

A Troubling Analysis of Reproducibility and Progress in Recommender Systems Algorithms Research - Online Appendix

MAURIZIO FERRARI DACREMA, SIMONE BOGLIO, and PAOLO CREMONESI, Politecnico di Milano, Italy

DIETMAR JANNACH, University of Klagenfurt, Austria

The design of algorithms that generate personalized ranked item lists is a central topic of research in the field of recommender systems. In the past few years, in particular, approaches based on deep learning (neural) techniques have become dominant in the literature. For all of them, substantial progress over the state-of-the-art is claimed. However, indications exist of certain problems in today's research practice, e.g., with respect to the choice and optimization of the baselines used for comparison, raising questions about the published claims. In order to obtain a better understanding of the actual progress, we have tried to reproduce recent results in the area of neural recommendation approaches based on collaborative filtering. The worrying outcome of the analysis of these recent works—all were published at prestigious scientific conferences between 2015 and 2018—is that 11 out of the 12 reproducible neural approaches can be outperformed by conceptually simple methods, e.g., based on the nearest-neighbors heuristics. None of the computationally complex neural methods was actually consistently better than already existing learning-based techniques, e.g., using matrix factorization or linear models. In our analysis, we discuss common problematic issues in today's research practice, which, despite the many papers that are published on the topic, has apparently led the field to a certain level of stagnation.¹

CCS Concepts: • **Information systems** → **Recommender systems**; *Collaborative filtering*; • **General and reference** → Evaluation.

Additional Key Words and Phrases: Recommender Systems, Deep Learning, Evaluation; Reproducibility

ACM Reference Format:

Maurizio Ferrari Dacrema, Simone Boglio, Paolo Cremonesi, and Dietmar Jannach. 2019. A Troubling Analysis of Reproducibility and Progress in Recommender Systems Algorithms Research - Online Appendix. *ACM Transactions on Information Systems* 1, 1, Article 1 (January 2019), 127 pages. <https://doi.org/10.1145/1122445.1122456>

¹This paper significantly extends or own previous work presented in [12].

Authors' addresses: Maurizio Ferrari Dacrema, maurizio.ferrari@polimi.it; Simone Boglio, simone.boglio@mail.polimi.it; Paolo Cremonesi, paolo.cremonesi@polimi.it, Politecnico di Milano, Italy, Milano; Dietmar Jannach, University of Klagenfurt, Klagenfurt, Austria, dietmar.jannach@aau.at.

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. Copyrights for components of this work owned by others than ACM must be honored. Abstracting with credit is permitted. To copy otherwise, or republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee. Request permissions from permissions@acm.org.

© 2019 Association for Computing Machinery.

1046-8188/2019/1-ART1 \$15.00

<https://doi.org/10.1145/1122445.1122456>

CONTENTS

| | |
|---|-----|
| Abstract | 1 |
| Contents | 2 |
| A Overview | 3 |
| B Baselines | 5 |
| C KDD: Collaborative Deep Learning | 9 |
| D SIGIR: Collaborative Memory Networks | 10 |
| E KDD: Collaborative Variational Autoencoders | 20 |
| F RecSys: Spectral Collaborative Filtering | 34 |
| G KDD: Leveraging Meta-path based Context for Top-N Recommendation with a Neural Co-Attention Model | 50 |
| H WWW: Neural Collaborative Filtering | 57 |
| I WWW: Variational Autoencoders for Collaborative Filtering | 64 |
| J IJCAI: Outer Product-based Neural Collaborative Filtering | 69 |
| K IJCAI: NeuRec: On Nonlinear Transformation for Personalized Ranking | 76 |
| L IJCAI: Deep Matrix Factorization Models for Recommender Systems | 89 |
| M IJCAI: CoupledCF: Learning Explicit and Implicit User-item Couplings in Recommendation for Deep Collaborative Filtering | 102 |
| N IJCAI: DELF: A Dual-Embedding based Deep Latent Factor Model for Recommendation | 117 |
| O Hyperparameter Range | 124 |
| References | 126 |

A OVERVIEW

This is the additional material associated with our article [11]. This material contains the full results of our experiments of which, due to space reasons and for the sake of improving readability, only the most representative ones are reported in the paper. In Appendix B the complete list of all baselines is presented as long as a brief description and references for each of them. The following Appendices from C to N report the results of the evaluation of each deep learning algorithm, ordered by year of publication from 2015 to 2018. Lastly in Appendix O all hyperparameters for all baselines are listed with the relative search space.

The results for each deep learning algorithm we analysed are reported in a separate section. Each section is composed of three parts, a comparison of the recommendation accuracy of the algorithms, the list of all optimal hyperparameters, and a comparison of the computation time they required.

Recommendation accuracy. Compares the recommendation accuracy of all baselines and of the deep learning model in the evaluation scenario chosen by the original authors. Different tables will therefore report different metrics and cutoffs depending on the original paper. Values in bold refer to either the deep learning algorithm outperforming *all* baselines or any baseline outperforming the deep learning algorithm. In some cases the results for EASE^R and SLIM BPR may be missing, this is due to the memory requirement exceeding instance capacity as the implementations we used did not optimize memory requirements.

Optimal hyperparameters. Reports the optimal hyperparameters for all baselines and datasets. Due to the stochastic nature of the Bayesian optimization and on how many local optima the model exhibits for that dataset, multiple optimization runs may yield equivalent results but different hyperparameters.

Computation time. Compares the computation time of all algorithms on a specific Amazon AWS instance.² The tables are composed by three columns. The first column (*Train time*), reports the mean and standard deviation of the time required to fit the models during the Bayesian hyperparameter optimization. In case of machine learning models requiring the selection of the number of epochs via early-stopping, the time required by the validation steps is included as it constitutes an integral part of the training procedure. The last two columns report the time required by each evaluation of the model during the Bayesian hyperparameter optimization³ (*Recommendation Time*) and the number of recommendation lists the algorithm is able to generate per second (*Recommendation [usr/s]*). For deep learning algorithms the train and evaluation time refer to the only hyperparameter configuration we report, therefore they are not associated to any standard deviation.

It should be noted that all algorithms implemented in our repository compute a score for each item but do not directly generate the recommended items list. The sorting of such items and generation of the recommended items list is done independently from the specific recommendation model. Due to the fixed cost of ranking the items based on their score, for each user, non personalized

²The computation time refers to the total instance time for one AWS instance p3.2xlarge, with 8 vCPU, 30GB RAM, and one Tesla V100-SXM2-16GB GPU.

³Note that the evaluation time refers to an evaluation performed on the test data. During the Bayesian optimization every time a new optimal set of hyperparameters is found, using the validation data, an additional evaluation is performed on the test data. No information from the test data is ever used. For this reason, it may happen that a baseline is not associated to a standard deviation in Recommendation Time, this means that the Bayesian optimization found an optimal solution which was not improved upon and therefore only one evaluation was performed.

algorithms, i.e., TopPop, will appear to generate the same number of recommendation per second as much more complex models.

Furthermore, the implementations of the baseline algorithms vary in terms of efficiency. Some use standard solvers (PureSVD, NMF, SLIM ElasticNet), others are written in Cython⁴ and compiled (KNNs, MF BPR, FunkSVD, SLIM BPR), others are written in plain Python with vectorized operations ($P^3\alpha$, $RP^3\beta$, iALS), some are single-core others take advantage of multithreading. Similarly the deep learning models are implemented in Tensorflow or Keras and with varying degrees of efficiency. Due to this heterogeneity the computational time measurements should not be taken as exact measurements but rather as a qualitative comparison.

⁴<https://cython.org/>

B BASELINES

Over the last 25 years, a multitude of algorithms of different types were proposed. In order to obtain a picture that is as broad as possible, we selected algorithms of different families for inclusion in our measurements. An overview of all used baselines is given in Table 1 and the relative hyperparameter ranges are reported in Appendix O.

Table 1. Overview of Baseline Methods

| <i>Family</i> | <i>Method</i> | <i>Description</i> |
|-----------------------------|-------------------|--|
| Non-personalized | TopPopular | Recommends the most popular items to everyone [9] |
| Nearest-Neighbor | UserKNN | User-based k-nearest neighbors [23] |
| | ItemKNN | Item-based k-nearest neighbors [24] |
| Graph-based | $P^3\alpha$ | A graph-based method based on random walks [8] |
| | $RP^3\beta$ | An extension of $P^3\alpha$ [20] |
| Content-Based and Hybrid | ItemKNN-CBF | ItemKNN with content-based similarity [17] |
| | ItemKNN-CFCBF | A simple item-based hybrid CBF/CF approach [18] |
| | UserKNN-CBF | UserKNN with content-based similarity |
| | UserKNN-CFCBF | A simple user-based hybrid CBF/CF approach |
| Non-Neural Machine Learning | iALS | Matrix factorization for implicit feedback data [13] |
| | PureSVD | A basic matrix factorization method [9] |
| | NFM | A basic non-negative matrix factorization method [7] |
| | FunkSVD | Matrix factorization for rating prediction [14] |
| | MF BPR | Matrix factorization optimized for ranking [22] |
| | SLIM ElasticNet | A scalable linear model [15, 19] |
| | SLIM BPR | A variation of SLIM optimizing ranking [3] |
| | EASE ^R | A recent linear model, similar to auto-encoders [25] |

B.0.1 Popularity-Based Ranking. Recommending the most popular items to everyone is a common strategy in practice. The method **TopPopular** implements this non-personalized recommendation approach. The popularity of an item is determined by its number of implicit or explicit ratings in the given dataset.

B.0.2 Nearest-Neighbor Methods. Nearest-neighbor techniques were used in the early GroupLens system [23] and first successful reports of collaborative filtering systems also used nearest-neighbor techniques [16]. We consider both *user-based* and *item-based* variants, **UserKNN** and **ItemKNN**.

Many variants of the basic nearest-neighbor prediction scheme were proposed over the years, see [6] for an early performance comparison. In this work, we therefore consider different variations of the nearest-neighbor techniques as well. For both UserKNN and ItemKNN, the following hyperparameters can be set and were optimized in our experiments, their ranges are reported in Appendix O.

- *Neighborhood Size*: This main parameter determines how many neighbors are considered for prediction.
- *Similarity Measure*: We made experiments with the Jaccard coefficient [21] as well as Cosine [24], Asymmetric Cosine [1], Dice-Sørensen [10] and Tversky [27] similarities. Some of these similarity measures also have their own parameters, as reported in Appendix O, which we optimized as well.
- *Shrinkage*: As proposed in [4], we used a parameter (the *shrink term*) to lower the similarity between items that have only few interactions in common. The shrinkage is applied to all similarities.
- *Feature Weighting*: Using feature weighting for ratings was proposed in [28]. In our experiments, we both tested configurations with no weighting and weighting with either the TF-IDF or the BM25 scheme.
- *Normalization*: This setting determines if we should consider the denominator in the similarity measure as normalization. Only some of the similarity measures have this parameter.

B.0.3 Graph-based Methods. Traditional nearest-neighbor models consider “direct” neighborhoods by computing similarities between pairs of objects. Graph-based models can help to overcome this possible limitation relying on a broader interpretation of neighborhoods. In our study, we consider two such graph-based methods called $P^3\alpha$ [8] and $RP^3\beta$ [20]. Both methods often lead to good recommendation quality at low computational cost. Interestingly, these two methods appear to be almost unknown in the community and seldom used as baselines, despite the fact that they are very simple, effective and have been published in top-tier venues.

- $P^3\alpha$: This method implements a two-steps random walk from users to items and vice-versa, where the probabilities to jump between users and items are computed from the normalized ratings raised to the power of α . The method is equivalent to a KNN item-based CF algorithm, with the similarity matrix being computed as the dot-product of the probability vectors [8]. In addition to what described in the original algorithm, we normalize each row of the similarity matrix with its $l1$ norm. The hyperparameters of the algorithm include the size of the neighborhood and the value for α .
- $RP^3\beta$: This is an improved version of $P^3\alpha$ proposed in [20]. In $RP^3\beta$, each similarity between two items is computed with $P^3\alpha$ and divided by the popularity of the items raised to the power of β . Again, we normalize each row of the similarity matrix with its $l1$ norm. If β is 0, $RP^3\beta$ is equivalent to $P^3\alpha$. The hyperparameters of the algorithm are the size of the neighborhood and the values for α and β .

B.0.4 Content-based and hybrid Methods. Some of the neural methods investigated in this paper include side information about items or users. We have therefore included two simple baselines that make usage of content information.

- **ItemKNN-CBF, UserKNN-CBF**: A neighborhood-based content-based-filtering (CBF) approach, where we compute the item (or user) similarities based on the items’ (or user’s) content features (attributes) [17]. We tested the same set of similarity measures described for the collaborative KNN methods (Jaccard coefficient, Cosine, Asymmetric Cosine, Dice-Sørensen and Tversky similarity). The hyperparameters are the same as for the ItemKNN and UserKNN methods.
- **ItemKNN-CFCBF, UserKNN-CFCBF**: A hybrid algorithm based on item-item (or user-user) similarities and described in [18]. The similarity between items is computed by first concatenating, for each item, the vector of implicit ratings (collaborative features) and the vector of item attributes (content features) and by later computing the similarity between

the concatenated vectors. In case of user-user similarities the algorithm operates in a similar way, concatenating the vector of implicit ratings of each user with the user's content feature vector. The hyperparameters and similarity measures are the same as for ItemKNN, plus a parameter w that controls the relative importance of the content features with respect to the collaborative features. When w is 0, this algorithm is equivalent to the pure collaborative versions, either ItemKNN or UserKNN.

B.0.5 Non-Neural Machine Learning Approaches. Countless machine learning models were proposed for *top-n* recommendation tasks in the literature. In our experiments, we included a number of comparably basic models from the literature as representatives of which methods were often considered the state-of-the-art in pre-neural times.

- **Matrix Factorization (MF) Techniques:** The application of matrix decomposition methods for collaborative filtering problems was investigated already in the early years of recommender systems [5], and became a de-facto standard after the Netflix prize competition (2006-2009). We made experiments with many variants, but will limit our discussion to two main techniques which proved to consistently lead to competitive results among the different MF techniques.
 - **iALS:** In their seminal work [13], Hu et al. proposed an *Alternating Least Squares* approach for implicit feedback datasets, which turns implicit feedback signals into confidence values. The authors also proposed a particular optimization method that has the advantage of scaling well on larger datasets. A number of hyperparameters can be tuned for the method, including the number of latent factors, the confidence scaling and the regularization factor.
 - **PureSVD:** This method corresponds to a basic matrix factorization approach as proposed in [9]. To implement PureSVD, we used a standard SVD decomposition method provided in the scikit-learn package for Python.⁵ The only hyperparameter of this method is the number of latent factors.
 - **NMF:** This method performs a *Non Negative Matrix Factorization*, which is described in [7]. As opposed to PureSVD, NFM guarantees all latent factors to be positive. We used a standard NMF decomposition method provided in the scikit-learn package for Python.⁶ The only hyperparameter of this method is the number of latent factors.
 - **FunkSVD:** This matrix factorization algorithm was proposed by Simon Funk in his well known online article⁷ during the Netflix Prize. This method optimises rating prediction via MSE. The embeddings of users and items are regularised with a Frobenius norm. In order to ensure the suitability of FunkSVD for a *top-n* recommendation task we added a hyperparameter which ensures a certain quota of the samples used during training are randomly sampled among the unseen items and are associated with a rating of 0. Another hyperparameter controls whether the model should include the global bias, user bias and item bias. Other hyperparameters include the learning rate, the regularisation coefficients, and the number of latent factors.
 - **MF BPR:** This algorithm was presented in the well known article from Rendle et al. [22] as a matrix factorization model optimizing ranking accuracy via a BPR loss. MF BPR is a widely used baseline in the article we surveyed. This method, as opposed to FunkSVD, PureSVD and NFM, has been explicitly designed for implicit interactions. Furthermore, as opposed to iALS it is trained using gradient ascent. Hyperparameters of this method include the number of latent factor, the learning rate and the regularization coefficients.

