IE400 Quiz II – Fall 2018

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Question 1. [16 points]

Consider the following linear programming formulation:

Maximize $2x_1 + x_2 + 4x_3 \quad + 5x_5 + x_6$

s.t.

$3x_1 + 6x_2 + 3x_3 + 2x_4 + 3x_5 + 4x_6 \leq 60$

$x_1, \quad x_2, \quad x_3, \quad x_4, \quad x_5, \quad x_6 \geq 0$

a) Write the problem in *standard* form. [2 points]

Maximize Z = $\mathbf{2x_1 + x_2 + 4x_3 \quad + 5x_5 + x_6}$

s.t.

$\mathbf{3x_1 + 6x_2 + 3x_3 + 2x_4 + 3x_5 + 4x_6 + s_1 = 60}$

$\mathbf{x_1, \quad x_2, \quad x_3, \quad x_4, \quad x_5, \quad x_6, s_1 \geq 0}$

b) Determine and state *all* the basic feasible solutions (BFS) to the problem above. For each basic feasible solution, specify which variables are *basic*. Determine the optimal solution and optimal objective function value. [6 points]

Since the number of constraints m = 1 and number of variables is 7, the number of basic solutions are $\binom{n}{m}$ = 7. Each basic feasible solution consists of 1 basic variable and the rest non-basic.

<u>List of Basic Feasible Solutions</u>:

1) **x_1 is basic. Therefore, 3x1 = 60. $x_1 = 20$, $x_2 = x_3 = x_4 = x_5 = x_6 = s_1 = 0$, Obj. Fn. Value = 40.**
2) **x_2 is basic. $6x_2 = 60$. $x_2 = 10$, $x_1 = x_3 = x_4 = x_5 = x_6 = s_1 = 0$, Obj. Fn. Value = 10.**

3) **x_3 is basic. $3x_3 = 60$. $x_3 = 20$, $x_1 = x_2 = x_4 = x_5 = x_6 = s_1 = 0$, Obj. Fn. Value = 80.**
4) **x_4 is basic. $2x_4 = 60$. $x_4 = 30$, $x_1 = x_2 = x_3 = x_5 = x_6 = s_1 = 0$, Obj. Fn. Value = 0.**
5) **x_5 is basic. $3x_5 = 60$. $x_5 = 20$, $x_1 = x_2 = x_3 = x_4 = x_6 = s_1 = 0$, Obj. Fn. Value = 100.**
6) **x_6 is basic. $4x_6 = 60$. $x_5 = 15$, $x_1 = x_2 = x_3 = x_4 = x_5 = s_1 = 0$, Obj. Fn. Value = 15.**
7) **s_1 is basic. $s_1 = 60$. $s_1 = 60$, $x_1 = x_2 = x_3 = x_4 = x_5 = x_6 = 0$, Obj. Fn. Value = 0.**

The optimal basic feasible solution is: x_5 is basic. $3x_5 = 60$. $x_5 = 20$, $x_1 = x_2 = x_3 = x_4 = x_6 = s_1 = 0$, Obj. Fn. Value = 100.

c) To the problem above, add constraints $x_3 \leq 10$, $x_5 \leq 10$ and $x_1+x_3 \leq 20$. With the addition of these 3 constraints, perform one iteration of the simplex method using the simplex tableau. State the solution obtained after one iteration of the simplex method. Is this solution optimal? Why or Why not? Is the solution degenerate? Why or Why not? [8 points].

After adding the 3 constraints, the problem in standard form is:

$$\text{Maximize } Z = 2x_1 + x_2 + 4x_3 + 5x_5 + x_6$$

s.t.

$$\begin{aligned}
3x_1 + 6x_2 + 3x_3 + 2x_4 + 3x_5 + 4x_6 + s_1 &= 60 \\
x_3 + s_2 &= 10 \\
x_5 + s_3 &= 10 \\
x_1 + x_3 + s_4 &= 10 \\
x_1,\ x_2,\ x_3,\ x_4,\ x_5,\ x_6, s_1, s_2, s_3, s_4 &\geq 0
\end{aligned}$$

Initial Simplex Tableau:

Z	x_1	x_2	x_3	x_4	x_5	x_6	s_1	s_2	s_3	s_4	RHS
1	-2	-1	-4	0	-5	-1	0	0	0	0	0
0	3	6	3	2	3	4	1	0	0	0	60
0	0	0	1	0	0	0	0	1	0	0	10
0	0	0	0	0	1	0	0	0	1	0	10
0	1	0	1	0	0	0	0	0	0	1	10

Entering Variable x_5 (most -negative coefficient in row 0). Row 1 ratio = 60/3 = 20. Row 3 ratio = 10/1 = 10. Min Ratio = Min {20, 10} = 10. Therefore the pivot row is 3. Leaving Variable is s_3.

