Q1) Ford has four automobile plants. Each is capable of producing the Taurus, Lincoln or Escort but it can only produce one of these cars. The fixed cost of operating each plant for a year and the variable cost of producing a car of each type at each plant are given in the table below. Ford faces the following restrictions:

- Each plant can produce one type of car.
- The total production of each type of car must be at a single plant; that is, for example, if any Tauruses are made at plant 1, then all Tauruses must be made there.
- If plants 3 and 4 are used, then plant 1 must also be used.

Formulate the problem in order to minimize the total cost.

	Fixed Cost	Variable Costs		
		Taurus	Lincoln	Escort
Plant 1	7 billion	12000	16000	9000
Plant 2	6 billion	15000	18000	11000
Plant 3	4 billion	17000	19000	12000
Plant 4	2 billion	19000	22000	14000

Q2) A power plant has three boilers. If a given boiler is operated, it can be used to produce a quantity of steam (in tons) between the minimum and maximum given in the first table below. The cost of producing a ton of steam on each boiler is also given. Steam from the boilers is used to produce power on three turbines. If operated, each turbine can process an amount of steam (in tons) between the minimum and maximum given in the second table. The cost of processing

a ton of steam and the power produced by each turbine is also given. Formulate an IP that can be used to minimize the cost of producing 8000 kWh of power.

Boiler Number	Minimum Steam	Maximum Steam	Cost/Ton
1	500	1000	10
2	300	900	8
3	400	800	6

Turbine Number	Minimum	Maximum	KWh per Ton of Steam	Processing Cost per Ton
1	300	600	4	2
2	500	800	5	3
3	600	900	6	4

Q3) A Tourism Company provides a pick-up service for Italian and German tourist groups who are coming to Istanbul Ataturk Airport. They send translators who are fluent in Italian and German to pick the incoming groups and assist them. The company divided a day into six 4 hour periods and translators work in these 4 hour periods. Each translator works 2 periods in a row. The numbers of required Italian and German speaking translators for each period are given below. The hourly wage of an Italian speaking translator is $40/hour and the hourly wage of a German speaking translator is $35/hour. If necessary, the company also sends extra translators who are fluent in both Italian and German to the airport in order to assist the regular translators. These extra translators are hired for single periods only. The hourly wage of employing these extra translators is $60/hour. Formulate an integer linear programming model to allocate the translators for each period with minimum total cost.

Period	Italian	German
1	5	4
2	3	4
3	8	7
4	8	7
5	11	13
6	4	4

Q4) In the planning of the monthly production for the next six months a company must, in each month, operate either a normal shift or an extended shift (if it produces at all). A normal shift costs £100,000 per month and can produce up to 5,000 units per month. An extended shift costs £180,000 per month and can produce up to 7,500 units per month. Note here that, for either type of shift, the cost incurred is fixed by a union guarantee agreement and so is independent of the amount produced.

It is estimated that changing from a normal shift in one month to an extended shift in the next month costs an extra £15,000. No extra cost is incurred in changing from an extended shift in one month to a normal shift in the next month.

The cost of holding stock is estimated to be £2 per unit per month (based on the stock held at the end of each month) and the initial stock is 3,000 units (produced by a normal shift). The amount in stock at the end of month 6 should be at least 2,000 units. The demand for the company's product in each of the next six months is estimated to be as shown below:

Month	1	2	3	4	5	6
Demand	6,000	6,500	7,500	7,000	6,000	6,000

Production constraints are such that if the company produces anything in a particular month it must produce at least 2,000 units. If the company wants a production plan for the next six months that avoids stock outs, formulate their problem as an integer program.

Q5) NASA has 4 astronauts to be employed at a space mission for 4 posts on a space ship. The posts require different qualifications (competencies) but each astronaut can perform each one of these tasks, albeit with varying degree of proficiency. The proficiency rating of astronaut i assigned to post j is measured as p_{ij}. It is asked to find the optimal assignment of astronauts to posts which gives the highest total proficiency rating while meeting the following requirements:

    i. Each astronaut should be assigned to a single post.

    ii. Each post should have a single astronaut.

Q6) A paper manufacturer produces rolls of standard fixed width w and of standard length l. Customers order rolls of width w but varying lengths. In particular, d_k rolls with length l_k and width w are ordered for customer $k=1...n$ (assume $l_k \leq l$ *forall* $k=1..n$). What is the minimum number of rolls that should be cut to meet the demand?