⁵https://scikit-learn.org/stable/modules/generated/sklearn.utils.extmath.randomized_svd.html

⁶<https://scikit-learn.org/stable/modules/generated/sklearn.decomposition.NMF.html>

⁷<http://sifter.org/~simon/journal/20061211.html>

- *Sparse Linear Models (SLIM)*: SLIM was proposed as a well-performing regression-based method for *top-n* recommendation tasks in [19]. In our work, we use the more scalable variant proposed in [15] (**SLIM ElasticNet**) which learns the item similarity matrix one item at a time (e.g. one column w at a time) by solving a regression problem in such a way that the interactions for the target item y are learned by using all other interactions as training data. To implement *SLIM ElasticNet* we used a standard ElasticNet solver provided in the `scikit-learn` package for Python.⁸ The hyperparameters of this method include the ratio of $l1$ and $l2$ regularizations as well as a regularization magnitude coefficient.
- *Sparse Linear Models BPR*: This algorithm is a variant of the previously mentioned SLIM ElasticNet which optimizes ranking accuracy rather than prediction error (**SLIM BPR**) [2, 3, 26]. The algorithm learns an item-item similarity matrix by optimizing the BPR loss function, described in [22], via gradient ascent. The hyperparameters of this method include the number of neighbours as described in the Nearest-Neighbor Methods, the regularization coefficients and whether the learned similarity matrix should be symmetric or not.
- *EASE^R*: In a recent article [25] the author showed that an “embarrassingly shallow” linear model, which shares similarities with an auto-encoder, can produce highly-accurate recommendations that often outperform existing and much more complex techniques. A peculiarity of this model is the existence of a closed-form solution for the training objective which results in very fast training. The only hyperparameter is the choice of the regularization factor. This algorithm has been published in 2019 and, as such, the papers covered by our study could not include EASE^R as a baseline. However, we include EASE^R to investigate whether shallow auto-encoders are able to provide, on average, more accurate recommendations with respect to complex deep-learning architectures.

⁸https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.ElasticNet.html

C KDD: COLLABORATIVE DEEP LEARNING

This algorithm is evaluated in the same experimental conditions and on the same data as *CVAE*. For the full results please refer to Section E.

D SIGIR: COLLABORATIVE MEMORY NETWORKS

Relevant statistics on the dataset, which we mentioned in the paper, are reported in Table 2 and 3. The results of our evaluation can be seen in Table 4 (CiteULike), Table 5 (Epinions) and Table 6 (Pinterest). The corresponding optimal hyperparameters are reported in Table 10 (collaborative KNNs), Table 11 (non-neural machine learning and graph based) and Table 12 (CMN).

Lastly, the time required to train and evaluate the models is reported in Table 7 (CiteULike), Table 8 (Epinions) and Table 9 (Pinterest).

Table 2. Dataset characteristics.

| Dataset | Interactions | Items | Users | Sparsity |
|-------------|--------------|---------|--------|----------|
| Epinions | 664.8 k | 139.7 k | 40.1 k | 99.98% |
| CiteULike-a | 204.9 k | 16.9 k | 5.5 k | 99.78% |
| Pinterest | 1.5 M | 9.9 k | 55.1 k | 99.73% |

Table 3. Dataset popularity bias characteristics.

| | Max pop | Min pop | Avg pop | Gini Index | Shannon | Herfindahl |
|-----------|---------|---------|---------|------------|---------|------------|
| Citeulike | 321.00 | 1.00 | 12.07 | 0.37 | 13.65 | 1.00 |
| Pinterest | 1636.00 | 1.00 | 147.60 | 0.45 | 12.77 | 1.00 |
| Epinions | 2026.00 | 1.00 | 4.76 | 0.69 | 15.11 | 1.00 |

Table 4. Experimental results for the CMN method for the Citeulike dataset.

| | @ 5 | | @ 10 | |
|-----------------------|---------------|---------------|---------------|---------------|
| | HR | NDCG | HR | NDCG |
| Random | 0.0503 | 0.0293 | 0.0960 | 0.0439 |
| TopPopular | 0.1810 | 0.1226 | 0.2774 | 0.1537 |
| UserKNN CF cosine | 0.8231 | 0.7027 | 0.8962 | 0.7265 |
| UserKNN CF dice | 0.8099 | 0.6839 | 0.8836 | 0.7079 |
| UserKNN CF jaccard | 0.8116 | 0.6880 | 0.8838 | 0.7115 |
| UserKNN CF asymmetric | 0.8226 | 0.7039 | 0.8959 | 0.7279 |
| UserKNN CF tversky | 0.8121 | 0.6892 | 0.8867 | 0.7135 |
| ItemKNN CF cosine | 0.8247 | 0.7045 | 0.8925 | 0.7267 |
| ItemKNN CF dice | 0.8089 | 0.6823 | 0.8863 | 0.7075 |
| ItemKNN CF jaccard | 0.8065 | 0.6793 | 0.8861 | 0.7053 |
| ItemKNN CF asymmetric | 0.8233 | 0.7041 | 0.8944 | 0.7274 |
| ItemKNN CF tversky | 0.8081 | 0.6796 | 0.8874 | 0.7055 |
| $P^3\alpha$ | 0.8272 | 0.7144 | 0.8971 | 0.7370 |
| $RP^3\beta$ | 0.8326 | 0.7227 | 0.9002 | 0.7447 |
| EASE ^R | 0.8107 | 0.6966 | 0.8771 | 0.7182 |
| SLIM BPR | 0.8099 | 0.6916 | 0.8861 | 0.7164 |
| SLIM ElasticNet | 0.8265 | 0.7168 | 0.8908 | 0.7376 |
| MF BPR | 0.7316 | 0.6053 | 0.8245 | 0.6356 |
| MF FunkSVD | 0.7860 | 0.6488 | 0.8672 | 0.6752 |
| PureSVD | 0.7233 | 0.6020 | 0.7954 | 0.6254 |
| NMF | 0.7161 | 0.5534 | 0.8245 | 0.5887 |
| iALS | 0.8308 | 0.7085 | 0.9006 | 0.7313 |
| CMN | 0.7874 | 0.6505 | 0.8746 | 0.6790 |

Table 5. Experimental results for the CMN method for the Epinions dataset.

| | @ 5 | | @ 10 | |
|-----------------------|---------------|---------------|---------------|---------------|
| | HR | NDCG | HR | NDCG |
| Random | 0.0496 | 0.0293 | 0.0987 | 0.0449 |
| TopPopular | 0.5492 | 0.4204 | 0.6672 | 0.4587 |
| UserKNN CF cosine | 0.4282 | 0.3631 | 0.4764 | 0.3787 |
| UserKNN CF dice | 0.4108 | 0.3475 | 0.4589 | 0.3630 |
| UserKNN CF jaccard | 0.4108 | 0.3473 | 0.4589 | 0.3628 |
| UserKNN CF asymmetric | 0.4294 | 0.3642 | 0.4767 | 0.3795 |
| UserKNN CF tversky | 0.4207 | 0.3571 | 0.4700 | 0.3731 |
| ItemKNN CF cosine | 0.4309 | 0.3584 | 0.4854 | 0.3760 |
| ItemKNN CF dice | 0.4088 | 0.3426 | 0.4631 | 0.3601 |
| ItemKNN CF jaccard | 0.4088 | 0.3427 | 0.4631 | 0.3602 |
| ItemKNN CF asymmetric | 0.4149 | 0.3437 | 0.4761 | 0.3635 |
| ItemKNN CF tversky | 0.4179 | 0.3476 | 0.4757 | 0.3662 |
| $P^3\alpha$ | 0.4008 | 0.3411 | 0.4389 | 0.3533 |
| $RP^3\beta$ | 0.3928 | 0.3329 | 0.4341 | 0.3462 |
| EASE ^R | - | - | - | - |
| SLIM BPR | 0.3988 | 0.3393 | 0.4422 | 0.3533 |
| SLIM ElasticNet | 0.4133 | 0.3471 | 0.4667 | 0.3643 |
| MF BPR | 0.4668 | 0.3662 | 0.5594 | 0.3962 |
| MF FunkSVD | 0.5427 | 0.4196 | 0.6567 | 0.4566 |
| PureSVD | 0.4073 | 0.3069 | 0.5045 | 0.3384 |
| NMF | 0.4055 | 0.3218 | 0.4951 | 0.3508 |
| iALS | 0.0519 | 0.0316 | 0.1003 | 0.0470 |
| CMN | 0.4699 | 0.3781 | 0.5399 | 0.4008 |

Table 6. Experimental results for the CMN method for the Pinterest dataset.

| | @ 5 | | @ 10 | |
|-----------------------|---------------|---------------|---------------|---------------|
| | HR | NDCG | HR | NDCG |
| Random | 0.0499 | 0.0296 | 0.0984 | 0.0450 |
| TopPopular | 0.1665 | 0.1064 | 0.2740 | 0.1409 |
| UserKNN CF cosine | 0.7017 | 0.5050 | 0.8614 | 0.5570 |
| UserKNN CF dice | 0.7026 | 0.5053 | 0.8634 | 0.5578 |
| UserKNN CF jaccard | 0.7034 | 0.5062 | 0.8639 | 0.5585 |
| UserKNN CF asymmetric | 0.7005 | 0.5037 | 0.8630 | 0.5567 |
| UserKNN CF tversky | 0.7024 | 0.5047 | 0.8636 | 0.5572 |
| ItemKNN CF cosine | 0.7132 | 0.5116 | 0.8781 | 0.5653 |
| ItemKNN CF dice | 0.7095 | 0.5091 | 0.8766 | 0.5635 |
| ItemKNN CF jaccard | 0.7094 | 0.5086 | 0.8764 | 0.5630 |
| ItemKNN CF asymmetric | 0.7126 | 0.5110 | 0.8776 | 0.5648 |
| ItemKNN CF tversky | 0.7095 | 0.5086 | 0.8761 | 0.5629 |
| $P^3\alpha$ | 0.6990 | 0.5034 | 0.8596 | 0.5559 |
| $RP^3\beta$ | 0.7147 | 0.5150 | 0.8772 | 0.5680 |
| EASE ^R | 0.7050 | 0.5106 | 0.8559 | 0.5599 |
| SLIM BPR | 0.7120 | 0.5151 | 0.8733 | 0.5678 |
| SLIM ElasticNet | 0.7084 | 0.5107 | 0.8683 | 0.5628 |
| MF BPR | 0.6924 | 0.4886 | 0.8694 | 0.5463 |
| MF FunkSVD | 0.7088 | 0.5037 | 0.8686 | 0.5559 |
| PureSVD | 0.6619 | 0.4721 | 0.8146 | 0.5219 |
| NMF | 0.6550 | 0.4618 | 0.8287 | 0.5183 |
| iALS | 0.7219 | 0.5175 | 0.8677 | 0.5652 |
| CMN | 0.7013 | 0.5005 | 0.8674 | 0.5547 |

Table 7. Computation time for the algorithms in the selected results for the CMN method on the Citeulike dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|-----------------------|---|------------------------|---------------------------|
| Random | 0.00 [sec] | 4.83 [sec] | 1150 |
| TopPopular | 0.01 [sec] | 5.46 [sec] | 1017 |
| UserKNN CF cosine | 0.52 ± 0.04 [sec] | 9.76 ± 0.23 [sec] | 566 |
| UserKNN CF dice | 0.52 ± 0.04 [sec] | 9.41 ± 0.39 [sec] | 575 |
| UserKNN CF jaccard | 0.52 ± 0.04 [sec] | 9.69 ± 0.38 [sec] | 572 |
| UserKNN CF asymmetric | 0.51 ± 0.04 [sec] | 9.80 ± 0.07 [sec] | 574 |
| UserKNN CF tversky | 0.50 ± 0.04 [sec] | 9.58 ± 0.02 [sec] | 580 |
| ItemKNN CF cosine | 3.10 ± 0.31 [sec] | 9.75 ± 0.41 [sec] | 564 |
| ItemKNN CF dice | 3.06 ± 0.21 [sec] | 9.70 ± 0.38 [sec] | 554 |
| ItemKNN CF jaccard | 3.06 ± 0.21 [sec] | 9.87 ± 0.16 [sec] | 575 |
| ItemKNN CF asymmetric | 3.24 ± 0.21 [sec] | 9.73 ± 0.44 [sec] | 553 |
| ItemKNN CF tversky | 3.02 ± 0.24 [sec] | 9.70 ± 0.17 [sec] | 581 |
| $P^3\alpha$ | 13.78 ± 2.87 [sec] | 9.56 ± 0.13 [sec] | 583 |
| $RP^3\beta$ | 15.82 ± 3.05 [sec] | 9.51 ± 0.26 [sec] | 576 |
| EASE ^R | 102.76 [sec] / 1.71 ± 0.01 [min] | 8.99 ± 0.05 [sec] | 612 |
| SLIM BPR | 645.01 [sec] / 10.75 ± 4.22 [min] | 10.16 ± 0.22 [sec] | 538 |
| SLIM ElasticNet | 236.77 [sec] / 3.95 ± 1.56 [min] | 9.79 ± 0.66 [sec] | 559 |
| MF BPR | 776.37 [sec] / 12.94 ± 8.09 [min] | 6.51 ± 1.04 [sec] | 879 |
| MF FunkSVD | 1057.07 [sec] / 17.62 ± 12.82 [min] | 6.12 ± 0.37 [sec] | 881 |
| PureSVD | 1.23 ± 0.47 [sec] | 7.32 ± 0.21 [sec] | 744 |
| NMF | 153.39 [sec] / 2.56 ± 2.07 [min] | 6.71 ± 0.50 [sec] | 870 |
| iALS | 593.57 [sec] / 9.89 ± 4.71 [min] | 5.92 ± 0.21 [sec] | 911 |
| CMN | 6818.32 [sec] / 1.89 [hour] | 20.18 [sec] | 275 |

Table 8. Computation time for the algorithms in the selected results for the CMN method on the Epinions dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|-----------------------|--|--------------------------------------|---------------------------|
| Random | 0.01 [sec] | 56.42 [sec] | 712 |
| TopPopular | 0.02 [sec] | 91.41 [sec] / 1.52 [min] | 439 |
| UserKNN CF cosine | 12.81 ± 0.45 [sec] | 120.93 [sec] / 2.02 ± 0.02 [min] | 330 |
| UserKNN CF dice | 12.51 ± 0.39 [sec] | 119.91 [sec] / 2.00 ± 0.03 [min] | 329 |
| UserKNN CF jaccard | 12.51 ± 0.41 [sec] | 120.24 [sec] / 2.00 ± 0.02 [min] | 331 |
| UserKNN CF asymmetric | 13.04 ± 0.37 [sec] | 121.49 [sec] / 2.02 ± 0.03 [min] | 325 |
| UserKNN CF tversky | 12.66 ± 0.36 [sec] | 121.45 [sec] / 2.02 ± 0.01 [min] | 331 |
| ItemKNN CF cosine | 125.68 [sec] / 2.09 ± 0.14 [min] | 128.99 [sec] / 2.15 ± 0.05 [min] | 305 |
| ItemKNN CF dice | 122.99 [sec] / 2.05 ± 0.01 [min] | 127.09 [sec] / 2.12 ± 0.04 [min] | 311 |
| ItemKNN CF jaccard | 123.08 [sec] / 2.05 ± 0.01 [min] | 128.41 [sec] / 2.14 ± 0.03 [min] | 306 |
| ItemKNN CF asymmetric | 126.35 [sec] / 2.11 ± 0.02 [min] | 129.97 [sec] / 2.17 ± 0.07 [min] | 303 |
| ItemKNN CF tversky | 125.31 [sec] / 2.09 ± 0.01 [min] | 127.61 [sec] / 2.13 ± 0.06 [min] | 306 |
| $P^3\alpha$ | 367.87 [sec] / 6.13 ± 0.19 [min] | 116.08 [sec] / 1.93 ± 0.03 [min] | 341 |
| $RP^3\beta$ | 395.01 [sec] / 6.58 ± 0.20 [min] | 116.68 [sec] / 1.94 ± 0.03 [min] | 339 |
| EASE ^R | - | - | - |
| SLIM BPR | 42149.10 [sec] / 11.71 ± 5.47 [hour] | 124.94 [sec] / 2.08 ± 0.07 [min] | 323 |
| SLIM ElasticNet | 14201.25 [sec] / 3.94 ± 1.31 [hour] | 127.63 [sec] / 2.13 ± 0.14 [min] | 310 |
| MF BPR | 10857.32 [sec] / 3.02 ± 1.65 [hour] | 98.43 [sec] / 1.64 ± 0.28 [min] | 440 |
| MF FunkSVD | 3409.08 [sec] / 56.82 ± 68.92 [min] | 105.37 [sec] / 1.76 ± 0.19 [min] | 327 |
| PureSVD | 2.36 ± 3.67 [sec] | 88.22 [sec] / 1.47 ± 0.04 [min] | 464 |
| NMF | 1754.00 [sec] / 29.23 ± 18.12 [min] | 100.15 [sec] / 1.67 ± 0.18 [min] | 448 |
| iALS | 4470.54 [sec] / 1.24 ± 0.79 [hour] | 87.28 [sec] / 1.45 ± 0.00 [min] | 459 |
| CMN | 33203.75 [sec] / 9.22 [hour] | 292.74 [sec] / 4.88 [min] | 137 |

