal{O}$ is measured directly on the first system in the state $|\phi\rangle$ and

II. A game in which $\mathcal{O} \otimes \mathcal{I}$ is measured on the combined system in the state $|\Phi\rangle$, the gains and losses are distributed, and then *subsequently* $\mathcal{I} \otimes \mathcal{P}$ is measured on the second system.

However, Deutsch claims that because of additivity the latter game is identical in expected value to a single game on the composite system in the state $|\Phi\rangle$ in which the observable $\mathcal{O}\otimes\mathcal{I}+\mathcal{I}\otimes\mathcal{P}$ is measured. Using the formula for the expected payoff of games with equal amplitudes we get altogether

$$V(|\phi\rangle,\mathcal{O},\Pi)=V\left(\Phi,\mathcal{O}\otimes\mathcal{I}+\mathcal{I}\otimes\mathcal{P},\Pi\right)= \tag{15}$$
$$\frac{1}{3}(x_1+y_1)+\frac{1}{3}(x_2+y_2)+\frac{1}{3}(x_2+y_3)=\frac{1}{3}x_1+\frac{2}{3}x_2$$

where x_i are the payoffs of the original game. Therefore, the value of the game is again determined uniquely by the Born probabilities, as required. The proof of (10) is then generalized for superpositions with arbitrary complex amplitudes and for mixed states.

5 Whence probabilities?

We now proceed to argue that Deutsch's theorem does not solve the probability problem in the many worlds theory (and does not establish the self interpretation claim, i. e. that a separate probability axiom is not needed in quantum mechanics). Several authors (Barnum et al. 2000, Gill 2003, Lewis 2003) argued that Deutsch's theorem is *non sequitur* at various points of the proof since it requires accepting some nontrivial assumptions that go beyond decision theory. Wallace (2003a, 2005) and Saunders (2004, 2005) defend Deutsch's approach. We summarize below the arguments in the literature, and focus on what we think are the main problems in the derivation.

1. Deutsch assumes that classical decision theory is applicable to the many worlds theory. This is of course not obvious since the latter is a deterministic theory in which the (pure) quantum state of a closed system is supposed to give a complete description of it. On the other hand, decision theory is a theory of rational action in the face of uncertainty. It is not clear exactly what is the nature of the uncertainty faced by an agent in the many worlds universe. For example, in a single game $G =:< |\phi\rangle,\mathcal{O},\Pi >$, with $|\phi\rangle$ given by (11), the agent knows the quantum state of the system and therefore if she accepts the many worlds theory she knows with complete certainty that both outcomes will occur. Hence, she has no prior notion of probability (nor does she have grounds to assign likelihood ordering on the set of outcomes). By contrast, all the fundamental approaches to classical decision theory presuppose a prior notion of uncertainty, which is either directly represented in terms of probability, or else is represented as a likelihood order that *a posteriori* gives rise to probability.[9] So, it seems that a notion of uncertainty which is required in order to apply decision theory is missing, and without it Deutsch's proof cannot get off the ground.

[9]The various representation theorems in decision theory describe the agent's decisions between games as maximizing expected utility, where the probabilities of the outcomes in each game are known to the agent in advance. Savage (1972) derives the (subjective) probabilities, assuming that the agent has a preference order on the set of possible acts.

Nevertheless, it has been argued (Saunders 1998, Vaidman 1998, Wallace 2003a) that in the many worlds theory some sort of subjective uncertainty is compatible with the total knowledge of the quantum state and its deterministic dynamics. In the game G above, for example, although the agent knows that she will split into two copies after the measurement, she doesn't know which copy she will become (and expects to become only *one* of them; see Saunders 1998 for the argument) since before the measurement there is no fact of the matter as to which branch of the post-measurement state she will track. We do not think, however, that this in itself is enough to generate uncertainty in the sense relevant to probability. If the agent believes that the many worlds theory is true, and she knows that the quantum state in the game G is $|\phi\rangle$ in (11), then she knows with certainty that she will split into two copies, and there is no further fact of the matter about which she is ignorant (see Greaves 2004 for expanding on this). However, it has been proposed (Greaves 2004) that decision theory might be applicable in the many worlds theory even if the world splitting involves no uncertainty. This view is based on Parfit's account of caring for the future according to which in splitting situations the agent ought to care for *all* her future copies. The claim is that this is just enough in order to apply Deutsch's theorem to show that a rational agent cares about her future copies in proportion to their amplitude squared measure. Greaves (2004) argues that this shows that in the many worlds theory her caring measure plays the same role as genuine chance in stochastic theories. We think that this is wrong for precisely the same reasons argued below.

