MYD-Fall 2020-Physics1-Lecture 1

General Physics 1

Istanbul Medipol University
Fall 2020

Instructor: Asst. Prof. Merve Yüsra Doğan

Welcome and Introduction

- Course Webpage
  MS Teams
  Google classroom, class code: 72jbrpy
  Use Medipol email

- Syllabus

Syllabus

- **Instructor:** Asst. Prof. Merve Yüsra Doğan, mydogan@medipol.edu.tr
- **Office Hours:** Monday, 13:00 - 14:00, Teams
- **Textbook:** Young & Freedman, University Physics with Modern Physics, 13th Edition

- **Lecture:** Monday 14:30 - 17:15

- **Recitation:** TBA

- **Laboratory: on MS Teams, a different class.**

Syllabus - Teaching Assistants

TA	email
Ömer Kartlı	okartli@st.medipol.edu.tr
Hakan Yücelten	hyucelten@st.medipol.edu.tr
Mehmet Emin Bayrak	mebayrak@st.medipol.edu.tr
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Syllabus - Calendar

Tentative Calendar			
Month	**Day**	**Chapter**	**Subject**
October	5		Cancelled
	12	1	Units, Physical Quantities and Vectors
	19	2	Motion along a straight line
	26	3	Motion in two or three dimensions
November	2	4	Newton's Laws of Motion
	9	5	Applying Newton's Laws
	16	6	Work and Kinetic Energy
	23	7	Potential Energy and Energy Conservation
	30		**Midterm Week**
December	7	8	Momentum, Impulse and Collisions
	14	9	Rotation of Rigid Bodies
	21	10	Dynamics of Rotational Motion
	28	11	Equilibrium and Elasticity
January	4	14	Periodic Motion
	11	12	Fluid Mechanics
	18	13	Gravitation
	25	15	Mechanical Waves
February	1		**Finals Week**

Syllabus - Grading

- Quiz 30 %
- Midterm 30 %
- Final 40 %

Syllabus - Exams

Exams

- You will need to upload files for quiz and exams.
- Please do have an easy access method to scan and upload a pdf file. The easiest method is to have a scanning app on your smartphone.
- We may ask you to turn on your camera during any exam.
- We may have Zoom meetings for exams

Quiz

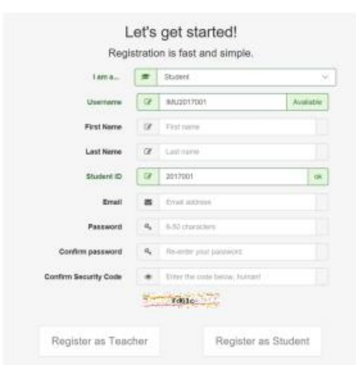
- Quizzes will be held during class.
- The quiz platform will vary as Quizevolved / MS Teams / Google Classroom to be announced during lecture.
- Quiz weights may vary according to content.
- **If we suspect any wrong-doing during any quiz/exam, we will give you an oral exam to replace the respective exam.**

Midterm and Final

- The midterm day and final day will be set during midterm/finals week.
- Final will cover the entire course material.

Syllabus – Online Quiz

- http://quizevolved.com
- Join this class: ACCESS CODE: 11649953
- **Write your name as it appears on your records when you sign up**
- During lecture, I will give you an access code for the week's quiz



Syllabus - Homework

- Several problems will be assigned
- Homework will not be collected
- It is good practice to work on these problems to fully comprehend the subject matter.

Syllabus - Written Homework Format

- Guidelines to keep your work neat
- Important to develop good problem solving skills
- Organize your thoughts
- Representative sketches
- Correct formulas
- Thought process

Syllabus - Recitation

- Problem solving sessions
- Recitations are biweekly. Time TBA
- Recitation weeks are highlighted in green in the calendar.
- Although recitations are not mandatory, it will be wise to attend them

Syllabus - Laboratory Information

- Labs will be online – virtual labs
- You will be assigned a lab section/time.
- You must attend the section you are assigned.
- You cannot miss an experiment and make up in another section.
- Any excuses need written approval.
- Your TAs are in full charge of the laboratory session

- You need to have a copy of the Lab Manual
- Bring the manual with you to every lab.

