CS473 ASSIGNMENT-1

Boran Yildirim Yigit Bediskan

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1

(a) ***Loop Invariant:*** $gcd(a,b) = gcd(x,y) \quad and \quad x >= 0 \quad and \quad y >= 0$

(b) ***Initialization:*** At the beginning of the first iteration x is still equal to a, and y is still equal to b, therefore,

$$gcd(a,b) = gcd(x,y) \quad and \quad x >= 0 \quad and \quad y >= 0$$

Maintanence: Assume that at the beginning of the ith iteration,

$$gcd(a,b) = gcd(x,y) \quad and \quad x >= 0 \quad and \quad y >= 0$$

In the ith iteration, if $(x > y)$, then $x_1 = x - y$, x_1 is positive, and $gcd(x_1, y) = gcd(a,b)$, because any number that divides x and y also divides $x - y$, else if $(x < y)$, then $y_1 = y - x$, y_1 is positive, and $gcd(x, y_1) = gcd(a,b)$, because any number that divides x and y also divides $y - x$.

(c) ***Termination:*** The loop terminates when x = y. At this point, since $gcd(a,b) = gcd(x,y) \quad and \quad x >= 0 \quad and \quad y >= 0$ Thus the procedure returns x.

2

(a)

Require: A, B, n
  $i \leftarrow 0$
  $j \leftarrow n - 1$
  while $i < n$ **and** $j > 0$ **do**
    $tmp \leftarrow A[i] + B[j]$
    if $tmp == x$ **then**
      return FOUND
    else if $tmp > x$ **then**
      $j \leftarrow j - 1$
    else if $tmp < x$ **then**
      $i \leftarrow i + 1$
    end if
  end while

  return NOTFOUND

(b)

(c) ***Loop Invariant:*** At the beginning of each iteration of the while loop,

$$i + j = n - 1$$

Initialization: At the beginning of the first iteration,

$$i = 0 \quad and \quad j = n - 1,$$

$$therefore \quad i + j = n - 1$$

Maintenance: Assume that at the beginning of the ith iteration,

$$i + j = n - 1$$

In the ith iteration, if x is grater than (A[i] + B[j]) then i is incremented by 1, else if x is less than (A[i] + B[j]) then j is decremented by 1, thus i + j does not change and remains equal to n - 1 at the beginning of the $(i + 1)$th iteration.

3

(a)

n	Insertion Sort	Merge Sort
2^4	1e-06	6e-06
2^8	4.9e-05	3.4e-05
2^{12}	0.012053	0.0006336
2^{16}	3.03937	0.012187
2^{20}	∞	0.259265

(b)

n	Insertion Sort	Merge Sort
2^4	1e-06	6e-06
2^8	9.7e-05	2.2e-05
2^{12}	0.028046	0.00036
2^{16}	6.31218	0.009998
2^{20}	∞	0.138445