2. Deutsch claims to derive some of his conclusions from the additivity axiom, but he implicitly assumes stronger axioms. For example, take the derivation of the expected utility rule (10) in the case of unequal amplitudes. Deutsch identifies a game in which the observable $\mathcal{O}$ is measured on a single system in state $|\phi\rangle$ in (13), with one in which $\mathcal{O}\otimes\mathcal{I}$ and $\mathcal{I}\otimes\mathcal{P}$ are measured one after the other on a combined system in state $|\Phi\rangle$ in (14). However, additivity alone cannot deliver the goods here for one has to postulate also some *non-contextuality* axiom. For example, an axiom that states the following: *If the payoff for the outcomes* $|\psi_i\rangle$ *in the game* $G_1 =< |\phi\rangle, \mathcal{O}, \Pi >$*, and the payoff for the outcomes* $|\psi_i\rangle|\chi\rangle$ *in the game* $G_2 =< |\phi\rangle|\chi\rangle, \mathcal{O}\otimes\mathcal{I}, \Pi^{'} >$ *are the same, then* G_1*and* G_2 *are equivalent, no matter what the state* $|\chi\rangle$ *is.* This certainly does not follow from additivity which says nothing about joint measurements on composite systems, nor about whether or not the *expected* payoff of one game might be changed or unchanged by playing it *jointly* with another game on another system. The agent may think that the chances of getting $|\psi_i\rangle$ may vary between the two contexts.

3. In the case of equal amplitudes Deutsch is also implicitly using an axiom which is stronger than additivity. Consider two games $G_1 =< |\phi\rangle, \mathcal{O}, \Pi >$ in which the payoffs are x_i for the outcome $|\psi_i\rangle$, and a second game $G_2 =< |\phi\rangle, \mathcal{O}, \Pi^* >$ with the same quantum state and measurement but in which for the outcome $|\psi_i\rangle$, the payoff is $x_i + k$, where k is a constant. (As before $|\psi_i\rangle$ are the eigenstates of $\mathcal{O}$.) Deutsch maintains that additivity implies

$$V(G_2) = V(G_1) + k \tag{16}$$

However, additivity alone will not do the job. All it does in this case is to constrain the agent to be indifferent between receiving a single payoff $x_i + k$ and receiving a sure payoff k followed by x_i. The relation (16) on the other hand implies more, namely that the expected payoff of the game G_1 is unchanged by playing it jointly with a game with a sure constant payoff k. As it turns out, there are infinitely many different "expected payoff" formulas which do not reassemble (10), but are consistent with additivity. None of Deutsch's equivalences (for both equal and unequal amplitudes) are satisfied for these rules. For example, for games with arbitrary amplitudes take the expected payoff of a game to be the arithmetic mean of the payoffs with nonzero amplitudes; and for equal amplitude-games take the arithmetic sum of the payoffs (see Barnum et al. 2000, Lewis 2003).[10]

4. As can be seen from the above it is implicit in Deutsch's definition of a quantum game that the same triple $< |\psi\rangle, \mathcal{O}, \Pi >$ may represent different games, which are then assigned the same value. In order to justify this, Wallace (2003a) introduced an explicit condition called *Measurement Neutrality* which says that the expected payoff V cannot differ between two games that agree on the triple $< |\psi\rangle, \mathcal{O}, \Pi >$ but disagree on *how* $\mathcal{O}$ is measured. In a later version Wallace (2005) relies on a weaker set of decision theoretic axioms but adds to them an extra constraint, called *Equivalence*, which says that the agent should assign equal likelihoods to the outcomes of any two measurements that have equal quantum probability.[11] He then proceeds to argue that in the many worlds universe this is a principle of *rationality*.