- I will post your section assignment on Google classroom/MS Teams

Syllabus - Laboratory Information

- Lab Manual: Webpage

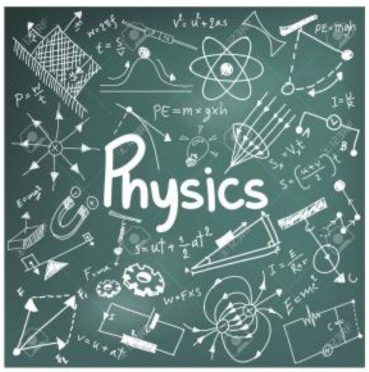
- Lab Orientation: In the lab, at the beginning of your first lab

- 6 Experiments, days and times TBA

Experiments
1. Motion Along a Straight Line 2. Projectile Motion
3. Hooke's Law
4. Conservation of Momentum
5. Rotational Motion
6. Simple Pendulum

What you must do this week

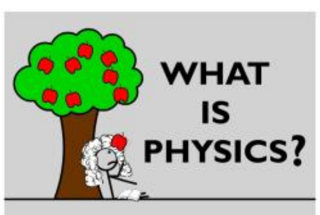
- Add class on Google Classroom
- Sign up on Quizevolved.com
- Get a copy of the Laboratory Manual



PHYSICS Intro

- An experimental science
- Aims to understand the complex universe based on simple ideas
- Finds patterns that relate the phenomena of nature.
- Definition:

The science that deals with **matter, energy, motion, and force**

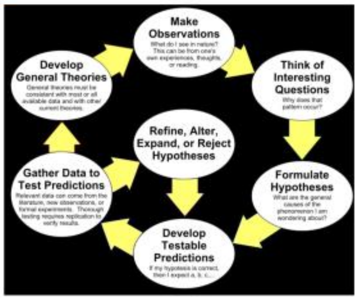


The scientific method

- Finds patterns that relate the phenomena of nature.
- All sciences use

  the scientific method

  1. Observation
  2. Question
  3. Hypothesis
  4. Prediction
  5. Test --> experiments
  6. Iterate
  7. Formulate a general rule



Physical laws

- The general rules or patterns arrived at are called

  physical theories

- A very well established or widely used theory is called

  a *physical law* or *principle*.

Fundamental quantities of physics

- Some physical quantities are so fundamental that we can define them only by describing how to measure them.

Fundamental Quantities	**Units**	
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	Kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	Cd

Some Derived Quantities	**Units**
Speed (ratio of a length by a time)	m/s
Acceleration (ratio of a speed by a time)	m/s^2
Force (multiplication of acceleration by mass)	$kg \cdot m/s^2$

SI Units: International System of Units
(Système International d'Unités)

- Reference standard
- Such a standard defines a unit of the quantity
- A physical quantity is described by a number and a unit
- The 'INTERNATIONAL SYSTEM OF UNITS', abbreviated SI, defines the seven quantities and a physical standard for each (*UNIT*)
- Quantities of the same type can be compared against the same unit

Some SI derived quantities and units

Quantity	Units
area	m^2
volume	m^3
mass density	kg/m^3
speed, velocity	m/s
angular velocity	rad/s
acceleration	m/s^2
angular acceleration	rad/s^2
kinematic viscosity	m^2/s

Quantity	Units		
frequency	Hertz	H	1/s
force	Newton	N	$kg \cdot m/s^2$
pressure	Pascal	Pa	$kg / (m \cdot s^2)$
work, energy	Joule	J	$kg \cdot m^2/s^2$
power	Watt	W	$kg \cdot m^2/s^3$
quantity of electricity	Coulomb	C	$A \cdot s$
potential difference	Volt	V	$kg \cdot m^2/(A \cdot s^3)$

The British System of Units

English Unit	SI Unit	Conversion
Mile	Kilometer	1 mile = 1.609 km
Foot	Meter	1 ft = 0.305 m
Inch	Centimeter	1 in = 2.54 cm
Pound	Grams	1 lb = 453.59 g
Ounce	Grams	1 oz = 28.35 g
Gallon	Liter	1 gallon = 3.79 l
Celsius	Kelvin	0° C = 273.15 K

