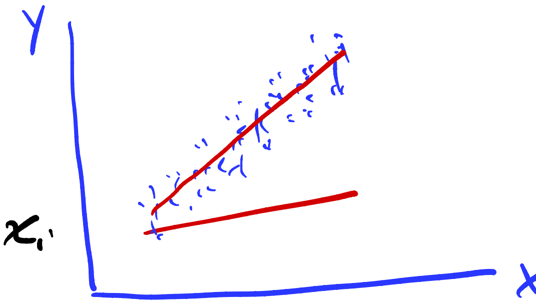
Lecture Note 6: Maximum Likelihood

Least squares vs. MLE



Random var. X with pdf $f(x;\theta)$

Sample X_i iid: $\{X_i\}_{i=1}^{N} \to$ each i has x_i

$$\max_{\tilde{\theta}} L = \max_{\tilde{\theta}} \prod_{i=1}^{N} f(x_i;\tilde{\theta})$$

$$\max_{\tilde{\theta}} \ln L = \max_{\tilde{\theta}} \sum_{i=1}^{N} \ln\left[f(x_i;\tilde{\theta})\right]$$

Solution: $\hat{\theta}$ satisfies $\frac{\partial \ln L}{\partial \hat{\theta}} = 0$

Properties: ① Consistency: $\hat{\theta} \xrightarrow{p} \theta$

② Asymptotic normality (CLT): $\hat{\theta} \xrightarrow{d} N(\theta, \Sigma)$

③ Asymptotic efficiency

Bernoulli $X_i = \begin{cases} 1 & \text{w.p. } p \\ 0 & \text{w.p. } 1-p \end{cases}$

Sample $N=3$, iid, $(1, 1, 0)$

$$L = Pr[X_1=1] \cdot Pr[X_2=1] \cdot Pr[X_3=0]$$

$$= p \cdot p \cdot (1-p)$$

$$= p^2(1-p)$$

MLE: $\max_{\tilde{p}} L = \max_{\tilde{p}} \tilde{p}^2(1-\tilde{p})$

$$\max_{\tilde{p}} \ln L = \max_{\tilde{p}} 2\ln(\tilde{p}) + \ln(1-\tilde{p})$$

FOC: $\frac{d \ln L}{d\hat{p}} = \frac{2}{\hat{p}} - \frac{1}{1-\hat{p}} = 0 \quad \Rightarrow \hat{p} = \frac{2}{3}$

N Bernoulli variables, S successes, $N-S$ failures

$$L = p^S (1-p)^{N-S}$$

$$\ln L = S \ln(p) + (N-S) \ln(1-p)$$

$$\hat{p} = \frac{S}{N}$$

Methods for finding MLE

1. Analytic optimization
2. (Undirected) grid search
3. (Directed) grid search