5. Deutsch's decision theoretic program stands or falls depending on whether either of the above assumptions could be justified *independently* of the quantum probabilities (for such attempts, see Wallace 2003a, 2005 and Saunders 2004, 2005). We do not think, however, that this can be done. Our argument consists of two parts: Firstly, both *Measurement Neutrality* and *Equivalence* are inherently probabilistic assumptions of the kind that can only be justified empirically. Secondly, among the many worlds there is a multitude of branches in which observers (who update their probabilities in a Bayesian way) are utterly *ir*rational if they believe in either axiom. These branches, however, cannot be rendered a-typical. Hence neither axiom is a rationality principle.

Let us spell out the argument. At the heart of both axioms there is a statement about *identity of events* (that is, measurement outcomes). *Measurement*

[10]Note that alternative probability rules have to be consistent with all the axioms of decision theory and not only with additivity, and with the structure of Hilbert space (including various richness axioms). By Gleason's (1957) theorem all such alternative probability rules are *contextual* (see below). The point, however, is that they exist.

[11]These conditions imply that the agent is indifferent between two games that agree on $< |\psi\rangle, \mathcal{O}, \Pi >$ but differ in the branch structure induced by the measurements. As a general rule, branching indifference does not seem plausible in the many worlds theory. See Lewis (2003) for the argument and Wallace (2005) for a counter-argument.

Neutrality and *Equivalence* each implies that the events to which subjective probability is assigned should ultimately be identified with (closed) subspaces of a Hilbert space. Once this is established Born's rule follows from Gleason's theorem (Gleason 1957) and the result becomes quite obvious from a mathematical point of view. The reason why Wallace's derivation seems to take a longer route is that the identity assumption is dressed in a decision theoretic clothes.[12] Consider, for example, *Equivalence* which states:

If E and F are events and $\mathcal{W}_M(E) = \mathcal{W}_N(F)$, then $E|M \simeq F|N$.

Here M and N are measurements, E is an event in the Boolean algebra generated by the possible outcomes of M, and F an event in the algebra generated by the possible outcomes of N. The relation $E|M \simeq F|N$ means that the event E in the M measurement is (subjectively) equally likely as the event F in the N measurement. $\mathcal{W}_M(E)$ is the quantum mechanical probability associated with E, and similarly for $\mathcal{W}_N(F)$. So *Equivalence* is the statement that identity of quantum probabilities implies equality in the subjective likelihood order. Now, Wallace claims that in the many worlds universe this is a principle of rationality.

The quantity $\mathcal{W}_M(E)$ is defined only when E is a subspace of the Hilbert space, so in this sense the identity is assumed by *Equivalence*. Now, as to the non-contextuality, if E is an event in the algebra of two distinct measurements M and N, then by *Equivalence* $\mathcal{W}_M(E) = \mathcal{W}_N(E)$, since quantum mechanics assigns non-contextual probabilities. Hence, *Equivalence* implies the non-contextuality of the subjective likelihood order, and the events "outcome E in the M measurement" and "outcome E in the N measurement" are recognized by the agent as having the same likelihood. All that is left to do is to prove that the likelihood order relation can be numerically represented. (In fact, *any* likelihood relation defined on pairs $E|M$, which is non-contextual and numerically representable satisfies Born's rule. This is just Gleason's theorem). To argue that Equivalence is a rationality principle, Wallace compares two quantum states (representing two games) in each of which the amplitudes are all equal, but the payoff in each is associated with a different component of the state. He then argues that a rational agent (i.e. who only cares about the payoff) won't prefer one of the games over the other, exactly as implied by Equivalence. This, however, presupposes that the agent's likelihood ordering is non-contextual, as we just argued. The agent might prefer one of the games, if she happens to believe that she has better chances to win it, no matter what the amplitudes in each game are, as long as her preference ordering is contextual. Moreover, this is perfectly consistent with the agent's being rational in the sense that she only cares about the payoff.

So the crucial question before us now is this: Can the non-contextuality of probabilities, in one or another formulation, be a principle of rationality? The easiest way to see why not is to consider two classical "experiments" a toss of a coin and a throw of a die. A priori there are two logical possibilities.