Table 9. Computation time for the algorithms in the selected results for the CMN method on the Pinterest dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|-------------------------|---|-------------------------------------|---------------------------|
| Random | 0.02 [sec] | 47.56 [sec] | 1160 |
| TopPopular | 0.04 [sec] | 52.75 [sec] | 1046 |
| UserKNN CF cosine | 28.98 ± 1.55 [sec] | 94.03 [sec] / 1.57 ± 0.03 [min] | 586 |
| UserKNN CF dice | 29.42 ± 0.98 [sec] | 94.33 [sec] / 1.57 ± 0.02 [min] | 578 |
| UserKNN CF jaccard | 29.45 ± 1.21 [sec] | 94.86 [sec] / 1.58 ± 0.01 [min] | 582 |
| UserKNN CF asymmetric | 30.05 ± 1.40 [sec] | 94.93 [sec] / 1.58 ± 0.06 [min] | 567 |
| UserKNN CF tversky | 28.91 ± 1.58 [sec] | 95.05 [sec] / 1.58 ± 0.02 [min] | 571 |
| ItemKNN CF cosine | 1.82 ± 0.19 [sec] | 92.88 [sec] / 1.55 ± 0.02 [min] | 592 |
| ItemKNN CF dice | 1.76 ± 0.21 [sec] | 91.06 [sec] / 1.52 ± 0.04 [min] | 594 |
| ItemKNN CF jaccard | 1.77 ± 0.17 [sec] | 91.28 [sec] / 1.52 ± 0.04 [min] | 597 |
| ItemKNN CF asymmetric | 1.78 ± 0.17 [sec] | 90.51 [sec] / 1.51 ± 0.05 [min] | 593 |
| ItemKNN CF tversky | 1.74 ± 0.16 [sec] | 90.53 [sec] / 1.51 ± 0.04 [min] | 595 |
| P ³ α | 8.71 ± 2.25 [sec] | 88.71 [sec] / 1.48 ± 0.02 [min] | 627 |
| RP ³ β | 9.23 ± 2.85 [sec] | 90.04 [sec] / 1.50 ± 0.03 [min] | 608 |
| EASE ^R | 22.30 ± 0.27 [sec] | 76.12 [sec] / 1.27 ± 0.02 [min] | 721 |
| SLIM BPR | 3594.20 [sec] / 59.90 ± 28.93 [min] | 91.58 [sec] / 1.53 ± 0.03 [min] | 597 |
| SLIM ElasticNet | 433.57 [sec] / 7.23 ± 2.50 [min] | 91.23 [sec] / 1.52 ± 0.04 [min] | 595 |
| MF BPR | 6439.39 [sec] / 1.79 ± 1.12 [hour] | 64.56 [sec] / 1.08 ± 0.18 [min] | 755 |
| MF FunkSVD | 8220.55 [sec] / 2.28 ± 1.76 [hour] | 58.83 ± 10.08 [sec] | 1006 |
| PureSVD | 2.33 ± 1.89 [sec] | 56.22 ± 0.27 [sec] | 984 |
| NMF | 686.16 [sec] / 11.44 ± 9.74 [min] | 72.56 [sec] / 1.21 ± 0.26 [min] | 937 |
| iALS | 2694.24 [sec] / 44.90 ± 36.27 [min] | 57.41 ± 1.73 [sec] | 955 |
| CMN | 28100.23 [sec] / 7.81 [hour] | 354.04 [sec] / 5.90 [min] | 156 |

Table 10. Hyperparameter values for our collaborative KNN baselines on all datasets.

| Algorithm | Hyperparameter | CiteULike | Pinterest | Epinions |
|-----------------------|-------------------|------------|------------|------------|
| UserKNN CF cosine | topK | 578 | 668 | 1000 |
| | shrink | 0 | 0 | 0 |
| | similarity | cosine | cosine | cosine |
| | normalize | True | True | True |
| | feature weighting | BM25 | none | TF-IDF |
| UserKNN CF dice | topK | 627 | 818 | 1000 |
| | shrink | 0 | 0 | 0 |
| | similarity | dice | dice | dice |
| | normalize | False | True | False |
| UserKNN CF jaccard | topK | 637 | 807 | 1000 |
| | shrink | 0 | 0 | 0 |
| | similarity | jaccard | jaccard | jaccard |
| | normalize | False | True | False |
| UserKNN CF asymmetric | topK | 690 | 1000 | 1000 |
| | shrink | 1000 | 0 | 163 |
| | similarity | asymmetric | asymmetric | asymmetric |
| | normalize | True | True | True |
| | asymmetric alpha | 1.0291 | 0.4622 | 0.4379 |
| | feature weighting | BM25 | BM25 | TF-IDF |
| UserKNN CF tversky | topK | 533 | 940 | 935 |
| | shrink | 35 | 0 | 9 |
| | similarity | tversky | tversky | tversky |
| | normalize | True | True | True |
| | tversky alpha | 1.4634 | 2.0000 | 0.1591 |
| | tversky beta | 0.0885 | 0.0000 | 1.9682 |
| ItemKNN CF cosine | topK | 594 | 942 | 1000 |
| | shrink | 999 | 1000 | 448 |
| | similarity | cosine | cosine | cosine |
| | normalize | True | True | False |
| | feature weighting | TF-IDF | BM25 | TF-IDF |
| ItemKNN CF dice | topK | 996 | 981 | 1000 |
| | shrink | 11 | 0 | 1000 |
| | similarity | dice | dice | dice |
| | normalize | False | False | True |
| ItemKNN CF jaccard | topK | 480 | 983 | 1000 |
| | shrink | 3 | 0 | 1000 |
| | similarity | jaccard | jaccard | jaccard |
| | normalize | True | True | False |
| ItemKNN CF asymmetric | topK | 1000 | 1000 | 1000 |
| | shrink | 649 | 845 | 850 |
| | similarity | asymmetric | asymmetric | asymmetric |
| | normalize | True | True | True |
| | asymmetric alpha | 0.2742 | 0.2281 | 1.5411 |
| | feature weighting | TF-IDF | BM25 | none |
| ItemKNN CF tversky | topK | 421 | 1000 | 1000 |
| | shrink | 28 | 0 | 555 |
| | similarity | tversky | tversky | tversky |
| | normalize | True | True | True |
| | tversky alpha | 0.0103 | 1.9767 | 0.0000 |
| | tversky beta | 0.9612 | 2.0000 | 0.0000 |

Table 11. Hyperparameter values for our non-neural machine learning and graph based baselines on all datasets.

| Algorithm | Hyperparameter | CiteULike | Pinterest | Epinions |
|-----------------|----------------------|------------------|----------------|----------------|
| $P^3\alpha$ | topK | 653 | 453 | 1000 |
| | alpha | 0.6310 | 1.1895 | 0.1164 |
| | normalize similarity | False | True | False |
| $RP^3\beta$ | topK | 764 | 816 | 1000 |
| | alpha | 0.7110 | 1.1916 | 0.0000 |
| | beta | 0.2297 | 0.4365 | 0.0000 |
| | normalize similarity | True | True | False |
| $EASE^R$ | l2 norm | 4.60E+02 | 1.72E+03 | - |
| SLIM BPR | topK | 803 | 726 | 1000 |
| | epochs | 165 | 235 | 370 |
| | symmetric | False | True | False |
| | sgd mode | adam | adagrad | adagrad |
| | lambda i | 1.00E-02 | 1.00E-05 | 1.00E-02 |
| | lambda j | 1.00E-02 | 3.06E-05 | 1.00E-02 |
| | learning rate | 1.00E-04 | 1.00E-01 | 1.00E-04 |
| SLIM ElasticNet | topK | 1000 | 705 | 1000 |
| | l1 ratio | 4.21E-05 | 1.55E-04 | 1.00E-05 |
| | alpha | 0.0265 | 0.0316 | 0.2911 |
| MF BPR | sgd mode | adam | adagrad | adagrad |
| | epochs | 1045 | 935 | 995 |
| | num factors | 175 | 146 | 200 |
| | batch size | 512 | 128 | 16 |
| | positive reg | 9.89E-03 | 7.72E-03 | 1.00E-02 |
| | negative reg | 7.25E-03 | 1.00E-02 | 1.00E-02 |
| | learning rate | 2.80E-03 | 4.63E-02 | 1.00E-01 |
| MF FunkSVD | sgd mode | adam | adam | adam |
| | epochs | 300 | 500 | 75 |
| | use bias | True | False | True |
| | batch size | 16 | 8 | 4 |
| | num factors | 55 | 37 | 1 |
| | item reg | 4.02E-05 | 1.00E-05 | 1.00E-05 |
| | user reg | 1.00E-02 | 1.00E-02 | 9.01E-03 |
| | learning rate | 2.44E-03 | 5.99E-04 | 1.58E-04 |
| | negative quota | 0.2792 | 0.0941 | 0.4998 |
| PureSVD | num factors | 320 | 77 | 1 |
| NMF | num factors | 122 | 77 | 45 |
| | solver | mult. update | coord. descent | coord. descent |
| | init type | nndsvda | nndsvda | random |
| | beta loss | kullback-leibler | frobenius | frobenius |
| iALS | num factors | 115 | 52 | 49 |
| | confidence scaling | linear | linear | log |
| | alpha | 15.4014 | 50.0000 | 9.8676 |
| | epsilon | 0.4163 | 0.0052 | 0.0013 |
| | reg | 1.00E-05 | 1.00E-05 | 6.20E-03 |
| | epochs | 60 | 90 | 100 |

Table 12. Hyperparameter values for the neural algorithm on all datasets.

| Algorithm | Hyperparameter | CiteULike | Pinterest | Epinions |
|-----------|----------------|-----------|-----------|----------|
| CMN | epochs | 50 | 5 | 45 |
| | epochs gmf | 100 | 100 | 100 |
| | hops | 3 | 3 | 3 |
| | neg samples | 4 | 4 | 4 |
| | reg l2 cmn | 1.00E-01 | 1.00E-01 | 1.00E-01 |
| | reg l2 gmf | 1.00E-04 | 1.00E-04 | 1.00E-04 |
| | pretrain | True | True | True |
| | learning rate | 1.00E-03 | 1.00E-03 | 1.00E-03 |
| | verbose | False | False | False |
| | batch size | 128 | 256 | 128 |
| | embed size | 50 | 50 | 40 |

E KDD: COLLABORATIVE VARIATIONAL AUTOENCODERS

Relevant statistics on the dataset, which we mentioned in the paper, are reported in Table 13 and 14. The results of our evaluation can be seen in Table 15 (CiteULike-a, $P=1$), Table 16 (CiteULike-a, $P=10$), Table 17 (CiteULike-t, $P=1$), Table 18 (CiteULike-t, $P=10$). The corresponding optimal hyperparameters are reported in Table 23 (collaborative KNNs), Table 24 (non-neural machine learning and graph based), Table 25 (content-based KNNs), Table 26 (hybrid KNNs) and Table 27 (CVAE and CDL).

Lastly, the time required to train and evaluate the models is reported in Table 19 (CiteULike-a, $P=1$), Table 20 (CiteULike-a, $P=10$), Table 21 (CiteULike-t, $P=1$), Table 22 (CiteULike-t, $P=10$).

Table 13. Dataset characteristics.

| Dataset | Interactions | Items | Users | Sparsity | Item features |
|--------------|--------------|--------|---------|----------|---------------|
| CiteULike-a | 204.9 k | 16.9 k | 5.5 k | 99.78% | 8.0 k |
| CiteULike-t | 134.8 k | 25.9 k | 7.9 k | 99.93% | 20.0 k |
| NetflixPrize | 15.3 M | 9.2 k | 407.2 k | 99.59% | 20.0 k |

Table 14. Train data density for the different experimental settings of CDL.

| Dataset | Experiment | Interactions | Density |
|-------------|------------|--------------|------------|
| CiteULike-a | $P = 1$ | 5.5 k | $5.8e - 5$ |
| CiteULike-a | $P = 10$ | 55.5 k | $5.8e - 4$ |
| CiteULike-t | $P = 1$ | 7.9 k | $3.8e - 5$ |
| CiteULike-t | $P = 10$ | 53.3 k | $2.5e - 4$ |

Table 15. Experimental results for the CVAE method for the CiteULike-a P=1 dataset.

