IE400 Quiz I – Fall 2018

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Question 1. An automobile tire company has the ability to produce both nylon and fiberglass tires. The company has two presses, a Wheeling machine and a Regal machine and appropriate moulds that can be used to produce these tires. The production hours available in the next three months are given in the table below. Also provided below are the number of tires they have agreed to deliver during the next three months.

MONTH	**AVAILABLE HOURS**		**DELIVERY REQUIRED**	
	Wheeling	**Regal**	**Nylon**	**Fibreglass**
JUNE	700	1,500	4,000	1,000
JULY	300	400	8,000	5,000
AUGUST	1,000	300	3,000	5,000

Each machine can make the two types of tires independently. The production rate for each machine-tire combination is as follows:

TYRE	**Wheeling hr./Tire**	**Regal hr/Tire**
NYLON	0.15	0.16
FIBREGLASS	0.12	0.14

The variable cost of producing tires are Tl 250 per operating hour, regardless of which machine is used or which tire is produced. The inventory carrying costs are Tl. 5 per month. Material costs for the nylon and fiberglass tyres are Tl 155 and Tl. 196 respectively. Prices have been set at Tl. 350 per nylon tire and Tl. 450 per fiberglass tire.

What should be the production schedule for the three months so as to maximize the profit? Develop an appropriate linear programming model that when solved will provide the optimal production schedule. Define the decision variables needed to develop your model precisely as well as state the meaning or purpose of each constraint. [4 points]

Decision Variables:

XNWi: # of Nylon tires produced by Wheeling machine in month i = 1, 2, 3. i = 1(June), i=2(July), i=3(August).

XFWi: # of Fibreglass tires produced by Wheeling machine in month i = 1, 2, 3. i = 1(June), i=2(July), i=3(August).

XNRi: # of Nylon tires produced by Regal machine in month i = 1, 2, 3. i = 1(June), i=2(July), i=3(August).

XFRi: # of Fibreglass tires produced by Regal machine in month i = 1, 2, 3. i = 1(June), i=2(July), i=3(August).

YNi: # of Nylon tires sold in month i = 1, 2, 3. i = 1(June), i=2(July), i=3(August).

YFi: # of Fibreglass tires sold in month i = 1, 2, 3. i = 1(June), i=2(July), i=3(August).

INi: Ending inventory of Nylon tires in month i = 1, 2, 3. i = 1(June), i=2(July), i=3(August).

IFi: Ending inventory of Fibreglass tires in month i = 1, 2, 3. i = 1(June), i=2(July), i=3(August).

Per unit cost of producing Nylon tires in Wheeling machine = 250*0.15+155 = Tl 192.5

Per unit cost of producing Nylon tires in Regal machine = 250*0.16+155 = Tl 195.0

Per unit cost of producing Fibreglass tires in Wheeling machine = 250*0.12+196 = Tl 226

Per unit cost of producing Fibreglass tires in Regal machine = 250*0.14+196 = Tl 231

Model:

Max Profit = 350 YN1 + 350 YN2 + 350 YN3 + 450 YF1 + 450 YF2 + 450 YF3 – 192.5 XNW1 – 192.5 XNW2 – 192.5 XNW3 - 195 XNR1 – 195 XNR2 – 195 XNR3 - 226 XFW1 – 226 XFW2 – 226 XFW3 - 231 XFR1 – 231 XFR2 – 231 XFR3 – 5 IN1 – 5 IN2 – 5 IN3 – 5 IF1 – 5 IF2 – 5 IF3

s.t.

INi = INi-1 + XNWi + XNRi – YNi, i = 1, 2, 3 {Ending Inventory of Nylon Tires}

IN0 = 0 {Beginning Inventory of Nylon Tires}

IFi = IFi-1 + XFWi + XFRi – YFi, i = 1, 2, 3 {Ending Inventory of Fibreglass Tires}

IF0 = 0 {Beginning Inventory of Fibreglass Tires}

$0.15\ XNW1 + 0.12\ XFW1 \le 700$ {# of hours used in Wheeling machine in month 1 <= 700}

$0.15\ XNW2 + 0.12\ XFW2 \le 300$ {# of hours used in Wheeling machine in month 2 <= 300}

$0.15\ XNW3 + 0.12\ XFW3 \le 1000$ {# of hours used in Wheeling machine in month 3 <= 1000}

$0.16\ XNR1 + 0.14\ XFR1 \le 1500$ {# of hours used in Regal machine in month 1 <= 1500}

$0.16\ XNR2 + 0.14\ XFR2 \le 400$ {# of hours used in Regal machine in month 2 <= 400}

$0.16\ XNR3 + 0.14\ XFR3 \le 300$ {# of hours used in Regal machine in month 3 <= 300}

