Solution

General Physics 1
Fall 2017

Name:____________
Student ID:____________

Question 1

The position of a particle is given as a function of time as $\vec{r} = \left(2t^2\,\hat{i} - 6t\,\hat{j}\right)$ meters.

5 a) Find the displacement vector $\Delta\vec{r}$ of the particle between $t = 1$ and $t = 3$ seconds.

5 b) Find the average velocity vector $\vec{v}_{avg}$ of the particle between $t = 1$ and $t = 3$ seconds.

5 c) Find the angle between the average velocity vector $\vec{v}_{avg}$ of the particle and the x-axis between $t = 1$ and $t = 3$ seconds.

5 d) Find the instantaneous velocity $\vec{v}$ of the particle at $t = 2$ seconds.

a) $\boxed{\vec{r}(t=3) = 2(3^2)\,\hat{i} - 6(3)\,\hat{j}\ m}^{1} = (18\,\hat{i} - 18\,\hat{j})\,m$

$-\ \boxed{\vec{r}(t=1) = (2(1)\,\hat{i} - 6(1)\,\hat{j})\,m}^{1} = (2\,\hat{i} - 6\,\hat{j})\,m$

$\boxed{\Delta\vec{r}_{3-1} = \vec{r}(t=3) - \vec{r}(t=1)}_{2} = \boxed{(16\,\hat{i} - 12\,\hat{j})\,m}^{1}$

b) $\boxed{\vec{v}_{avg} = \frac{\Delta\vec{r}_{3-1}}{\Delta t}}^{2} = \frac{\Delta\vec{r}_{(3-1)}}{\boxed{(3-1)s}_{1}} = \boxed{\left(8\,\hat{i} - 6\,\hat{j}\right)\frac{m}{s}}^{2}$

c) $\boxed{\arctan\left(\frac{v_{avg\,y}}{v_{avg\,x}}\right)}^{2} = \theta = \arctan\left(\frac{-6}{8}\right)^{1\ 2} = \arctan\left(-\frac{3}{4}\right) = -36.9^\circ$

d) $\boxed{\vec{v} = \frac{d\vec{r}}{dt}}^{2} = \boxed{\left(4t\,\hat{i} - 6\,\hat{j}\right)\frac{m}{s}}^{2}$

$\boxed{\vec{v}(t=2) = \left(8\,\hat{i} - 6\,\hat{j}\right)\frac{m}{s}}^{1}$

Alternative: c) $\boxed{\vec{v}_{avg} \cdot \hat{i} = |v_{avg}| \cos\theta}^{2}$

$\boxed{|v_{avg}| = \sqrt{8^2 + 6^2} = 10\ m/s}^{1}$

$\boxed{8\hat{i} \cdot \hat{i} - 6\hat{j} \cdot \cancel{0} = (10\,m/s)\cos\theta}^{1}$

$8 = 10\cos\theta$

$\boxed{\theta = \arccos\left(\frac{4}{5}\right) = \pm 36.9^\circ}^{1}$

Because $\vec{v}$ is in 4th quadrant (region), $\theta = -37^\circ$

Solution

General Physics 1
Fall 2017

Name:____________
Student ID:____________

Question 2

A runner starts from rest and finishes a 75 meter race in 9 seconds. For the first 15 meters, she runs with constant acceleration and then with constant velocity.

a) How long does it take for her to run the first 15 meters?
b) How long does it take for her to run the last 30 meters?
c) What is her final velocity?
d) What is her acceleration in the first 15 meters?

Another runner is in the race. She starts from rest at the same time and runs with constant acceleration of 1.5 m/s².

