IE400-Fall 2018-2019

Study Set 1

Q1) The Southern Confederation of Kibbutzim is a group of three kibbutzim (farming communities). Overall planning for this group is done by an office and this office currently is planning agricultural production for the coming year. The agricultural output of each kibbutz is limited by both the amount of available irrigable land and water allocated for irrigation. These data are given in Table 1. The crops suited for this region include sugar beets, cotton and sorghum. These crops differ primarily in their return per acre and their consumption of water. In addition to that there is a maximum quota for the total acreage that can be devoted to each of these crops (Table 2). To ensure equity between the three kibbutzim, it has been agreed that every kibbutz will plant the same proportion of its available irrigable land. For example, if kibbutz 1 will plant 200 of its available 400 acres, then kibbutz 2 must plant 300 of its 600 acres while kibbutz 3 plants 150 acres of its 300 acres. Formulate an LP whose solution will maximize the total net return to the Southern Confederation of Kibbutzim as a whole.

Kibbutz	Usable Land(Acres)	Water Allocation(Acre Feet)
1	400	600
2	600	800
3	300	375

Table 1

Crop	Maximum Quota(Acres)	Water Consumption(Acre Feet/Acre)	Net Return($/Acre)
Sugar Beets	600	3	1000
Cotton	500	2	750
Sorghum	325	1	250

Table 2

Q2) Chemco produces two chemicals: A and B. These chemicals are produced via two manufacturing processes. Process 1 requires 2 hours of labor and 1 lb of raw material to produce 2 oz of A, and 1 oz of B. Process 2 requires 3 hours of labor and 2 lb of raw material to produce 3 oz of A, and 2 oz of B. 60 hours of labor and 40 lb of raw material are available. Demand for A is unlimited but only 20 oz of B can be sold. A sells for $16/oz and B sells for $14/oz. Any B that is unsold must be disposed of at a cost of $2/oz. Formulate an LP to maximize Chemco’s revenue less disposal cost.

Q3) Alexis Cornby makes her living buying and selling corn. On January 1, she has 50 tons of corn and $1000. On the first day of each month, Alexis can buy corn at the following prices per ton: January, $300; February, $350; March, $400; April, $500. On the last day of each month, Alexis can sell corn at the following prices per ton: January, $250; February, $400; March, $350; April, $550. Alexis stores her corn in a warehouse that can hold at most 100 tons of corn. She must be able to pay cash for all corn at the time of purchase.

Use linear programming how Alexis can maximize her cash on hand at the end of April.

Q4) Willy Wonka's Candy Company produces three types of candy: Wonka Bars, Bottle Caps and Giant Sweet Tarts. In order to produce the different type of candies, Willy can run three different production processes as described below. Each process involves blending different types of sugars in the Magical Factories Mixer.

Process 1: Running Process 1 for one hour:

Costs: $5
Requires: Two barrels of sugar type A and three barrels of sugar type B
Output: Two Wonka Bars and one packet of Bottle Caps

Process 2: Running Process 2 for one hour:

Costs: $4
Requires: One barrel of sugar type A and three barrels of sugar type B
Output: Three packets of Bottle Caps

Process 3: Running Process 3 for one hour:

Costs: $1
Requires: Two barrels of sugar type B and three packets of Bottle Caps
Output: Two Wonka Bars and one packet of Giant Sweet tarts

Each week we can purchase:
200 barrels of sugar type A at $2 per barrel
300 barrels of sugar type B at $3 per barrel

Assume that they can sell everything that they can produce.
Wonka Bars are sold at $9 per bar
Bottle Caps are sold at $10 per packet
Giant Sweet Tarts are sold at $24 per packet.

Assume that 100 hours of mixing are available.

a) Formulate an LP whose solution will maximize Willy Wonka's profit. **Hint:** Use decision variable x_i: number of hours process i runs, i=1, 2, 3.

b) Assume that instead of having 200 barrels of sugar type A and 300 barrels of sugar type B available that you can order a total of 500 Barrels. Show how to modify your LP formulation in part a to account for this revised problem.

c) Suppose that instead of selling the tree candies separately, they can only be sold as part of a box consisting of one Wonka Bar, two packets of Bottle Caps and one pack of Giant Sweet Tarts. Each Wonka Box sells for $54. Modify your LP formulation in part a to model this new scenario. (You may need to add another decision variable.)

Q5) A company wishes to plan its production of two products with seasonal demands over a 12 month period from the beginning of January. The monthly demand of product 1 is 100,000 kg during the months of October, November and December; 10,000 kg during the months of January, February, March and April; and 30,000 kg during the remaining months. The demand of product 2 is 50,000 kg during the months of October through February and 15,000 kg during the remaining months. Suppose that cost of producing a kg of product 1 and 2 is $5 and $8, respectively, provided that these were produced prior to June. After June, the costs are reduced to $4.5/kg and $7/kg because of the installation of an improved production system. The total amount of products 1 and 2 that can be produced during any particular month cannot exceed 120,000 kg for Jan-Sept and 150,000 kg for Oct-Dec. Furthermore, at the end of each month products left at the hand are carried to the inventory space in order to be used in the coming months. Each kg of product 1 occupies 2 cubic-feet and each kg of product 2 4 cubic-feet of inventory. Suppose that the maximum inventory space allocated to these products is 150,000 cubic-feet and that the holding cost per cubic foot during any month is $0.10. Formulate the production scheduling problem so that total production and inventory costs are minimized.

