# CS473 Assignment 2

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## 1

a) The functions $f(n)$ and $g(n)$ are non negative. There exists $n_0$ such that $f(n) \geq 0$ and $g(n) \geq 0$ for all $n \geq n_0$.
Thus, we have that for all $n \geq n_0$,

$$f(n) + g(n) \geq f(n) \geq 0 \quad and \quad f(n) + g(n) \geq g(n) \geq 0$$

Adding both inequalities, since the functions are non negative, we get $f(n) + g(n) \geq max(f(n), g(n))$ for all $n \geq n_0$.
This proves that,

$$max(f(n), g(n)) \leq c(f(n) + g(n)) \quad for \quad all \quad n \geq n_0 \quad with \quad c = 1$$

In other words, $max(f(n), g(n)) = O(f(n) + g(n))$.
Similarly, we can see that

$$max(f(n), g(n)) \geq f(n) \quad and \quad max(f(n), g(n)) \geq g(n) \quad for \quad all \quad n \geq n_0$$

Adding these two inequalities, we can see that,

$$2max(f(n), g(n)) \geq (g(n) + f(n))$$

or

$$max(f(n), g(n)) \geq 1/2(g(n) + f(n)) \quad for \quad all \quad n \geq n_0.$$

Thus $max(f(n), g(n)) = \Omega(g(n) + f(n))$ with constant $c = 1/2$.

c1) Given $\exists c > 0$ such that $f(n) \leq cg(n)$ for sufficiently large $n$.

$$log(f(n)) \leq log(c(g(n))) = log(c) + log(g(n))$$

$$(log(c) \div log(g(n)) + log(g(n)) \div log(g(n))) * log(g(n))$$

which is bounded by a constant.
Counter example: $f(n) = 2^{1/n}$ and $g(n) = 2^{1/n^2}$
Thus, $log(f(n)) \neq O(log(g(n)))$

d1) Let $f(n) = 2^n$. The function $2^{2n}$ grows far faster than $2^n$. Thus, this is false.

## 2

Sorted functions in asymptotically increasing order. Asymptotically equivalent functions are shown in same lines.

1. $\epsilon^n$
2. $n^{-a}$
3. $n^{-\epsilon}$
4. $log(n^a)$, $log(n^b)$, $log(n^\epsilon)$, $log_{1/\epsilon}(n)$, $log(bn)$
5. $(log(n))^a$
6. $n^\epsilon$
7. $n/a$, $a^{log_a(n)}$, $\epsilon n$
8. $n^a$, $(n+b)^a$
9. $(n+a)^b$
10. $n^{a+b}$
11. $a^{\epsilon n}$
12. $a^n$
13. $b^n$