| | REC@50 | REC@100 | REC@150 | REC@200 | REC@250 | REC@300 |
|--------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| Random | 0.0027 | 0.0057 | 0.0084 | 0.0113 | 0.0142 | 0.0171 |
| TopPopular | 0.0253 | 0.0389 | 0.0486 | 0.0589 | 0.0651 | 0.0704 |
| UserKNN CF cosine | 0.0026 | 0.0053 | 0.0069 | 0.0102 | 0.0127 | 0.0154 |
| UserKNN CF dice | 0.0026 | 0.0053 | 0.0069 | 0.0102 | 0.0127 | 0.0154 |
| UserKNN CF jaccard | 0.0026 | 0.0053 | 0.0069 | 0.0102 | 0.0127 | 0.0154 |
| UserKNN CF asymmetric | 0.0026 | 0.0053 | 0.0069 | 0.0102 | 0.0127 | 0.0154 |
| UserKNN CF tversky | 0.0026 | 0.0053 | 0.0069 | 0.0102 | 0.0127 | 0.0154 |
| ItemKNN CF cosine | 0.0026 | 0.0053 | 0.0069 | 0.0102 | 0.0127 | 0.0154 |
| ItemKNN CF dice | 0.0026 | 0.0053 | 0.0069 | 0.0102 | 0.0127 | 0.0154 |
| ItemKNN CF jaccard | 0.0026 | 0.0053 | 0.0069 | 0.0102 | 0.0127 | 0.0154 |
| ItemKNN CF asymmetric | 0.0026 | 0.0053 | 0.0069 | 0.0102 | 0.0127 | 0.0154 |
| ItemKNN CF tversky | 0.0026 | 0.0053 | 0.0069 | 0.0102 | 0.0127 | 0.0154 |
| $P^3\alpha$ | 0.0026 | 0.0053 | 0.0069 | 0.0102 | 0.0127 | 0.0154 |
| $RP^3\beta$ | 0.0026 | 0.0053 | 0.0069 | 0.0102 | 0.0127 | 0.0154 |
| EASE ^R | 0.0043 | 0.0073 | 0.0120 | 0.0160 | 0.0203 | 0.0235 |
| SLIM BPR | 0.0027 | 0.0052 | 0.0071 | 0.0102 | 0.0130 | 0.0155 |
| SLIM ElasticNet | 0.0026 | 0.0053 | 0.0069 | 0.0102 | 0.0127 | 0.0154 |
| MF BPR | 0.0046 | 0.0082 | 0.0119 | 0.0154 | 0.0188 | 0.0223 |
| MF FunkSVD | 0.0047 | 0.0087 | 0.0125 | 0.0161 | 0.0194 | 0.0227 |
| PureSVD | 0.0055 | 0.0111 | 0.0168 | 0.0226 | 0.0289 | 0.0356 |
| NMF | 0.0036 | 0.0067 | 0.0090 | 0.0121 | 0.0142 | 0.0167 |
| iALS | 0.0050 | 0.0102 | 0.0149 | 0.0190 | 0.0235 | 0.0279 |
| ItemKNN CBF cosine | 0.0242 | 0.0267 | 0.0284 | 0.0317 | 0.0341 | 0.0367 |
| ItemKNN CBF dice | 0.0210 | 0.0235 | 0.0253 | 0.0287 | 0.0310 | 0.0336 |
| ItemKNN CBF jaccard | 0.0253 | 0.0282 | 0.0301 | 0.0335 | 0.0360 | 0.0386 |
| ItemKNN CBF asymmetric | 0.0256 | 0.0295 | 0.0316 | 0.0350 | 0.0379 | 0.0405 |
| ItemKNN CBF tversky | 0.0173 | 0.0200 | 0.0217 | 0.0251 | 0.0275 | 0.0300 |
| ItemKNN CFCBF cosine | 0.0034 | 0.0061 | 0.0076 | 0.0110 | 0.0135 | 0.0161 |
| ItemKNN CFCBF dice | 0.0236 | 0.0262 | 0.0279 | 0.0313 | 0.0336 | 0.0362 |
| ItemKNN CFCBF jaccard | 0.0553 | 0.0614 | 0.0639 | 0.0670 | 0.0691 | 0.0717 |
| ItemKNN CFCBF asymmetric | 0.0029 | 0.0055 | 0.0071 | 0.0104 | 0.0130 | 0.0156 |
| ItemKNN CFCBF tversky | 0.0448 | 0.0512 | 0.0547 | 0.0583 | 0.0612 | 0.0639 |
| CVAE | 0.0768 | 0.1171 | 0.1485 | 0.1744 | 0.1973 | 0.2168 |
| CDL | 0.0855 | 0.1208 | 0.1445 | 0.1623 | 0.1767 | 0.1901 |

Table 16. Experimental results for the CVAE method for the CiteULike-a P=10 dataset.

| | REC@50 | REC@100 | REC@150 | REC@200 | REC@250 | REC@300 |
|--------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| Random | 0.0027 | 0.0057 | 0.0086 | 0.0112 | 0.0140 | 0.0172 |
| TopPopular | 0.0040 | 0.0078 | 0.0103 | 0.0204 | 0.0230 | 0.0258 |
| UserKNN CF cosine | 0.0769 | 0.1174 | 0.1443 | 0.1670 | 0.1859 | 0.2010 |
| UserKNN CF dice | 0.0788 | 0.1186 | 0.1463 | 0.1689 | 0.1875 | 0.2030 |
| UserKNN CF jaccard | 0.0806 | 0.1207 | 0.1480 | 0.1705 | 0.1887 | 0.2034 |
| UserKNN CF asymmetric | 0.0769 | 0.1173 | 0.1441 | 0.1671 | 0.1859 | 0.2013 |
| UserKNN CF tversky | 0.0799 | 0.1192 | 0.1466 | 0.1696 | 0.1880 | 0.2025 |
| ItemKNN CF cosine | 0.0989 | 0.1441 | 0.1752 | 0.1982 | 0.2156 | 0.2300 |
| ItemKNN CF dice | 0.0945 | 0.1373 | 0.1675 | 0.1912 | 0.2092 | 0.2233 |
| ItemKNN CF jaccard | 0.0917 | 0.1340 | 0.1642 | 0.1876 | 0.2062 | 0.2207 |
| ItemKNN CF asymmetric | 0.0890 | 0.1334 | 0.1631 | 0.1865 | 0.2065 | 0.2215 |
| ItemKNN CF tversky | 0.0990 | 0.1428 | 0.1736 | 0.1972 | 0.2143 | 0.2281 |
| $P^3\alpha$ | 0.0907 | 0.1341 | 0.1636 | 0.1865 | 0.2055 | 0.2206 |
| $RP^3\beta$ | 0.0963 | 0.1408 | 0.1692 | 0.1908 | 0.2090 | 0.2239 |
| EASE ^R | 0.0835 | 0.1242 | 0.1528 | 0.1771 | 0.1956 | 0.2100 |
| SLIM BPR | 0.0876 | 0.1308 | 0.1583 | 0.1821 | 0.2005 | 0.2165 |
| SLIM ElasticNet | 0.0869 | 0.1281 | 0.1561 | 0.1789 | 0.1970 | 0.2115 |
| MF BPR | 0.0680 | 0.1011 | 0.1225 | 0.1402 | 0.1542 | 0.1663 |
| MF FunkSVD | 0.0483 | 0.0866 | 0.1157 | 0.1412 | 0.1636 | 0.1816 |
| PureSVD | 0.0715 | 0.1079 | 0.1313 | 0.1491 | 0.1636 | 0.1759 |
| NMF | 0.0628 | 0.1013 | 0.1285 | 0.1505 | 0.1679 | 0.1843 |
| iALS | 0.0779 | 0.1388 | 0.1834 | 0.2186 | 0.2472 | 0.2706 |
| ItemKNN CBF cosine | 0.2235 | 0.3180 | 0.3829 | 0.4283 | 0.4651 | 0.4950 |
| ItemKNN CBF dice | 0.1734 | 0.2495 | 0.3035 | 0.3455 | 0.3798 | 0.4076 |
| ItemKNN CBF jaccard | 0.1752 | 0.2522 | 0.3045 | 0.3457 | 0.3794 | 0.4062 |
| ItemKNN CBF asymmetric | 0.2234 | 0.3186 | 0.3835 | 0.4288 | 0.4641 | 0.4945 |
| ItemKNN CBF tversky | 0.1748 | 0.2507 | 0.3040 | 0.3466 | 0.3814 | 0.4097 |
| ItemKNN CFCBF cosine | 0.1858 | 0.2816 | 0.3445 | 0.3930 | 0.4335 | 0.4642 |
| ItemKNN CFCBF dice | 0.1803 | 0.2600 | 0.3126 | 0.3558 | 0.3876 | 0.4126 |
| ItemKNN CFCBF jaccard | 0.1855 | 0.2650 | 0.3175 | 0.3598 | 0.3924 | 0.4181 |
| ItemKNN CFCBF asymmetric | 0.1712 | 0.2690 | 0.3355 | 0.3845 | 0.4237 | 0.4565 |
| ItemKNN CFCBF tversky | 0.1832 | 0.2618 | 0.3159 | 0.3577 | 0.3899 | 0.4162 |
| CVAE | 0.0805 | 0.1569 | 0.2232 | 0.2760 | 0.3250 | 0.3687 |
| CDL | 0.0580 | 0.1108 | 0.1546 | 0.1946 | 0.2314 | 0.2640 |

Table 17. Experimental results for the CVAE method for the CiteULike-t P=1 dataset.

| | REC@50 | REC@100 | REC@150 | REC@200 | REC@250 | REC@300 |
|--------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| Random | 0.0026 | 0.0043 | 0.0065 | 0.0082 | 0.0101 | 0.0121 |
| TopPopular | 0.0134 | 0.0179 | 0.0247 | 0.0395 | 0.0456 | 0.0511 |
| UserKNN CF cosine | 0.0012 | 0.0038 | 0.0065 | 0.0084 | 0.0104 | 0.0123 |
| UserKNN CF dice | 0.0012 | 0.0038 | 0.0065 | 0.0084 | 0.0104 | 0.0123 |
| UserKNN CF jaccard | 0.0012 | 0.0038 | 0.0065 | 0.0084 | 0.0104 | 0.0123 |
| UserKNN CF asymmetric | 0.0012 | 0.0038 | 0.0065 | 0.0084 | 0.0104 | 0.0123 |
| UserKNN CF tversky | 0.0012 | 0.0038 | 0.0065 | 0.0084 | 0.0104 | 0.0123 |
| ItemKNN CF cosine | 0.0012 | 0.0038 | 0.0065 | 0.0084 | 0.0104 | 0.0123 |
| ItemKNN CF dice | 0.0012 | 0.0038 | 0.0065 | 0.0084 | 0.0104 | 0.0123 |
| ItemKNN CF jaccard | 0.0012 | 0.0038 | 0.0065 | 0.0084 | 0.0104 | 0.0123 |
| ItemKNN CF asymmetric | 0.0012 | 0.0038 | 0.0065 | 0.0084 | 0.0104 | 0.0123 |
| ItemKNN CF tversky | 0.0012 | 0.0038 | 0.0065 | 0.0084 | 0.0104 | 0.0123 |
| $P^3\alpha$ | 0.0012 | 0.0038 | 0.0065 | 0.0084 | 0.0104 | 0.0123 |
| $RP^3\beta$ | 0.0012 | 0.0038 | 0.0065 | 0.0084 | 0.0104 | 0.0123 |
| EASE ^R | 0.0034 | 0.0064 | 0.0096 | 0.0139 | 0.0160 | 0.0192 |
| SLIM BPR | 0.0013 | 0.0038 | 0.0066 | 0.0084 | 0.0102 | 0.0122 |
| SLIM ElasticNet | 0.0012 | 0.0038 | 0.0065 | 0.0084 | 0.0104 | 0.0123 |
| MF BPR | 0.0054 | 0.0094 | 0.0125 | 0.0153 | 0.0176 | 0.0203 |
| MF FunkSVD | 0.0029 | 0.0057 | 0.0083 | 0.0107 | 0.0130 | 0.0151 |
| PureSVD | 0.0053 | 0.0094 | 0.0140 | 0.0193 | 0.0246 | 0.0283 |
| NMF | 0.0037 | 0.0054 | 0.0074 | 0.0089 | 0.0107 | 0.0129 |
| iALS | 0.0061 | 0.0100 | 0.0137 | 0.0173 | 0.0206 | 0.0245 |
| ItemKNN CBF cosine | 0.0858 | 0.1248 | 0.1549 | 0.1790 | 0.2000 | 0.2180 |
| ItemKNN CBF dice | 0.1133 | 0.1566 | 0.1887 | 0.2122 | 0.2312 | 0.2478 |
| ItemKNN CBF jaccard | 0.1136 | 0.1567 | 0.1874 | 0.2116 | 0.2283 | 0.2433 |
| ItemKNN CBF asymmetric | 0.0916 | 0.1274 | 0.1493 | 0.1633 | 0.1743 | 0.1813 |
| ItemKNN CBF tversky | 0.1135 | 0.1566 | 0.1881 | 0.2125 | 0.2315 | 0.2490 |
| ItemKNN CFCBF cosine | 0.0944 | 0.1349 | 0.1647 | 0.1864 | 0.2059 | 0.2243 |
| ItemKNN CFCBF dice | 0.1129 | 0.1552 | 0.1867 | 0.2105 | 0.2300 | 0.2463 |
| ItemKNN CFCBF jaccard | 0.1133 | 0.1559 | 0.1848 | 0.2066 | 0.2218 | 0.2338 |
| ItemKNN CFCBF asymmetric | 0.0448 | 0.0525 | 0.0554 | 0.0575 | 0.0590 | 0.0609 |
| ItemKNN CFCBF tversky | 0.1133 | 0.1555 | 0.1855 | 0.2085 | 0.2249 | 0.2389 |
| CVAE | 0.0430 | 0.0639 | 0.0803 | 0.0950 | 0.1076 | 0.1200 |
| CDL | 0.0351 | 0.0573 | 0.0715 | 0.0822 | 0.0915 | 0.0989 |

Table 18. Experimental results for the CVAE method for the CiteULike-t P=10 dataset.

| | REC@50 | REC@100 | REC@150 | REC@200 | REC@250 | REC@300 |
|--------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| Random | 0.0016 | 0.0040 | 0.0054 | 0.0069 | 0.0088 | 0.0106 |
| TopPopular | 0.0578 | 0.0862 | 0.1100 | 0.1257 | 0.1416 | 0.1568 |
| UserKNN CF cosine | 0.2141 | 0.2661 | 0.2964 | 0.3169 | 0.3320 | 0.3437 |
| UserKNN CF dice | 0.2138 | 0.2661 | 0.2958 | 0.3171 | 0.3325 | 0.3444 |
| UserKNN CF jaccard | 0.2139 | 0.2648 | 0.2954 | 0.3154 | 0.3308 | 0.3426 |
| UserKNN CF asymmetric | 0.2134 | 0.2656 | 0.2963 | 0.3170 | 0.3341 | 0.3458 |
| UserKNN CF tversky | 0.2120 | 0.2651 | 0.2955 | 0.3172 | 0.3336 | 0.3462 |
| ItemKNN CF cosine | 0.2133 | 0.2658 | 0.2964 | 0.3173 | 0.3342 | 0.3457 |
| ItemKNN CF dice | 0.2157 | 0.2681 | 0.2995 | 0.3206 | 0.3366 | 0.3492 |
| ItemKNN CF jaccard | 0.2167 | 0.2685 | 0.2994 | 0.3205 | 0.3366 | 0.3491 |
| ItemKNN CF asymmetric | 0.2027 | 0.2616 | 0.2958 | 0.3197 | 0.3381 | 0.3525 |
| ItemKNN CF tversky | 0.2015 | 0.2606 | 0.2949 | 0.3190 | 0.3372 | 0.3521 |
| $P^3\alpha$ | 0.2276 | 0.2769 | 0.3069 | 0.3280 | 0.3450 | 0.3571 |
| $RP^3\beta$ | 0.2073 | 0.2636 | 0.2975 | 0.3210 | 0.3398 | 0.3538 |
| EASE ^R | 0.2056 | 0.2532 | 0.2821 | 0.3025 | 0.3171 | 0.3307 |
| SLIM BPR | 0.2187 | 0.2681 | 0.2988 | 0.3196 | 0.3383 | 0.3516 |
| SLIM ElasticNet | 0.2102 | 0.2612 | 0.2930 | 0.3129 | 0.3315 | 0.3446 |
| MF BPR | 0.1551 | 0.1990 | 0.2279 | 0.2482 | 0.2649 | 0.2824 |
| MF FunkSVD | 0.1231 | 0.1613 | 0.1857 | 0.2019 | 0.2155 | 0.2276 |
| PureSVD | 0.1329 | 0.1730 | 0.1994 | 0.2215 | 0.2393 | 0.2547 |
| NMF | 0.1082 | 0.1429 | 0.1771 | 0.2002 | 0.2199 | 0.2420 |
| iALS | 0.2338 | 0.3107 | 0.3566 | 0.3925 | 0.4175 | 0.4374 |
| ItemKNN CBF cosine | 0.1625 | 0.2237 | 0.2682 | 0.3001 | 0.3269 | 0.3493 |
| ItemKNN CBF dice | 0.1665 | 0.2323 | 0.2832 | 0.3206 | 0.3512 | 0.3756 |
| ItemKNN CBF jaccard | 0.1681 | 0.2342 | 0.2851 | 0.3210 | 0.3505 | 0.3761 |
| ItemKNN CBF asymmetric | 0.1630 | 0.2259 | 0.2689 | 0.3031 | 0.3314 | 0.3562 |
| ItemKNN CBF tversky | 0.1599 | 0.2291 | 0.2791 | 0.3170 | 0.3469 | 0.3727 |
| ItemKNN CFCBF cosine | 0.2675 | 0.3490 | 0.3939 | 0.4246 | 0.4519 | 0.4740 |
| ItemKNN CFCBF dice | 0.2166 | 0.2868 | 0.3361 | 0.3738 | 0.4024 | 0.4284 |
| ItemKNN CFCBF jaccard | 0.2172 | 0.2880 | 0.3363 | 0.3741 | 0.4026 | 0.4271 |
| ItemKNN CFCBF asymmetric | 0.2412 | 0.3160 | 0.3663 | 0.4051 | 0.4321 | 0.4548 |
| ItemKNN CFCBF tversky | 0.2178 | 0.2872 | 0.3383 | 0.3758 | 0.4053 | 0.4279 |
| CVAE | 0.2387 | 0.3274 | 0.3849 | 0.4263 | 0.4606 | 0.4854 |
| CDL | 0.2231 | 0.3019 | 0.3565 | 0.4031 | 0.4351 | 0.4618 |

