CS-473: Algorithms I Spring 2018

Assignment 6: Greedy Algorithms and Elementary Graph Algorithms

Instructor: Mehmet Koyutürk

Due Date and Instructions

Please return hard-copies to the instructor's office or mailbox by the end of business (17:00) on **Tuesday, May 15, 2018**.

- Hand-writing is accepted but the course personnel reserves the right to reject grading an assignment if the hand-writing is not legible.
- Assignments written in LaTeX will receive 5 bonus points.
- You can submit the assignment as a team of 2 students, however no pair of students are allowed to work together on more than 3 assignments.
- You are also allowed to talk to other students and the course personnel about the solutions, but you must write the answers yourself. Answers copied from other students or resources will be detected, and appropriate action will be taken.

Problem 1

[30 pts.] We have n activities. Each activity requires t_i time to complete and has deadline d_i. We would like to schedule the activities to minimize the maximum delay in completing any activity; that is, we would like to assign starting times s_i to all activities so that $\max_{1 \le i \le n}\{\Delta_i\}$ is minimized, where $\Delta_i = f_i - d_i$ is the delay for activity i and $f_i = s_i + t_i$ is the finishing time for activity i. Note that we can only perform one activity at a given time (if activity i starts at time s_i, the next scheduled activity has to start at time f_i).

For example, if $t =< 10, 5, 6, 2 >$ and $d =< 11, 6, 12, 20 >$, then the optimal solution is to schedule the activities in the order $< 2, 1, 3, 4 >$ to obtain starting/finishing times $s/f =< 5/15, 0/5, 15/21, 21/23 >$ and achieve a maximum delay of 9 (for the third activity).

State and prove the greedy choice property for this problem.

Problem 2

[30 pts] (Problem 22.2-7 from the textbook) There are two types of professional wrestlers: "babyfaces" ("good guys") and "heels" ("bad guys"). Between any pair of professional wrestlers, there may or may not be a rivalry. Suppose we have n professional wrestlers and we have a list of r pairs of wrestlers who are rivals with each other. Give an $O(n+r)$-time algorithm that determines whether it is possible to designate some of the wrestlers as babyfaces and the remainder as heels such that each rivalry is between a babyface and a heel. If it is possible to perform such a designation, your algorithm should produce it.

You do not need to provide pseudo-code, a succinct description of your algorithm, possibly referring to algorithms we have seen in class, will suffice.

Problem 3

[20 pts.] Prove or disprove: Let $G = (V, E)$ be a directed graph. For any $uv \in E$, if some run of DFS on G results in $v.f > u.f$, then uv must be on a cycle.

Problem 4

[20 pts.] Prove or disprove: Consider any run of DFS on a directed graph $G = (V, E)$. For any edge $uv \in E$, if there is a path from v to u in G, then uv cannot be a cross edge.