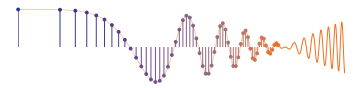


PROBLEM 5.3:



$$y[n] = 2x[n] - 3x[n-1] + 2x[n-2]$$

(a) MAKE A TABLE:

n	< 0	0	1	2	3	4	5	6	7	≥ 8
$x[n]$	0	1	2	3	2	1	1	1	1	1
$y[n]$	0	2	1	2	-1	2	3	1	1	1

$$y[0] = 2x[0] - 3x[-1] + 2x[-2] = 2(1) = 2$$

$$y[1] = 2x[1] - 3x[0] + 2x[-1] = 2(2) - 3(1) = 1$$

$$y[2] = 2x[2] - 3x[1] + 2x[0] = 2(3) - 3(2) + 2(1) = 2$$

$$y[3] = 2(2) - 3(3) + 2(2) = -1$$

$$y[4] = 2(1) - 3(2) + 2(3) = 2$$

$$y[5] = 2(1) - 3(1) + 2(2) = 3$$

$$y[6] = 2(1) - 3(1) + 2(1) = 1$$

$$y[7] = 2(1) - 3(1) + 2(1) = 1$$

$$y[8] = 2(1) - 3(1) + 2(1) = 1$$

(c) Impulse Response

$$h[0] = 2(1) - 3(0) + 2(0) = 2$$

$$h[1] = 2(0) - 3(1) + 2(0) = -3$$

$$h[2] = 2(0) - 3(0) + 2(0) = 2$$

Notice $h[n]$ just "reads out" the filter coefficients:

$$\text{i.e., } h[n] = b_n$$

PROBLEM 5.8:



Use convolution

$$\begin{array}{ccccccccccc} n: & \dots & -2 & -1 & 0 & 1 & 2 & 3 & 4 & \dots \\ x[n]: & & 0 & 1 & 0 & 1 & 0 & 1 & 0 & \dots \\ h[n]: & & & & 13 & -13 & 13 & & & \dots \end{array}$$

$$\begin{array}{ccccccccccc} \dots & 0 & 13 & 0 & 13 & 0 & 13 & 0 & \dots \\ \dots & -13 & 0 & -13 & 0 & -13 & 0 & -13 & \dots \\ \dots & 0 & 13 & 0 & 13 & 0 & 13 & 0 & \dots \end{array}$$

$$\begin{array}{cccccccc} -13 & 26 & -13 & 26 & -13 & 26 & -13 \end{array}$$

\uparrow
 $n=0$

$$\Rightarrow y[n] = \begin{cases} -13 & \text{for } n \text{ even} \\ 26 & \text{for } n \text{ odd} \end{cases}$$

PROBLEM 5.9:

Linearity ?

(a) YES.

$$\text{Let } x[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n]$$

$$x_1[n] \rightarrow y_1[n]$$

$$x_2[n] \rightarrow y_2[n]$$

$$\Rightarrow y[n] = (\alpha_1 x_1[n] + \alpha_2 x_2[n]) \cos(0.2\pi n)$$

$$= \underbrace{\alpha_1 x_1[n] \cos(0.2\pi n)}_{y_1[n]} + \underbrace{\alpha_2 x_2[n] \cos(0.2\pi n)}_{y_2[n]}$$

(b) YES.

$$y[n] = (\alpha_1 x_1[n] - \alpha_2 x_2[n]) - (\alpha_1 x_1[n-1] + \alpha_2 x_2[n-1])$$

$$= \alpha_1 \underbrace{(x_1[n] - x_1[n-1])}_{y_1[n]} + \alpha_2 \underbrace{(x_2[n] - x_2[n-1])}_{y_2[n]}$$

(c) NO.

$$\text{Let } x_1[n] = \delta[n] \text{ and } x_2[n] = -2\delta[n].$$

$$\hookrightarrow y_1[n] = \delta[n]$$

$$\hookrightarrow y_2[n] = |x_2[n]| = 2\delta[n]$$

$$\text{Let } x[n] = x_1[n] + x_2[n] = \delta[n] - 2\delta[n] = -\delta[n]$$

$$\hookrightarrow y[n] = |x[n]| = \delta[n] \quad \longleftrightarrow \quad y_1[n] + y_2[n] = \delta[n] + 2\delta[n] = 3\delta[n]$$

NOT EQUAL!

(d) NO! if $B \neq 0$

if $x_1[n] \rightarrow y_1[n]$, test $2x_1[n] \rightarrow 2y_1[n]$.

$$A(2x_1[n]) + B = 2(Ax_1[n] + B) - B \neq 2y_1[n]$$

TIME - INVARIANT?

(a) NO!

$$\text{Let } x[n] = \delta[n], \text{ then } y[n] = \delta[n] \cos(0.2\pi n) = \delta[n]$$

EVAL AT $n=0$

Try $x[n-1] = \delta[n-1]$, then output is

$$\delta[n-1] \cos(0.2\pi n) = \cos(0.2\pi) \delta[n-1].$$

$$\text{BUT } \cos(0.2\pi) \delta[n-1] \neq y[n-1] = \delta[n-1]$$



PROBLEM 5.9 (more):

TIME-INVARIANT?

(b) Yes.

If $x[n] \rightarrow y[n]$, Let $v[n] = x[n-n_0]$

$$\text{OUTPUT} = v[n] - v[n-1] = x[n-n_0] - x[n-n_0-1]$$

→ This is the same as $y[n-n_0] = x[n-n_0] - x[n-n_0-1]$

(c) YES.

