Lecture Note 5

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 \cdots + \beta_k X_k + U$$

$$= \beta_0 + \sum_{k=1}^{K} \beta_k X_k + U$$

$$= \underset{\begin{bmatrix} 1 \\ X_1 \\ X_2 \\ \vdots \\ X_k \end{bmatrix}}{X'} \underset{\begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_k \end{bmatrix}}{\beta} + U$$

$$Y = \alpha_0 + \alpha_1 X_1 + U$$

$\updownarrow$ see OVB formula in Lecture Note 5

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + U$$

$$\begin{tikzcd}
X_1 \arrow[r] & Y \\
X_2 \arrow[u] \arrow[ur] & U \arrow[u]
\end{tikzcd}$$

confounder

$$\begin{tikzcd}
X_1 \arrow[r] \arrow[d] & Y \\
X_2 \arrow[ur] & U \arrow[u]
\end{tikzcd}$$

mediator

Linear combination of coefficients

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + U$$

$$a\hat{\beta}_1 + b\hat{\beta}_2 \leftrightarrow a\beta_1 + b\beta_2$$

$$V[a\hat{\beta}_1 + b\hat{\beta}_2] = a^2 V[\hat{\beta}_1] + b^2 V[\hat{\beta}_2] + 2ab\,\mathrm{cov}(\hat{\beta}_1, \hat{\beta}_2)$$

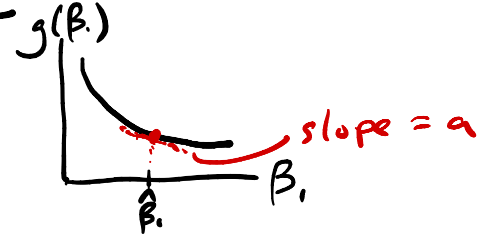
$$= a^2 \underset{\substack{\uparrow \\ SE[\hat{\beta}_1]^2}}{\sigma^2_{\hat{\beta}_1}} + b^2 \underset{\substack{\uparrow \\ SE[\hat{\beta}_2]^2}}{\sigma^2_{\hat{\beta}_2}} + 2ab\,\sigma_{\hat{\beta}_1, \hat{\beta}_2}$$

Example: null $\beta_1 = \beta_2 \Rightarrow \beta_1 - \beta_2 = 0 \Rightarrow a = 1, \; b = -1$

Nonlinear functions of coefficients

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + U$$

$g(\beta_1, \beta_2) = \dfrac{\beta_1}{\beta_2}$ $\qquad$ $g(\beta_1) = \dfrac{1}{\beta_1}$



Delta method: 1st-order Taylor approximation

$$V[a\hat{\beta}_1 + b\hat{\beta}_2] = \overset{\partial g/\partial \beta_1}{a^2} V[\hat{\beta}_1] + \overset{\partial g/\partial \beta_2}{b^2} V[\hat{\beta}_2] + 2ab\,\mathrm{cov}(\hat{\beta}_1, \hat{\beta}_2)$$

marginaleffects::hypotheses()

$$Y = \beta_0 + \beta_1 \text{white} + u$$

$$\frac{\mu_w}{\mu_{NW}} = \frac{\beta_0 + \beta_1}{\beta_0} = g(\beta_0, \beta_1)$$

$$\partial g / \partial \beta_0 = -\frac{\beta_1}{\beta_0} \qquad \partial g / \partial \beta_1 = \frac{1}{\beta_0}$$