Table 19. Computation time for the algorithms in the selected results for the CVAE method on the CiteULike-a P=1 dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|--------------------------|--------------------------------------|------------------------|---------------------------|
| Random | 0.00 [sec] | 15.06 [sec] | 369 |
| TopPopular | 0.02 [sec] | 15.06 [sec] | 369 |
| UserKNN CF cosine | 0.17 ± 0.02 [sec] | 14.92 [sec] | 372 |
| UserKNN CF dice | 0.17 ± 0.01 [sec] | 15.25 [sec] | 364 |
| UserKNN CF jaccard | 0.18 ± 0.01 [sec] | 14.90 [sec] | 372 |
| UserKNN CF asymmetric | 0.19 ± 0.00 [sec] | 15.04 [sec] | 369 |
| UserKNN CF tversky | 0.20 ± 0.01 [sec] | 14.92 [sec] | 372 |
| ItemKNN CF cosine | 1.10 ± 0.13 [sec] | 14.92 [sec] | 372 |
| ItemKNN CF dice | 1.16 ± 0.01 [sec] | 14.97 [sec] | 371 |
| ItemKNN CF jaccard | 1.20 ± 0.01 [sec] | 14.82 [sec] | 374 |
| ItemKNN CF asymmetric | 1.31 ± 0.02 [sec] | 14.90 [sec] | 372 |
| ItemKNN CF tversky | 1.42 ± 0.01 [sec] | 14.93 [sec] | 372 |
| P ³ α | 4.04 ± 0.09 [sec] | 14.90 [sec] | 372 |
| RP ³ β | 4.11 ± 0.03 [sec] | 14.86 [sec] | 374 |
| EASE ^R | 105.21 [sec] / 1.75 ± 0.04 [min] | 14.56 [sec] | 381 |
| SLIM BPR | 28.74 ± 8.04 [sec] | 14.85 ± 0.14 [sec] | 371 |
| SLIM ElasticNet | 574.75 [sec] / 9.58 ± 0.11 [min] | 17.19 [sec] | 323 |
| MF BPR | 47.23 ± 71.90 [sec] | 16.61 [sec] | 334 |
| MF FunkSVD | 37.78 ± 39.27 [sec] | 17.44 ± 0.36 [sec] | 314 |
| PureSVD | 0.74 ± 0.44 [sec] | 15.48 [sec] | 359 |
| NMF | 19.72 ± 25.84 [sec] | 16.45 ± 0.15 [sec] | 340 |
| iALS | 100.51 [sec] / 1.68 ± 0.89 [min] | 16.76 ± 0.13 [sec] | 333 |
| ItemKNN CBF cosine | 7.75 ± 0.62 [sec] | 14.41 ± 0.67 [sec] | 405 |
| ItemKNN CBF dice | 8.01 ± 0.62 [sec] | 14.27 ± 0.72 [sec] | 403 |
| ItemKNN CBF jaccard | 7.98 ± 0.59 [sec] | 14.42 ± 0.63 [sec] | 402 |
| ItemKNN CBF asymmetric | 7.94 ± 0.47 [sec] | 14.28 ± 0.53 [sec] | 401 |
| ItemKNN CBF tversky | 8.24 ± 0.59 [sec] | 14.31 ± 0.52 [sec] | 408 |
| ItemKNN CFCBF cosine | 7.86 ± 0.55 [sec] | 14.40 ± 0.80 [sec] | 411 |
| ItemKNN CFCBF dice | 7.99 ± 0.58 [sec] | 14.54 ± 0.70 [sec] | 404 |
| ItemKNN CFCBF jaccard | 8.03 ± 0.57 [sec] | 14.55 ± 0.47 [sec] | 394 |
| ItemKNN CFCBF asymmetric | 8.14 ± 0.59 [sec] | 14.39 ± 0.65 [sec] | 412 |
| ItemKNN CFCBF tversky | 8.22 ± 0.54 [sec] | 14.70 ± 0.59 [sec] | 396 |
| CVAE | 2151.48 [sec] / 35.86 [min] | 23.07 [sec] | 241 |
| CDL | 5461.42 [sec] / 1.52 [hour] | 17.32 [sec] | 321 |

Table 20. Computation time for the algorithms in the selected results for the CVAE method on the CiteULike-a P=10 dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|--------------------------|---------------------------------------|------------------------|---------------------------|
| Random | 0.02 [sec] | 14.45 [sec] | 356 |
| TopPopular | 0.00 [sec] | 13.93 [sec] | 369 |
| UserKNN CF cosine | 0.25 ± 0.03 [sec] | 14.19 ± 0.09 [sec] | 363 |
| UserKNN CF dice | 0.26 ± 0.01 [sec] | 14.12 ± 0.09 [sec] | 364 |
| UserKNN CF jaccard | 0.26 ± 0.01 [sec] | 14.29 ± 0.11 [sec] | 358 |
| UserKNN CF asymmetric | 0.27 ± 0.01 [sec] | 14.15 ± 0.07 [sec] | 361 |
| UserKNN CF tversky | 0.29 ± 0.01 [sec] | 14.08 ± 0.05 [sec] | 365 |
| ItemKNN CF cosine | 1.38 ± 0.07 [sec] | 14.16 ± 0.04 [sec] | 362 |
| ItemKNN CF dice | 1.37 ± 0.02 [sec] | 14.13 ± 0.04 [sec] | 364 |
| ItemKNN CF jaccard | 1.40 ± 0.02 [sec] | 14.10 ± 0.02 [sec] | 364 |
| ItemKNN CF asymmetric | 1.49 ± 0.02 [sec] | 14.13 ± 0.05 [sec] | 362 |
| ItemKNN CF tversky | 1.63 ± 0.02 [sec] | 14.18 ± 0.09 [sec] | 359 |
| $P^3\alpha$ | 4.36 ± 0.05 [sec] | 14.26 ± 0.14 [sec] | 364 |
| $RP^3\beta$ | 4.54 ± 0.09 [sec] | 14.24 ± 0.05 [sec] | 362 |
| EASE ^R | 107.59 [sec] / 1.79 ± 0.09 [min] | 14.36 ± 0.04 [sec] | 359 |
| SLIM BPR | 241.65 [sec] / 4.03 ± 1.30 [min] | 14.46 ± 0.14 [sec] | 356 |
| SLIM ElasticNet | 651.80 [sec] / 10.86 ± 0.57 [min] | 13.86 ± 0.19 [sec] | 367 |
| MF BPR | 717.46 [sec] / 11.96 ± 8.87 [min] | 15.18 ± 0.48 [sec] | 338 |
| MF FunkSVD | 546.76 [sec] / 9.11 ± 6.29 [min] | 14.83 ± 0.33 [sec] | 350 |
| PureSVD | 1.29 ± 0.54 [sec] | 15.01 ± 0.57 [sec] | 347 |
| NMF | 184.83 [sec] / 3.08 ± 3.41 [min] | 15.62 ± 0.19 [sec] | 328 |
| iALS | 325.86 [sec] / 5.43 ± 3.66 [min] | 15.55 ± 0.18 [sec] | 326 |
| ItemKNN CBF cosine | 8.25 ± 0.60 [sec] | 14.59 ± 0.24 [sec] | 346 |
| ItemKNN CBF dice | 8.36 ± 0.40 [sec] | 14.33 ± 0.13 [sec] | 358 |
| ItemKNN CBF jaccard | 8.33 ± 0.42 [sec] | 14.55 ± 0.16 [sec] | 353 |
| ItemKNN CBF asymmetric | 8.61 ± 0.52 [sec] | 14.66 ± 0.26 [sec] | 347 |
| ItemKNN CBF tversky | 8.66 ± 0.41 [sec] | 14.41 ± 0.12 [sec] | 355 |
| ItemKNN CFCBF cosine | 8.47 ± 0.54 [sec] | 14.32 ± 0.32 [sec] | 355 |
| ItemKNN CFCBF dice | 8.49 ± 0.41 [sec] | 14.23 ± 0.20 [sec] | 360 |
| ItemKNN CFCBF jaccard | 8.50 ± 0.41 [sec] | 14.23 ± 0.10 [sec] | 357 |
| ItemKNN CFCBF asymmetric | 8.59 ± 0.54 [sec] | 14.22 ± 0.32 [sec] | 356 |
| ItemKNN CFCBF tversky | 8.77 ± 0.45 [sec] | 14.31 ± 0.20 [sec] | 361 |
| CVAE | 4555.65 [sec] / 1.27 [hour] | 21.33 [sec] | 241 |
| CDL | 5443.56 [sec] / 1.51 [hour] | 15.26 [sec] | 337 |

Table 21. Computation time for the algorithms in the selected results for the CVAE method on the CiteULike-t P=1 dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|--------------------------|--|------------------------|---------------------------|
| Random | 0.00 [sec] | 22.92 [sec] | 347 |
| TopPopular | 0.00 [sec] | 22.42 [sec] | 355 |
| UserKNN CF cosine | 0.33 ± 0.03 [sec] | 21.79 [sec] | 365 |
| UserKNN CF dice | 0.34 ± 0.01 [sec] | 21.92 [sec] | 363 |
| UserKNN CF jaccard | 0.35 ± 0.01 [sec] | 21.82 [sec] | 364 |
| UserKNN CF asymmetric | 0.37 ± 0.01 [sec] | 21.68 [sec] | 367 |
| UserKNN CF tversky | 0.40 ± 0.01 [sec] | 21.71 [sec] | 366 |
| ItemKNN CF cosine | 2.42 ± 0.35 [sec] | 21.79 [sec] | 365 |
| ItemKNN CF dice | 2.59 ± 0.02 [sec] | 21.84 [sec] | 364 |
| ItemKNN CF jaccard | 2.68 ± 0.02 [sec] | 21.71 [sec] | 366 |
| ItemKNN CF asymmetric | 2.93 ± 0.03 [sec] | 21.75 [sec] | 365 |
| ItemKNN CF tversky | 3.21 ± 0.02 [sec] | 21.87 [sec] | 363 |
| $P^3\alpha$ | 10.31 ± 0.07 [sec] | 21.70 [sec] | 366 |
| $RP^3\beta$ | 10.26 ± 0.07 [sec] | 21.73 [sec] | 366 |
| EASE ^R | 328.17 [sec] / 5.47 ± 2.29 [min] | 15.31 ± 9.68 [sec] | 939 |
| SLIM BPR | 108.13 [sec] / 1.80 ± 0.55 [min] | 22.03 ± 0.02 [sec] | 361 |
| SLIM ElasticNet | 1223.53 [sec] / 20.39 ± 0.20 [min] | 21.75 [sec] | 365 |
| MF BPR | 158.20 [sec] / 2.64 ± 3.88 [min] | 25.31 ± 0.10 [sec] | 315 |
| MF FunkSVD | 101.31 [sec] / 1.69 ± 1.52 [min] | 25.15 ± 0.07 [sec] | 315 |
| PureSVD | 1.53 ± 0.65 [sec] | 22.63 ± 0.22 [sec] | 347 |
| NMF | 18.22 ± 45.02 [sec] | 24.81 [sec] | 320 |
| iALS | 118.47 [sec] / 1.97 ± 1.22 [min] | 25.19 ± 0.12 [sec] | 314 |
| ItemKNN CBF cosine | 5.86 ± 1.07 [sec] | 20.60 [sec] | 386 |
| ItemKNN CBF dice | 6.12 ± 0.97 [sec] | 20.89 [sec] | 381 |
| ItemKNN CBF jaccard | 6.18 ± 0.97 [sec] | 20.93 [sec] | 380 |
| ItemKNN CBF asymmetric | 6.36 ± 1.04 [sec] | 20.34 [sec] | 391 |
| ItemKNN CBF tversky | 6.70 ± 0.94 [sec] | 21.09 [sec] | 377 |
| ItemKNN CFCBF cosine | 5.84 ± 1.01 [sec] | 20.89 [sec] | 380 |
| ItemKNN CFCBF dice | 6.12 ± 0.94 [sec] | 20.95 [sec] | 379 |
| ItemKNN CFCBF jaccard | 6.17 ± 0.92 [sec] | 20.62 [sec] | 385 |
| ItemKNN CFCBF asymmetric | 6.38 ± 1.02 [sec] | 19.40 [sec] | 410 |
| ItemKNN CFCBF tversky | 6.71 ± 0.94 [sec] | 20.67 [sec] | 385 |
| CVAE | 3560.74 [sec] / 59.35 [min] | 31.19 [sec] | 255 |
| CDL | 22823.11 [sec] / 6.34 [hour] | 24.51 [sec] | 324 |

Table 22. Computation time for the algorithms in the selected results for the CVAE method on the CiteULike-t P=10 dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|--------------------------|---|-----------------------|---------------------------|
| Random | 0.00 [sec] | 7.69 [sec] | 338 |
| TopPopular | 0.00 [sec] | 7.51 [sec] | 346 |
| UserKNN CF cosine | 0.40 ± 0.04 [sec] | 7.48 ± 0.02 [sec] | 348 |
| UserKNN CF dice | 0.41 ± 0.01 [sec] | 7.52 ± 0.05 [sec] | 346 |
| UserKNN CF jaccard | 0.42 ± 0.01 [sec] | 7.51 ± 0.01 [sec] | 346 |
| UserKNN CF asymmetric | 0.45 ± 0.01 [sec] | 7.50 ± 0.11 [sec] | 343 |
| UserKNN CF tversky | 0.48 ± 0.01 [sec] | 7.49 ± 0.01 [sec] | 346 |
| ItemKNN CF cosine | 2.17 ± 0.46 [sec] | 7.51 ± 0.03 [sec] | 346 |
| ItemKNN CF dice | 3.01 ± 0.06 [sec] | 7.47 ± 0.02 [sec] | 349 |
| ItemKNN CF jaccard | 3.10 ± 0.07 [sec] | 7.40 ± 0.11 [sec] | 347 |
| ItemKNN CF asymmetric | 3.35 ± 0.07 [sec] | 7.53 ± 0.07 [sec] | 341 |
| ItemKNN CF tversky | 3.65 ± 0.09 [sec] | 7.47 ± 0.01 [sec] | 348 |
| $P^3\alpha$ | 10.63 ± 0.07 [sec] | 7.58 [sec] | 343 |
| $RP^3\beta$ | 10.71 ± 0.10 [sec] | 7.47 ± 0.08 [sec] | 345 |
| EASE ^R | 383.01 [sec] / 6.38 ± 2.54 [min] | 8.09 [sec] | 321 |
| SLIM BPR | 529.68 [sec] / 8.83 ± 4.30 [min] | 7.65 ± 0.10 [sec] | 337 |
| SLIM ElasticNet | 1454.18 [sec] / 24.24 ± 1.19 [min] | 7.46 ± 0.19 [sec] | 339 |
| MF BPR | 1143.88 [sec] / 19.06 ± 13.39 [min] | 8.27 ± 0.14 [sec] | 308 |
| MF FunkSVD | 1004.38 [sec] / 16.74 ± 8.98 [min] | 8.59 ± 0.09 [sec] | 299 |
| PureSVD | 2.03 ± 0.80 [sec] | 7.74 ± 0.01 [sec] | 336 |
| NMF | 478.36 [sec] / 7.97 ± 10.31 [min] | 8.57 ± 0.15 [sec] | 300 |
| iALS | 570.10 [sec] / 9.50 ± 6.63 [min] | 8.57 ± 0.02 [sec] | 303 |
| ItemKNN CBF cosine | 6.24 ± 0.87 [sec] | 7.58 ± 0.07 [sec] | 345 |
| ItemKNN CBF dice | 6.34 ± 0.62 [sec] | 7.62 ± 0.02 [sec] | 340 |
| ItemKNN CBF jaccard | 6.43 ± 0.64 [sec] | 7.62 ± 0.17 [sec] | 339 |
| ItemKNN CBF asymmetric | 7.03 ± 0.77 [sec] | 7.61 ± 0.20 [sec] | 335 |
| ItemKNN CBF tversky | 7.04 ± 0.67 [sec] | 7.69 ± 0.16 [sec] | 333 |
| ItemKNN CFCBF cosine | 6.38 ± 0.76 [sec] | 7.71 ± 0.22 [sec] | 329 |
| ItemKNN CFCBF dice | 6.33 ± 0.65 [sec] | 7.57 ± 0.11 [sec] | 346 |
| ItemKNN CFCBF jaccard | 6.38 ± 0.65 [sec] | 7.54 ± 0.04 [sec] | 346 |
| ItemKNN CFCBF asymmetric | 7.31 ± 0.76 [sec] | 7.65 ± 0.32 [sec] | 317 |
| ItemKNN CFCBF tversky | 6.87 ± 0.60 [sec] | 7.49 ± 0.05 [sec] | 346 |
| CVAE | 9969.70 [sec] / 2.77 [hour] | 11.68 [sec] | 222 |
| CDL | 22343.75 [sec] / 6.21 [hour] | 8.65 [sec] | 300 |