$YN1 \ge 4000$ {# of Nylon tires sold and delivered in month 1 >= 4000}

$YN2 \ge 8000$ {# of Nylon tires sold and delivered in month 2 >= 8000}

$YN3 \ge 3000$ {# of Nylon tires sold and delivered in month 3 >= 3000}

$YF1 \ge 1000$ {# of Fibreglass tires sold and delivered in month 1 >= 1000}

$YF2 \ge 5000$ {# of Fibreglass tires sold and delivered in month 2 >= 5000}

$YF3 \ge 5000$ {# of Fibreglass tires sold and delivered in month 3 >= 5000}

$IN1, IN2, IN3, IF1, IF2, IF3 \ge 0$ {Ending inventory of tires must be >= 0}

$XNWi, XFWi, XNRi, XFRi \ge 0$ for $i = 1, 2, 3$ {All tires produced in each month are >=0}

Question 2: Recall the Blending (1) problem that was formulated in class. In this problem, a refinery distills crude petroleum obtained from two sources, Saudi Arabia and Venezuela, into three main products: gasoline, jet fuel and lubricants.

The two crudes differ in chemical composition and, therefore, yield different product mixes. Each ton of Saudi crude yields 0.3 ton of gasoline, 0.4 ton of jet fuel and 0.2 ton of lubricants. Each ton of Venezuelan crude yields 0.4 ton of gasoline, 0.2 ton of jet fuel and 0.3 ton of lubricants. The remaining 10% of each ton is lost in refining.

The crudes also differ in cost and availability. The refinery can purchase up to 90 tons per day from Saudi Arabia at $400 per ton. Up to 60 tons per day of Venezuelan crude is available at $300 per ton.

The refinery's contracts with independent distributors require that it produce at least 20 tons per day of gasoline, 15 tons per day of jet fuel, and 5 tons per day of lubricants. How can these requirements be fulfilled most efficiently?

The refinery's decision problem is formulated as a linear programming problem as follows:

Let, x1 and x2 denote the tons of Saudi and Venezuelan crude refined per day.

The model:

Min 400 x1 + 300 x2

s.t. 0.3 x1 + 0.4 x2 >= 20.0 (gasoline requirement) (1)

0.4 x1 + 0.2 x2 >= 15 (jet fuel requirement) (2)

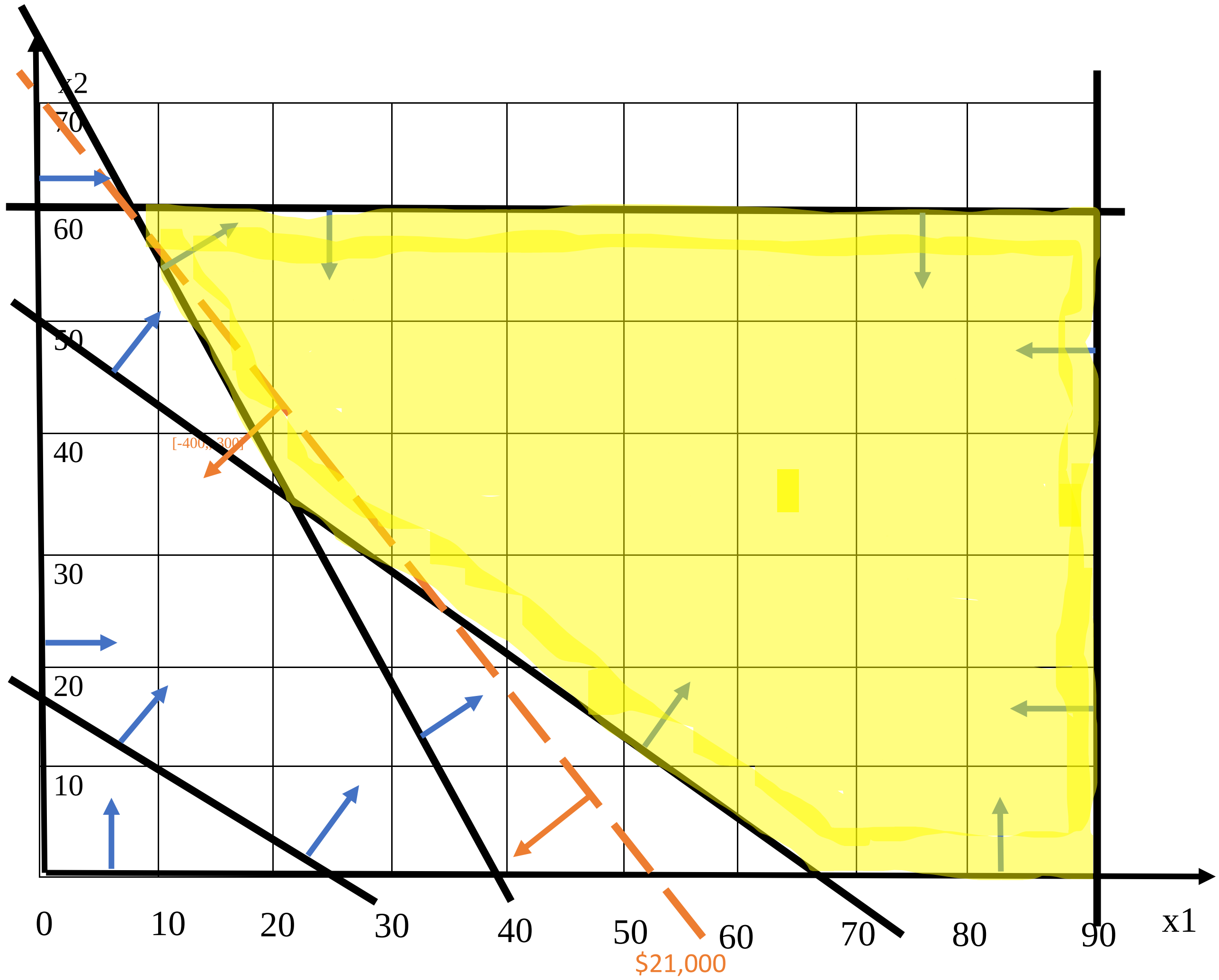
0.2 x1 + 0.3 x2 >= 5 (lubricant requirement) (3)