2 e) Who wins the race?
2 f) At time t = 2 seconds, what is her velocity relative to the first runner?

a) 15 m , const a , 1 $\boxed{v_1 = at}$ at end of 15 m , $t = t_1$

60 m , const v, $\boxed{v_2 = a t_1 \text{ at end of race}}$ $t = 9s$ (const) 1

1 $\boxed{\Delta t = \Delta t_1 + \Delta t_2 = 9 \quad 9 = t_1 + t_2 \quad t_2 = 9 - t_1}$

If they use it w/o writing it's OK 1 $\boxed{75m = \Delta x_1 + \Delta x_2}$

1 $\boxed{\Delta x_1 : 15\ m = \frac{1}{2} a t_1^2}$

1 $\boxed{\Delta x_2 : 60\ m = v_1 (9s - t_1)} = (a t_1)(9s - t_1) = (9s) a t_1 - a t_1^2 =$

$$\boxed{\begin{aligned} 60\ m &= (9s) a t_1 - a t_1^2 \\ (2\times)\quad 15\ m &= \tfrac{1}{2} a t_1^2 \\ + \end{aligned}} \quad \text{or} \quad \boxed{\frac{60}{30} = \frac{\cancel{v} t_2}{\cancel{v} t_1}} \quad 4$$

$90\ m = (9s) a t_1$

2 $\boxed{10 \frac{m}{s} = a t_1 = v_1 \text{ @ end of 15 m}} = \boxed{v_2 = 10 \frac{m}{s}}$

$15m = \frac{1}{2} (a t_1) t_1$

$15m = \frac{1}{2} \left(10 \frac{m}{s}\right) t_1$

1 $\boxed{t_1 = 3s}$

$t_2 = 9s - 3s = 6s$

$\boxed{v_2 = a t_1}$

2 $10 \frac{m}{s} = a (3s) \rightarrow \boxed{a = \frac{10}{3} \frac{m}{s^2}}$

Solution

1 $\boxed{\begin{array}{l}\text{Last 30 m, } v \text{ const} = v_2 \\ 30\ m = v_2 t\end{array}}$

$$30\ m = 10\ \frac{m}{s}\ t$$

$\boxed{t = 3s}$ for the last 30 m

Second runner, const $a = 1.5\ m/s^2$

$$x = \frac{1}{2} a t^2$$

$$75m = \frac{1}{2}\left(1.5\ \frac{m}{s^2}\right) t^2$$

2 $$\frac{150}{1.5}\ s^2 = t^2$$

$$t = 10s$$

First runner finished in 9 seconds. First runner wins.

no partial credit here

$$v_{\text{Second Runner}}(t=2) = a_2(2sec) = \left(1.5\ \frac{m}{s^2}\right)(2s) = 3\ m/s$$

$$v_{\text{First Runner}}(t=2) = a_1(2sec) = \left(\frac{10}{3}\ \frac{m}{s^2}\right)(2s) = \frac{20}{3}\ m/s$$

no partial credit here

2

$$v_{\text{second}/\text{First}} = v_{\text{second}/\text{earth}} + v_{\text{earth}/\text{First}} = v_{s/E} - v_{F/E} = 3\ \frac{m}{s} - \frac{20}{3}\frac{m}{s}$$

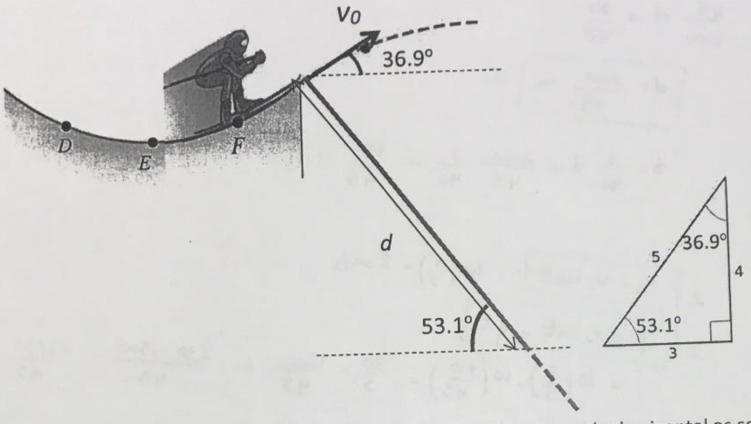
$$= -\frac{11}{3}\ m/s$$

Solution

General Physics 1
Fall 2017

Name:____________
Student ID:____________

Question 3



A skier leaves the ramp with initial velocity 10 m/s and 36.9° above the horizontal as seen in the picture. The slope below is inclined at 53.1°. Assume g = 10 m/s².

a) What is the maximum height that the skier reaches ? (define your coordinate system first)
b) What is the distance *d* from the ramp to where the skier lands ? (plug in numbers early on)
c) What are the velocity components just before the landing ?