[12] Wallace's (2002, 2003a) proof does not make use of Gleason's theorem (as a lemma), but it assumes much stronger assumptions than Gleason's theorem.

1. The limit of relative frequency of "heads" in a sequence of coin tosses remains the same (say, converges to 0.5) whether the die is thrown or not. (Note, the issue here is not whether the outcomes of the coin toss and die throw are independent or correlated, either case is possible).

2. The limit of relative frequency of "heads" in the sequence of coin tosses converges to 0.5 when the die in not thrown. However, when the die is cast, the limiting frequency of "heads" goes down to 0.3.

The fact that case **2** does not occur in our world is contingent and not *a priori*. The non-contextuality of the probability of "heads" allows us to formalize the joint experiment of coin toss and die cast on the product space. Otherwise, we would have to distinguish between two different mass events, "coin toss when die is not cast" on the one hand, and "coin toss when die is cast" on the other. In the product space these events are taken as one, which means that *identities of events in a probability space represent empirical knowledge*. Or, to put it differently, *identities encode information that is part of the prior, and not part of the rules of rationality*. If we were in a position of total ignorance, with no empirical knowledge of coins and dice, we might have rationally treated the two events "coin toss when die is not cast" and "coin toss when die is cast" as distinct, and assign them different prior probabilities.

Ironically, this point becomes even more acute in the many worlds universe, simply because all the possibilities are real. In particular, there are branches of the universe in which cases like **2** do occur. To give an example, it is enough to replace the classical coin and die by quantum measurements. Consider pairs of particles in the state

$$|\Psi\rangle = \frac{1}{2}|+\rangle_1|+\rangle_2 + \frac{1}{2}|+\rangle_1|-\rangle_2 + \frac{1}{2}|-\rangle_1|+\rangle_2 + \frac{1}{2}|-\rangle_1|-\rangle_2 \tag{17}$$

Now perform a sequence of measurements on n such pairs. On the "left" particle of each pair measure the z-component of the spin. On the "right" particle measure the z-component of the spin of the first, third, fifth,... pair, and do nothing on the second, fourth, sixth, ... pair. Let $\Phi^L(s)$, $s = \pm 1$ be the record state of the left hand side apparatus, $\Phi^R(s)$ the record state of the right hand side apparatus, and let $\Phi^R(0)$ be the state of the right hand side apparatus when it is not operated. After n measurements the combined state of the pairs and the measurement records is (without collapse):

$$|\Psi^n\rangle = \sum_{s_1,t_1,s_2,t_2,\ldots s_n,t_n} \frac{1}{2^n}|s_1\rangle_1|t_1\rangle_2\Phi^L(s_1)\Phi^R(t_1) \otimes |s_2\rangle_1\,|t_2\rangle_2\,\Phi^L(s_2)\Phi^R(0) \otimes \ldots \tag{18}$$

Here s_i, t_j range over all choices of $+1$ or -1, and they stand for the z-component of the spin of the left and right particles respectively. There are sequences $s_1, s_2, \ldots s_n$ of possible outcomes on the left side, such that the relative frequency of $+1$ in the subsequence $s_1, s_3, s_5, \ldots$ with odd indices is quite different from the relative frequency of $+1$ in the subsequence $s_2, s_4, s_6, \ldots$ with even indices. An agent inhabiting this branch is likely to conclude that the act of measuring the z-component of spin of the right particle changes the probability of the outcome

+1 of the left hand side measurement. If n is large enough the agent will be utterly irrational if she chooses to ignore this evidence. There are also more branches (counting-wise) on which Born's rule fails.

But how can a principle that *necessarily* contradicts the experience of some observers be a mark of rational choice? We stress once again that our argument does not depend on the counting measure (and on the Everettian assumption that the number of branches is equal to the number of all possible outcomes). All we need are the two premises (from Section 3): (i) For any sequence of measurements there is at least one branch in which Born's rule fails (or worse, a branch in which non contextuality of probability fails); (ii) The many worlds theory by itself does not provide a probability measure over branches. Without such measure, branches in which Born's rule fails cannot be considered a-typical or improbable (in the sense of having diminishing probability). Note how the many worlds theory differs on this issue from the standard collapse theory. In the many worlds universe branches with frequencies that match contextual probabilities *must* occur with certainty, while in the collapse theory they may occur with diminishing probability.

