

# Multivariate Linear Regression

Linear regression with multiple variables is also known as "multivariate linear regression".

We now introduce notation for equations where we can have any number of input variables.

$x_j^{(i)}$  = value of feature  $j$  in the  $i^{th}$  training example

$x^{(i)}$  = the input (features) of the  $i^{th}$  training example

$m$  = the number of training examples

$n$  = the number of features

The multivariable form of the hypothesis function accommodating these multiple features is as follows:

$$h_{\theta}(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \dots + \theta_n x_n$$

In order to develop intuition about this function, we can think about  $\theta_0$  as the basic price of a house,  $\theta_1$  as the price per square meter,  $\theta_2$  as the price per floor, etc.  $x_1$  will be the number of square meters in the house,  $x_2$  the number of floors, etc.

Using the definition of matrix multiplication, our multivariable hypothesis function can be concisely represented as:

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$$h_{\theta}(x) = [\theta_0 \quad \theta_1 \quad \dots \quad \theta_n] \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_n \end{bmatrix} = \theta^T x$$

## Gradient Descent for Multiple Variables

The gradient descent equation itself is generally the same form; we just have to repeat it for our 'n' features:

$$\begin{aligned} &\text{repeat until convergence: } \{ \\ &\quad \theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) \cdot x_0^{(i)} \\ &\quad \theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) \cdot x_1^{(i)} \\ &\quad \theta_2 := \theta_2 - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) \cdot x_2^{(i)} \\ &\quad \dots \\ &\quad \} \end{aligned}$$

In other words:

repeat until convergence: {  
 $\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) \cdot x_j^{(i)}$  for  $j := 0 \dots n$   
}

### Gradient Descent

Previously ( $n=1$ ):

Repeat {

→  $\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})$

$\frac{\partial}{\partial \theta_0} J(\theta)$

→  $\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_1^{(i)}$

(simultaneously update  $\theta_0, \theta_1$ )

}

New algorithm ( $n \geq 1$ ):

Repeat {

→  $\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_j^{(i)}$

(simultaneously update  $\theta_j$  for  $j = 0, \dots, n$ )

}

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→  $\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_0^{(i)}$

→  $\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_1^{(i)}$

→  $\theta_2 := \theta_2 - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_2^{(i)}$

...

*Handwritten notes in red:*

- Arrow from "New algorithm" to the first equation:  $\frac{\partial}{\partial \theta_j} J(\theta)$
- Arrow from "simultaneously update" to the index  $j$  in the loop.
- Red box around  $x_0^{(i)}$  in the first equation of the new algorithm, with a note  $x_0^{(i)} = 1$ .

## Feature Scaling

We can speed up gradient descent by having each of our input values in roughly the same range. This is because  $\theta$  will descend quickly on small ranges and slowly on large ranges, and so will oscillate inefficiently down to the optimum when the variables are very uneven.

The way to prevent this is to modify the ranges of our input variables so that they are all roughly the same. Ideally:

$$-1 \leq x^{(i)} \leq 1$$

or

$$-0.5 \leq x^{(i)} \leq 0.5$$

Two techniques to help with this are **feature scaling** and **mean normalization**. Feature scaling involves dividing the input values by the range (i.e. the maximum value minus the minimum value) of the input variable, resulting in a new range of just 1. Mean normalization involves subtracting the average value for an input variable from the values for that input variable resulting in a new average value for the input variable of just zero. To implement both of these techniques, adjust your input values as shown in this formula:

$$x_i := \frac{x_i - \mu_i}{s_i}$$

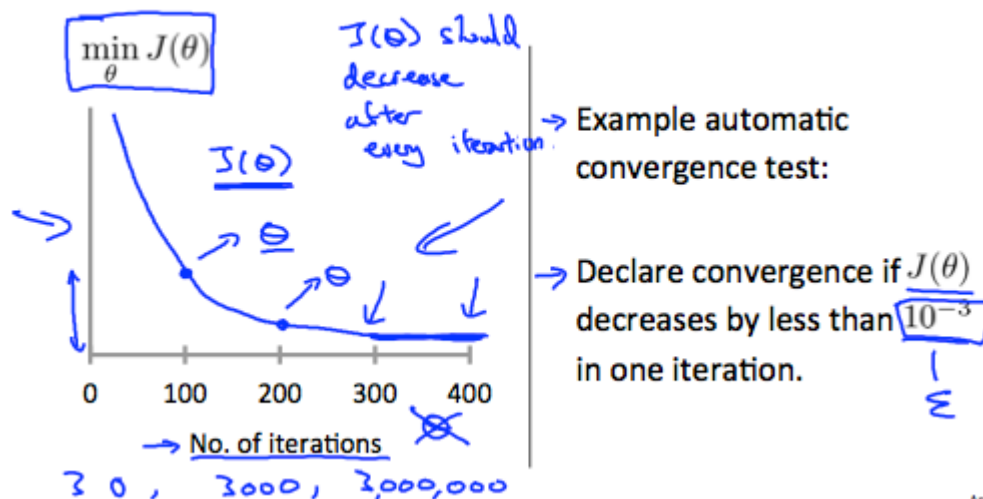
Where  $\mu_i$  is the **average** of all the values for feature (i) and  $s_i$  is the range of values (max - min), or  $s_i$  is the standard deviation.

## Learning Rate

**Debugging gradient descent.** Make a plot with *number of iterations* on the x-axis. Now plot the cost function,  $J(\theta)$  over the number of iterations of gradient descent. If  $J(\theta)$  ever increases, then you probably need to decrease  $\alpha$ .

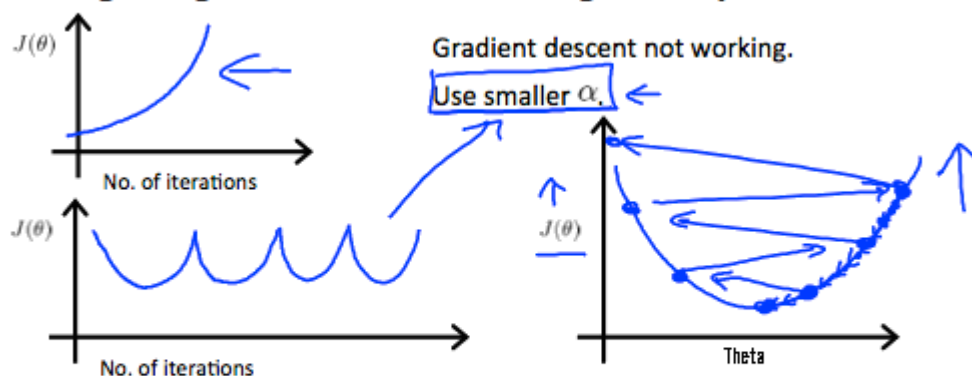
**Automatic convergence test.** Declare convergence if  $J(\theta)$  decreases by less than  $E$  in one iteration, where  $E$  is some small value such as  $10^{-3}$ . However in practice it's difficult to choose this threshold value.

**Making sure gradient descent is working correctly.**



It has been proven that if learning rate  $\alpha$  is sufficiently small, then  $J(\theta)$  will decrease on every iteration.

## Making sure gradient descent is working correctly.



- For sufficiently small  $\alpha$ ,  $J(\theta)$  should decrease on every iteration.
- But if  $\alpha$  is too small, gradient descent can be slow to converge.

To summarize:

If  $\alpha$  is too small: slow convergence.

If  $\alpha$  is too large: may not decrease on every iteration and thus may not converge.

## Polynomial Linear Regression

Our hypothesis function need not be linear (a straight line) if that does not fit the data well.

We can **change the behavior or curve** of our hypothesis function by making it a quadratic, cubic or square root function (or any other form).

For example, if our hypothesis function is  $h_{\theta}(x) = \theta_0 + \theta_1 x_1$  then we can create additional features based on  $x_1$ , to get the quadratic function  $h_{\theta}(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_1^2$  or the cubic function  $h_{\theta}(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_1^2 + \theta_3 x_1^3$

In the cubic version, we have created new features  $x_2$  and  $x_3$  where  $x_2 = x_1^2$  and  $x_3 = x_1^3$ .

To make it a square root function, we could do: 
$$h_{\theta}(x) = \theta_0 + \theta_1 x_1 + \theta_2 \sqrt{x_1}$$

One important thing to keep in mind is, if you choose your features this way then feature scaling becomes very important