MYD-Fall 2020-Physics1-Lecture 3

Sunday, November 1, 2020 10:28 PM

MYD-Fall 2020-Physics1-Lecture 3

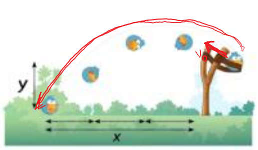
General Physics 1

Istanbul Medipol University
Fall 2020

Instructor: Dr. Merve Yüsra Doğan

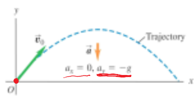
MOTION IN 2 AND 3 DIMENSIONS

Projectile Motion



Projectile motion

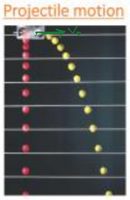
- A projectile is any body thrown that then follows a curved path
- moves in a vertical plane (x-y plane) **(2-D)**
- has initial velocity ($\vec{v}_0$)
- determined by the effects of gravity and air resistance.



Projectile motion

$v_{y_0}=0$

g



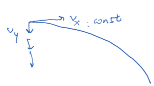
$v_y = \cancel{v_{y_0}}^{\,0} - gt$

$v_x = v_{x_0} + \cancel{a}^{\,0}t$

$v_x = v_{x_0}$

- The red ball is dropped
- The yellow ball is fired horizontally
- White lines mark equal time intervals
- Both projectiles fall the same distance in the same time.

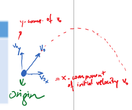
- We can analyze projectile motion as
  - horizontal motion with constant velocity and
  - vertical motion with constant acceleration:
  - $a_x = 0$ and $a_y = -g$.



Equations for projectile motion

- Considering x- and y- motions separately

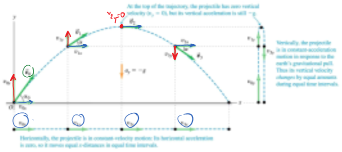
x-direction: constant velocity motion	y-direction: constant acceleration
$a_x = 0$ $v_x = \text{constant} = v_{0x}$ $x = x_0 + v_{0x}t$	$a_y = -g$ $v_y = v_{0y} - gt$ $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$



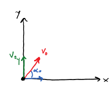
- It is convenient to take the origin at the initial point of the projectiel

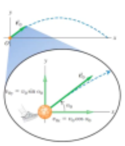
$$x_0 = y_0 = 0$$

Velocity vector during projectile motion



Trajectory of projectile motion





- Consider $x_0 = y_0 = 0$, $v_{0x} = v_0 \cos\alpha_0$, $v_{0y} = v_0 \sin\alpha_0$
- Then, for projectile motion

$$x = v_{0x}t = (v_0\cos\alpha_0)t \quad \Rightarrow \boxed{t = \frac{x}{v_0\cos\alpha_0}}$$

$$y = v_{0y}t - \frac{1}{2}gt^2 = (v_0\sin\alpha_0)t - \frac{1}{2}gt^2$$

substitute for t in y − equation

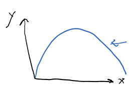
$$y = \frac{v_0\sin\alpha_0}{v_0\cos\alpha_0}x - \frac{g}{2v_0^2\cos^2\alpha_0}x^2$$

const = c_1 const = c_2

$$y = \frac{v_0\sin\alpha_0}{v_0\cos\alpha_0}x - \frac{g}{2v_0^2\cos^2\alpha_0}x^2$$

$y = c_1 x - c_2 x^2$, where c_1 and c_2 are constants Equation for a parabola

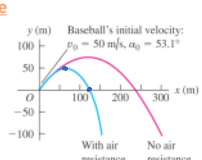
$y = c_1 x - c_2 x^2$



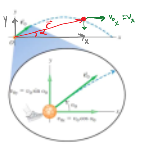
trajectory has shape of a parabola

Effects of air resistance

- Calculations become more complicated.
- Acceleration is not constant.
- Effects can be very large.
- Maximum height and range decrease.
- Trajectory is no longer a parabola.



