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EEE391

Name:

# EEE391
# Basics of Signals and Systems
**Midterm 1, 15 March 2006**

110 minutes. Closed Books and Notes

**IMPORTANT:** **FULLY JUSTIFY ALL ANSWERS**

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Q1 10pts a) Find the signal obtained by summing x(t) and y(t), where, $x(t)=6\cos(20t+5)$ and $y(t)=3\sin(20t)$. Is the result periodic? If so, find its period and frequency.

5pts b) Write y(t) of part (a) in terms of superposition of complex exponential functions.

Q2 A signal $x_a(t)$ is defined as: $x_a(t) = 3j + \cos(3t+\pi) + 8\cos(10t+\pi/3)+je^{-j6t}$

15pts a) Find the spectrum of $x_a(t)$. Present your answer both as a table and a plot.

5pts b) Is $x_a(t)$ periodic? If so, find its period and frequency.

5pts c) The variable of $x_a(t)$ is time in seconds. $x_a(t)$ is sampled with a sampling period of $T_s = 0.4$ sec. to obtain a discrete signal $x[n] = x_a(nT_s)$. Find x[n].

15pts d) x[n] is converted to an analog signal y(t) by an ideal D/C converter with $f_s=1/T_s$. Find y(t).

Q3 A discrete-time system is defined as $y[n] = (n-1)x[n]+x[n-1]$

20pts a) Is this system linear? Is it time-invariant? Is it causal?

Q4 A discrete-time linear and time-invariant FIR system has an impulse response $h[n] = \delta[n+1]+3\delta[n]+\delta[n-1]$.

20pts a) If $x[n] = u[n]$ is applied as an input to this system, where u[n] is the unit step function, find and plot its output. (Provide numerical results for each element of the output.) Indicate the transient and steady state ranges of the output.

5pts b) Is this system causal?

**END**

RESERVE 1

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MIDTERM 1 (15 March 2006) SOLUTIONS

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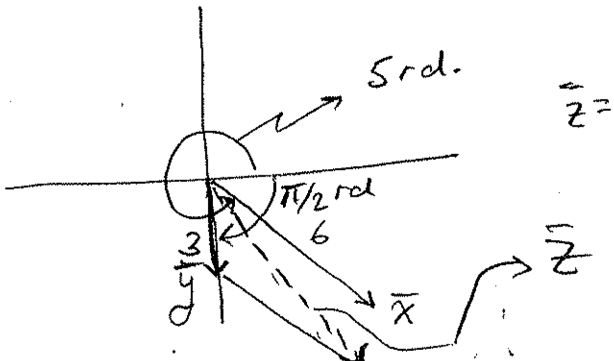
1-a) $x(t) = 6\cos(20t+5)$

$y(t) = 3\sin(20t) = 3\cos(20t - \frac{\pi}{2})$

Using phasors :

$\bar{z} = \bar{x} + \bar{y}$




$\bar{x} = 6e^{j5}$

$\bar{y} = 3e^{-j\pi/2} = -3j$

$\therefore \bar{z} = 6\cos 5 + j6\sin 5 - 3j$

$= 6\cos 5 + j(6\sin 5 - 3) = \underbrace{\sqrt{36\cos^2 5 + (6\sin 5 - 3)^2}}_{A}\ e^{j\phi}$

$$\boxed{\phi = \tan^{-1}\left\{\frac{6\sin 5 - 3}{6\cos 5}\right\}}$$

Note that $\tan^{-1}(\cdot)$ is a multivalued function. Take the value at the fourth quadrant as shown in the figure.

A can be further simplified as.

