CS473-6

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May 2018

1

Greedy Choice: Scheduling the activity that has the closest (earliest) deadline first.
Proof: Let's say there is an optimal solution S for this algorithm and say activity a_1 has the earliest deadline among all the activities for the problem. If a_1 is the first activity in S, then proof is completed. Otherwise, we will construct another optimal solution S' that has the a_1 as the first activity in the set.

Initially, we will bring a_1 to the first place by delaying all other activities, that normally finishes before a_1, t_{a_1} times. Let's say there is another activity a_2 that is affected by that. If this activity normally delays Δa_2, then it is now delayed Δa_2 + t_{a_1} = f_{a_2} - d_{a_2} + t_{a_1} times. And since the finishing time of a_2 will definitely before than the starting time of a_1 (since a_2 comes before a_1 in S and we cannot process 2 activities at the same time) and deadline of a_1 will be smaller than the deadline of a_2 (since we assumed a_1 has the earliest deadline at the beginning), following conversions hold:

$$f_{a_2} - d_{a_2} + t_{a_1} \leq s_{a_1} - d_{a_2} + t_{a_1}$$

$$(since \quad s_{a_1} + t_{a_1} = f_{a_1})$$

$$f_{a_1} - d_{a_2} \leq f_{a_1} - d_{a_1}$$

$$(since \quad f_{a_1} - d_{a_1} = \Delta a_1)$$

$$\Delta a_2 + t_{a_1} \leq \Delta a_1$$

So, since a_1 is not the latest activity in S, there is at least one activity that has bigger delay than a_1. Thus, we can say that, the maximum delay in S' cannot be larger than the maximum delay in S. Also, since S is optimal, S' is optimal too. So, proof is completed.

2

Create a graph G where each vertex represents a wrestler and each edge represents a rivalry. The graph will contain V vertices and E edges. Perform as many BFS's as needed to visit all vertices. Assign all wrestlers whose distance is even to be $babyfaces$ and all wrestlers whose distance is odd to be $heels$. Then check each edge to verify that it goes between a $babyface$ and a $heel$. For the BFS, $O(V)$ time to designate each wrestler as a $babyface$ or $heel$, and $O(E)$ time to check edges, which is $O(V + E)$ time overall.

3

Statement is partially true. Since by the parenthesis theorem, there are 2 possibilities:

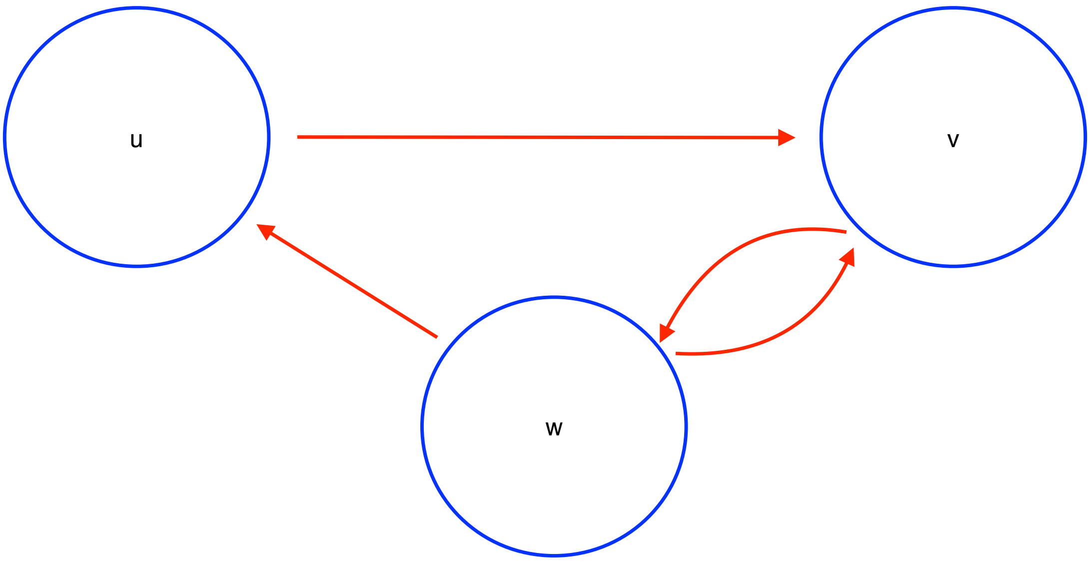
(a) If $u.d < u.f < v.d < v.f$, then none of them is the descendant of the other one. So, there is no cycle.

(b) If $v.d < u.d < u.f < v.f$, then v is the descendant of u. So, uv is on the cycle.

Therefore we can conclude that, $uv \in E$ may or may not be on a cycle by the parenthesis theorem.

4

The statement is false. As a counter-example, consider the following graph:



Assume that a DFS run on this graph discovers w before it discovers u and v (which is always possible since the outer for loop of generic DFS considers the vertices in arbitrary order). Then u and v will be white at time $w.d$. Now assume that DFS explores edge wv before edge wu. Then wv and wu will be tree edges, which will make uv a cross edge.