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EEE 391
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EEE 391

Basics of Signals and Systems
Final Exam
16 May 2006, Tuesday

closed book and notes

Given Time: 110 min

Instructor: Billur Barshan

Last Name : ....

First Name : ....

ID number : ....

Signature : ....

Exam Part	Total Points	Points Received
Q1	25	
Q2	25	
Q3	25	
Q4	25	
Total	100	

Allocation of points:

1) 25 pts (a) 13 pts (b) 12 pts
2) 25 pts (a) 13 pts (b) 12 pts
3) 25 pts (a) 20 pts (b) 5 pts
4) 25 pts (a) 10 pts (b) 15 pts

Attention: Read all the questions carefully and show your work for full or partial credit. Please put your answer for each part in the box provided.

Given Formulas:

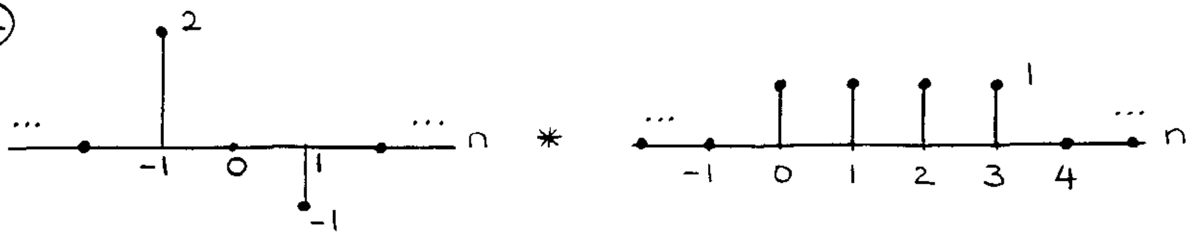
$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

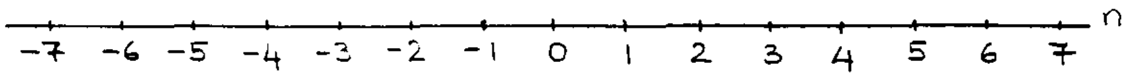
(1-8)

① Evaluate the convolutions:

a)



<u>Answer:</u>

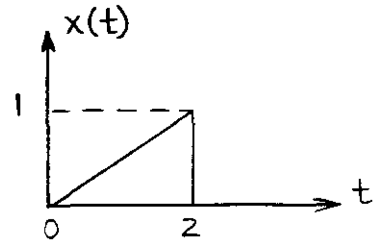


b) $\cos(2\pi t) * \cos(4\pi t)$

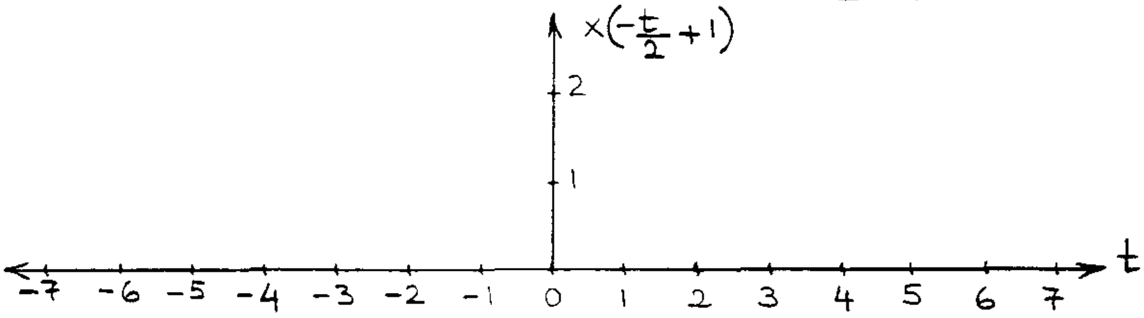
<u>Answer:</u>

Please show your work here.

2



a) Given the signal $x(t)$ above, plot the signal $x(-\frac{t}{2}+1)$ in the space provided below.



b) If the Fourier transform of $x(t)$ is $X(j\omega)$, find the Fourier transform of $3x(-\frac{t}{2}+1)$ in terms of $X(j\omega)$.

Answer:

Please show your work here.

(3) The signal $x(t) = \cos 200\pi t + 0.2 \cos 700\pi t$ is sampled ideally at a rate of 400 samples per second. The sampled signal is then passed through an ideal low-pass filter with a cut-off frequency of 200 Hz and bandwidth of 400 Hz.

a) Write the expression for $y(t)$, the output of the low-pass filter.

b) Briefly discuss whether (i) aliasing (ii) folding occur in this system.