$A = \left(36\cos^2 5 + 36\sin^2 5 - 36\sin 5 + 9\right)^{1/2}$

$= (45 - 36\sin 5)^{1/2} = \boxed{9\sqrt{5 - 4\sin 5}}$

Periodic
freq $= \frac{20}{2\pi}$ Hz
Period $= \frac{\pi}{10}$ sec.

b) $\boxed{y(t) = 3\,\frac{e^{j20t} - e^{-j20t}}{2j}}$

RESERVE

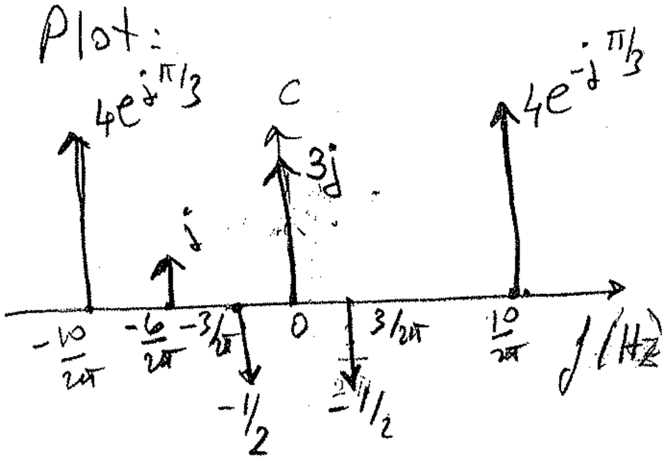
2. a) First write $x_a(t)$ as a superposition of complex sinusoids:

$$x_1(t) = 3j e^{j0 \cdot t} + \frac{1}{2} e^{j(3t+\pi)} + \frac{1}{2} e^{-j(3t+\pi)} + 4 e^{j(10t+\pi/3)} + 4 e^{-j(10t+\pi/3)} + j e^{-j6t}$$

Then make a table of amplitudes vs existing frequencies:

| $f_k$ (Hz) | Amplitude $c_k$ |
|---|---|
| $0$ | $3j$ |
| $\frac{3}{2\pi}$ | $\frac{1}{2} e^{j\pi} = -\frac{1}{2}$ |
| $-\frac{3}{2\pi}$ | $\frac{1}{2} e^{-j\pi} = -\frac{1}{2}$ |
| $\frac{10}{2\pi}$ | $4 e^{j\pi/3}$ |
| $-\frac{10}{2\pi}$ | $4 e^{-j\pi/3}$ |
| $-\frac{6}{2\pi}$ | $j$ |

Plot:



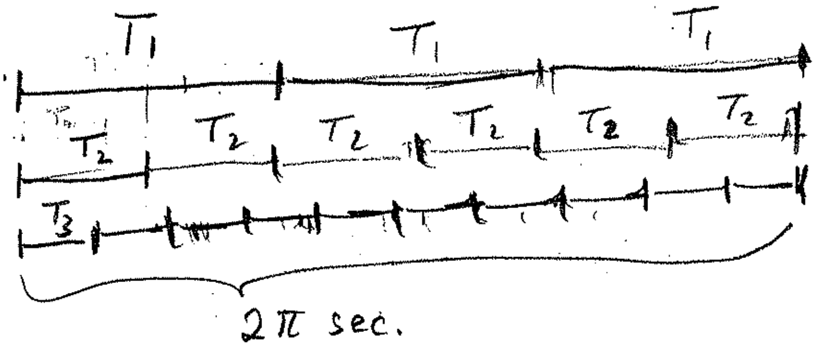
b) $x_a(t)$ has four components: DC, cos with freq. $\frac{3}{2\pi}$ Hz, another cosine with freq. $\frac{10}{2\pi}$ Hz and a complex sinusoidal with freq $-\frac{6}{2\pi}$ Hz.

Therefore, the periods are

$$T_1 = \frac{2\pi}{3}\text{ sec}, \quad T_3 = \frac{2\pi}{10}\text{ sec}, \quad T_2 = \frac{2\pi}{6}\text{ sec}.$$

a DC signal is periodic with <u>any</u> $T \neq 0$.

$\therefore$




$T_1 \cdot 3 = T_3 \cdot 10 = T_2 \cdot 6$ = common period

$= \boxed{2\pi \text{ sec}}$ (LCM of periods)

Yes $x_a(t)$ is periodic.