One might argue that "deviant" branches (in which the relative frequencies do not match the Born probabilities) are not relevant to *our* decision making since it is highly improbable that our branch is a deviant one. Or, in decision theoretic terms, it is highly irrational to believe that our branch is a deviant one (see Wallace 2002, Sec. 8 & 9). But how can we assume that an agent knows this without begging the question? Of course, citing Deutsch's theorem to justify this belief would be flatly circular since this is exactly what the theorem is meant to establish.[13] But one might argue that we do not need Deutsch's theorem for this. The fact that quantum amplitude determines probability (or some similar constraint such as *Measurement Neutrality* or *Equivalence*) can be read off the statistics we observe in our branch. And what Deutsch's theorem shows is that this is all we need in order to derive the Born probabilities (if we are rational in the sense of decision theory). In other words, the argument might go, the empirical findings in our branch lead us, by updating our probabilities in a Bayesian way and applying decision theory, to *discover* as a matter of fact the Born probabilities.

However, this is a far cry from solving the original probability problem for the following reason. What we wanted is an argument to the effect that branches with relative frequencies that match the quantum probabilities are typical. In decision theoretic terms, this just means that it is rational for an agent to adopt the Born probabilities unconditionally on some choice of a branch. But since the above argument relies heavily on the empirical data accessible in some branches, what we get from Deutsch's theorem is the much weaker result that on a branch in which the relative frequencies conform to Born's rule it is rational to adopt Born's rule.

But even this is not entirely true. Let us go slowly to see what exactly

[13] Note that on pain of circularity Wallace's (2002) Everettian Principal Principle cannot help here either, since it presupposes Deutsch's theorem.

this means. In decision theory applied to the context of a splitting universe we have to be careful about how to understand expressions like “our branch”, “branches like ours”, etc. We know (by empirical reasoning) that our branch belongs to a subset of branches with finite segments in which the frequencies seem to approximately depend on the quantum amplitude. But we cannot infer from our empirical data that our branch belongs to a subset of branches in which the future segments have the same property, *nor even that this is highly probable*.[14] In fact, and this is the crucial point, such an inference would be flatly *inconsistent* with the splitting picture itself.[15] So, as it stands, Deutsch's theorem shows that it is rational for an agent to adopt the Born probabilities only in *some* future segments of the branch we happen to track. But this comes down to a triviality, since in the many worlds theory we know anyway (and with certainty) that the frequencies match the Born probabilities in some future segments of our branch (and in other branches match other “rules”, whichever they are). The only way to avoid this triviality is by injecting into Deutsch's theorem nothing shorter of an assumption to the effect that the Born probabilities are typical of the *total* set of branches. In decision theoretic terms this is just equivalent to assuming that it is rational for us to believe that our branch is not a deviant one (as discussed above). But in the context of deriving the quantum probabilities from non-probabilistic axioms this would be circular.

Bayesian approaches to quantum probability have become popular lately also in standard quantum theory with collapse. Such approaches are consistent to the extent that they *assume* that the lattice of subspaces of the Hilbert space is the event space, and two events which are identical in the lattice have equal probabilities.[16] Such assumptions are common place also in the classical Bayesian approach. An example is furnished by modeling the joint experiment of coin toss and die throw on the product space. We have previously noted that this model implicitly assumes the non-contextuality of the probabilities. One can justify this assumption in the same way that one justifies any element of the prior distribution: as an empirical conjecture. Subsequently one proceeds to perform the usual Bayesian updating and statistical tests to see the extent to which the non-contextuality assumption is justified. In most classical cases it is, and counterexamples are highly contrived. In quantum mechanics with collapse the situation is even better since the event structure determines the probability rule (Gleason's theorem), so the agent usually needs no tiresome updating. He or she are then justified to the extent that quantum mechanics is empirically adequate. By contrast, in the many worlds picture, there are agents that will learn the hard way that Born's rule is simply wrong.