Unit prefixes

Number	Multiplication Factor	Prefix	Prefix Symbol
1 000 000 000 000 000	10^{15}	peta	P
1 000 000 000 000	10^{12}	tera	T
1 000 000 000	10^{9}	giga	G
1 000 000	10^{6}	mega	M
1 000	10^{3}	kilo	k
100	10^{2}	hecto	h
10	10^{1}	deka	da
0.1	10^{-1}	deci	d
0.01	10^{-2}	centi	c
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	µ
0.000 000 001	10^{-9}	nano	n
0.000 000 000 001	10^{-12}	pico	p
0.000 000 000 000 001	10^{-15}	femto	f

Some size scales

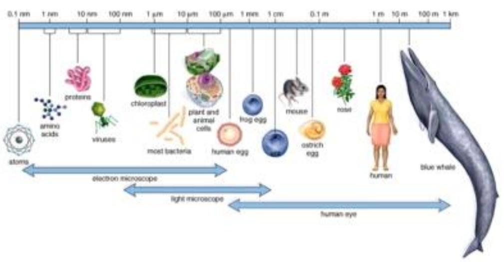


image from Michigan Nanotechnology Institute

Unit consistency and calculating with units

- Physical quantities are represented by both a number and a unit
- Example: d represent a distance of 100 meters, d = 100 m

- **An equation must always be dimensionally consistent.**

- *Example: How many minutes does it take to travel 50 km in a car traveling with constant speed at 100 km/hr ?*

$$duration = (50\,\mathrm{km})\left(\frac{\mathrm{hr}}{100\,\mathrm{km}}\right)\left(\frac{60\,\mathrm{min}}{\mathrm{hr}}\right)$$

$$duration = 30\,\mathrm{min}$$

Unit consistency and calculating with units

- *Example: Speed limit on the highway is 65 mph. What is this in meters per second?*

$$65\text{ mph} = \left(\frac{65\text{ miles}}{\text{hour}}\right)\left(\frac{1.609\text{ km}}{\text{miles}}\right)\left(\frac{1000\text{ m}}{\text{km}}\right)\left(\frac{\text{hour}}{60\text{ min}}\right)\left(\frac{\text{min}}{60\text{ s}}\right)$$

$$65\text{ mph} = 29.05\text{ m/s}$$

- *Don't write:* $65\text{ mph} = \dfrac{65 \times 1.609 \times 1000}{60 \times 60}\text{ m/s}$
- **Always include unit conversions in your calculations**

Scientific notation and significant figures

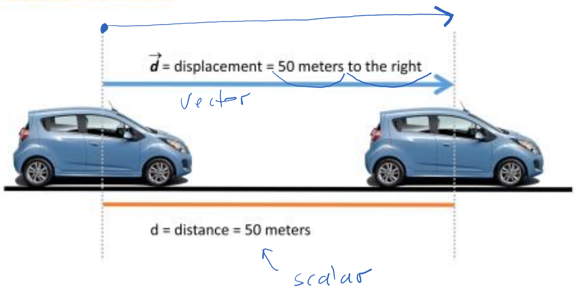
- In scientific notation, the number is represented with a single digit before decimals, multplied by a power of 10.
- Example:
  - $572000000 = 5.72 \times 10^{8}$
  - $0.0000408 = 4.08 \times 10^{-5}$

    (digit term) (exponential term)
- Significant figures of a number are digits that carry meaning contributing to its measurement resolution:
  - 0.00045 (two significant figures) becomes 4.5×10^{-4}
  - 0.00452500 (six significant figures) becomes 4.52500×10^{-3}.
  - 1300 to four significant figures is written as 1.300×10^{3}
  - 1300 to two significant figures is written as 1.3×10^{3}

Scalars and vectors

- A ***scalar quantity*** can be described by a ***single number (magnitude)***.
- A ***vector quantity*** has both a ***magnitude*** and a ***direction*** in space.
- A vector quantity is represented with an arrow over it: $\vec{A}$. $\vec{A}$
- The magnitude of $\vec{A}$ is written as A or $|\vec{A}|$.