Fundamental period $= 2\pi$ sec. $= T$

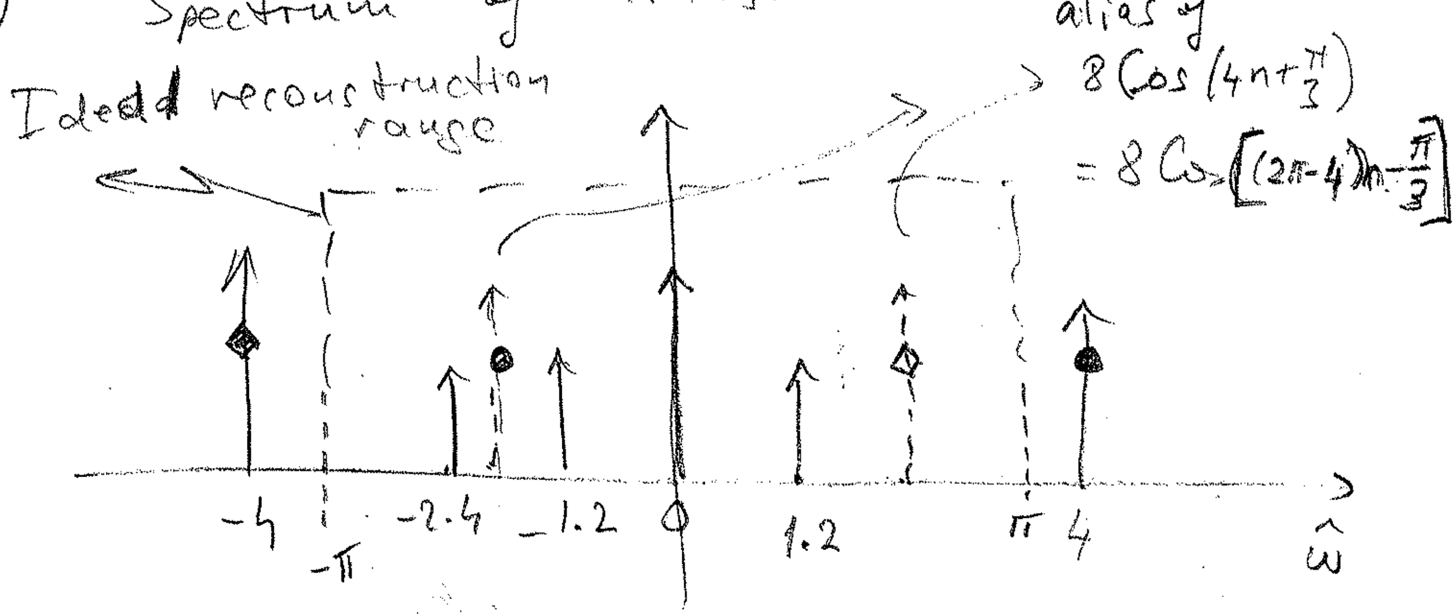
Frequency $= \frac{1}{T} = \frac{1}{2\pi}$ Hz. $= 1$ rd/sec.

RESERVE

c) $x[n] = x_a(nT_s) = 3j + \cos(3\cdot 0.4n + \pi) + 8\cos(10\cdot 0.4n + \frac{\pi}{3}) + je^{-j6\cdot 0.4n}$

$= 3j + \cos(1.2n + \pi) + 8\cos(4n + \frac{\pi}{3}) + je^{-j2.4n}$

d) Spectrum of $x[n]$:




But $|4|$ is larger than $2\pi$ ∴ it has an alias with $\hat{\omega} \in [-\pi, \pi)$, $\hat{\omega} = 4 + 2\pi p$

For $p = -1$, $\hat{\omega} = 4 - 2\pi \approx -2.28$ rd.