Table 23. Hyperparameter values for our collaborative KNN baselines on all datasets.

| Algorithm | Hyperparameter | CiteULike-a P=1 | CiteULike-a P=10 | CiteULike-t P=1 | CiteULike-t P=10 |
|-----------------------|-------------------|--------------------|---------------------|--------------------|---------------------|
| UserKNN CF cosine | topK | 844 | 455 | 945 | 347 |
| | shrink | 998 | 490 | 229 | 1000 |
| | similarity | cosine | cosine | cosine | cosine |
| | normalize | False | False | False | True |
| | feature weighting | BM25 | TF-IDF | TF-IDF | none |
| UserKNN CF dice | topK | 396 | 317 | 203 | 359 |
| | shrink | 551 | 1000 | 832 | 1000 |
| | similarity | dice | dice | dice | dice |
| | normalize | False | True | False | False |
| UserKNN CF jaccard | topK | 436 | 348 | 660 | 327 |
| | shrink | 918 | 1000 | 205 | 1000 |
| | similarity | jaccard | jaccard | jaccard | jaccard |
| | normalize | False | False | True | True |
| UserKNN CF asymmetric | topK | 748 | 483 | 584 | 777 |
| | shrink | 733 | 1000 | 709 | 314 |
| | similarity | asymmetric | asymmetric | asymmetric | asymmetric |
| | normalize | True | True | True | True |
| | asymmetric alpha | 1.9382 | 2.0000 | 1.5064 | 1.9573 |
| | feature weighting | none | none | BM25 | none |
| UserKNN CF tversky | topK | 808 | 870 | 264 | 892 |
| | shrink | 917 | 967 | 705 | 981 |
| | similarity | tversky | tversky | tversky | tversky |
| | normalize | True | True | True | True |
| | tversky alpha | 1.3044 | 0.0119 | 0.2812 | 0.1122 |
| | tversky beta | 1.5023 | 1.9836 | 1.1578 | 0.0128 |
| ItemKNN CF cosine | topK | 484 | 423 | 139 | 419 |
| | shrink | 555 | 936 | 127 | 235 |
| | similarity | cosine | cosine | cosine | cosine |
| | normalize | False | True | True | False |
| | feature weighting | none | TF-IDF | none | TF-IDF |
| ItemKNN CF dice | topK | 472 | 540 | 610 | 760 |
| | shrink | 292 | 61 | 402 | 991 |
| | similarity | dice | dice | dice | dice |
| | normalize | False | False | False | True |
| ItemKNN CF jaccard | topK | 114 | 612 | 548 | 754 |
| | shrink | 784 | 93 | 679 | 378 |
| | similarity | jaccard | jaccard | jaccard | jaccard |
| | normalize | True | True | False | True |
| ItemKNN CF asymmetric | topK | 700 | 900 | 848 | 1000 |
| | shrink | 430 | 1000 | 797 | 0 |
| | similarity | asymmetric | asymmetric | asymmetric | asymmetric |
| | normalize | True | True | True | True |
| | asymmetric alpha | 1.5472 | 0.0000 | 0.1235 | 0.0000 |
| | feature weighting | TF-IDF | TF-IDF | BM25 | none |
| ItemKNN CF tversky | topK | 831 | 728 | 317 | 749 |
| | shrink | 810 | 44 | 32 | 0 |
| | similarity | tversky | tversky | tversky | tversky |
| | normalize | True | True | True | True |
| | tversky alpha | 0.6361 | 1.4249 | 0.9626 | 0.0000 |
| | tversky beta | 1.4516 | 1.0858 | 1.8163 | 2.0000 |

Table 24. Hyperparameter values for our non-neural machine learning and graph based baselines on all datasets.

| Algorithm | Hyperparameter | CiteULike-a P=1 | CiteULike-a P=10 | CiteULike-t P=1 | CiteULike-t P=10 |
|-------------------|----------------------|-------------------------|-------------------------|------------------------|--------------------------|
| $P^3\alpha$ | topK | 688 | 662 | 314 | 961 |
| | alpha | 1.1735 | 0.5112 | 0.7234 | 0.3851 |
| | normalize similarity | True | False | False | False |
| $RP^3\beta$ | topK | 54 | 710 | 537 | 1000 |
| | alpha | 1.1242 | 0.0000 | 0.4656 | 0.8688 |
| | beta | 1.3282 | 0.0000 | 0.2366 | 0.0000 |
| | normalize similarity | True | True | True | True |
| EASE ^R | l2 norm | 1.25E+06 | 6.98E+06 | 8.80E+02 | 5.41E+03 |
| SLIM BPR | topK | 402 | 458 | 537 | 818 |
| | epochs | 45 | 160 | 50 | 135 |
| | symmetric | False | False | False | False |
| | sgd mode | sgd | adagrad | adagrad | adagrad |
| | lambda i | 2.35E-03 | 1.00E-05 | 1.44E-03 | 1.00E-02 |
| | lambda j | 7.93E-03 | 1.00E-05 | 1.09E-04 | 1.00E-02 |
| | learning rate | 2.65E-04 | 1.00E-04 | 1.58E-04 | 1.00E-04 |
| SLIM ElasticNet | topK | 795 | 450 | 644 | 949 |
| | l1 ratio | 1.72E-01 | 2.51E-04 | 1.23E-02 | 3.61E-05 |
| | alpha | 0.4724 | 0.0179 | 0.8273 | 1.0000 |
| MF BPR | sgd mode | adam | adam | adagrad | adagrad |
| | epochs | 25 | 660 | 20 | 1005 |
| | num factors | 90 | 200 | 73 | 200 |
| | batch size | 64 | 256 | 16 | 64 |
| | positive reg | 7.61E-04 | 1.00E-02 | 1.27E-04 | 1.00E-02 |
| | negative reg | 6.05E-04 | 9.89E-03 | 3.37E-04 | 1.00E-02 |
| | learning rate | 6.15E-04 | 4.56E-03 | 1.93E-02 | 1.00E-01 |
| MF FunkSVD | sgd mode | adagrad | adam | adam | adagrad |
| | epochs | 45 | 500 | 35 | 485 |
| | use bias | False | False | False | True |
| | batch size | 128 | 128 | 256 | 8 |
| | num factors | 48 | 28 | 77 | 200 |
| | item reg | 2.36E-05 | 3.45E-03 | 2.22E-04 | 1.00E-02 |
| | user reg | 6.40E-04 | 9.10E-04 | 2.28E-04 | 1.00E-02 |
| | learning rate | 8.37E-02 | 3.98E-03 | 9.13E-04 | 1.00E-01 |
| | negative quota | 0.1630 | 0.1334 | 0.4428 | 0.5000 |
| PureSVD | num factors | 284 | 350 | 350 | 350 |
| NMF | num factors | 245 | 188 | 127 | 301 |
| | solver | mult. update nndsvda | mult. update nndsvda | mult. update random | coord. descent random |
| | init type | kullback-leibler | frobenius | kullback-leibler | frobenius |
| | beta loss | | | | |
| iALS | num factors | 195 | 50 | 195 | 49 |
| | confidence scaling | log | log | linear | log |
| | alpha | 34.4360 | 50.0000 | 16.8297 | 50.0000 |
| | epsilon | 0.0055 | 0.0010 | 3.4947 | 0.0032 |
| | reg | 3.02E-04 | 1.00E-05 | 2.90E-04 | 1.00E-02 |
| | epochs | 5 | 60 | 5 | 105 |

Table 25. Hyperparameter values for our content based KNN baselines on all datasets.

| Algorithm | Hyperparameter | CiteULike-a P=1 | CiteULike-a P=10 | CiteULike-t P=1 | CiteULike-t P=10 |
|------------------------|-------------------|--------------------|---------------------|--------------------|---------------------|
| ItemKNN CBF cosine | topK | 5 | 849 | 692 | 563 |
| | shrink | 0 | 825 | 955 | 50 |
| | similarity | cosine | cosine | cosine | cosine |
| | normalize | True | True | True | True |
| | feature weighting | none | BM25 | BM25 | TF-IDF |
| ItemKNN CBF dice | topK | 5 | 571 | 626 | 636 |
| | shrink | 0 | 0 | 474 | 7 |
| | similarity | dice | dice | dice | dice |
| | normalize | True | True | True | True |
| | | | | | |
| ItemKNN CBF jaccard | topK | 13 | 527 | 429 | 543 |
| | shrink | 637 | 0 | 736 | 9 |
| | similarity | jaccard | jaccard | jaccard | jaccard |
| | normalize | True | False | True | False |
| | | | | | |
| ItemKNN CBF asymmetric | topK | 10 | 801 | 219 | 976 |
| | shrink | 14 | 1000 | 895 | 92 |
| | similarity | asymmetric | asymmetric | asymmetric | asymmetric |
| | normalize | True | True | True | True |
| | asymmetric alpha | 1.7300 | 0.4022 | 1.3276 | 0.5989 |
| | feature weighting | TF-IDF | BM25 | TF-IDF | TF-IDF |
| ItemKNN CBF tversky | topK | 5 | 572 | 849 | 1000 |
| | shrink | 1000 | 0 | 491 | 0 |
| | similarity | tversky | tversky | tversky | tversky |
| | normalize | True | True | True | True |
| | tversky alpha | 0.0000 | 2.0000 | 1.4789 | 2.0000 |
| | tversky beta | 2.0000 | 2.0000 | 1.7951 | 2.0000 |

Table 26. Hyperparameter values for our hybrid KNN baselines on all datasets.

| Algorithm | Hyperparameter | CiteULike-a P=1 | CiteULike-a P=10 | CiteULike-t P=1 | CiteULike-t P=10 |
|--------------------------|-------------------|--------------------|---------------------|--------------------|---------------------|
| ItemKNN CFCBF cosine | topK | 19 | 807 | 962 | 1000 |
| | shrink | 140 | 1000 | 151 | 1000 |
| | similarity | cosine | cosine | cosine | cosine |
| | normalize | True | False | True | False |
| | feature weighting | BM25 | BM25 | TF-IDF | TF-IDF |
| | ICM weight | 0.0101 | 1.1447 | 3.7971 | 1.5675 |
| ItemKNN CFCBF dice | topK | 7 | 492 | 918 | 332 |
| | shrink | 194 | 50 | 944 | 1000 |
| | similarity | dice | dice | dice | dice |
| | normalize | False | False | False | False |
| | ICM weight | 0.8195 | 72.9657 | 0.5285 | 0.0100 |
| ItemKNN CFCBF jaccard | topK | 34 | 554 | 418 | 347 |
| | shrink | 51 | 0 | 708 | 1000 |
| | similarity | jaccard | jaccard | jaccard | jaccard |
| | normalize | True | True | False | True |
| | ICM weight | 2.1492 | 0.0100 | 0.0121 | 100.0000 |
| ItemKNN CFCBF asymmetric | topK | 26 | 976 | 342 | 1000 |
| | shrink | 22 | 1000 | 252 | 1000 |
| | similarity | asymmetric | asymmetric | asymmetric | asymmetric |
| | normalize | True | True | True | True |
| | asymmetric alpha | 0.1370 | 0.0000 | 1.5682 | 0.0000 |
| | feature weighting | BM25 | TF-IDF | BM25 | BM25 |
| | ICM weight | 0.0145 | 1.1144 | 0.0119 | 6.7370 |
| ItemKNN CFCBF tversky | topK | 59 | 585 | 530 | 283 |
| | shrink | 991 | 0 | 733 | 672 |
| | similarity | tversky | tversky | tversky | tversky |
| | normalize | True | True | True | True |
| | tversky alpha | 1.7260 | 1.3555 | 1.3336 | 0.0000 |
| | tversky beta | 0.6061 | 2.0000 | 0.8886 | 2.0000 |
| | ICM weight | 27.7050 | 100.0000 | 1.1171 | 0.0100 |

Table 27. Hyperparameter values for the neural algorithm on all datasets.

| Algorithm | Hyperparameter | CiteULike-a P=1 | CiteULike-a P=10 | CiteULike-t P=1 | CiteULike-t P=10 |
|-----------|--------------------|--------------------|---------------------|--------------------|---------------------|
| CVAE | epochs | 5 | 35 | 5 | 60 |
| | learning rate vae | 1.00E-02 | 1.00E-02 | 1.00E-02 | 1.00E-02 |
| | learning rate cvae | 1.00E-03 | 1.00E-03 | 1.00E-03 | 1.00E-03 |
| | num factors | 50 | 50 | 50 | 50 |
| | dimensions vae | [200, 100] | [200, 100] | [200, 100] | [200, 100] |
| | epochs vae | [50, 50] | [50, 50] | [50, 50] | [50, 50] |
| | batch size | 128 | 128 | 128 | 128 |
| | lambda u | 1.00E-01 | 1.00E-01 | 1.00E-01 | 1.00E-01 |
| | lambda v | 10 | 10 | 10 | 10 |
| | lambda r | 1 | 1 | 1 | 1 |
| | a | 1 | 1 | 1 | 1 |
| | b | 0.0100 | 0.0100 | 0.0100 | 0.0100 |
| | M | 300 | 300 | 300 | 300 |
| CDL | para lv | 10 | 10 | 10 | 10 |
| | para lu | 1 | 1 | 1 | 1 |
| | para ln | 1000.0000 | 1000.0000 | 1000.0000 | 1000.0000 |
| | batch size | 128 | 128 | 128 | 128 |
| | epoch sdae | 200 | 200 | 200 | 200 |
| | epoch dae | 200 | 200 | 200 | 200 |

F RECSYS: SPECTRAL COLLABORATIVE FILTERING

Relevant statistics on the dataset, which we mentioned in the paper, are reported in Table 28 and 29. The results of our evaluation can be seen in Table 30 (Amazon Instant Video), Table 31 (Hetrec), Table 32 (Movielens 1M original split) and Table 33 (Movielens 1M our split). The results for beyond-accuracy metrics can be seen in Table 34 (Amazon Instant Video), Table 35 (Hetrec), Table 36 (Movielens 1M original split) and Table 37 (Movielens 1M our split). The corresponding optimal hyperparameters are reported in Table 42 (collaborative KNNs), Table 43 (non-neural machine learning and graph based) and 44 (SpectralCF original hyperparameters and ours).