RESERVE

4

a) The input x(t) and the output y(t) of a causal linear time invariant system are related by the differential equation:

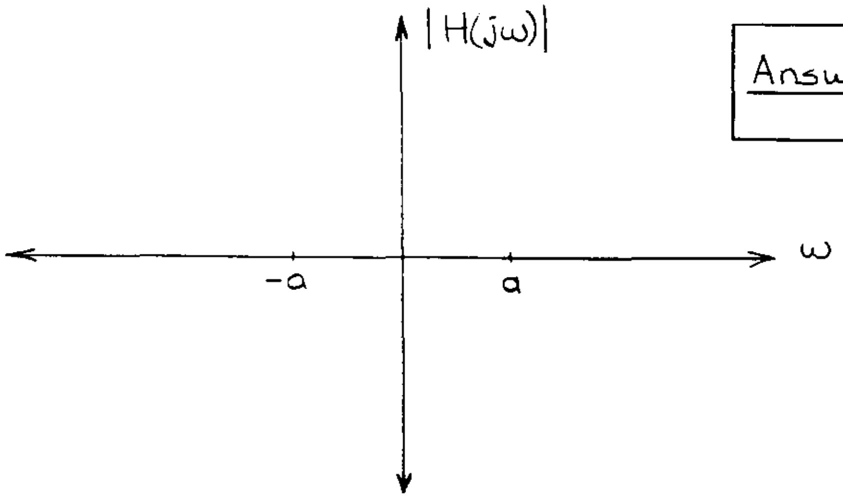
$$\frac{d^2y(t)}{dt^2} + \sqrt{2}\,\frac{dy(t)}{dt} + y(t) = 2\,\frac{dx^2(t)}{dt^2} - 3\,x(t)$$

Find the frequency response function $H(j\omega)$ for this system.

<u>Answer</u>:

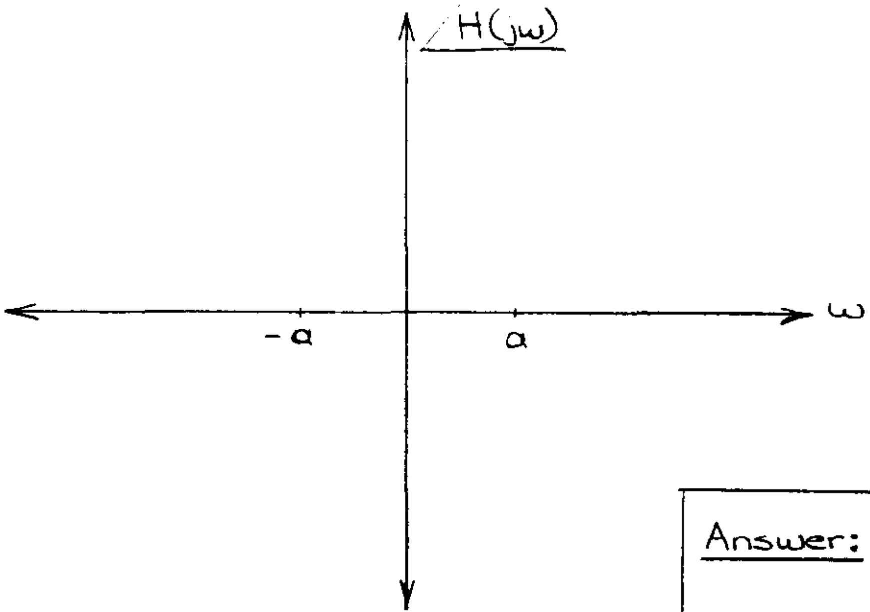
b) Given the frequency response function $H(j\omega) = \dfrac{j\omega}{a+j\omega}\, e^{-j\pi/2}$ $(a>0)$

i) Find the magnitude response $|H(j\omega)|$ and sketch it below.



<u>Answer</u>:

ii) Find the phase response $\angle H(j\omega)$ and sketch it below.