Position and velocity along the trajectory

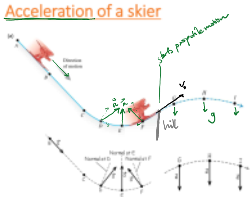


$v_y = v_{0_y} - gt$

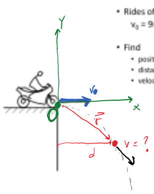
$\alpha = \tan \frac{y}{x}$

$|\vec{r}| = \sqrt{x^2 + y^2}$

Acceleration of a skier



Example



- Rides off the edge of a cliff with horizontal velocity $v_0 = 9\,m/s$
- Find
  - position, $\vec{r}$
  - distance from the edge of cliff, d
  - velocity at 0.50 seconds after jump v

$v_{0_x} = v_0 = 9\,m/s \qquad v_x = v_{0_x} = 9\,m/s$

$v_{0_y} = 0 \qquad v_y = -gt$

$d = v_{0_x} t$

$v = v_x \hat{i} + v_y \hat{j}$

$= v_{0_x} \hat{i} - gt \hat{j}$

$\vec{r} = x \hat{i} + y \hat{j}$

$= (v_{0_x} t)\hat{i} + \underbrace{\left(-\frac{1}{2} g t^2\right)}_{y_0 = 0 \quad v_y = 0} \hat{j}$

$d = (9\,m/s)(0.5\,s) = 4.5\,m$

$v = (9\,m/s)\hat{i} - (10\,\frac{m}{s^2})(0.5)\hat{j} = (9\,\frac{m}{s})\hat{i} - (5\,\frac{m}{s})\hat{j}, \quad |\vec{v}| = \sqrt{9^2 + 5^2}\,\frac{m}{s}$

$\vec{r} = (4.5\,m)\hat{i} - (1.25\,m)\hat{j}$

$\frac{1}{2}(10)\left(\frac{1}{2}\right)^2$

Example (3.6)

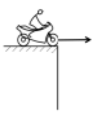
- Rides off the edge of a cliff with horizontal velocity $v_0 = 9\,m/s$
- Find

$y_0 = 0$ $v_y = 0$

$$\frac{1}{2}(10)\left(\frac{1}{2}\right)^2$$

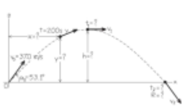
Example (3.6)

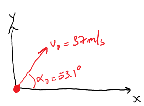
- Rides off the edge of a cliff with horizontal velocity $v_0 = 9$ m/s
- Find
  - position,
  - distance from the edge of cliff,
  - velocity at 0.50 seconds after jump



Example (3.7)

- A baseball leaves the bat at speed $v_0 = 37$ m/s at an angle $\alpha_0 = 53.1°$
- Find position and velocity at t = 2.00 s
- Find when the ball reaches the heightest point and its height at that time
- Find the horizontal range





$v_0 = 37$ m/s , $\alpha_0 = 53.1°$

$v_{0x} = v_0 \cos\alpha_0 =$

$v_{0y} = v_0 \sin\alpha_0$

$$v_x(t = 2s) = (v_0 \cos\alpha_0)$$

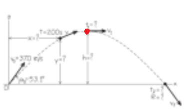
$$v_y(t = 2s) = (v_0 \sin\alpha_0) - gt$$

$$x = x_0 + (v_0 \cos\alpha_0)t$$

$$y = y_0 + (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$$

Example (3.7)

- A baseball leaves the bat at speed $v_0 = 37$ m/s at an angle $\alpha_0 = 53.1°$
- Find position and velocity at t = 2.00 s
- Find when the ball reaches the heightest point and its height at that time
- Find the horizontal range



@ highest point

$v_y = 0$: $v_y = v_{0y} - gt$

$= (v_0 \sin\alpha_0) - gt = 0$

$$\Rightarrow t = \frac{v_0 \sin\alpha_0}{g}$$

$h = ?$

$$y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$$

$$h_{max} = (v_0 \sin\alpha_0)\frac{v_0 \sin\alpha_0}{g} - \frac{1}{2}g\frac{v_0^2 \sin^2\alpha_0}{g^2}$$

$$h_{max} = \frac{v_0^2 \sin^2\alpha_0}{2g}$$

max possible height: $\alpha_0 = 90°$ $\Rightarrow \sin\alpha_0 = 1$

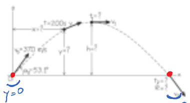
max possible height $= \frac{v_0^2}{2g}$

Example (3.7)

- A baseball leaves the bat at speed $v_0 = 37$ m/s at an angle $\alpha_0 = 53.1°$
- Find position and velocity at t = 2.00 s
- Find when the ball reaches the heightest point and its height at that time
- Find the horizontal range

Example (3.7)

- A baseball leaves the bat at speed v_0 = 37 m/s at an angle α_0 =53.1°
  - Find position and velocity at t = 2.00 s
  - Find when the ball reaches the heightest point and its height at that time
  - Find the horizontal range → max x position



$x = v_{0x} t$?

