CS476: Automata Theory and Formal Languages

Homework 1

Due: 18/03/2014 17.00

Questions

1. (20pts) State whether the following statements are true or not. You must give a BRIEF explanation or show a counter example to receive full credit.

   (a) (5pts) If a subset of a regular language is regular then all of its subsets are regular.

   (b) (5pts)$(0111 + 1011 + 1101 + 1110)^*$ is the regular expression for the set of all strings with $3n_0 = n_1$ where n_0 and n_1 respectively denote the numbers of 0s and 1s in w.

   (c) (5pts) Let $R_1 = aba^+$ and $R_2 = ab^+a$ be two regular expressions then $R_1 \cup R_2 = ab(a + b)^*b$.

   (d) (5pts) With pumping lemma, we can prove that $L = \{w : w = 1000\}$ is a regular language.

2. (20pts) The following table shows a non-associative operation closed under set $A = \{a, b, c, d\}$. The left-to-right and right-to-left evaluation values of a string w, when interpreted as an expression, are represented as w_{lr} and w_{rl}, respectively. As an example if $w = adb$ then $w_{lr} = (ad)b = ab = c$ and $w_{rl} = a(db) = ac = b$.

	a	b	c	d
a	d	c	b	a
b	b	c	a	d
c	c	d	a	a
d	b	c	a	b

   (a) (6pts) Give a DFA for the following language with the alphabet A, the first input to the DFA will be the left-most letter of the string:

$$L = \{w : w \in A^*, w_{lr} = d\}. \tag{1}$$

   (b) (14pts) Give an NFA with minimum number of states for the following language with the alphabet A, the first input to the NFA will be the left-most letter of the string:

$$L = \{w : w \in A^*, w_{rl} = d\}. \tag{2}$$

3. (10pts) Give a regular expression for the set of all strings from $\{0, 1\}^*$, which are base 2 representations of a number which is a multiple of 5. For example, the string 1111 which represents $(1111)_2$ and equals 15, must be generated by the regular expression since it is a multiple of 5.

4. (20pts) If L is a regular language prove that the following languages are also regular.

   (a) (8pts) $L_1 = \{wv : w \in L, |v| = 2\}$

   (b) (12pts) $L_2 = \{w : w^r w \in L\}$

5. (30pts) Prove or disprove that the following languages are regular.

   (a) (15pts) $L = \{x\$y : x, y \in \{0, 1\}^*, n_0(x) = n_1(y)\}$.

   (b) (15pts) $L = \{xy : x, y \in \{0, 1\}^*, n_0(x) = n_1(y)\}$.