<u>Answer</u>:

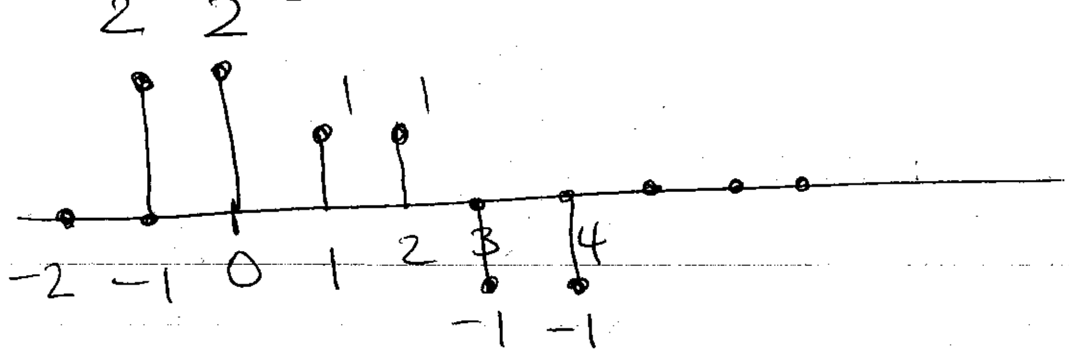
Please show your work at the back.

EEE 391 Spring 05-06

Final Exam Solutions:

1)

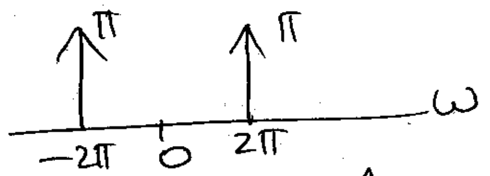
a)

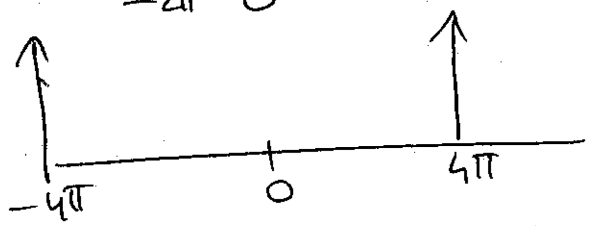


b) $\cos 2\pi t \xleftrightarrow{F} X_1(j\omega) = \pi\,\delta(\omega-2\pi) + \pi\,\delta(\omega+2\pi)$

$\cos 4\pi t \xleftrightarrow{F} X_2(j\omega) = \pi\,\delta(\omega-4\pi) + \pi\,\delta(\omega+4\pi)$

$\cos 2\pi t * \cos 4\pi t \longleftrightarrow X_1(j\omega)X_2(j\omega) = 0$

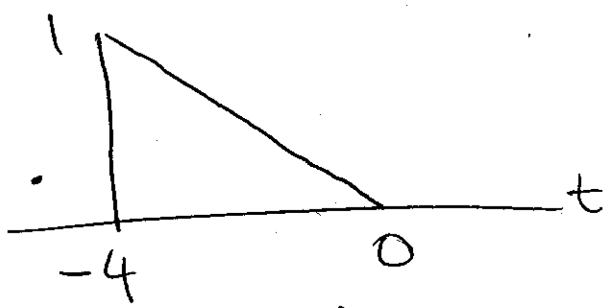




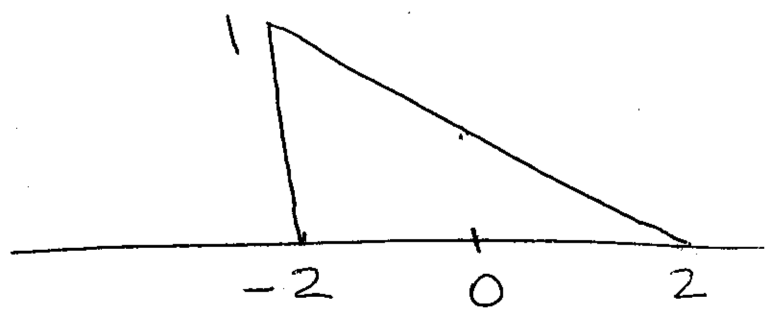
2)

a) $x\left(-\frac{1}{2}(t-2)\right)$

$x\left(-\frac{t}{2}\right)$



$x\left(-\frac{1}{2}(t-2)\right)$



b) $\cancel{3}x(t) \xleftrightarrow{F} X(j\omega)$

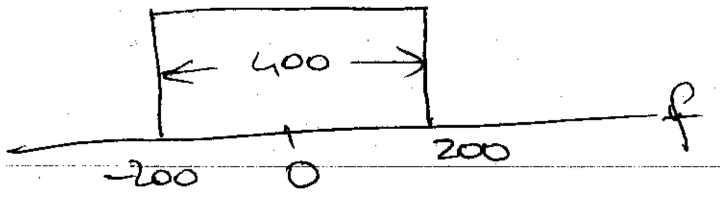
$3x\left(-\frac{t}{2}+1\right) \xleftrightarrow{F} 3$

