CS473 Assignment 2

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a) The functions $f(n)$ and $g(n)$ are non negative. There exists n_0 such that $f(n) \geq 0$ and $g(n) \geq 0$ for all $n \geq n_0$.
Thus, we have that for all $n \geq n_0$,

$$f(n) + g(n) \geq f(n) \geq 0 \quad and \quad f(n) + g(n) \geq g(n) \geq 0$$

Adding both inequalities, since the functions are non negative, we get $f(n) + g(n) \geq max(f(n), g(n))$ for all $n \geq n_0$.
This proves that,

$$max(f(n), g(n)) \leq c(f(n) + g(n)) \quad for \quad all \quad n \geq n_0 \quad with \quad c = 1$$

In other words, $max(f(n), g(n)) = O(f(n) + g(n))$.
Similarly, we can see that

$$max(f(n), g(n)) \geq f(n) \quad and \quad max(f(n), g(n)) \geq g(n) \quad for \quad all \quad n \geq n_0$$

Adding these two inequalities, we can see that,

$$2max(f(n), g(n)) \geq (g(n) + f(n))$$

or

$$max(f(n), g(n)) \geq 1/2(g(n) + f(n)) \quad for \quad all \quad n \geq n_0.$$

Thus $max(f(n), g(n)) = \Omega(g(n) + f(n))$ with constant $c = 1/2$.

c1) Given $\exists c > 0$ such that $f(n) \leq cg(n)$ for sufficiently large n.

$$log(f(n)) \leq log(c(g(n))) = log(c) + log(g(n))$$

$$(log(c) \div log(g(n)) + log(g(n)) \div log(g(n))) * log(g(n))$$

which is bounded by a constant.
Counter example: $f(n) = 2^{1/n}$ and $g(n) = 2^{1/n^2}$
Thus, $log(f(n)) \neq O(log(g(n)))$

d1) Let $f(n) = 2^n$. The function 2^{2n} grows far faster than 2^n. Thus, this is false.

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Sorted functions in asymptotically increasing order. Asymptotically equivalent functions are shown in same lines.

1. ϵ^n
2. $n^{-\epsilon}$, n^{-a}
3. $log(n^a)$, $log(n^b)$, $log(n^\epsilon)$, $log(n^a)$, $log_{1/\epsilon}(n)$, $(log(n))^a$, $log(bn)$
4. n^ϵ, $a^{log_a(n)}$, n/a, ϵn, $(n+a)^b$, n^{a+b}, $(n+b)^a$, n^a
5. a^n, b^n, $a^{\epsilon n}$