→ come back to ground : $y=0$

$y = v_{0y} t - \frac{1}{2} g t^2 = 0$ $\quad t_1 = 0$

$\Rightarrow t_2 = \frac{2 v_{0y}}{g} = \frac{2 v_0 \sin\alpha_0}{g}$

range : max x : $v_{0x} t_2 = (v_0 \cos\alpha_0)\left(\frac{2 v_0 \sin\alpha_0}{g}\right) = \frac{2 v_0^2}{g} \cos\alpha \sin\alpha$ $\qquad 2\cos\alpha \sin\alpha = \sin(2\alpha)$

$= \frac{v_0^2}{g} \sin(2\alpha)$

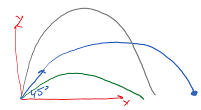
What α gives max range?

$\sin(2\alpha)$: max

$\sin(2\alpha) = 1$

$2\alpha = 90°$

$\alpha = 45°$



Maximum height, maximum range

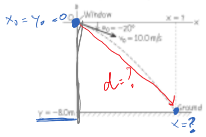
- For a projectile, where the origin and endpoint have the same height
  - Find at what angle you achieve max height → 90°
  - Find at what angle you achieve max range → 45°

Maximum height, maximum range

- For a projectile, where the origin and endpoint have the same height
  - Find at what angle you achieve max height
  - Find at what angle you achieve max range

Different initial and final heights

- You throw a ball from your window 8 m above ground
- The ball leaves your hand with v_0 = 10 m/s and α_0 = 20° below the horizontal
- Far far from your building does the ball land?



$x = v_{0x}\, t$

$y = -8m = v_{0y}\, t - \frac{1}{2} g t^2$ → solve for t, substitute

$v_{0x} = v_0 \cos\alpha_0 \quad v_{0y} = v_0 \sin\alpha_0$

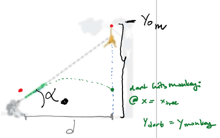
$d = \sqrt{x^2 + y^2}$

Tranquilizing a monkey

- A zookeeper fires a dart directly at the monkey
- Dart leaves the gun at the instant the monkey lets go of the tree
- Show that the dart always hits the monkey

Tranquilizing a monkey

- A zookeeper fires a dart directly at the monkey
- Dart leaves the gun at the instant the monkey lets go of the tree
- Show that the dart always hits the monkey



$x_m = d$

$Y_{0m} = d \tan\alpha_0$

$Y_m = (d \tan\alpha_0) - \frac{1}{2} g t^2$

$x_{dart} = (v_{0\,dart} \cos\alpha_0)\, t$

when dart hits the monkey:

$x_{dart} = d = v_{0\,dart} \cos\alpha_0\, t$

$t = \frac{d}{v_{0\,dart} \cos\alpha_0} = t_{hit}$

@ t_{hit}, is $Y_m = Y_{dart}$?

substitute $t_{hit} = \frac{d}{v_{0\,dart} \cos\alpha_0}$

$Y_{dart} = (v_0 \sin\alpha_0)\, t - \frac{1}{2} g t^2 \Rightarrow (v_0 \sin\alpha_0) \frac{d}{v_0 \cos\alpha_0} - \frac{g}{2} \left(\frac{d}{v_0 \cos\alpha_0}\right)^2 = d \tan\alpha_0 - \frac{g}{2} \left(\frac{d}{v_0 \cos\alpha_0}\right)^2$

$Y_{monkey} = (d \tan\alpha_0) - \frac{1}{2} g t^2 \Rightarrow \left(d \frac{\sin\alpha_0}{\cos\alpha_0}\right) - \frac{g}{2} \left(\frac{d}{v_0 \cos\alpha_0}\right)^2 = d \tan\alpha_0 - \frac{g}{2} \left(\frac{d}{v_0 \cos\alpha_0}\right)^2$

same!

