CS476: Automata Theory and Formal Languages
Homework 2
Due: 15/04/2014, 17:00

Questions

1. (40pts) Give context free grammars (CFG) and construct pushdown automata (PDA) for the following languages. Argue that the grammar you gave generates the language correctly. Explain how each PDA works and why it is correct.

    (a) $L = \{a^m b^n c^p d^q \mid m + n = p + q\}$

    (b) $L = \{w_1 c w_2 c ... c w_k c c w_j^R \mid k \geq 1, 1 \leq j \leq k, w_i \in \{a, b\}^+ \text{ for } i = 1, ..., k\}$

    (c) $L = \{a^k b^l c^m \mid k = |l - m|\}$

    (d) $L = \{w \in \{a, b, c, \}^* \mid |w| = 3n_a(w)\}$

2. (32pts) Prove that the following languages are not context free

    (a) $L = \{a^i b^j c^k | k = ij\}$

    (b) $L = \{a^i b^j | j \leq i^2\}$

    (c) $L = \{www | w \in \{a, b\}^*\}$

    (d) $L = \{a^p \mid p \text{ is prime}\}$

3. (13pts) Consider the following grammar: (Apply the following procedures using the algorithms discussed in class)

$$S \rightarrow AB \mid aB \mid aC \mid bD$$
$$A \rightarrow aab \mid \epsilon$$
$$B \rightarrow bbA$$
$$C \rightarrow aD \mid bD$$
$$D \rightarrow Ca \mid Cb$$

    (a) Eliminate any useless symbols.

    (b) Eliminate any ϵ productions in the resulting grammar.

    (c) Eliminate any unit productions in the resulting grammar.

    (d) Put the resulting grammar into Chomsky Normal Form.

4. (15pts) Consider the following grammar:

$$S \rightarrow aSb \mid bA \mid Ba$$
$$A \rightarrow bA \mid B$$
$$B \rightarrow aB \mid A \mid \epsilon$$

    (a) Describe the language of this grammar in English.

    (b) Prove that the grammar is ambiguous.

    (c) Give an unambiguous grammar for the language.

    (d) Explain that the grammar you gave in (c) is unambiguous and it generates the language correctly.