Scalar vs vector

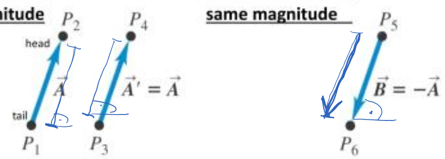


Vectors

- head, tail, magnitude, direction

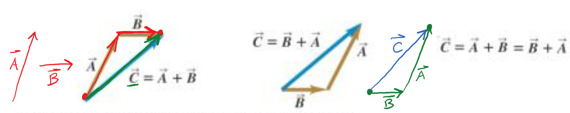
same direction
same magnitude

opposite direction
same magnitude

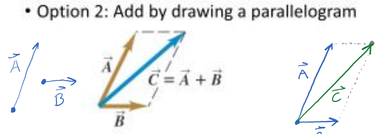


Vector addition: two vectors

- Option 1: Add by placing them head to tail



- Option 2: Add by drawing a parallelogram

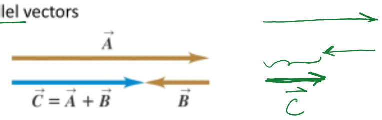


$\vec{C} = \vec{A} + \vec{B} = \vec{B} + \vec{A}$

Vector addition

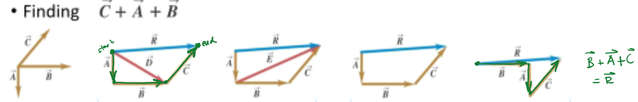
- Adding two parallel vectors

- Adding two antiparallel vectors



Vector addition: multiple vectors

- Add with head-to-tail method in any order
- Finding $\vec{C} + \vec{A} + \vec{B}$



Put them head to tail in any order

Algebraically, add one by one

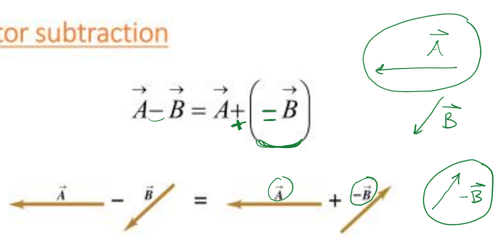
$\vec{A}+\vec{B}=\vec{D}$ $\quad$ $\vec{B}+\vec{C}=\vec{E}$

$\vec{D}+\vec{C}=\vec{R}$ $\quad$ $\vec{A}+\vec{E}=\vec{R}$

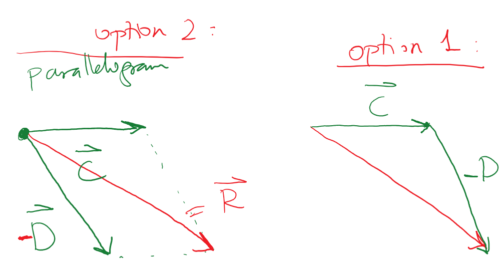
Graphically, draw the resultant vector from the first tail to the last head.

Vector subtraction

$$\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$$



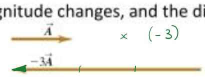
$\vec{C} - \vec{D} = ?$



Multiplying a vector by a scalar

- When a vector $\vec{A}$ is multiplied by a scalar c, the product $c\vec{A}$ has magnitude $|c|A$.
- If c is positive, only the magnitude of $\vec{A}$ changes.

- If c is negative, the magnitude changes, and the direction of $\vec{A}$ is reversed



$\vec{A}$ vector

c scalar

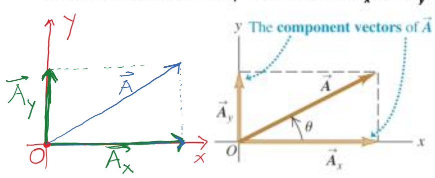
$\longrightarrow$ 2

$= 2\vec{A}$

$\longrightarrow$ (-3)

Vector components

- Consider a two-dimensional coordinate system and the vector $\vec{A}$ placed with its the tail at the origin
- Projecting $\vec{A}$ onto the axes gives its component vectors
- $\vec{A}$ is the sum of its component vectors $\vec{A}_x$ and $\vec{A}_y$



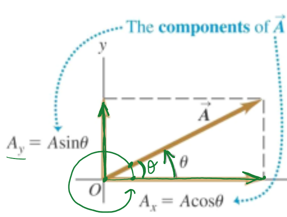
$\vec{A}_x + \vec{A}_y = \vec{A}$

Vector components

- Components are found using trigonometry

$$A_x = A\cos\theta$$
$$A_y = A\sin\theta$$

θ is measured from the +x-axis toward the +y-axis.