$v_0 = 10\ m/s$ $\qquad v_x = v_0 \cos\theta$

$y = y_0 + v_0 \sin\theta\, t - \frac{1}{2} g t^2$ $\qquad v_y = v_0 \sin\theta - gt$

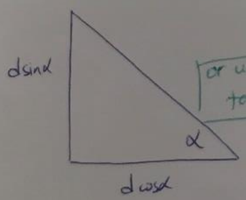
@ h_{max} : $v_y = 0$: $t = \frac{v_0 \sin\theta}{g}$

$y_{max} = \frac{v_0^2 \sin^2\theta}{g} - \frac{1}{2} g \frac{v_0^2 \sin^2\theta}{g^2}$ (correct θ : 3)

$\boxed{h_{max} = \frac{1}{2} \frac{v_0^2 \sin^2\theta}{g}} = \frac{1}{2} \frac{(100)}{10} \left(\frac{3}{5}\right)^2 = \boxed{\frac{9}{5}\ m}$ 2

5

some add Δh of slope, ignore, accept if correct coord. sys.



4

$\boxed{x = (v_0 \cos\theta) t = d \cos\alpha}$ $\qquad \boxed{t = \frac{d \cos\alpha}{v_0 \cos\theta}}$ 2

or using tan α $\qquad \boxed{y = (v_0 \sin\theta) t - \frac{1}{2} g t^2 = -d \sin\alpha}$ 4

$x: \ 10\left(\frac{4}{5}\right) t = d\left(\frac{3}{5}\right) \rightarrow t = \frac{3}{40} d$

$y: \ 10\left(\frac{3}{5}\right) t - \frac{1}{2} g t^2 = -d\left(\frac{4}{5}\right)$

Solution

$$6t - 5t^2 = -\frac{4}{5}d \qquad t = \frac{3}{40}d$$

$$6\left(\frac{3}{40}d\right) - 5\left(\frac{9}{1600}\right)d^2 = -\frac{4}{5}d$$

$$\frac{18}{40}d - \frac{45}{1600}d^2 = -\frac{4}{5}d$$

$$\frac{45}{1600}d = \frac{50}{40}$$

$$\boxed{d = \frac{2000}{45}\ m}\ 2$$

$$t = \frac{3}{40}d = \frac{2000}{45}\,\frac{3}{40} = \frac{150}{45} = \frac{10}{3}$$

$$1\ \boxed{v_x = v_0\cos\theta} = 10\left(\frac{4}{5}\right) = 8\ m/s$$

$$2\ \boxed{v_y = v_0\sin\theta - gt}$$

$$= 10\left(\frac{3}{5}\right) - 10\left(\frac{150}{45}\right) = \frac{30}{5} - \frac{1500}{45} = \frac{270 - 1500}{45} = -\frac{1230}{45}\ \frac{m}{s}$$

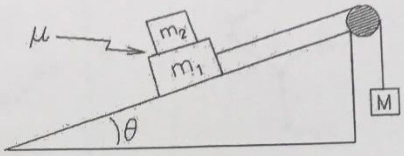
$$6 - \frac{100}{3} = \frac{-82}{3}$$

Solution

General Physics 1
Fall 2017

Name:____________
Student ID:____________

Question 4

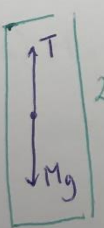


A mass M is suspended by a rope, which goes around a pulley and is connected to mass m_1 which sits on an inclined plane. A mass m_2 kg sits on top of m_1 , as shown. The incline angle is θ . The pulley is frictionless, the rope is massless, and there is no friction between m_1 and the incline. However, there *is* friction between m_1 and m_2 , with coefficients μ_s and μ_k .