$3x\left(-\frac{1}{2}(t-2)\right) \xleftrightarrow{F} 3\cdot e^{-j\omega 2}\cdot\frac{1}{|-1/2|}X\left(j\frac{\omega}{-1/2}\right) = 3e^{-j2\omega}\cdot 2X(-j2\omega)$

$= 6e^{-j2\omega}X(-j2\omega)$

③ $x(t) = \cos 200\pi t + 0.2 \cos 700\pi t$

$f_s = 400$ Hz.



a) $\hat{\omega} = \dfrac{\omega}{f_s} = \dfrac{2\pi f}{f_s}$

$\omega_1 = 200\pi$

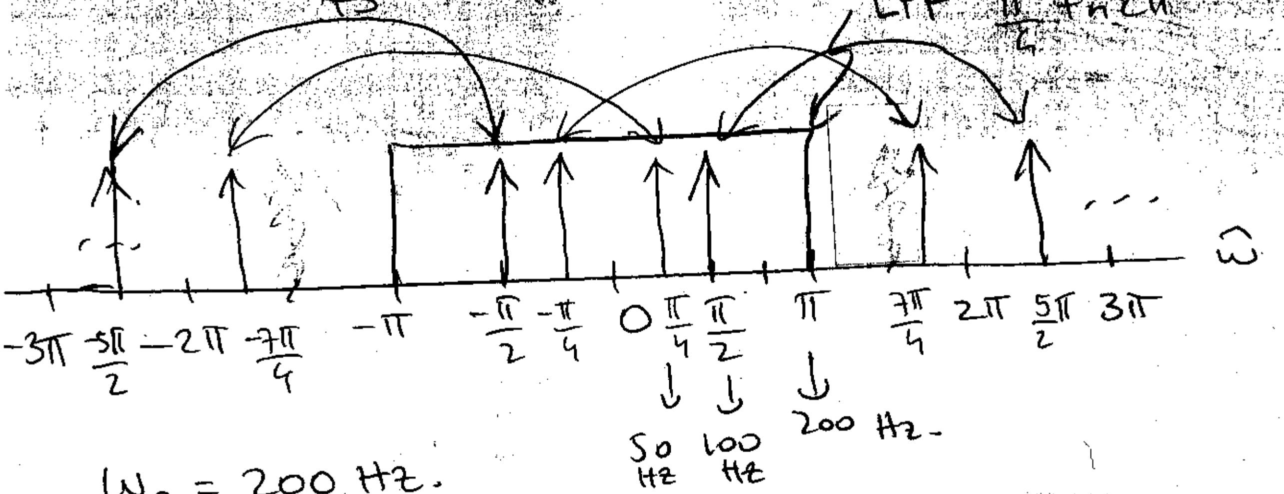
$\hat{\omega}_1 = 200\pi \cdot T_s = \dfrac{200\pi}{f_s} = \dfrac{200\pi}{400} = \dfrac{\pi}{2} + n2\pi \quad n = 0, \pm1, \pm2, \dots$

$-\dfrac{\pi}{2} + n2\pi$ "

$\omega_2 = \underline{700\pi}$ aliases:

$\hat{\omega}_2 = \dfrac{700\pi}{f_s} = \dfrac{700\pi}{400} = \dfrac{7}{4}\pi \qquad -\dfrac{\pi}{4} + n2\pi \quad n = 0, \pm1, \pm2, \dots$

$\dfrac{\pi}{4} + n2\pi$ "



$W_c = 200$ Hz.

$\hat{\omega}_c = \dfrac{200 \cdot 2\pi}{f_s} = \dfrac{200 \cdot 2\pi}{400} = \pi$

$y(t) = \cos 200\pi t + 0.2 \cos 100\pi t$

b) aliasing, yes, for $0.2 \cos 700\pi t$

folding " " " "

RESERVE

(4) a) $\left[(j\omega)^2 + \sqrt{2}\, j\omega + 1\right] Y(j\omega) = \left[2(j\omega)^2 - 3\right] X(j\omega)$