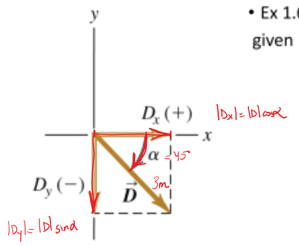
$|A_x| = |A|\cos\theta$

$|A_y| = |A|\sin\theta$

$\vec{A} = \vec{A}_x + \vec{A}_y$

$|A_x|\hat{x} + |A_y|\hat{y} =$

Finding vector components



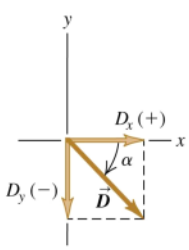
- Ex 1.6 : Find x- and y- components of $\vec{D}$ given

$$\left|\vec{D}\right| = 3.00\,\text{m} \qquad \alpha = -45^\circ$$

$\vec{D}$: $|\vec{D}| = 3m$

$\alpha = 45^\circ$

Finding vector components



- Ex 1.6 : Find x- and y- components of $\vec{D}$ given

$$\left|\vec{D}\right| = 3.00\,\text{m} \qquad \alpha = -45^\circ$$

Answer:

$$D_x = D\cos\theta = (3.00\,m)(\cos(-45^\circ)) = 2.1\,m$$

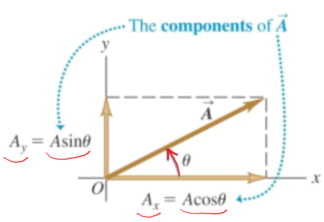
$$D_y = D\sin\theta = (3.00\,m)(\sin(-45^\circ)) = -2.1\,m$$

Vector components

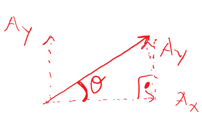
- Similarly, we can find the magnitude and direction of a vector from its components

$$A = \sqrt{A_x^2 + A_y^2}$$

$$\tan\theta = \frac{A_y}{A_x}$$



$$\frac{|A_y|}{|A_x|} = \tan\theta$$



$$|\vec{A}| = \sqrt{|A_x|^2 + |A_y|^2}$$

Calculating with components

- The sum of two or more vectors can be calculated using components

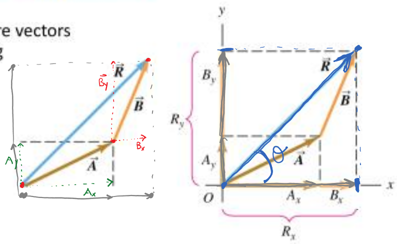
$R_x = A_x + B_x$

$R_y = A_y + B_y$

- more generally,

$R_x = A_x + B_x + C_x + D_x + \ldots$

$R_y = A_y + B_y + C_y + D_y + \ldots$



$\vec{A} = A_x \vec{x} + A_y \hat{y}$

$+\ \vec{B} = B_x \vec{x} + B_y \hat{y}$

$\vec{R} = (A_x + B_x)\vec{x} + (A_y + B_y)\vec{y}$

$\vec{R} = R_x \vec{x} + R_y \vec{y}$

$|R| = \sqrt{R_x^2 + R_y^2}$

$\tan\theta = \frac{R_y}{R_x}$, $\theta = \arctan \frac{R_y}{R_x}$

$\vec{A} = A_x \hat{x} + A_y \hat{y}$

$\vec{B} = B_x + B_y$

$\vec{C} = C_x + C_y$

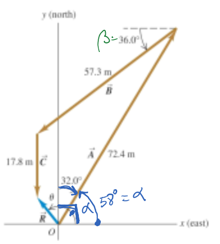
$+\ \vec{D} = D_x + D_y$

$\vec{R} = \underbrace{(A_x + B_x + C_x + D_x)}_{R_x} \hat{x} + \underbrace{(A_y + B_y + C_y + D_y)}_{R_y} \hat{y}$