Similarly $p=1$, $\hat{\omega} = -4 + 2\pi \approx 2.28$ rd.

$$\therefore \boxed{y(t) = 3j + \cos(3t + \pi) + 8\cos\left[\frac{(2\pi - 4)}{0.4}t - \frac{\pi}{3}\right] + je^{-j6t}}$$

aliased component

3) a)

$$x_1[n] \longrightarrow \boxed{T} \longrightarrow (n-1)x_1[n] + x_1[n-1] = y_1[n]$$

$$x_2[n] \qquad (n-1)x_2[n] + x_2[n-1] = y_2[n]$$

$$x_3[n] \qquad (n-1)x_3[n] + x_3[n-1] = y_3[n]$$

$$x_3[n] = a x_1[n] + b x_2[n]$$

$$\therefore \quad y_3[n] = (n-1)\, x_3[n] + x_3[n-1]$$

$$= (n-1)\left[a x_1[n] + b_2 x_2[n]\right] + a x_1(n-1) + b\, x_2[n-1]$$

$$= a\left[(n-1)\, x_1[n] + x_1(n-1)\right]$$

$$+ b\left[(n-1) x_2[n] + x_2[n-1]\right]$$

$$= a\, y_1[n] + b\, y_2[n] \quad \therefore \quad \boxed{\text{Linear}}$$

$$x[n] \longrightarrow \boxed{T} \longrightarrow (n-1)\, x[n] + x[n-1] = y[n]$$

$$x[n-n_0] \qquad (n-1)\, x[n-n_0] + x[n-n_0-1] = T\left[x[n-n_0]\right]$$

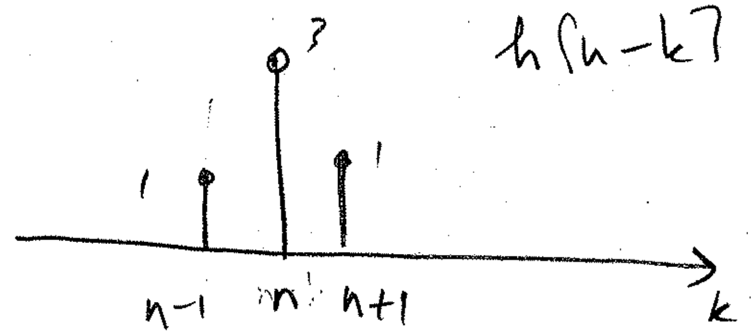
But $y[n-n_0] = (n-n_0-1)\, x[n-n_0] + x[n-n_0-1]$

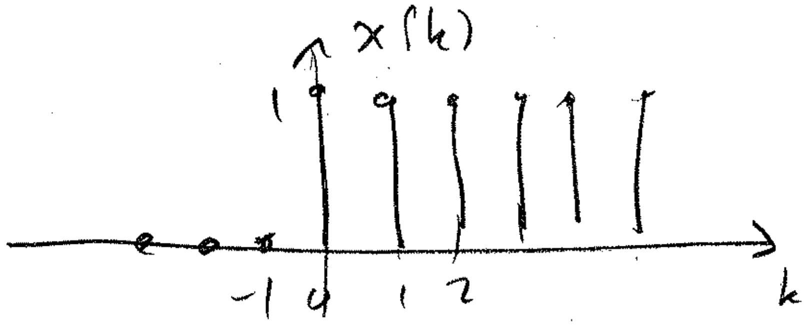
So: $y[n-n_0] \neq T\left[x[n-n_0]\right] \Rightarrow \boxed{\text{Not TI}}$

RESERVE

Since $y[n]$ is computed only by using $x[n]$ & $x[n-1]$, no future values of input is needed to compute the output at a given time $n$. $\Rightarrow$ $\boxed{\text{CAUSAL}}$

4-)




$$\therefore \quad y[n] = \sum x[k]\, h[n-k]$$

$$y[n] = \begin{cases} 0 & n < -1 \\ 1 & n = -1 \\ 4 & n = 0 \\ 5 & n \geq 1 \end{cases}$$

RESERVE