CS473 Assignment 3

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Solution is in the last page.

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(a) $\Theta(n^{log_a b})$ Case 1

(b) $\Theta(n^2 log n)$ Case 2

(c) $\Theta(n^\epsilon)$ Case 3

(d) $\Theta(n^\epsilon (log n)^b)$ Case 3

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(a) Given:

$\exists$ a constant $c > 0$, $T(\alpha n) + T((1 - \alpha)n) + cn$

Guess:

$\exists$ a constant $d \geq 0$ such that $T(n) \leq dn\log n$

Substitute:

$d\alpha n\log(\alpha n) + d(1 - \alpha)n\log((1 - \alpha)n) + cn \leq dn\log n$

(b) Given:

$\exists$ a constant $c > 0$, $cn + \sum\limits_{i=1}^{k} T(n/2^i)$

Guess:

$\exists$ a constant $d \geq 0$ such that $T(n) \leq dn$

Substitute:

$cn + kdn \sum\limits_{i=1}^{k} 1/2^i \leq dn$

$cn + kdn\frac{(2^k - 1)}{2^k} \leq dn$

$c + kd\frac{(2^k - 1)}{2^k} \leq d$

$\frac{c}{1 - k(2^k - 1)/2^k} \leq d$

Thus, $T(n) = O(n)$.

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Require: A, p, r
  $x \leftarrow A[p]$
  $i \leftarrow p$
  $j \leftarrow r$
  $flag \leftarrow false$
  while $i \neq j$ **do**
    if flag **then**
      if $A[j] > x$ **then**
        exchange $A[i]$ with $A[j]$
        exchange i with j
        $flag \leftarrow false$
        $j \leftarrow j - 1$
      else
        $j \leftarrow j + 1$
      end if
    else
      if $A[i] > x$ **then**
        exchange $A[i]$ with $A[j]$
        exchange i with j
        $flag \leftarrow true$
        $j \leftarrow j + 1$
      else
        $j \leftarrow j - 1$
      end if
    end if
  end while