Q6) I now have $100 and three choices of investment during the next four years. For every dollar invested: *Investment A* yields $0.10 a year after the investment and yields $1.30 three years after the investment, *Investment B* yields $0.20 a year after the investment and yields $1.10 two years after the investment, *Investment C* yields $1.50 three years after the investment. Investments A and C are available in the beginning of year 1 and 2 while B is available in the beginning of years 1, 2 and 3. During each year, uninvested cash can be placed in money market funds, which yield 6% interest per year. At most $50 may be placed in each of investments A, B, and C in each year. Formulate an LP to maximize my cash on hand four years from now.

Q7) Consider the following simplex tableau of a given maximization LP problem.

	z	x_1	x_2	x_3	x_4	x_5	RHS
z	1	0	**P**	0	0	2	**R**
x_1	0	1	**A**	0	0	1	**D**
x_3	0	0	**B**	1	0	3	**E**
x_4	0	0	**C**	0	1	0	**F**

a) Give the current bfs and the current objective value in terms of given letters.

b) Suppose that x_2 enters and x_3 leaves. Find the new bfs and the corresponding objective function value (in terms of given letters).

c) Suppose that x_2 enters and x_4 leaves, and **F**=0. Does the solution change? If yes, find the new bfs and the corresponding objective function value (in terms of given letters). If no, explain.

Q8) Determine whether the statements below are true or false. For each of your answer, you need to provide a proof or counterexample, whichever is necessary.

a) Set $S = \{x: f(x) \leq 10\}$ where f is a continuous function, is a convex set.

b) Set $S = \{x \in \mathbb{R}^n: Ax \leq b\}$ where A is $m \times n$, b is $m \times 1$, is a convex set.

c) Set $S = \{x \in \mathbb{R}^n: Ax \leq b\}$ where A is $m \times n$, b is $m \times 1$, always has at least one extreme point.

d) Set $S = \{x \in \mathbb{R}: |x| \geq 4\}$ where $|.|$ is absolute value sign, is convex.

e) Consider $S_1 = \{x \in \mathbb{R}^n: Ax \leq b\}$ and $S_2 = \{x \in \mathbb{R}^n: Cx \leq d\}$ where A and C are $m \times n$, b and d are $m \times 1$, are convex sets in $\mathbb{R}^n$. Set $S_1 \cap S_2$ is also convex.

Q9) For the LP given in the below:

$$\begin{aligned} \max \quad & ax_1 + bx_2 \\ \text{s.t.} \quad & 4x_1 + x_2 \leq 12 \\ & x_1 - x_2 \leq 4 \\ & x_1 \geq 0 \end{aligned}$$

a) Find **all possible relationship(s)** between a and b that characterizes the set of all alternative optimal solutions for the LP. Also, indicate their signs.

b) Give an **example** a and b which makes point (3.2, -0.8) unique optimal.

c) Give an **example** a and b which makes point (0, 12) unique optimal.

Q10) Consider the following two linear programming problems:

LP1	LP2
$\min\ c_1x+c_2y$	$\max\ c_1x+c_2y$
$x-2y \le 4$	$x-2y \le 4$
$-x+y \le 3$	$-x+y \le 3$
$x \ge 0$	$x \ge 0$
$y \ge 0$	$y \ge 0$

a) Find coefficients c_1 and c_2 such that both LP1 and LP2 will be unbounded. (Note that the same coefficients will be used in both models.)

b) Find coefficients c_1 and c_2 such that LP1 will have alternative optimal solutions and LP2 will be unbounded. (Note that the same coefficients will be used in both models.)

c) Find coefficients c_1 and c_2 such that LP1 will have a unique optimal solution and LP2 will be unbounded. (Note that the same coefficients will be used in both models.)

Q11) Convert the following LP to standard form:

$$\min z = 3x_1 + x_2$$
$$s.t \quad x_2 - 3x_1 \ge 3$$
$$x_1 + x_2 \le 4$$
$$2x_1 - x_1 + 3x_3 = 3$$
$$x_1 \ge 0 \ \ x_2 \le 0$$
$$x_3\ free$$

Q12) Identify all the basic solutions in the following linear system.

$$3x_1 - x_2 + 5x_3 + 3x_4 = 15$$
$$x_1 + 3x_2 + x_4 = 10$$
$$x_i \ge 0\ ;\ \ \forall\, i \in \{1,2,3,4\}$$

Q13) Consider the problem

$$\min z = 5x_1 + 3x_2$$
$$s.t\ \ 4x_1 + 2x_2 \le 12$$
$$4x_1 + x_2 \le 10$$

$$x_1 + x_2 \leq 4$$

$$x_1, x_2 \geq 0$$

a) Convert the problem into standard form and construct a bfs at which $(x_1, x_2) = (0,0)$.

b) Carry out the full tableau implementation of the simplex method, starting with the basic feasible solution of part (a).

c) Draw a graphical representation of the problem in terms of the original variables x_1, x_2 and indicate the path taken by the simplex algorithm.