CS476: Automata Theory and Formal Languages

Homework 3

Assigned: 23/04/2014
Due: 12/05/2014, 17:00

Questions

1. (20pts) State whether the following statements are true or not. You must give a BRIEF explanation or show a counter example to receive full credit.

   (a) If a language L is decidable then the language $\bar{L}$ is also decidable.

   (b) Every deterministic Turing machine (DTM) has an equivalent nondeterministic Turing machine.

   (c) There exists a polynomial-time reduction from Boolean Satisfiability Problem (SAT) to Subset Sum Problem (SS).

   (d) If a problem P is NP-complete then the language induced by problem P is undecidable.

2. (20pts) Give a Turing Machine that decides language $L = \{w1^n : |w| = n, w \in \{0,1\}^*\}$.

3. (30pts) Disprove (by reduction) or prove that the following languages are decidable.

   (a) $L = \{\langle A \rangle : A \text{ is a DFA and } L(A) = L'(A)\}$ where $L'(A) = \{\bar{w} : w \in L(A)\}$

   (b) $L = \{\langle A, S \rangle : A \text{ is a TM and } L(A) \subseteq S\}$

   (c) $L = \{\langle M_1, M_2 \rangle : M_1 \text{ and } M_2 \text{ are TMs such that } M_1 \text{ accepts } \langle M_2 \rangle \text{ and } M_2 \text{ accepts } \langle M_1 \rangle\}$

4. (30pts) Number 3 is a lucky number (this is the reason why you have 3 assignments). So, we have two problems (both are related with number "3"): `3-SET-PARTITION` and `3-GRAPH-PARTITION`. Prove that they are NP-Complete.

   (a) `3-SET-PARTITION`: Let A, B, C be three finite, disjoint sets and let T be a subset of $A \times B \times C$. That is, T consists of triples (a, b, c) such that $a \in A$, $b \in B$ and $c \in C$. Given A, B, C, T and an integer k, the `3-SET-PARTITION` problem decides whether there is a subset M of T (i.e., $M \subseteq$ T), such that $|M| \geq k$ and for any two distinct triples $(a_1, b_1, c_1) \in M$ and $(a_2, b_2, c_2) \in M$, we have $a_1 \neq a_2$, $b_1 \neq b_2$ and $c_1 \neq c_2$.

   (b) `3-GRAPH-PARTITION`: Given an undirected graph $G = (V, E)$, the `3-GRAPH-PARTITION` problem decides whether the vertex set V can be partitioned into three disjoint subsets V_1, V_2 and V_3, such that for any two vertices $v_a \in V$ and $v_b \in V$, if $(v_a, v_b) \in E$, then v_a and v_b cannot be placed into the same partition V_i, $i \in \{1, 2, 3\}$.