$= R_x \hat{x} + R_y \hat{y}$

Calculating with components

- Ex 1.7: Find R and θ for the given configuration



$\vec{A}\vec{B} + \vec{C} = \vec{R}$

$\vec{A}$: $|\vec{A}| = 72.4\,m$ $\alpha = 58°$

$A_x = (72.4) \cos 58°$

$A_y = (72.4) \sin 58°$

$\vec{B}$: $|\vec{B}| = 57.3\,m$ $\beta = 36° + 180° = 216°$

$B_x = (57.3) \cos(216°)$

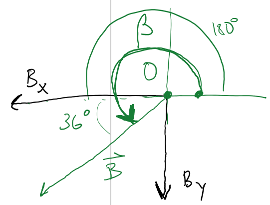
$B_y = (57.3) \sin 216°$

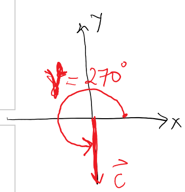
$\vec{C}$: $|\vec{C}| = 17.8\,m$

$\gamma = 270°$

$C_x = 0 = 17.8 \underbrace{\cos 270°}_{0}$

$C_y = -17.8 = 17.8 \sin 270°$





$\vec{R} = R_x \vec{x} + R_y \hat{y}$

$R_x = A_x + B_x + C_x$

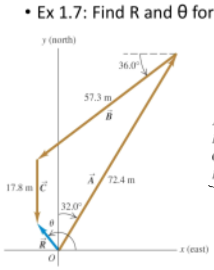
$R_y = A_y + B_y + C_y$

$|R| = \sqrt{R_x^2 + R_y^2}$

$\theta = \arctan \frac{R_y}{R_x}$

Calculating with components

- Ex 1.7: Find R and θ for the given configuration



$R_x = A_x + B_x + C_x$

$R_y = A_y + B_y + C_y$

$R = \sqrt{R_x^2 + R_y^2}$

$\theta = \arctan \frac{R_y}{R_x}$

$A_x = (72.4\,m)(\cos 58) = 38.37\,m$

$B_x = (57.3\,m)(\cos 216) = -46.36\,m$

$C_x = (17.8\,m)(\cos 270) = 0$

$R_x = -7.99\,m$

$A_y = (72.4\,m)(\sin 58) = 61.4\,m$

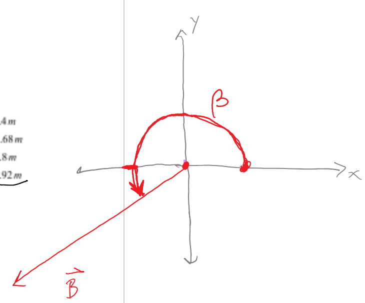
$B_y = (57.3\,m)(\sin 216) = -33.68\,m$

$C_y = (17.8\,m)(\sin 270) = -17.8\,m$

$R_y = 9.92\,m$

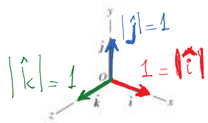
$R = \sqrt{(-7.99\,m)^2 + (9.92\,m)^2} = 12.7\,m$

$\theta = \arctan \frac{9.92\,m}{-7.99\,m} = -51°$



Unit vectors

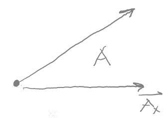
- A *unit vector* has a magnitude of 1 with no units.



$\hat{\imath}$ points in the +*x*-direction

$\hat{\jmath}$ points in the +*y*-direction

$\hat{k}$ points in the +*z*-direction



Any vector can be expressed in terms of its components as $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$

such as $displacement, \vec{D} = (20\hat{\imath} - 40\hat{\jmath} + 60\hat{k})\, m$

meters

Products of vectors

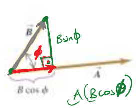
- Two types of products of vectors:
  - Dot product
  - Cross product

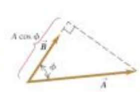
The dot product (scalar product)

$\vec{A} \bullet \vec{B} = c$

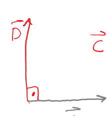
vector • vector = scalar

$\vec{A} \bullet \vec{B} = A(B\cos\phi) = B(A\cos\phi) = \vec{B} \bullet \vec{A}$

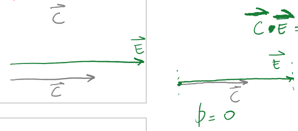




- For perpendicular vectors, $\phi = 90^\circ, \cos\phi = 0;\ \vec{A} \bullet \vec{B} = 0$
- For parallel vectors, $\phi = 0^\circ,\ \cos\phi = 1;\ \vec{A} \bullet \vec{B} = (A)(B)$



