CS-473/573: Algorithms I Example

Quiz 6: Depth-First-Search (DFS)

Name: Solution

Let u and v be two vertices in a directed graph $G(V, E)$. Prove or disprove the following claim: If uv and vu are both in E, then in any run of DFS on G, exactly one of these two edges will be a back edge.

Note: If you will disprove, you must provide a counter-example. If you will prove, you must provide a rigorous argument proving that this will always be the case. You can refer to the theorems in the book (Parentheses Theorem, White Path Theorem) as appropriate (e.g., "we know by the parentheses theorem that...").

Solution:

The statement is true.

Assume without loss of generality that $u.d < v.d$. Then, at the time u is discovered, v is white, and since $uv \in E$, there is a white path from u to v. It follows by the white path theorem that v is a descendent of u, hence vu is a back edge and uv is either a tree edge or front edge.

Similarly, if $v.d < u.d$, then uv is a back edge and vu is either a tree edge or a front edge. Therefore, in either case, exactly one of uv or vu is a back edge.