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Section 1

IE400 2018 Fall
Project
Meghdut Telecom

Below are the necessary decision variables for building the constraints and writing objective function:

$C_i = binary\ variable\ taking\ value\ of\ 1\ when\ city\ i\ is\ linked\ in\ year\ 1, for\ each\ i$

$X_{ij} =\ binary\ variable\ taking\ value\ of\ 1\ when\ city\ i\ and\ city\ j\ is\ linked\ in\ year\ 1,\ for\ i < j$

$F_i =\ binary\ variable\ taking\ value\ of\ 1\ when\ city\ i\ is\ linked\ in\ year\ 1\ and$

$in\ positions\ 2\ or\ 4\ in\ the\ exhibit\ 1,\ for\ each\ i$

$Y_{ij} = \ =\ binary\ variable\ taking\ value\ of\ 1\ when\ city\ i\ and\ city\ j\ is\ linked\ in\ year\ 2,\ for\ i < j$

$K_{ij} = 1\ if\ revenues\ for\ connection\ between\ city\ i\ and\ j\ are\ realized\ in\ first\ year$

Our parameters are C_{ij}, $R1_{ij}$ and $R2_{ij}$ for costs of links, first year revenues and second year revenues. (The question does not state explicitly, but I assume that revenues from pairs of connected cities in year 1 will also be collected in year 2, since they will continue to be linked).

Here, i takes values between 1 and 7. After designing these decision variables, our three Integer Programming problems are:

1) Revenue maximization

Max $$\sum_{i=1}^{6} \sum_{j>i}^{7} K_{ij} R1_{ij} + \sum_{i=1}^{6} \sum_{j>i}^{7} R2_{ij}$$

Subject to:

1) $\sum_{i=1}^{7} C_i = 4$ Only four cities linked in year1

2) $\sum_{i=1}^{7} F_i = 2$ Only two cities in position 2 and 4

3) $F_i \leq C_i\ for\ each\ i$ Cities in position 2 and 4 must be in year 1 linked cities

4) $\left(\sum_{j>i}^{7} X_{ij} + \sum_{j=1}^{j<i} X_{ij} \right) = 2C_i\ for\ each\ i$ every connected city in year 1 has two links in year 1

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5) $\sum_{i=1}^{6} \sum_{j>i}^{7} X_{ij} = 4$ total of four links in year 1

6) $\left(\sum_{j>i}^{7} Y_{ij} + \sum_{j=1}^{j<i} Y_{ij} \right) \leq 2\left(1 - C_i\right) + F_i \, for\ each\ i$ A link in year 2 can be across cities that are not linked in year 1 and cities in position 2 and 4. A city has one link in second year if it is in position 2 and 4. If it is not connected in year 1, then in year 2 it must have two links.

7) $\sum_{i=1}^{6} \sum_{j>i}^{7} Y_{ij} = 4$ total of four links in year 2

8) $X_{ij} + Y_{ij} \leq 1 \, for\ each\ i < j$ A link can be built in first year or second year or not built at all

9) $K_{ij} \leq \frac{1}{2} C_i + \frac{1}{2} C_j \, for\ each\ i < j$ First Year revenues for any pair will be collected only when cities i and j are in year 1 connected cities

10) $\sum_{i=1}^{6} \sum_{j>i}^{7} K_{ij} = 6$ 6 pairs in year 1 for revenue collection

11) $X_{ij} + F_i + F_j \leq 2 \, for\ each\ i < j$ Cities in position 2 and 4 are not connected in first year

All decision variables are binary.

