CS 476
Automata Theory and Formal Languages
Instructor: Ali Aydın Selçuk
Department of Computer Engineering, Bilkent University

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# Homework #2

Due 17.00, November 19, 2012

1. (16pts) Give a PDA for each of the following languages. Explain how each PDA works and why it is correct.
   (a) $L = \{a^i b^j c^k \mid i \neq j \text{ or } j \neq k\}$
   (b) $L = \{w \in \{a,b\}^* \mid n_a(w) \leq n_b(w) \leq 2n_a(w)\}$

2. (24pts) Give a CFG for each of the languages in Question 1. Prove that each grammar generates the desired language correctly.

3. (20pts) Which of the following languages are context free and which are not? Justify your answer by either giving CFG or PDA, or using pumping lemma.
   (a) $L = \{w\overline{w} \mid w \in \{0,1\}^*\}$, where, $\overline{w}$ denotes the string obtained from $w$ by replacing the 0s in $w$ by 1s and the 1s by 0s.
   (b) $L = \{a^i b^j \mid j \leq i^2\}$
   (c) $L = \{a^n b^m c^m d^n \mid n, m \geq 0\}$
   (d) $L = \{a^n b^m c^n d^m \mid n, m \geq 0\}$

4. (20pts) Consider the grammar below and apply the following procedures using the algorithms discussed in class.
   $S \to A \mid ABa \mid AbA \quad A \to Aa \mid \epsilon \quad B \to Bb \mid BC \quad C \to CB \mid CA \mid bB$
   (a) Eliminate any $\epsilon$ productions.
   (b) Eliminate any unit productions in the resulting grammar.
   (c) Eliminate any useless symbols in the resulting grammar.
   (d) Put the resulting grammar into Chomsky Normal Form.

5. (20pts) What languages are generated by the following CFGs? Explain your answers.
   (a) (10pts)
   $$S \to bS \mid Sa \mid aSb \mid \epsilon.$$
   (b) (10pts)
   $$S \to aAB \mid aBA \mid bAA \mid \epsilon$$
   $$A \to aS \mid bAAA$$
   $$B \to aABB \mid aBAB \mid aBBA \mid bS.$$

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**Bonus Question**

Consider the language $L = (a+b)^* - \{(a^i b^i)^i \mid i \geq 1\}$. Show that this language is context-free by building a PDA that accepts this language. Make sure that you justify your answer by carefully explaining why your PDA is correct (that it accepts *exactly* $L$).