4.2

$$\exp(M) \triangleq I + M + \frac{M^2}{2!} + \cdots + \frac{M^k}{k!} = \sum_{k=0}^{\infty} \frac{M^k}{k!}$$

Take $M = iAx$. Odd, even terms behave differently!

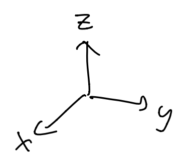
$$M^{2k} = (-1)^k A^{2k} x^{2k}$$

$$M^{2k+1} = i(-1)^k A^{2k+1} x^{2k+1}$$

$$\rightarrow \exp(M) = \sum_{k=0}^{\infty} \frac{M^{2k}}{(2k)!} + \sum_{k=0}^{\infty} \frac{M^{2k+1}}{(2k+1)!} = \sum_{k=0}^{\infty} \frac{(-1)^k \overset{I}{\cancel{A^{2k}}} x^{2k}}{(2k)!} + i \sum_{k=0}^{\infty} \frac{(-1)^k \overset{A}{\cancel{A^{2k+1}}} x^{2k+1}}{(2k+1)!}$$

$$= \cos(x)\, I + i \sin(x)\, A$$

4.4



Intuition: what does the Hadamard gate do?

$$|z;+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \xrightarrow{H} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = |x;+\rangle$$

$$|z;-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \longrightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = |x;-\rangle$$

$=$ $\frac{\pi}{2}$ rotation about y axis

CLAIM: $R_y(\frac{\pi}{2}) = R_x(\frac{\pi}{2})\, R_z(\frac{\pi}{2})\, R_x(\frac{\pi}{2})\, e^{-i\pi/2}$

verify this rigorously (but geometrically this is correct!)

4.47

Zassenhaus formula:

$$e^{t(X+Y)} = e^{tX} e^{tY} e^{-\frac{t^2}{2}[X,Y]} e^{\frac{t^3}{6}(2[Y,[X,Y]] + [X,[X,Y]])} \cdots$$

$$H = \sum_{k=1}^{L} H_k$$

$$e^{-iHt} = e^{-it[H_1 + \sum_{k=2}^{L} H_k]} = e^{-itH_1} e^{-it\sum_{k=2}^{L} H_k} e^{\frac{t^2}{2}\underbrace{[H_1, \sum_{k=2}^{L} H_k]}_{=0}} \prod e^{f(\underbrace{[H_1, \sum H_k]}_{=0})}$$

$$= e^{-itH_1} e^{-it\sum_{k=2}^{L} H_k}$$

$\vdots$ repeating for $e^{-it\sum_{k=2}^{L} H_k}$

$$= \prod_{k=1}^{L} e^{-itH_k}$$

4.48

terms $\leq$ # subsets of n of size at most c

$$= \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{c}$$

$$\leq c \cdot \binom{n}{c}$$

$$\leq \frac{n^c}{(c-1)!} \quad \text{(polynomial in c)}$$

$$\binom{n}{c} = \frac{n!}{c!(n-c)!} = \frac{n(n-1)\cdots(n-c+1)}{c!} \leq \frac{n^c}{c!}$$