[14] Obviously we have no epistemic access to such facts without some probabilistic input (see below), and so expressions like “our branch” cannot mean “the full (past and future) history of our branch” for purposes of decision making. We exclude here information that requires time travel and similar science fiction stories.

[15] Note that this is much stronger than the usual problem in standard inductive inferences that the future is underdetermined by the past.

[16] See Pitowsky (2005). At least one has to derive the Hilbertian structure from other empirical assumptions (see Hardy 2005). For Bayesian approaches to quantum theory, see Pitowsky (2003) and references therein.

6 Conclusion

We have made a full circle. In the many worlds theory the Born probability rule *cannot* be derived from the non-probabilistic part of quantum mechanics, with or without (the non probabilistic part of) rational decision theory.[17] And the reason for this, as we just saw, boils down to the familiar problem we began with, namely that the branching is unrelated to the quantum mechanical probability. But, the problem goes even deeper. Not only is it the case that the Born probability cannot be derived in the many worlds theory, but also it is *unintelligible* to postulate as an additional empirical hypothesis any probability rule of the form: the probability of an observer to find herself on a post measurement branch is equal to the square of the amplitude of that branch in the universal quantum state.[18]

To see why, let us backtrack a bit and consider what it would mean to identify the quantum measure with probability in the many worlds theory as above. No matter how we understand probability (i. e. as measuring degrees of belief, limiting relative frequency, chance), if probability is supposed to do its job, it must be related at least *a-posteriori* to the statistical pattern in which events occur in our world in such a way that the relative frequencies that actually occur in our world turn out to be *typical.* We take this as a necessary condition on whatever it is that plays the role of probability in our physical theory. Now, the quantum probability rule cannot satisfy this condition in the many worlds theory (nor can any other non-trivial probability rule), since in this theory the dynamics logically *entails* that *any* combinatorially possible sequence of outcomes *occurs* with complete certainty, regardless of its quantum probability. As we argued in the previous sections, nothing in the unitary dynamics of the quantum state picks out the Born probability as related in such a way to the frequencies that occur in the worlds, and there are no other features of the many worlds theory which might pick out the Born probability in this sense as opposed to any other 'probability' rules. Even for agents like us, who observed up to now finite sequences which a-posteriori seem to conform to the quantum probability, adopting the quantum probability as our subjective probability for future action is completely arbitrary, since there are future copies of us who are bound to observe frequencies that don't match the quantum probabilities. We know now and with certainty that for some of our future copies the quantum probability rule will turn out to be false. So, if we believe that the many worlds theory is true, it will be utterly irrational for us to adopt the quantum probability rule as our subjective probability for future action (nor any other non-trivial probability rule). Viewed in this way, it is not even clear that rational decision theory is applicable in the many worlds theory.

[17] As we said above, we believe that in general no probability rule that describes the statistical pattern in which physical events occur in the world may be derived in any physical theory with pure *deterministic* dynamics for essentially the same reasons spelled out above. But this goes beyond the scope of this paper.

[18] This conclusion holds regardless of whether or not splitting is taken to involve subjective uncertainty in the sense relevant to probability (see Saunders 1998, Vaidman 1998, Wallace 2005 and Greaves 2004 for variations on this issue).

For these reasons we believe that in quantum mechanics the only way in which the subjective probabilities of observers could be guided by the quantum mechanical probabilities is by adding to the dynamics in the theory a genuine stochastic process. This can be done either by injecting chances into the dynamics (as in collapse theories) or by adding to the theory some stochastic (chance) process over and above the unitary dynamics of the state. However, as to the latter possibility, we have argued in our (2003) that it seems inevitable that adding a stochastic dynamics for the worlds or for the minds over and above the dynamics of the quantum state (e. g. of the kind proposed by Albert and Loewer (1988) in their many minds theory) would turn the many worlds theory into a hidden variables theory of a sort, in which subsets of branches (or minds) have quantitative properties that transcend, and cannot be inferred from quantum theory itself.

Acknowledgement We thank the Israel Science Foundation (grants numbers 879/02 and 240/06) for supporting this research.

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