CS476: Automata Theory and Formal Languages

Homework 3

Assigned: 19/12/2012

Due: 31/12/2012

Questions

1. (10pts) State whether the following statements are true or not. You must give a BRIEF explanation or show a counter example to receive full credit.

   (a) (2.5pts) If languages L_1 and L_2 are both undecidable then the language $L_1 \cup L_2$ is also undecidable.

   (b) (2.5pts) If languages L_1 and $L_1 - L_2$ are both decidable then the language L_2 is also decidable.

   (c) (2.5pts) If a language is undecidable then it is NP-complete.

   (d) (2.5pts) For $P = NP$ to be the case, Boolean Satisfiability Problem (SAT) has to be polynomial time solvable.

2. (15pts) A *k-head* Turing machine has k heads reading cells of one tape. A move of this TM depends on the state and on the symbol scanned by each head. In one move, the TM can change state, write a new symbol on the cell scanned by each head, and can move each head left, right, or keep it stationary. Since several heads may be scanning the same cell, we assume the heads are numbered 1 through k, and the symbol written by te highest numbered head scanning a given cell is the one that actually gets written there. Show that this variant of TM model is equivalent to ordinary TM model.

3. (30pts) Disprove (by reduction) or prove that the following languages are decidable.

   (a) (10pts) $PALINDROM_{DFA} = \{\langle A\rangle : A \text{ is a DFA and } L(A) = L^R(A)\}$, where;

   $$L^R(A) = \{w^R : w \in L(A)\}$$

   (b) (10pts) $SUPERSET_{TM} = \{\langle M, S\rangle : M \text{ is a TM, } S \text{ is a finite set and } L(M) \supseteq S\ \}$

   (c) (10pts) FRIENDS$_{TM}$ $= \{\langle M, M'\rangle : M, M' \text{ are TM's where both } M \text{ accepts } \langle M'\rangle \text{ and } M' \text{ accepts } \langle M\rangle.\}$

4. (15pts) Let us define *Glue* operation on languages as follows:

   $$Glue(L_1, L_2) = \{w_1 v w_2 : |v| \geq 1; w_1 v \in L_1; v w_2 \in L_2; v, w_1, w_2 \in \Sigma^*\}$$

   (a) Show that the class of decidable languages are closed under *Glue* operation.

   (b) Show that the class of recognizable languages are closed under *Glue* operation.

5. (30pts) Prove that the following problems are NP-complete.

   (a) (15pts) *Multi-Knapsack*: Given m knapsacks, a set $S = \{t_1, t_2, \ldots, t_n\}$ of n items each of which is associated with a weight $w_i \in Z^+$ and m values $v_{ij} \in Z^+$ which denotes the value of item t_i whenever it has been put into j^{th} knapsack and two positive integers W,V; can you find an assignment such that each item is assigned to exactly one of m knapsacks, total weight of each knapsack is smaller than or equal to W and total value of items with respect to the assignment is larger than or equal to V.

   (b) (15pts) *Vertex Clique Cover* (VCC): Given a graph $G = (V, E)$ and a positive integer q; can you find a set of cliques in G that together touch all vertices and the number of cliques in the set does not exceed q? (Hint: You can use *Graph Coloring* problem.)