$\vec{C} \bullet \vec{D} = CD\cos\phi = CD\cos 90^\circ = 0$



$\vec{C} \bullet \vec{E} = CE\underbrace{\cos 0}_{1} =$

$= CE$

The dot product in components

$\vec{A} \bullet \vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) \bullet (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$

$$\boxed{\begin{array}{l} \hat{\imath} \bullet \hat{\imath} = \hat{\jmath} \bullet \hat{\jmath} = \hat{k} \bullet \hat{k} = (1)(1)\cos 0^\circ = 1 \\ \hat{\imath} \bullet \hat{\jmath} = \hat{\jmath} \bullet \hat{k} = \hat{k} \bullet \hat{\imath} = (1)(1)\cos 90^\circ = 0 \end{array}}$$

$\vec{A} \bullet \vec{B}$

$(A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) \bullet (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$

$(A_x\hat{\imath} \cdot B_x\hat{\imath} + A_y\hat{\imath} \cdot B_x\hat{\imath} + A_z\hat{k} \cdot B_x\hat{\imath}) + (A_x\hat{\imath} \cdot B_y\hat{\jmath} + A_y\hat{\jmath} \cdot B_y\hat{\jmath} \cdots$

$\vec{A}\bullet\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\bullet(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$\boxed{\begin{array}{l}\hat{i}\bullet\hat{i} = \hat{j}\bullet\hat{j} = \hat{k}\bullet\hat{k} = (1)(1)\cos 0^\circ = 1 \\ \hat{i}\bullet\hat{j} = \hat{j}\bullet\hat{k} = \hat{k}\bullet\hat{i} = (1)(1)\cos 90^\circ = 0\end{array}}$

$\vec{A}\bullet\vec{B} = A_xB_x\hat{i}\hat{i} + A_yB_y\hat{j}\hat{j} + A_zB_z\hat{k}\hat{k} + A_xB_y\hat{i}\hat{j} + A_xB_z\hat{i}\hat{k} + \dots$

$\vec{A}\bullet\vec{B} = A_xB_x(1) + A_yB_y(1) + A_zB_z(1) + A_xB_y(0) + A_xB_z(0) + \dots$

$\boxed{\vec{A}\bullet\vec{B} = A_xB_x + A_yB_y + A_zB_z}$

$\begin{pmatrix} A_x\hat{i} \\ A_y\hat{j} \\ A_z\hat{k}\end{pmatrix}\bullet\begin{pmatrix} B_x\hat{i} \\ B_y\hat{j} \\ B_z\hat{k}\end{pmatrix} = A_xB_x + A_yB_y + A_zB_z$

example: $\begin{pmatrix} 3\hat{i} \\ -4\hat{j} \\ 5\hat{k}\end{pmatrix}\bullet\begin{pmatrix} 5\hat{i} \\ 6\hat{j} \\ -4\hat{k}\end{pmatrix} = (3)(5) + (-4)(6) + (5)(-4) = 15 - 24 - 20 = -29$ ← scalar

$(A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\bullet(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$(A_x\hat{i}\cdot B_x\hat{i} + A_y\hat{j}\cdot B_x\hat{i} + A_z\hat{k}\cdot B_x\hat{i}) + (A_x\hat{i}\cdot B_y\hat{j} + A_y\hat{j}\cdot B_y\hat{j} \dots \dots$

A_xB_x

$(\hat{i}\cdot\hat{i}) = |\hat{i}||\hat{i}|\cos 0 = 1\times 1\times 1 = 1$

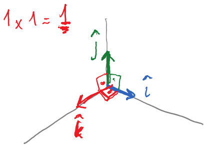
$\hat{i}\cdot\hat{j} = 0$ normal vectors

$\hat{i}\cdot\hat{k} = 0$ normal

$\hat{j}\cdot\hat{j} = 1$

$\hat{j}\cdot\hat{k} = 0$

$\hat{k}\cdot\hat{k} = 1$



The dot product in components

- Ex 1.11: Find the angle between the vectors

$\vec{A} = (2.00\hat{i} + 3.00\hat{j} + 1.00\hat{k})$ and $\vec{B} = (-4.00\hat{i} + 2.00\hat{j} - 1.00\hat{k})$