Hint: Let M be a large number such that M number of rolls are enough to satisfy all demand (e.g. $M=\sum_{k=1}^{n} d_k$ is such value) Assume the manufacturer has M number of unmanufactured rolls in the stock. Use decision variables:

$$x_i = \begin{cases} 1 \ if \ roll \ i \ is \ used \\ 0 \qquad\qquad\quad o.w. \end{cases}$$

y_{ik} = number of rolls of type k (of length l_k) cut from roll i, $i=1,...,M$, $k=1,...,n$

Q7) Use the two-phase method to find the optimal solution to the following LP:

$$
\begin{aligned}
\text{Min} \quad & -3 \, x_1 + 4 \, x_2 \\
\text{s.t.} \quad & x_1 + x_2 \leq 4 \\
& 2 \, x_1 + 3 \, x_2 \geq 18 \\
& x_1, \quad x_2 \geq 0
\end{aligned}
$$

Q8) Consider the following simplex tableau of a given **minimization** LP problem.

	z	x_1	x_2	x_3	s_1	s_2	s_3	RHS
z	1	0	0	**A**	-10	-1	**B**	32
s_3	0	0	0	**C**	4	-2	1	**D**
x_2	0	0	1	-2	**F**	-1	0	6
x_1	0	**E**	0	-1	1	2	0	4

Give **general** conditions on each of the unknowns **A-F** such that each of the following statements is true. Even if the statement holds independently of the values of a specific variable, you should still mention that variable.

a) The tableau is final and there exists a unique optimal solution.
b) The simplex method determines an unbounded solution from this tableau.
c) The current bfs is degenerate (not necessarily optimal).
d) The current solution is optimal, there are alternative optimal solutions but no alternative optimal bfs.

Q9) Identify the basic solutions, basic feasible solutions and degenerate basic feasible solutions (if exists) in the following linear system.

$$x_1 - x_2 + 3x_3 + 5x_4 = 15$$
$$x_1 + x_2 + x_4 = 9$$
$$x_i \geq 0 \quad i = 1..4$$

Q10) Convert the following LP to standard form and solve with the Big-M method.

$$\max \; -x_1 + 3x_2 + x_3$$

$$s.t \; -x_1 + x_2 - x_3 \geq -4$$

$$2x_1 + x_2 + x_3 \geq 10$$

$$x_1 \geq 0 \; x_2 \leq 0 \; x_3 \; free$$

Q11) Read each of the following statements and indicate whether it is TRUE or FALSE. For each statement, if the statement is TRUE explain why, if FALSE provide a counter example.

a) Every LP with an unbounded feasible region is unbounded.

b) A linear programming problem cannot have exactly one feasible solution.

c) Phase I LP will always stop with an optimal tableau.

d) If Big M method stops with unboundedness, then 2-phase method will definitely move to second phase.

e) If during some iteration of simplex method, a bfs is degenerate, then the LP will stop with alternative optimum solutions.

f) If during some iteration of simplex method, a bfs is degenerate, after one iteration of simplex method, it will stay degenerate.

g) Any convex combination of two optimal solutions (if exist) to an LP is also optimal.

h) Consider an LP that has a finite optimum solution. It is possible to change the objective function so that the LP becomes unbounded.

Q12) Use the simplex algorithm to find two optimal solutions to the following LP:

$$
\begin{aligned}
\text{Max} \quad & 5x_1 + 3x_2 + x_3 \\
\text{s.t.} \quad & x_1 + x_2 + 3x_3 \le 6 \\
& 5x_1 + 3x_2 + 6x_3 \le 15 \\
& x_i \ge 0 \quad i = 1..3
\end{aligned}
$$

Q13) Consider the following linear program

$$\text{Max } cx$$

$$\text{st. } Ax = b$$

$$x \ge 0$$

where A is a matrix of order mxn, rank m and c, b, and x are vectors of appropriate dimensions. Let P denote the feasible region for this LP i.e. P={ x € R^n : Ax=b, x ≥ 0 } Read each of the following statements carefully and indicate whether it is TRUE or FALSE and explain your answer.

a) If a new constraint is added to P then the objective function value increases.

b) If n = m + 1 then the number of basic feasible solutions of P is at most m + 1

c) At an iteration of the simplex algorithm, the solution found may have more than m positive variables.

d) It is possible for this LP to have exactly two optimal solutions.

e) Every extreme point of P corresponds to exactly one basis matrix B.

Q14)

Consider the following problem:

$$
\begin{aligned}
\text{max} \quad & 2x_1 + x_2 + 6x_3 - 4x_4 \\
\text{s.t.} \quad & x_1 + 2x_2 + 4x_3 - x_4 && \le 6 \\
& 2x_1 + 3x_2 - x_3 + x_4 && \le 12 \\
& x_1 + x_3 + x_4 && \le 6 \\
& x_1, x_2, x_3, x_4 \ge 0
\end{aligned}
$$

Convert this LP to standard form. Construct the simplex tableau for the basis: x_1, x_2 and x_4. Indicate whether the tableau is optimal, if not state the entering and the leaving variables. You do not need to perform further iterations.