EEE 391

Basics of Signals and Systems
Announced Quiz I
16 March 2010, Tuesday
closed book and notes
no calculators
Given Time: 60 min
Instructors: Billur Barshan and Haldun Özaktaş

Last Name : ....

First Name : ....

Section : ....

ID number : ....

Signature : ....

Exam Part	Total Points	Points Received
Q1	30	
Q2	34	
Q3	36	
Total	100	

Allocation of points:

1) 30 pts (a) 12 pts (b) 18 pts
2) 34 pts (a) 12 pts (b) 22 pts
3) 36 pts

Attention:

Read all the questions carefully and show your work for full or partial credit. Justify all your answers.

Given Formulas:

$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-j\left(\frac{2\pi}{T_0}\right)kt}\, dt$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k\, e^{j\left(\frac{2\pi}{T_0}\right)kt}$$

$$\int_0^{T_0} e^{-j\frac{2\pi k}{T_0}t}\, dt = \begin{cases} T_0 & k=0 \\ 0 & k\neq 0 \end{cases}$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\sin\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$\sin\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\cos 0 = 1$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$\cos\left(\frac{\pi}{2}\right) = 0 \qquad \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$\sin\theta = \cos(\theta - \pi/2)$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\cos\theta = \sin(\theta + \pi/2)$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$x(t) = \sum_{k=1}^{N} A_k \cos(\omega_0 t + \phi_k) = A\cos(\omega_0 t + \phi)$$

where $A e^{j\phi} = \sum_{k=1}^{N} A_k e^{j\phi_k}$

$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

$$\hat{\omega}_\ell = \hat{\omega}_0 + 2\pi\ell$$

$$\ell = 0, \pm 1, \pm 2, \cdots$$

$$x[n] = x(nT_s) = x\left(\frac{n}{f_s}\right)$$

$$\hat{\omega}_\ell = -\hat{\omega}_0 + 2\pi\ell$$

EEE 391 Announced Quiz 1:

① $x(t) = 3 - 4\sin(\omega t + \pi) + 5\cos\left[\omega t + \left(\tan^{-1}\frac{4}{3}\right)\right]$

a) Find the phasors representing each of the three terms in the signal.

b) Find the result using phasor addition and express $x(t)$ in the form $x(t) = A\cos(\omega t + \phi)$.

② Some of the Fourier series coefficients of a real and periodic signal $x(t)$ are given as:

$$a_1 = \frac{1}{j} \qquad a_2 = 2e^{j\frac{\pi}{3}} \qquad a_4 = j-1 \quad \text{where } j = \sqrt{-1}$$

It is known that the average value of the signal is -2. The Fourier series coefficients for $|k| \neq 0, 1, 2, 4$ are all zero

a) Write the Fourier series coefficients for

$k = -4, -3, -2, -1, 0, 1, 2, 3, 4$

in polar form.

b) Express the signal $x(t)$ in the form:

$$x(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos(\omega_k t + \phi_k)$$

where $\omega_k = k\omega_0$ and $\omega_0 = 4$ rad/s.

③ Suppose that a discrete-time signal $x[n]$ is given by:

$$x[n] = 2\cos\left(0.3\pi n + \frac{\pi}{4}\right)$$

and that it was obtained by sampling a continuous-time signal $x(t) = A\cos(2\pi f_0 t + \phi)$ at a sampling frequency of $f_s = 5$ kHz. Determine three different continuous-time signals that could have produced $x[n]$. All these continuous-time signals should have a frequency less than 6 kHz. Be careful in writing the phase part of the signals.