Lastly, the time required to train and evaluate the models is reported in Table 38 (Amazon Instant Video), Table 39 (Hetrec), Table 40 (Movielens 1M original split) and Table 41 (Movielens 1M our split).

Table 28. Dataset characteristics.

| Dataset | Interactions | Items | Users | Density |
|----------------------|--------------|-------|-------|---------|
| Movielens 1M | 226 K | 3706 | 6040 | 1.01% |
| Hetrec | 71 K | 10109 | 2113 | 0.33% |
| Amazon Instant Video | 22 K | 5860 | 3113 | 0.12% |

Table 29. The statistics compare the popularity bias of the two splits of Movielens 1M we report the results of, the original one provided by the authors and the split generated by us following the description provided in the paper. The three rows refer to the statistics of the dataset as a whole and those of the train and test data split. In a truly random data split the statistics of train and test data should mirror closely those of the full dataset. It is possible to see that the original test data (values in bold) has very different statistical properties than the original full dataset, hinting at a possible error in the splitting procedure. *Kendall Tau* counts the number of pairwise disagreements between two ranking lists, its result is the percentage of item pairs whose ordering is discordant between the two splits. This metric highlight how inconsistent is the original test data with respect to the original train data.

| | Max pop | Avg pop | Gini Index | Kendall Tau | Shannon |
|------------|--------------------|---------|-------------|-------------|-------------|
| | Movielens original | | | | |
| Full data | 1963.00 | 61.08 | 0.78 | 1.00 | 9.99 |
| Train data | 1936.00 | 48.37 | 0.79 | 0.87 | 9.89 |
| Test data | 1361.00 | 12.71 | 0.92 | 0.44 | 8.49 |
| | Movielens ours | | | | |
| Full data | 1963.00 | 58.08 | 0.79 | 1.00 | 9.99 |
| Train data | 1575.00 | 46.44 | 0.79 | 0.97 | 9.99 |
| Test data | 388.00 | 11.64 | 0.80 | 0.85 | 9.93 |

Table 30. Experimental results for the SpectralCF method for the Amazon Instant Video dataset.

| | @ 20 | | @ 40 | | @ 60 | | @ 80 | | @ 100 | |
|----------------------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| | REC | MAP | REC | MAP | REC | MAP | REC | MAP | REC | MAP |
| Random | 0.0038 | 0.0004 | 0.0066 | 0.0005 | 0.0115 | 0.0006 | 0.0136 | 0.0006 | 0.0172 | 0.0007 |
| TopPopular | 0.1134 | 0.0288 | 0.1687 | 0.0308 | 0.2067 | 0.0316 | 0.2448 | 0.0322 | 0.2716 | 0.0326 |
| UserKNN CF cosine | 0.2853 | 0.1200 | 0.3559 | 0.1228 | 0.3986 | 0.1238 | 0.4297 | 0.1243 | 0.4527 | 0.1246 |
| UserKNN CF dice | 0.2845 | 0.1198 | 0.3550 | 0.1225 | 0.3989 | 0.1236 | 0.4294 | 0.1240 | 0.4520 | 0.1243 |
| UserKNN CF jaccard | 0.2850 | 0.1198 | 0.3544 | 0.1225 | 0.3991 | 0.1235 | 0.4282 | 0.1240 | 0.4515 | 0.1243 |
| UserKNN CF asymmetric | 0.2850 | 0.1201 | 0.3552 | 0.1228 | 0.3991 | 0.1239 | 0.4297 | 0.1244 | 0.4526 | 0.1247 |
| UserKNN CF tversky | 0.2856 | 0.1196 | 0.3544 | 0.1223 | 0.3991 | 0.1233 | 0.4286 | 0.1238 | 0.4515 | 0.1241 |
| ItemKNN CF cosine | 0.2858 | 0.1209 | 0.3535 | 0.1236 | 0.3985 | 0.1246 | 0.4303 | 0.1252 | 0.4528 | 0.1255 |
| ItemKNN CF dice | 0.2968 | 0.1290 | 0.3628 | 0.1316 | 0.4010 | 0.1325 | 0.4262 | 0.1329 | 0.4487 | 0.1332 |
| ItemKNN CF jaccard | 0.2957 | 0.1296 | 0.3628 | 0.1322 | 0.3996 | 0.1331 | 0.4250 | 0.1335 | 0.4482 | 0.1338 |
| ItemKNN CF asymmetric | 0.3044 | 0.1309 | 0.3712 | 0.1336 | 0.4127 | 0.1346 | 0.4426 | 0.1351 | 0.4653 | 0.1354 |
| ItemKNN CF tversky | 0.2913 | 0.1195 | 0.3594 | 0.1221 | 0.4021 | 0.1231 | 0.4319 | 0.1236 | 0.4538 | 0.1239 |
| $P^3\alpha$ | 0.3019 | 0.1276 | 0.3721 | 0.1304 | 0.4151 | 0.1314 | 0.4433 | 0.1319 | 0.4648 | 0.1322 |
| $RP^3\beta$ | 0.3029 | 0.1354 | 0.3715 | 0.1382 | 0.4119 | 0.1391 | 0.4378 | 0.1396 | 0.4584 | 0.1398 |
| EASE ^R | 0.2898 | 0.1210 | 0.3577 | 0.1236 | 0.3961 | 0.1245 | 0.4230 | 0.1249 | 0.4416 | 0.1252 |
| SLIM BPR | 0.2973 | 0.1326 | 0.3671 | 0.1353 | 0.3993 | 0.1361 | 0.4279 | 0.1366 | 0.4501 | 0.1369 |
| SLIM ElasticNet | 0.2882 | 0.1224 | 0.3576 | 0.1250 | 0.3966 | 0.1260 | 0.4256 | 0.1264 | 0.4482 | 0.1267 |
| MF BPR | 0.2320 | 0.0951 | 0.2925 | 0.0974 | 0.3340 | 0.0983 | 0.3678 | 0.0989 | 0.3921 | 0.0992 |
| MF FunkSVD | 0.1800 | 0.0484 | 0.2615 | 0.0516 | 0.3129 | 0.0527 | 0.3488 | 0.0533 | 0.3752 | 0.0536 |
| PureSVD | 0.1555 | 0.0533 | 0.2079 | 0.0553 | 0.2466 | 0.0561 | 0.2758 | 0.0566 | 0.3031 | 0.0569 |
| NMF | 0.1406 | 0.0569 | 0.1850 | 0.0586 | 0.2147 | 0.0592 | 0.2454 | 0.0597 | 0.2654 | 0.0600 |
| iALS | 0.2741 | 0.0951 | 0.3526 | 0.0981 | 0.4003 | 0.0993 | 0.4330 | 0.0998 | 0.4582 | 0.1002 |
| SpectralCF | 0.1130 | 0.0246 | 0.1652 | 0.0266 | 0.2090 | 0.0275 | 0.2393 | 0.0280 | 0.2712 | 0.0283 |
| SpectralCF article default | 0.1063 | 0.0255 | 0.1542 | 0.0273 | 0.1969 | 0.0282 | 0.2332 | 0.0288 | 0.2670 | 0.0292 |

Table 31. Experimental results for the SpectralCF method for the Hetrec dataset.

| | @ 20 | | @ 40 | | @ 60 | | @ 80 | | @ 100 | |
|----------------------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| | REC | MAP | REC | MAP | REC | MAP | REC | MAP | REC | MAP |
| Random | 0.0028 | 0.0003 | 0.0051 | 0.0004 | 0.0070 | 0.0004 | 0.0093 | 0.0004 | 0.0113 | 0.0005 |
| TopPopular | 0.2044 | 0.0639 | 0.2918 | 0.0684 | 0.3484 | 0.0710 | 0.3927 | 0.0726 | 0.4291 | 0.0737 |
| UserKNN CF cosine | 0.2649 | 0.1081 | 0.3604 | 0.1132 | 0.4139 | 0.1159 | 0.4576 | 0.1177 | 0.4927 | 0.1189 |
| UserKNN CF dice | 0.2627 | 0.1081 | 0.3528 | 0.1132 | 0.4061 | 0.1158 | 0.4522 | 0.1176 | 0.4909 | 0.1189 |
| UserKNN CF jaccard | 0.2617 | 0.1077 | 0.3514 | 0.1127 | 0.4050 | 0.1153 | 0.4491 | 0.1170 | 0.4900 | 0.1184 |
| UserKNN CF asymmetric | 0.2667 | 0.1088 | 0.3606 | 0.1139 | 0.4142 | 0.1166 | 0.4571 | 0.1184 | 0.4934 | 0.1197 |
| UserKNN CF tversky | 0.2617 | 0.1079 | 0.3507 | 0.1129 | 0.4050 | 0.1156 | 0.4494 | 0.1173 | 0.4901 | 0.1187 |
| ItemKNN CF cosine | 0.2682 | 0.1092 | 0.3533 | 0.1136 | 0.4142 | 0.1165 | 0.4575 | 0.1182 | 0.4952 | 0.1196 |
| ItemKNN CF dice | 0.2549 | 0.1041 | 0.3464 | 0.1088 | 0.4061 | 0.1115 | 0.4504 | 0.1132 | 0.4860 | 0.1145 |
| ItemKNN CF jaccard | 0.2537 | 0.1051 | 0.3434 | 0.1098 | 0.4017 | 0.1125 | 0.4473 | 0.1143 | 0.4852 | 0.1156 |
| ItemKNN CF asymmetric | 0.2703 | 0.1003 | 0.3610 | 0.1058 | 0.4184 | 0.1087 | 0.4637 | 0.1107 | 0.4943 | 0.1121 |
| ItemKNN CF tversky | 0.2632 | 0.1038 | 0.3554 | 0.1091 | 0.4174 | 0.1121 | 0.4588 | 0.1139 | 0.4944 | 0.1153 |
| $P^3\alpha$ | 0.2572 | 0.0981 | 0.3532 | 0.1037 | 0.4139 | 0.1067 | 0.4608 | 0.1085 | 0.5002 | 0.1099 |
| $RP^3\beta$ | 0.2688 | 0.1058 | 0.3628 | 0.1114 | 0.4279 | 0.1146 | 0.4714 | 0.1165 | 0.5073 | 0.1178 |
| EASE ^R | 0.2707 | 0.1158 | 0.3580 | 0.1207 | 0.4188 | 0.1235 | 0.4634 | 0.1254 | 0.4945 | 0.1267 |
| SLIM BPR | 0.2566 | 0.1035 | 0.3451 | 0.1086 | 0.4016 | 0.1114 | 0.4452 | 0.1130 | 0.4840 | 0.1144 |
| SLIM ElasticNet | 0.2791 | 0.1214 | 0.3634 | 0.1261 | 0.4226 | 0.1291 | 0.4637 | 0.1311 | 0.4976 | 0.1324 |
| MF BPR | 0.1820 | 0.0586 | 0.2776 | 0.0638 | 0.3312 | 0.0662 | 0.3752 | 0.0678 | 0.4073 | 0.0689 |
| MF FunkSVD | 0.2010 | 0.0540 | 0.2976 | 0.0597 | 0.3575 | 0.0623 | 0.3988 | 0.0639 | 0.4300 | 0.0649 |
| PureSVD | 0.2560 | 0.1087 | 0.3438 | 0.1130 | 0.3995 | 0.1158 | 0.4417 | 0.1175 | 0.4823 | 0.1190 |
| NMF | 0.2065 | 0.0865 | 0.2834 | 0.0903 | 0.3388 | 0.0931 | 0.3790 | 0.0947 | 0.4179 | 0.0960 |
| iALS | 0.2726 | 0.1104 | 0.3660 | 0.1163 | 0.4289 | 0.1195 | 0.4842 | 0.1217 | 0.5188 | 0.1232 |
| SpectralCF | 0.1918 | 0.0660 | 0.2810 | 0.0707 | 0.3438 | 0.0735 | 0.3840 | 0.0749 | 0.4109 | 0.0757 |
| SpectralCF article default | 0.1450 | 0.0320 | 0.2261 | 0.0358 | 0.2849 | 0.0381 | 0.3366 | 0.0397 | 0.3866 | 0.0410 |

Table 32. Experimental results for the SpectralCF method for the Movielens 1M original dataset.

| | @ 20 | | @ 40 | | @ 60 | | @ 80 | | @ 100 | |
|----------------------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| | REC | MAP | REC | MAP | REC | MAP | REC | MAP | REC | MAP |
| Random | 0.0062 | 0.0013 | 0.0114 | 0.0014 | 0.0163 | 0.0015 | 0.0213 | 0.0016 | 0.0267 | 0.0017 |
| TopPopular | 0.0382 | 0.0065 | 0.0969 | 0.0092 | 0.1207 | 0.0101 | 0.1651 | 0.0113 | 0.2286 | 0.0127 |
| UserKNN CF cosine | 0.0917 | 0.0193 | 0.1558 | 0.0226 | 0.2100 | 0.0246 | 0.2514 | 0.0258 | 0.2868 | 0.0267 |
| UserKNN CF dice | 0.0993 | 0.0214 | 0.1683 | 0.0250 | 0.2223 | 0.0271 | 0.2623 | 0.0283 | 0.2978 | 0.0292 |
| UserKNN CF jaccard | 0.0998 | 0.0211 | 0.1691 | 0.0247 | 0.2241 | 0.0268 | 0.2659 | 0.0280 | 0.3012 | 0.0289 |
| UserKNN CF asymmetric | 0.0982 | 0.0214 | 0.1644 | 0.0248 | 0.2165 | 0.0268 | 0.2578 | 0.0280 | 0.2943 | 0.0290 |
| UserKNN CF tversky | 0.0999 | 0.0208 | 0.1687 | 0.0244 | 0.2239 | 0.0265 | 0.2672 | 0.0278 | 0.3021 | 0.0287 |
| ItemKNN CF cosine | 0.0676 | 0.0143 | 0.1217 | 0.0170 | 0.1632 | 0.0185 | 0.2004 | 0.0195 | 0.2326 | 0.0203 |
| ItemKNN CF dice | 0.0673 | 0.0154 | 0.1226 | 0.0181 | 0.1679 | 0.0197 | 0.2071 | 0.0208 | 0.2443 | 0.0218 |
| ItemKNN CF jaccard | 0.0670 | 0.0146 | 0.1218 | 0.0173 | 0.1664 | 0.0189 | 0.2051 | 0.0200 | 0.2438 | 0.0209 |
| ItemKNN CF asymmetric | 0.0988 | 0.0215 | 0.1745 | 0.0256 | 0.2311 | 0.0279 | 0.2783 | 0.0296 | 0.3189 | 0.0308 |
| ItemKNN CF tversky | 0.0730 | 0.0147 | 0.1375 | 0.0179 | 0.1908 | 0.0199 | 0.2384 | 0.0213 | 0.2798 | 0.0224 |
| $P^3\alpha$ | 0.1218 | 0.0256 | 0.2072 | 0.0306 | 0.2696 | 0.0335 | 0.3174 | 0.0353 | 0.3583 | 0.0367 |
| $RP^3\beta$ | 0.0916 | 0.0192 | 0.1628 | 0.0230 | 0.2216 | 0.0254 | 0.2738 | 0.0271 | 0.3171 | 0.0284 |
| EASE ^R | 0.0914 | 0.0184 | 0.1531 | 0.0216 | 0.2010 | 0.0234 | 0.2398 | 0.0246 | 0.2736 | 0.0254 |
| SLIM BPR | 0.1292 | 0.0283 | 0.2090 | 0.0329 | 0.2687 | 0.0355 | 0.3140 | 0.0371 | 0.3525 | 0.0384 |
| SLIM ElasticNet | 0.0915 | 0.0205 | 0.1521 | 0.0237 | 0.2014 | 0.0255 | 0.2392 | 0.0267 | 0.2692 | 0.0275 |
| MF BPR | 0.0863 | 0.0205 | 0.1532 | 0.0239 | 0.2062 | 0.0260 | 0.2518 | 0.0274 | 0.2874 | 0.0284 |
| MF FunkSVD | 0.1061 | 0.0262 | 0.1746 | 0.0301 | 0.2286 | 0.0323 | 0.2753 | 0.0339 | 0.3127 | 0.0351 |
| PureSVD | 0.0828 | 0.0172 | 0.1424 | 0.0204 | 0.1833 | 0.0219 | 0.2170 | 0.0229 | 0.2472 | 0.0236 |
| NMF | 0.0795 | 0.0199 | 0.1252 | 0.0223 | 0.1602 | 0.0236 | 0.1884 | 0.0245 | 0.2127 | 0.0251 |
| iALS | 0.0868 | 0.0188 | 0.1441 | 0.0218 | 0.1933 | 0.0235 | 0.2299 | 0.0245 | 0.2621 | 0.0252 |
| SpectralCF | 0.1567 | 0.0621 | 0.2098 | 0.0642 | 0.2470 | 0.0658 | 0.2899 | 0.0675 | 0.3337 | 0.0689 |
| SpectralCF article default | 0.1849 | 0.0838 | 0.2376 | 0.0857 | 0.2808 | 0.0879 | 0.3248 | 0.0894 | 0.3493 | 0.0902 |