2) Cost minimization

Min $$\sum_{i=1}^{6} \sum_{j>i}^{7} C_{ij} \left(X_{ij} + Y_{ij}\right)$$

Subject to:

1) $\sum_{i=1}^{7} C_i = 4$ Only four cities linked in year1

2) $\sum_{i=1}^{7} F_i = 2$ Only two cities in position 2 and 4

3) $F_i \leq C_i \, for \, each \, i$ Cities in position 2 and 4 must be in year 1 linked cities

4) $\left(\sum_{j>i}^{7} X_{ij} + \sum_{j=1}^{j<i} X_{ij} \right) = 2C_i \, for \, each \, i$ every connected city in year 1 has two links in year 1

5) $\sum_{i=1}^{6} \sum_{j>i}^{7} X_{ij} = 4$ total of four links in year 1

6) $\left(\sum_{j>i}^{7} Y_{ij} + \sum_{j=1}^{j<i} Y_{ij} \right) \leq 2(1 - C_i) + F_i \, for \, each \, i$ A link in year 2 can be across cities that are not linked in year 1 and cities in position 2 and 4. A city has one link in second year if it is in position 2 and 4. If it is not connected in year 1, then in year 2 it must have two links.

7) $\sum_{i=1}^{6} \sum_{j>i}^{7} Y_{ij} = 4$ total of four links in year 2

8) $X_{ij} + Y_{ij} \leq 1 \, for \, each \, i < j$ A link can be built in first year or second year or not built at all

9) $X_{ij} + F_i + F_j \leq 2 \, for \, each \, i < j$ Cities in position 2 and 4 are not connected in first year

All decision variables are binary.

3) Profit maximization

Max $$\sum_{i=1}^{6} \sum_{j>i}^{7} K_{ij} R1_{ij} + \sum_{i=1}^{6} \sum_{j>i}^{7} R2_{ij} - \sum_{i=1}^{6} \sum_{j>i}^{7} C_{ij} (X_{ij} + Y_{ij})$$

Subject to:

1) $\sum_{i=1}^{7} C_i = 4$ Only four cities linked in year1

2) $\sum_{i=1}^{7} F_i = 2$ Only two cities in position 2 and 4

3) $F_i \leq C_i \, for \, each \, i$ Cities in position 2 and 4 must be in year 1 linked cities

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4) $\left(\sum_{j>i}^{7} X_{ij} + \sum_{j=1}^{j<i} X_{ij} \right) = 2C_i \ for\ each\ i$ every connected city in year 1 has two links in year 1

5) $\sum_{i=1}^{6} \sum_{j>i}^{7} X_{ij} = 4$ total of four links in year 1

6) $\left(\sum_{j>i}^{7} Y_{ij} + \sum_{j=1}^{j<i} Y_{ij} \right) \leq 2(1 - C_i) + F_i for\ each\ i$ A link in year 2 can be across cities that are not linked in year 1 and cities in position 2 and 4. A city has one link in second year if it is in position 2 and 4. If it is not connected in year 1, then in year 2 it must have two links.

7) $\sum_{i=1}^{6} \sum_{j>i}^{7} Y_{ij} = 4$ total of four links in year 2

8) $X_{ij} + Y_{ij} \leq 1 \ for\ each\ i < j$ A link can be built in first year or second year or not built at all

9) $K_{ij} \leq \frac{1}{2} C_i + \frac{1}{2} C_j for\ each\ i < j$ First Year revenues for any pair will be collected only when cities i and j are in year 1 connected cities

10) $\sum_{i=1}^{6} \sum_{j>i}^{7} K_{ij} = 6$ 6 pairs in year 1 for revenue collection

11) $X_{ij} + F_i + F_j \leq 2 \ for\ each\ i < j$ Cities in position 2 and 4 are not connected in first year

All decision variables are binary.

Solution of these problems with Excel solver yields the following results:

(First Year links in blue, second year links in orange)

