IE400 – 2018-2019 Fall

Midterm Study Set

1) A cargo plane has three compartments for storing cargo: front, center and rear. These compartments have the following limits on both weight and space:

Compartment	Weight capacity (tons)	Space capacity (cubic meters)
Front	10	6800
Center	16	8700
Rear	8	5300

The following four cargoes are available for shipment on the next flight:

Cargo	Weight (tons)	Volume (cubic meters/ton)	Profit (TL/ton)
C1	18	480	310
C2	15	650	380
C3	23	580	350
C4	12	390	285

We assume

- that each cargo can be split into whatever proportions/fractions we desire for shipment
- that each cargo can be split between 2 or more compartments if we so desire

The aim is to determine how much (if any) of each cargo C1, C2, C3 and C4 should be accepted and how to distribute each among the compartments so that the total profit for the flight is maximized.

a) Let us define the utilization rate of a cargo compartment as the ratio of the total cargo weight carried in that compartment to the weight capacity of that compartment (for example, if a total of 8 tons of cargo is carried in the front compartment, the utilization rate will be 8/10=0.8). Formulate this problem as a linear programming problem under the constraint that the utilization rate of each compartment must be the same to maintain the balance of the plane.

b) Formulate this problem as a linear programming problem under the constraint that the differences between the utilization rates of any two compartments should not be more than 0.1 to maintain the balance of the plane.

c) Consider the two LPs in parts a) and b). Which one is more likely to have a larger optimal value? Please justify your answer.

d) If cargo can be delivered only in integer amounts (in tons) and if it is not possible to split the cargo between compartments, how would your model in part b) change? Your new model should still be a linear one.

2) Kocalar Delivery Company must make deliveries to 10 customers whose respective demands are *dj* for j = 1,...,10. Each customer can be visited by more than one truck to satisfy the demands. The company has four trucks available with capacities L_k and daily operating costs c_k for k = 1,...,4. Formulate the model to determine which trucks to use so as to minimize the cost of delivering to all the customers. Add the following requirements:

a) A single truck cannot deliver to more than five customers.

b) Customer 1 and customer 7 cannot be visited by the same truck.

c) Additionally, if a truck visits customer 3 then it has to visit customer 4 or customer 5. (Assume demands of these customers do not exceed truck capacities.)

d) Both customer 2 and customer 6 should be visited by the same truck(s).

e) Either at least one of the customers 1, 2 and 3 or at most 2 of customers 5, 6, 7 should be visited by truck 4.

3) Consider a set of 20 linear inequalities $\sum_{j=1}^{n} a_{ij} x_j \leq b_i$ for i=1,2…20. Formulate an integer program which represents that a point x satisfies the following:

- Point x should satisfy at least 10 of the 20 constraints.
- If point x satisfies constraint 1, then it should also satisfy constraint 4.
- If point x satisfies constraint 2, then it should not satisfy constraint 3.
- If point x satisfies either both of the constraints 5 and 6 or neither of them.
- Point x should satisfy either at least one of the constraints 7,8,9 or at least two of cosntraints 10,11,12,13.

4) Glueco produces three types of glue on two different production lines. Each line can be utilized by up to seven workers at a time. Workers are paid \$500 per week on production line 1, and \$900 per week on production line 2. A week of production costs \$1,000 to set up production line 1 and \$2,000 to set up production line 2. During a week on a production line, each worker produces the number of units of glue shown in the table below. Each week, at least 120 units of glue 1, at least 150 units of glue 2, and at least 200 units of glue 3 must be produced. Formulate an IP to minimize the total cost of meeting weekly demands.

	Glue		
Production Line	1	2	3
1	20	30	40
2	50	35	45

5) A product can be produced on four different machines. Each machine has a fixed setup cost, variable production costs per-unit-processed, and a production capacity given in the table below. A total of 2,000 units of the product must be produced. Formulate an IP to minimize total costs.

Machine	Fixed Cost ($)	Variable Cost per Unit ($)	Capacity
1	1,000	20	900
2	920	24	1,000
3	800	16	1,200
4	700	28	1,600

6) Model the problem of finding the minimum number of colors needed to color a map so that no neighboring countries have the same color.

7) Solve the following linear program by both the two-phase method and the big M-method:

$$
\begin{array}{llllll}
\min & 3x_1 & + 2x_2 & + 4x_3 & & \\
\text{s.t.} & & & & & \\
 & 2x_1 & + x_2 & + 3x_3 & = & 60 \\
 & 3x_1 & + 3x_2 & + 5x_3 & \geq & 120 \\
 & x_1 & , \; x_2 & , \; x_3 & \geq & 0
\end{array}
$$

8) Consider the following simplex tableaus of LP problems. First two are minimization problems while the last tableau belongs to a maximization problem.

x_1	x_2	e_1	e_2	x_3	x_4	RHS
-2	-3	0	0	-M	-M	0
2	1	-1	0	1	0	4
1	-1	0	-1	0	1	1

x_1	x_2	s_1	e_1	x_3	x_4	RHS
0	0	0	0	-1	-1	0
1	1	1	0	0	0	3
2	1	0	-1	1	0	4
1	1	0	0	0	1	3

x_1	x_2	x_3	s_1	s_2	e_1	RHS
-2	-1	-1	0	0	0	0
1/3	2/3	0	1	0	1/3	1
2	1	0	0	1	0	3
2/3	1/3	1	0	0	-1/3	1

Are these tableaus in proper format? If yes, indicate the entering and leaving variables. If not, explain why and show how can you convert them into proper format.

9) Consider the following simplex tableau of a given maximization LP problem.

	z	x_1	x_2	x_3	x_4	x_5	RHS
z	1	**A**	2	0	0	0	10
x_3	0	-1	**D**	1	0	0	4
x_4	0	**B**	-4	0	1	0	1
x_5	0	**C**	3	0	0	1	**E**

Give general conditions on each of the unknowns A-E such that each of the following statements is true. Even if the statement holds independently of the values of a specific variable, you should still mention that that variable can take any value. So, for each part you should present 5 conditions, one for each unknown.

a) The tableau is final and there exists a unique optimal solution.
b) The simplex method determines an unbounded solution from this tableau.
c) The current bfs is degenerate (not necessarily optimal).
d) The current solution is optimal, there are alternative optimal solutions but no alternative optimal bfs.
e) Find a specific set of values for unknowns A-E satisfying the general conditions in part d). Give the current optimal bfs and an alternative optimal solution that is not a bfs.

10) For the given LP in the below:

$$\min\ ax+by$$

$$\text{s.t. } x+2y \geq -4$$

$$-3x+y \leq 6$$

$$x \geq 0\ ,\ y \text{ urs}$$

a) Draw the feasible region.

b) Give an example for a and b such that LP will have a unique optimal solution. For the chosen a and b, what is the optimum solution and optimum solution value?
c) Give an example for a and b such that LP will be unbounded.
d) Give an example for a and b such that LP will have alternate optimal solutions but only one extreme point optimal solution.
e) Convert the LP to the standard form.
f) Give a basic feasible solution and identify the basic and nonbasic variables in the provided basic feasible solution.

11) Solve the following problem using simplex algorithm. You may guess the initial bfs graphically.

$$\begin{array}{llll} \max & 4x_1 & + 5x_2 & \\ \text{st.} & x_1 & + 3x_2 & \geq 22 \\ & & x_2 & \leq 6 \\ & 2x_1 & - 5x_2 & \leq 0 \\ & x_1\ , & x_2 & \geq 0 \end{array}$$