Output depends only on $x[\]$ at 'n', so $y[n-n_0] = |x[n-n_0]|$

(d) Yes

$y[n-n_0] = Ax[n-n_0] + B$ is always true.

CAUSAL?

(a) YES.

$y[n]$ at $n=n_0$ depends only on $x[n]$ at $n=n_0$, and not on past or future values.

(b) Yes.

$y[n]$ at $n=n_0$ depends only on $x[n]$ at $n=n_0$ & $n=n_0-1$
so it only uses the 'present' and the 'past.'

(c) YES

$y[n]$ at $n=n_0$ depends only on $x[n]$ at $n=n_0$.

$$y[n_0] = |x[n_0]|$$

(d) YES

$y[n]$ at $n=n_0$ depends only on $x[n]$ at $n=n_0$.

$$y[n_0] = Ax[n_0] + B$$



PROBLEM 5.12:

$$x_1[n] = u[n] \longrightarrow y_1[n] = \delta[n] + 2\delta[n-1] - \delta[n-2]$$

$$x_2[n] = 3u[n] - 2u[n-4]$$

Use linearity and time-invariance:

$$\begin{aligned} y_2[n] &= 3y_1[n] - 2y_1[n-4] \\ &= 3\delta[n] + 6\delta[n-1] - 3\delta[n-2] - 2\delta[n-4] \\ &\quad - 4\delta[n-5] + 2\delta[n-6]. \end{aligned}$$

List of values:

n	< 0	0	1	2	3	4	5	6	≥ 7
$y_2[n]$	0	3	6	-3	0	-2	-4	2	0

PROBLEM 5.14:



(a) $h[n] = \delta[n-2] \Rightarrow$ filter is a delay by 2

$$y[n] = u[n-3] - u[n-6]$$

To find $x[n]$ we need to "un-delay" $y[n]$.

$$\Rightarrow x[n] = u[n-1] - u[n-4]$$

(b) First-difference FIR $\Rightarrow h[n] = \delta[n] - \delta[n-1]$

The first-difference filter has a nonzero output at n when $x[n] \neq x[n-1]$ are not equal.

If $y[n] = \delta[n] - \delta[n-4]$, then the input $x[n]$ changes value at $n=0$ and $n=4$. At $n=0$, it jumps up by one; at $n=4$, it jumps down.

$$\Rightarrow x[n] = u[n] - u[n-4]$$

jump up
by one

jump
down

(c) 4-pt averager: $y[n] = \frac{1}{4}(x[n] + x[n-1] + x[n-2] + x[n-3])$

$$\text{If } y[n] = -5\delta[n] - 5\delta[n-2]$$

$$y[0] = -5 = \frac{1}{4}(x[0] + x[-1] + x[-2] + x[-3])$$

** if we assume $x[n] = 0$ for $n < 0$, then $x[0] = -20$

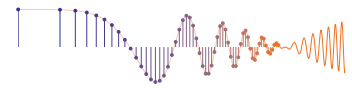
$$y[1] = 0 = \frac{1}{4}(x[1] + x[0] + x[-1] + x[-2]) = \frac{1}{4}x[1] - 5$$

$$\Rightarrow x[1] = 20$$

$$\left. \begin{aligned} y[2] &= -5 = \frac{1}{4}(x[2] + x[1] + x[0] + x[-1]) \\ &= \frac{1}{4}(x[2] + 20 - 20 + 0) = \frac{1}{4}x[2] \end{aligned} \right\} x[2] = -20$$

$$y[3] = 0 = \frac{1}{4}(x[3] + x[2] + x[1] + x[0]) \Rightarrow x[3] = -20$$

$$\Rightarrow x[n] = \begin{cases} 0 & \text{for } n < 0 \\ -20 & \text{for } n \text{ even} \\ 20 & \text{for } n \text{ odd} \end{cases}$$



PROBLEM 5.17:

$$\begin{aligned} \text{(a)} \quad h_1[n] &= \delta[n] - \delta[n-1] \\ h_2[n] &= \delta[n] + \delta[n-2] \\ h_3[n] &= \delta[n-1] + \delta[n-2] \end{aligned}$$

(b) The overall $h[n]$ is the convolution of the $h_i[n]$.

$$h[n] = h_1[n] * h_2[n] * h_3[n]$$

$$\begin{aligned} h_1[n] * h_2[n] &= (\delta[n] - \delta[n-1]) * (\delta[n] + \delta[n-2]) \\ &= \delta[n] - \delta[n-1] + \delta[n-2] - \delta[n-3] \end{aligned}$$

Now convolve with $h_3[n]$

$$\begin{array}{ccccccc} 1 & -1 & 1 & -1 & & & \\ 0 & 1 & 1 & & & & \\ \hline 0 & 0 & 0 & 0 & & & \\ & & 1 & -1 & 1 & -1 & \\ & & & 1 & -1 & 1 & -1 \\ \hline 0 & 1 & 0 & 0 & 0 & -1 & \\ n=0 & \uparrow n=1 & & & & \uparrow n=5 & \end{array}$$

$$h[n] = \delta[n-1] - \delta[n-5]$$

$$\begin{aligned} \text{(c)} \quad y[n] &= h[n] * x[n] \\ &= (\delta[n-1] - \delta[n-5]) * x[n] \\ y[n] &= x[n-1] - x[n-5] \end{aligned}$$