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Zassenhaus formula:

$$e^{t(X+Y)} = e^{tX} e^{tY} e^{-\frac{t^2}{2}[X,Y]} e^{\frac{t^3}{6}(2[Y,[X,Y]] + [X,[X,Y]])} \cdots$$

$$H = \sum_{k=1}^{L} H_k$$

$$e^{-iHt} = e^{-it[H_1 + \sum_{k=2}^{L} H_k]} = e^{-itH_1} e^{-it\sum_{k=2}^{L} H_k} e^{\frac{t^2}{2}\underbrace{[H_1, \sum_{k=2}^{L} H_k]}_{=0}} \prod e^{f(\underbrace{[H_1, \sum H_k]}_{=0})}$$

$$= e^{-itH_1} e^{-it\sum_{k=2}^{L} H_k}$$

$\vdots$ repeating for $e^{-it\sum_{k=2}^{L} H_k}$

$$= \prod_{k=1}^{L} e^{-itH_k}$$

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$\#$ terms $\leq$ $\#$ subsets of n of size at most c

$$= \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{c}$$

$$\leq c \cdot \binom{n}{c}$$

$$\leq \frac{n^c}{(c-1)!} \quad \text{(polynomial in } c\text{)}$$

$$\binom{n}{c} = \frac{n!}{c!(n-c)!} = \frac{n(n-1)\cdots(n-c+1)}{c!} \leq \frac{n^c}{c!}$$