Next Simplex Tableau:

Z	x_1	x_2	x_3	x_4	x_5	x_6	s_1	s_2	s_3	s_4	RHS
1	**-2**	**-1**	**-4**	**0**	**0**	**-1**	**0**	**0**	**5**	**0**	**50**
0	**3**	**6**	**3**	**2**	**0**	**4**	**1**	**0**	**-3**	**0**	**30**
0	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**1**	**0**	**0**	**10**
0	**0**	**0**	**0**	**0**	**1**	**0**	**0**	**0**	**1**	**0**	**10**
0	**1**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**1**	**10**

The solution above is: s_1 = 30, s_2 = 10, x_5 = 10, s_4 = 10, the rest of the variables are zero in value. Obj. Fn Value = 50.

The above solution is not optimal because there are 4 coefficients in row 0 with negative value.

Question 2: [16 points]

Consider the problem

Minimize $3x_1 + 2x_2$

s.t.

$-x_1 + x_2 \leq 1$

$5x_1 + 3x_2 \leq 15$

$2x_2 \geq 3$

$x_1, x_2 \geq 0$

a) Setup the initial simplex tableau using the Big M method. State the basic feasible solution revealed in the initial tableau. Is the solution revealed in the initial simplex tableau feasible to the original problem? [4 points]

The problem in standard form is:

Minimize $Z = 3x_1 + 2x_2$

s.t.

$$\begin{aligned} -x_1 + x_2 \quad + s_1 \quad &= 1 \\ 5x_1 + 3x_2 \quad + s_2 \quad &= 15 \\ 2x_2 \quad - s_3 \quad &= 3 \end{aligned}$$

$x_1, x_2, s_1, s_2, s_3 \geq 0$

An artificial variable a_3 is introduced as a slack in row 3 to make the problem in canonical form. Thus, the new problem with the artificial variable is:

Minimize $Z = 3x_1 + 2x_2 + Ma_3$

s.t.

$$\begin{aligned} -x_1 + x_2 \quad + s_1 \quad &= 1 \\ 5x_1 + 3x_2 \quad + s_2 \quad &= 15 \\ 2x_2 \quad - s_3 + a_3 \quad &= 3 \end{aligned}$$

$x_1, x_2, s_1, s_2, s_3, a_3 \geq 0$

The initial tableau without pricing out a_3 is:

Z	x_1	x_2	s_1	s_2	s_3	a_3	RHS
1	-3	-2	0	0	0	-M	0
0	-1	1	1	0	0	0	1
0	5	3	0	1	0	0	15
0	0	2	0	0	-1	1	3

Row 0 is priced out by replacing row 0 by multiplying row 3 by M and adding it to row 0.

The initial canonical tableau:

Z	x_1	x_2	s_1	s_2	s_3	a_3	RHS
1	-3	-2+2M	0	0	-M	0	3M
0	-1	1	1	0	0	0	1
0	5	3	0	1	0	0	15
0	0	2	0	0	-1	1	3

The solution reflected above is: $s_1 = 1$, $s_2 = 15$ and $a_3 = 3$ and rest of variables equal to 0. This solution is not feasible to the original problem since $a_3 > 0$, implying that constraint 3 is violated.

b) Is the initial basic feasible solution optimal? Why or Why not? If not, perform one iteration to obtain the next basic feasible solution. Identify clearly the entering and leaving variable. Is the new basic feasible solution feasible to the original problem? Why or Why not? [12 points].

The initial BFS is not optimal as the coefficient of x_2 in row 0 is +ve (because coefficient of M is +ve). The entering variable is x_2. Ratios by row is: (1/1, 15/3, 3/2). Minimum Ratio = Min (1/1, 15/3, 3/2) = 1. The leaving variable is s_1. The new simplex tableau is:

Z	x_1	x_2	s_1	s_2	s_3	a_3	RHS
1	2M-5	0	-2M+2	0	-M	0	M+2
0	-1	1	1	0	0	0	1
0	2	0	-3	1	0	0	12
0	2	0	-4	0	-1	1	1

The new basic feasible solution is: $x_2 = 1$, $s_2 = 12$, $a_3 = 1$ and the rest of the variables equal to 0. This solution is not feasible to the original problem since a_3 is positive, implying that constraint 3 is violated.