$\left.\begin{array}{l}\vec{A}\cdot\vec{B} = AB\cos\theta \\ \vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z\end{array}\right\} =$

$AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$

$AB\cos\theta = [(2)(-4) + (3)(2) + (1)(-1)] = -3$

$\cos\theta = \dfrac{-3}{|A||B|} = \dfrac{-3}{\sqrt{14}\sqrt{21}} \Rightarrow \theta = \arccos\dfrac{-3}{\sqrt{14}\sqrt{21}}$

$|A| = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{14}$, $|B| = \sqrt{(-4)^2 + 2^2 + (-1)^2} = \sqrt{21}$

The dot product in components

- Ex 1.11: Find the angle between the vectors

$\vec{A} = (2.00\hat{i} + 3.00\hat{j} + 1.00\hat{k})$ and $\vec{B} = (-4.00\hat{i} + 2.00\hat{j} - 1.00\hat{k})$

$\boxed{\vec{A}\bullet\vec{B} = A(B\cos\phi) \Rightarrow \phi = \arccos\dfrac{\vec{A}\bullet\vec{B}}{AB}} = \arccos\dfrac{-3}{\sqrt{14}\sqrt{21}} = 100^\circ$

$\boxed{\vec{A}\bullet\vec{B} = A_xB_x + A_yB_y + A_zB_z} = (2)(-4) + (3)(2) + (1)(-1) = -3$

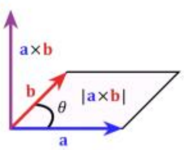
$\boxed{A = \sqrt{A_x^2 + A_y^2 + A_z^2}} = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{14}$

$\boxed{B = \sqrt{B_x^2 + B_y^2 + B_z^2}} = \sqrt{(-4)^2 + 2^2 + (-1)^2} = \sqrt{21}$

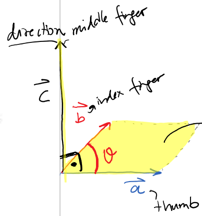
The cross product (vector product)

$\vec{C} = \vec{A} \times \vec{B}$

$\left|\vec{C}\right| = \left|\vec{A} \times \vec{B}\right| = AB \sin\phi$



- $\vec{C}$ always perpendicular to the plane
- Direction of $\vec{C}$ is determined by the right hand rule
- Magnitude of $\vec{C}$ corresponds to the area of the parallelogram



area = $|\vec{c}|$ = $AB\sin\theta$

$\vec{a} \times \vec{b} = \vec{c}$

Properties of the cross product

$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

- For perpendicular vectors, $\theta = 90^\circ, \sin\theta = 1;\ \vec{A} \times \vec{B} = AB$
- For parallel vectors, $\theta = 0^\circ,\ \sin\theta = 0;\ \vec{A} \times \vec{B} = 0$

$\left(\vec{A} + \vec{B}\right) \times \vec{C} = \left(\vec{A} \times \vec{C}\right) + \left(\vec{B} \times \vec{C}\right)$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

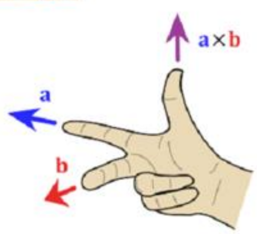
Properties of the cross product

- Calculating the determinant $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$ $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \vec{i} \cdot \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} - \vec{j} \cdot \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} + \vec{k} \cdot \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}$$

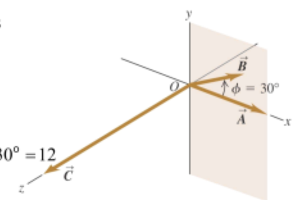
$$\vec{a} \times \vec{b} = (a_y b_z - a_z b_y) \cdot \vec{i} + (a_x b_z - a_z b_x) \cdot \vec{j} + (a_x b_y - a_y b_x) \cdot \vec{k}$$

The right hand rule



The cross product

- Ex 1.12: Magnitudes of $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ shown in figure are 6 and 4 units respectively. Find their cross product $\vec{\boldsymbol{C}}$.



- Magnitude :

$$\left|\vec{C}\right| = \left|\vec{A}\times\vec{B}\right| = AB\sin\phi = (6)(4)\sin 30^{\circ} = 12$$

- Direction: using right hand rule,

$$\hat{k}$$