APPLIED MATHEMATICS

Data-driven discovery of partial differential equations

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We propose a sparse regression method capable of discovering the governing partial differential equatior a given system by time series measurements in the spatial domain. The regression framework relies on sr promoting techniques to select the nonlinear and partial derivative terms of the governing equations tr accurately represent the data, bypassing a combinatorially large search through all possible candidat The method balances model complexity and regression accuracy by selecting a parsimonious model analysis. Time series measurements can be made in an Eulerian framework, where the sensors ar tially, or in a Lagrangian framework, where the sensors move with the dynamics. The method is com efficient, robust, and demonstrated to work on a variety of canonical problems spanning a numbr domains including Navier-Stokes, the quantum harmonic oscillator, and the diffusion equation. method is capable of disambiguating between potentially nonunique dynamical terms by usir series taken with different initial data. Thus, for a traveling wave, the method can distinguish wave equation and the Korteweg–de Vries equation, for instance. The method provides a pr nique for discovering governing equations and physical laws in parameterized spatiotempc first-principles derivations are intractable.

INTRODUCTION

Data-driven discovery methods, which have been enabled in the past decade by the plummeting cost of sensors, data storage, and computational resources, have a transformative impact on the sciences, facilitating a variety of innovations for characterizing high-dimensional data generated from experiments. Less well understood is how to uncover underlying physical laws and/or governing equations from time series data that exhibit spatiotemporal activity. Traditional theoretical methods for deriving the underlying partial differential equations (PDEs) are rooted in conservation laws, physical principles, and/or phenomenological behaviors. These first-principles derivations lead to many of the canonical models ubiquitous in physics, engineering,

recently, sparsity (*15*) efficient computati (*16*, *17*). Both the sion methods avo balance model method we pr linear, nonlin fication of P tive about The innov ciently h teristic