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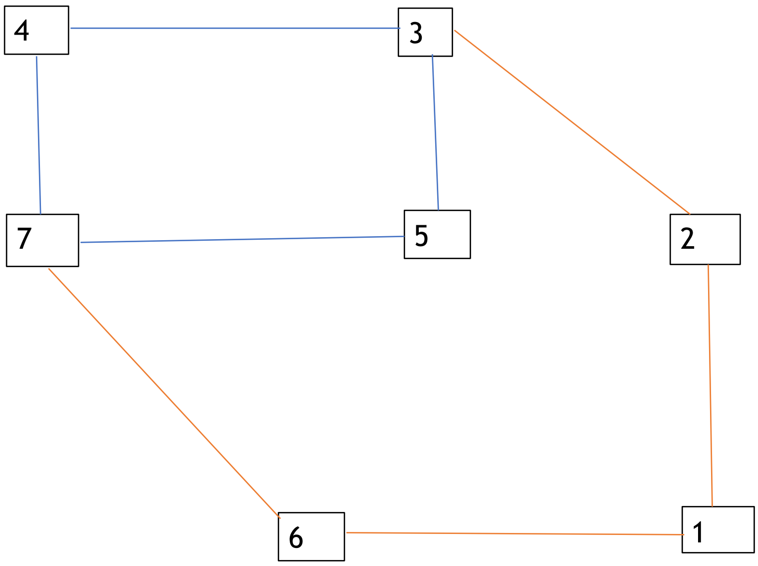
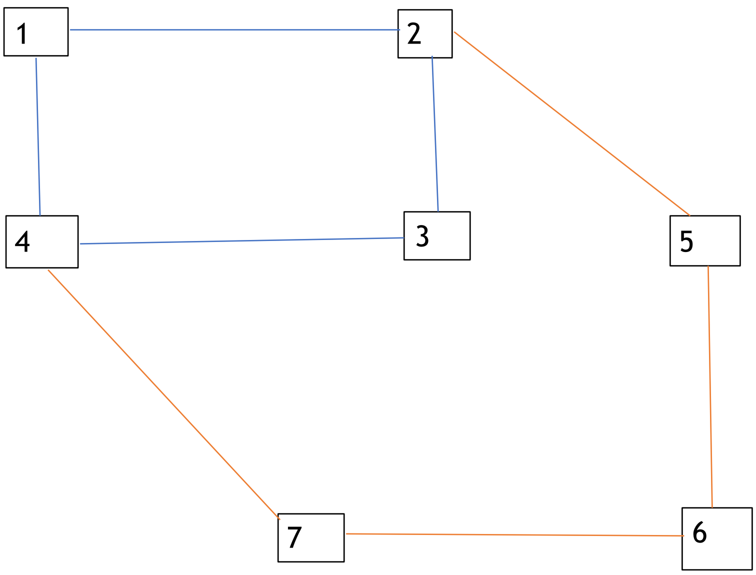
Revenue Maximization:

Maximum Revenue: 954



Cost Minimization:

Minimum Cost: 915



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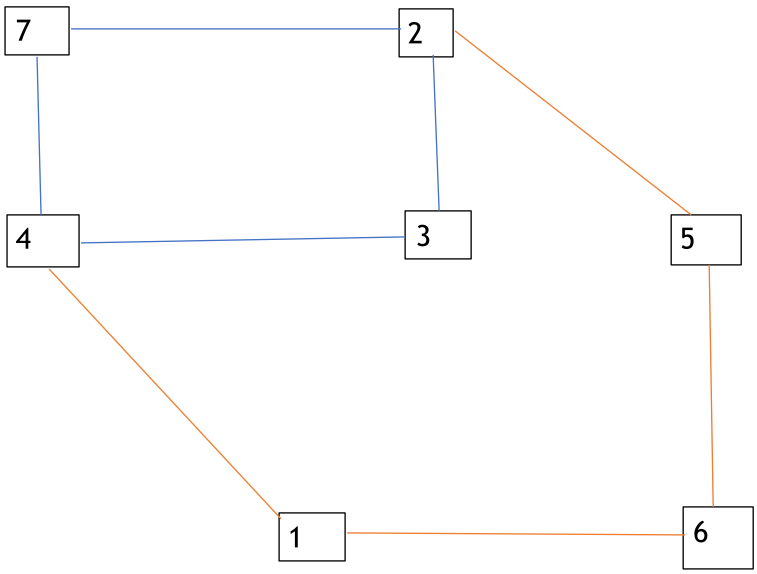
Profit Maximization:

Maximum Profit: -33



Since this is a long-term plan, considering only two years of revenues for revenue and profit calculations would be misleading. Minimizing costs would be more reasonable.