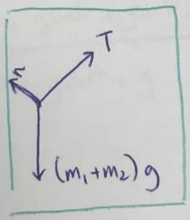
Case 1: System is in equilibrium.
a) Draw a free body diagram for M.
b) Draw a free body diagram for m_1 and m_2 as a composite body (together as one).
c) What is M ?

Case 2: M is now heavier and moves down with acceleration a.
d) What is the tension in the rope ?
e) What is the new M ?

Case 3: M is at its maximum value such that m_2 rides on top of m_1 without slipping.
f) Draw separate free body diagrams for m_1 and m_2.
g) What is the acceleration ?
h) What is the maximum value of M ?

a)


2

b)


2

c) $\boxed{T = Mg}$

$\boxed{T = (m_1+m_2)g\sin\theta}$, $\boxed{M = (m_1+m_2)\sin\theta}$ 2

d) M: $\boxed{\Sigma F_y = T - Mg = Ma \quad T = M(a+g)}$ or $\boxed{T = (m_1+m_2)(a+g\sin\theta)}$ 2

e) $\boxed{\Sigma F_x: -(m_1+m_2)g\sin\theta + T = (m_1+m_2)a}$

$-(m_1+m_2)g\sin\theta + M(a+g) = (m_1+m_2)a$

$\boxed{M = \frac{(m_1+m_2)(a+g\sin\theta)}{a+g}}$ 2

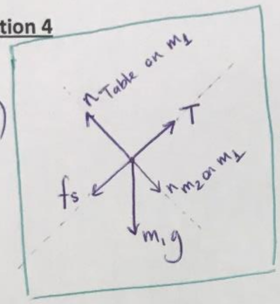
Solution

General Physics 1
Fall 2017

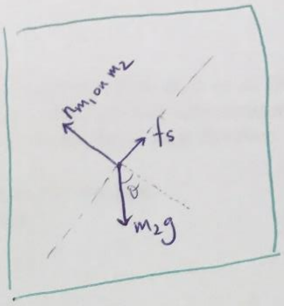
Name:____________
Student ID:____________

Question 4

f)



4



4

g) m_1 : $\sum F_x : -m_2 g\sin\theta + f_s = m_2 a$] 1

$\sum F_y : n - m_2 g\cos\theta = 0$] 1

$n = m_2 g\cos\theta$] 1 $\quad f_s = \mu_s n$] 1

$$-m_2 g\sin\theta + \mu_s m_2 g\cos\theta = m_2 a$$

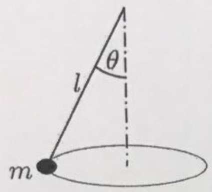
$$a = \mu_s g\cos\theta - g\sin\theta$$] 1

h) $$M = \frac{(m_1+m_2)(\mu_s g\cos\theta - g\sin\theta + g\sin\theta)}{\mu_s g\cos\theta - g\sin\theta + g} = \frac{(m_1+m_2)(\mu_s g\cos\theta)}{g(\mu_s\cos\theta - \sin\theta + 1)}$$] 2

Solution

Name:____________
Student ID:____________

Question 5



A conical pendulum. A bob with mass m at the end of a wire of length l moves in a horizontal circle with constant speed v. The wire makes a fixed angle θ with the vertical direction.

a) Find the tension F in the wire.
b) Find the period T.

FBD:

2 (if no solution)

$$\boxed{\sum F_y = F\cos\theta - mg = 0}$$

$$\boxed{F = \frac{mg}{\cos\theta}}$$

(+2) 4 (w/or w/o FBD)

$$\boxed{\sum F_x : F\sin\theta = ma}$$

$$\frac{mg}{\cos\theta}\sin\theta = ma$$

$$\boxed{a = g\tan\theta}$$

2

$$2\pi R = vT$$

$$\boxed{T = \frac{2\pi R}{v}} \quad \boxed{a = \frac{v^2}{R}}$$

$$T = \sqrt{\frac{4\pi^2 R^2}{aR}}$$

$$\boxed{= 2\pi\sqrt{\frac{R}{g\tan\theta}}} \quad \boxed{R = l\sin\theta}$$ 1

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}\cos\theta}}$$

→ if correct 6 even w/o previous