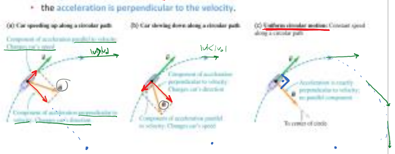
Uniform Circular Motion

same!
@ this t they are at same heights

Uniform Circular Motion

Uniform Circular Motion

- For *uniform circular motion*,
  - the speed is constant and
  - the acceleration is perpendicular to the velocity.



$\vec{v}$ has the same magnitude
it changes direction

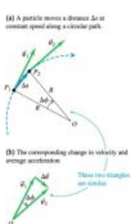
Average acceleration for uniform circular motion

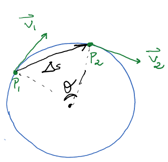
- Constant speed:
  - Magnitudes of v_1 and v_2 are the same
- Similar triangles:

$$\frac{|\Delta \vec{v}|}{v_1} = \frac{\Delta s}{R}$$

$$|\Delta \vec{v}| = v_1 \frac{\Delta s}{R}$$

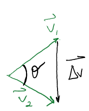
$$a_{average} = \frac{|\Delta \vec{v}|}{\Delta t} = \frac{v_1}{R}\frac{\Delta s}{\Delta t}$$





$|\vec{v}_1| = |\vec{v}_2|$

$\Delta \vec{v} = \vec{v}_2 - \vec{v}_1$



$$\frac{|\Delta \vec{v}|}{v_1} = \frac{\Delta s}{R}$$

$$|\Delta \vec{v}| = v_1 \frac{\Delta s}{R}$$

$$\text{avg acc} = \frac{|\Delta \vec{v}|}{\Delta t} = \frac{v_1}{R}\frac{\Delta s}{\Delta t}$$

Instantaneous acceleration for uniform circular motion

- always points toward the center of the circle and is called the *centripetal acceleration.*

$$a_{rad} = \lim_{\Delta t \to 0} a_{average} = \lim_{\Delta t \to 0} \frac{v_1}{R}\frac{\Delta s}{\Delta t} = \frac{v_1}{R}\lim_{\Delta t \to 0}\frac{\Delta s}{\Delta t} = \frac{v_1}{R}v_1 = \frac{v^2}{R}$$

- The magnitude of the acceleration is
  $a_{rad} = v^2/R.$
- The *period* T is the time for one revolution.
  $v = 2\pi R/T$ $\quad a_{rad} = 4\pi^2 R/T^2.$

$$\lim_{\Delta t \to 0} \frac{v_1}{R}\frac{\Delta s}{\Delta t} = \frac{v_1}{R}\underbrace{\lim_{\Delta t \to 0}\frac{\Delta s}{\Delta t}}_{v_1} = \frac{v_1}{R}v_1 = \frac{v_1^2}{R} = \frac{v^2}{R} = \text{inst. acc.}$$

one complete cycle $= 2\pi R$

$V = \frac{2\pi R}{}$, $a_{rad} = \frac{v^2}{R} = \frac{4\pi^2 R^2}{T^2 R} = \frac{4\pi^2 R}{T^2}$

- The *period* T is the time for one revolution.
  - $v = 2\pi R/T$ $\qquad$ $a_{rad} = 4\pi^2 R/T^2$.

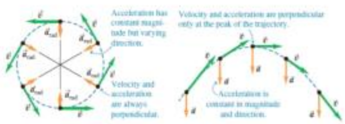
one complete cycle $= 2\pi R$

$V = \frac{2\pi R}{T}$, $a_{rad} = \frac{v^2}{R} = \frac{4\pi^2 R^2}{T^2 R} = \frac{4\pi^2 R}{T^2}$

T = period

Acceleration and velocity

- Uniform circular motion
- Projectile motion



Example

- A sports car has a lateral acceleration as its rounds a curve in the road
  - $a_{max} = 0.96g = 9.4\ m/s^2$ $\quad$ $v=40\ m/s$ $\qquad$ $R_{min} = ?$

Example

- A carnival ride moves in a circle of radius R = 5 m with constant speed completing the circle in 4 seconds.
  - a = ?