x1 <= 90 (Saudi availability) (4)

x2 <= 60 (Venezuelan availability) (5)

x1, x2 >= 0 (6), (7)

a) Use the graph below to show all the feasible solutions to the refinery's decision problem. Mark all the extreme points of the feasible solution in the graph clearly. [2 points]



b) Draw the **iso-cost** line with a cost of $21,000. Use this **iso-cost** line to identify the optimal solution, i.e., the optimal value of x1 and x2. What is the optimal cost? [1/2 point]

Based on the graph drawn above, the optimal solution lies at the intersection of constraint lines:

0.3x1 + 0.4x2 = 20 (1)

0.4x1 + 0.2x2 = 15. (2)

Multiplying (2) by -2 and adding (1) and (2), the equation obtained is:

-0.5x1 = -10. This results in, x1* = 20 and x2* = 35.

The Optimal Solution: [20, 35]. Optimal Cost = 20*400 + 35*300 = $18,500.

c) What is the minimum amount by which the cost of Venezuela crude should decrease for the optimal solution in b) to move to a new extreme point? Show all your calculations. [1/2 point]

Let the cost of Venezuelan crude be denoted as c2, while the cost of Saudi crude is fixed at 400. Then the objective function is denoted as:

$Z = 400x1 + c2x2$.

Therefore, the iso-cost line is described as:

$x2 = (-400/c2)*x1 + Z/c2$.

The slope of the iso-cost line is $(-400/c2)$.

Constraint line (1) is:

$0.3x1 + 0.4x2 = 20$, which can be rewritten as

$x2 = (-0.3/0.4)x1 + 20/0.4 \Rightarrow x2 = -0.75x1 + 50$

The slope of constraint line (1) is – 0.75.

Constraint line (2) is:

$0.4x1 + 0.2x2 = 15$, which can be rewritten as

$x2 = -2x1 + 37.5$

The slope of constraint line (2) is -2.

Thus, the current optimal solution remains optimal as long as:

$-2 \leq -400/c2 \leq -0.75$.

Clearly, as c2 decreases from 300 the current optimal solution will remain optimal till $-400/c2 = -2$. When $c2 = -400/-2 = 200$, we have alternate optimal solutions Thus the optimal solution will change only after c2 decreases below 200.

Therefore, the minimum amount by which the cost of Venezuelan crude will have to decrease by is \$400 - \$200 = \$200.

d) What is the minimum amount by which the cost of Saudi Arabian crude should increase for the optimal solution in b) to move to a new extreme point? Show all your calculations. [1/2 point]

Let the cost of Saudi Arabian crude be denoted as c1, while the cost of Venezuelan crude is fixed at \$300. Then the objective function is denoted as:

$Z = c1x1 + 300x2$.

Therefore, the iso-cost line is described as:

x2 = (-c1/300)*x1 + Z/300.

The slope of the iso-cost line is (-c1/300).

Thus, the current optimal solution remains optimal as long as:

-2 ≤ -c1/300 ≤ -0.75.

Clearly, as c1 increases from 400 the current optimal solution will remain optimal till -c1/300 = -2. When c1 = 600, we have alternate optimal solutions.

Therefore, the minimum amount by which the cost of Saudi Arabian crude will have to increase by is $600 - $400 = $200.

e) What is the minimum amount by which the daily requirement of lubricant (which is currently 5 tons) must increase to for the optimal solution to change? Show your calculations. [1/2 point]

Currently, the constraint: 0.2 x1 + 0.3 x2 >= 5 is <u>non-binding</u> at optimality.
That is because at x1* = 20 and x2* = 35, the tons of lubricant produced is: 0.2*20 + 0.3*35 = 14.5 tons.

Thus, the daily requirement for lubricants has to increase by (14.5 – 5.0) = <u>9.5 tons</u> before the optimal solution can change.