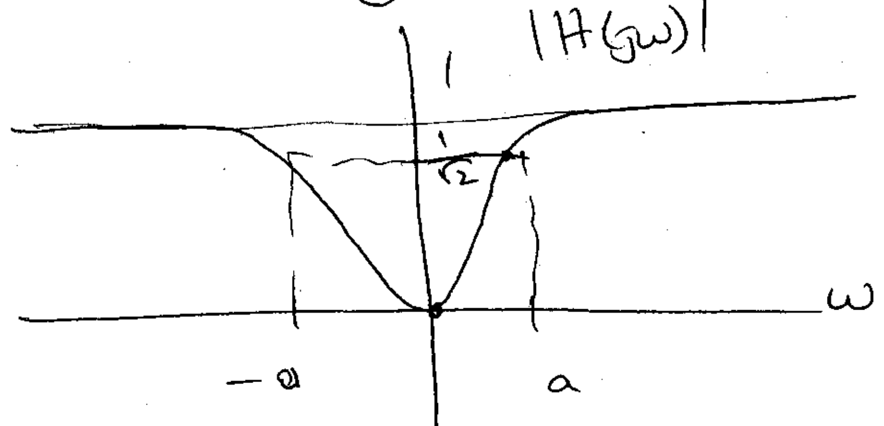
$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{2(j\omega)^2 - 3}{(j\omega)^2 + \sqrt{2}\, j\omega + 1} = \frac{-\left[2\omega^2+3\right]}{-\omega^2+\sqrt{2}j\omega+1}$$

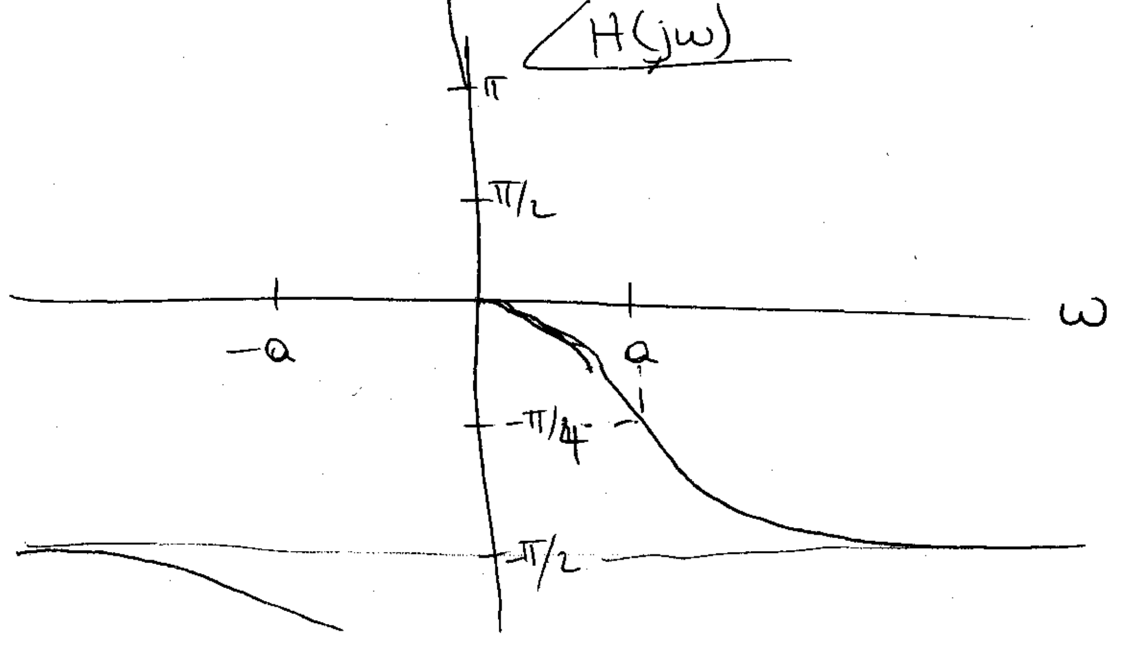
$$= \frac{2\omega^2+3}{\omega^2-\sqrt{2}j\omega-1}$$

b) $H(j\omega) = \dfrac{j\omega}{a+j\omega} e^{-j\pi/2} \quad a>0$

$$|H(j\omega)| = \frac{|\omega|}{\sqrt{a^2+\omega^2}}$$

$$\angle H(j\omega) = \begin{cases} \cancel{\frac{\pi}{2}} - \tan^{-1}\frac{\omega}{a} - \cancel{\frac{\pi}{2}} & \omega>0 \\ -\frac{\pi}{2} - \tan^{-1}\frac{\omega}{a} - \frac{\pi}{2} & \omega<0 \end{cases} = \begin{cases} -\tan^{-1}\frac{\omega}{a} & \omega>0 \\ -\pi - \tan^{-1}\frac{\omega}{a} & \omega<0 \end{cases}$$





RESERVE