Table 33. Experimental results for the SpectralCF method for the Movielens 1M ours dataset.

| | @ 20 | | @ 40 | | @ 60 | | @ 80 | | @ 100 | |
|----------------------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| | REC | MAP | REC | MAP | REC | MAP | REC | MAP | REC | MAP |
| Random | 0.0055 | 0.0010 | 0.0117 | 0.0012 | 0.0178 | 0.0013 | 0.0234 | 0.0014 | 0.0280 | 0.0014 |
| TopPopular | 0.1892 | 0.0584 | 0.2788 | 0.0636 | 0.3356 | 0.0666 | 0.3834 | 0.0687 | 0.4226 | 0.0702 |
| UserKNN CF cosine | 0.2978 | 0.1195 | 0.4108 | 0.1280 | 0.4866 | 0.1329 | 0.5444 | 0.1361 | 0.5868 | 0.1382 |
| UserKNN CF dice | 0.2960 | 0.1185 | 0.4111 | 0.1270 | 0.4872 | 0.1319 | 0.5443 | 0.1351 | 0.5873 | 0.1372 |
| UserKNN CF jaccard | 0.3001 | 0.1201 | 0.4134 | 0.1285 | 0.4901 | 0.1335 | 0.5457 | 0.1367 | 0.5884 | 0.1388 |
| UserKNN CF asymmetric | 0.2880 | 0.1156 | 0.4016 | 0.1236 | 0.4785 | 0.1285 | 0.5361 | 0.1316 | 0.5820 | 0.1338 |
| UserKNN CF tversky | 0.2903 | 0.1161 | 0.4061 | 0.1244 | 0.4818 | 0.1293 | 0.5394 | 0.1325 | 0.5842 | 0.1346 |
| ItemKNN CF cosine | 0.2929 | 0.1154 | 0.4067 | 0.1236 | 0.4843 | 0.1287 | 0.5401 | 0.1318 | 0.5829 | 0.1340 |
| ItemKNN CF dice | 0.2747 | 0.1053 | 0.3770 | 0.1125 | 0.4519 | 0.1170 | 0.5070 | 0.1200 | 0.5556 | 0.1222 |
| ItemKNN CF jaccard | 0.2731 | 0.1051 | 0.3783 | 0.1126 | 0.4539 | 0.1172 | 0.5099 | 0.1202 | 0.5582 | 0.1224 |
| ItemKNN CF asymmetric | 0.2876 | 0.1134 | 0.4000 | 0.1213 | 0.4768 | 0.1263 | 0.5367 | 0.1295 | 0.5820 | 0.1317 |
| ItemKNN CF tversky | 0.2760 | 0.1043 | 0.3877 | 0.1122 | 0.4659 | 0.1175 | 0.5250 | 0.1209 | 0.5713 | 0.1232 |
| $P^3\alpha$ | 0.2939 | 0.1141 | 0.4150 | 0.1233 | 0.4900 | 0.1285 | 0.5463 | 0.1318 | 0.5903 | 0.1342 |
| $RP^3\beta$ | 0.2737 | 0.1044 | 0.3879 | 0.1124 | 0.4664 | 0.1173 | 0.5234 | 0.1206 | 0.5726 | 0.1230 |
| EASE ^R | 0.2967 | 0.1176 | 0.4118 | 0.1261 | 0.4855 | 0.1312 | 0.5402 | 0.1345 | 0.5854 | 0.1367 |
| SLIM BPR | 0.2886 | 0.1086 | 0.4048 | 0.1170 | 0.4813 | 0.1219 | 0.5362 | 0.1249 | 0.5782 | 0.1269 |
| SLIM ElasticNet | 0.3069 | 0.1265 | 0.4246 | 0.1356 | 0.5010 | 0.1410 | 0.5564 | 0.1443 | 0.6001 | 0.1466 |
| MF BPR | 0.2616 | 0.0956 | 0.3662 | 0.1028 | 0.4377 | 0.1071 | 0.4890 | 0.1097 | 0.5307 | 0.1116 |
| MF FunkSVD | 0.2684 | 0.0875 | 0.3890 | 0.0963 | 0.4663 | 0.1015 | 0.5252 | 0.1049 | 0.5720 | 0.1072 |
| PureSVD | 0.2595 | 0.1008 | 0.3638 | 0.1083 | 0.4378 | 0.1131 | 0.4913 | 0.1161 | 0.5347 | 0.1182 |
| NMF | 0.2384 | 0.0908 | 0.3351 | 0.0972 | 0.4032 | 0.1014 | 0.4568 | 0.1041 | 0.4981 | 0.1060 |
| iALS | 0.3033 | 0.1183 | 0.4201 | 0.1273 | 0.4933 | 0.1326 | 0.5493 | 0.1360 | 0.5925 | 0.1383 |
| SpectralCF | 0.1813 | 0.0533 | 0.2643 | 0.0581 | 0.3274 | 0.0613 | 0.3823 | 0.0635 | 0.4261 | 0.0651 |
| SpectralCF article default | 0.1785 | 0.0540 | 0.2590 | 0.0586 | 0.3232 | 0.0614 | 0.3689 | 0.0632 | 0.4101 | 0.0646 |

Table 34. Experimental results for the SpectralCF method for the Amazon Instant Video dataset on beyond accuracy metrics.

| | Div. MIL | Div. HHI | @ 50 Cov. Item | Div. Gini | Div. Shannon |
|----------------------------|---------------|---------------|----------------------|---------------|-----------------|
| Random | 0.9915 | 0.9998 | 1.0000 | 0.8907 | 12.4892 |
| TopPopular | 0.0340 | 0.9807 | 0.0104 | 0.0089 | 5.7129 |
| UserKNN CF cosine | 0.8077 | 0.9961 | 0.7804 | 0.1230 | 9.3830 |
| UserKNN CF dice | 0.8102 | 0.9962 | 0.7812 | 0.1237 | 9.3986 |
| UserKNN CF jaccard | 0.8118 | 0.9962 | 0.7826 | 0.1247 | 9.4121 |
| UserKNN CF asymmetric | 0.8078 | 0.9962 | 0.7805 | 0.1224 | 9.3800 |
| UserKNN CF tversky | 0.8115 | 0.9962 | 0.7833 | 0.1250 | 9.4130 |
| ItemKNN CF cosine | 0.8078 | 0.9962 | 0.7797 | 0.1223 | 9.3794 |
| ItemKNN CF dice | 0.8542 | 0.9971 | 0.8176 | 0.1744 | 9.8970 |
| ItemKNN CF jaccard | 0.8664 | 0.9973 | 0.8183 | 0.1783 | 9.9675 |
| ItemKNN CF asymmetric | 0.8361 | 0.9967 | 0.8160 | 0.1521 | 9.6981 |
| ItemKNN CF tversky | 0.8169 | 0.9963 | 0.7881 | 0.1360 | 9.5093 |
| $P^3\alpha$ | 0.8276 | 0.9965 | 0.8181 | 0.1486 | 9.6327 |
| $RP^3\beta$ | 0.8685 | 0.9974 | 0.8340 | 0.1947 | 10.0877 |
| EASE ^R | 0.8369 | 0.9967 | 0.8232 | 0.1655 | 9.7498 |
| SLIM BPR | 0.8513 | 0.9970 | 0.8420 | 0.1723 | 9.8906 |
| SLIM ElasticNet | 0.8339 | 0.9967 | 0.7630 | 0.1384 | 9.6056 |
| MF BPR | 0.8488 | 0.9970 | 0.7971 | 0.1458 | 9.7015 |
| MF FunkSVD | 0.8778 | 0.9975 | 0.3333 | 0.0814 | 9.3079 |
| PureSVD | 0.7981 | 0.9960 | 0.1621 | 0.0434 | 8.4209 |
| NMF | 0.8787 | 0.9976 | 0.2560 | 0.0740 | 9.1913 |
| iALS | 0.8992 | 0.9980 | 0.3904 | 0.0956 | 9.5477 |
| SpectralCF | 0.3934 | 0.9879 | 0.2425 | 0.0270 | 7.1816 |
| SpectralCF article default | 0.0529 | 0.9811 | 0.0104 | 0.0089 | 5.7491 |

Table 35. Experimental results for the SpectralCF method for the Hetrec dataset on beyond accuracy metrics.

| | Div. MIL | Div. HHI | @ 50 Cov. Item | Div. Gini | Div. Shannon |
|----------------------------|---------------|---------------|----------------------|---------------|-----------------|
| Random | 0.9950 | 0.9999 | 0.9999 | 0.8202 | 13.2277 |
| TopPopular | 0.1661 | 0.9833 | 0.0113 | 0.0057 | 5.9971 |
| UserKNN CF cosine | 0.4982 | 0.9900 | 0.1013 | 0.0107 | 7.0928 |
| UserKNN CF dice | 0.5171 | 0.9903 | 0.1285 | 0.0121 | 7.2141 |
| UserKNN CF jaccard | 0.5092 | 0.9902 | 0.1267 | 0.0118 | 7.1816 |
| UserKNN CF asymmetric | 0.5058 | 0.9901 | 0.1021 | 0.0109 | 7.1269 |
| UserKNN CF tversky | 0.5122 | 0.9902 | 0.1284 | 0.0120 | 7.1977 |
| ItemKNN CF cosine | 0.5381 | 0.9908 | 0.1502 | 0.0133 | 7.3124 |
| ItemKNN CF dice | 0.5750 | 0.9915 | 0.1891 | 0.0170 | 7.5607 |
| ItemKNN CF jaccard | 0.6061 | 0.9921 | 0.2707 | 0.0237 | 7.8102 |
| ItemKNN CF asymmetric | 0.6072 | 0.9921 | 0.1466 | 0.0164 | 7.6400 |
| ItemKNN CF tversky | 0.5485 | 0.9910 | 0.1763 | 0.0151 | 7.4221 |
| $P^3\alpha$ | 0.4491 | 0.9890 | 0.1010 | 0.0097 | 6.9267 |
| $RP^3\beta$ | 0.5381 | 0.9908 | 0.2086 | 0.0166 | 7.4314 |
| EASE ^R | 0.5972 | 0.9919 | 0.1174 | 0.0138 | 7.4797 |
| SLIM BPR | 0.4518 | 0.9890 | 0.1490 | 0.0113 | 7.0062 |
| SLIM ElasticNet | 0.6657 | 0.9933 | 0.1419 | 0.0182 | 7.8591 |
| MF BPR | 0.5711 | 0.9914 | 0.2019 | 0.0160 | 7.4794 |
| MF FunkSVD | 0.6221 | 0.9924 | 0.1486 | 0.0219 | 7.9797 |
| PureSVD | 0.6808 | 0.9936 | 0.0542 | 0.0147 | 7.6309 |
| NMF | 0.7178 | 0.9943 | 0.0838 | 0.0176 | 7.9064 |
| iALS | 0.7224 | 0.9944 | 0.0799 | 0.0187 | 7.9915 |
| SpectralCF | 0.1971 | 0.9839 | 0.0163 | 0.0058 | 6.1024 |
| SpectralCF article default | 0.2351 | 0.9847 | 0.0112 | 0.0059 | 6.1460 |

Table 36. Experimental results for the SpectralCF method for the Movielens 1M original dataset on beyond accuracy metrics.

| | Div. MIL | Div. HHI | @ 50 Cov. Item | Div. Gini | Div. Shannon |
|----------------------------|---------------|---------------|----------------------|---------------|-----------------|
| Random | 0.9865 | 0.9997 | 1.0000 | 0.9344 | 11.8455 |
| TopPopular | 0.2035 | 0.9841 | 0.0367 | 0.0160 | 6.0744 |
| UserKNN CF cosine | 0.6877 | 0.9938 | 0.3420 | 0.0519 | 7.9763 |
| UserKNN CF dice | 0.7012 | 0.9940 | 0.3806 | 0.0554 | 8.0611 |
| UserKNN CF jaccard | 0.6800 | 0.9936 | 0.3501 | 0.0505 | 7.9359 |
| UserKNN CF asymmetric | 0.7100 | 0.9942 | 0.4062 | 0.0574 | 8.1084 |
| UserKNN CF tversky | 0.6692 | 0.9934 | 0.3412 | 0.0484 | 7.8764 |
| ItemKNN CF cosine | 0.6152 | 0.9923 | 0.4704 | 0.0433 | 7.6568 |
| ItemKNN CF dice | 0.6527 | 0.9931 | 0.4996 | 0.0493 | 7.8369 |
| ItemKNN CF jaccard | 0.6395 | 0.9928 | 0.5009 | 0.0478 | 7.7830 |
| ItemKNN CF asymmetric | 0.6811 | 0.9936 | 0.2337 | 0.0428 | 7.7403 |
| ItemKNN CF tversky | 0.6024 | 0.9920 | 0.4507 | 0.0407 | 7.5887 |
| $P^3\alpha$ | 0.7103 | 0.9942 | 0.3520 | 0.0535 | 8.0366 |
| $RP^3\beta$ | 0.7150 | 0.9943 | 0.6896 | 0.0742 | 8.3048 |
| EASE ^R | 0.7133 | 0.9943 | 0.2267 | 0.0489 | 7.9319 |
| SLIM BPR | 0.6920 | 0.9938 | 0.3765 | 0.0565 | 8.0698 |
| SLIM ElasticNet | 0.7685 | 0.9954 | 0.3093 | 0.0672 | 8.3712 |
| MF BPR | 0.6960 | 0.9939 | 0.3868 | 0.0577 | 8.0915 |
| MF FunkSVD | 0.7756 | 0.9955 | 0.1711 | 0.0590 | 8.1975 |
| PureSVD | 0.7769 | 0.9955 | 0.1447 | 0.0567 | 8.1242 |
| NMF | 0.8160 | 0.9963 | 0.2130 | 0.0754 | 8.5403 |
| iALS | 0.8394 | 0.9968 | 0.2753 | 0.0893 | 8.7898 |
| SpectralCF | 0.2662 | 0.9853 | 0.0470 | 0.0170 | 6.2529 |
| SpectralCF article default | 0.1761 | 0.9835 | 0.0286 | 0.0153 | 6.0031 |

Table 37. Experimental results for the SpectralCF method for the Movielens 1M ours dataset on beyond accuracy metrics.

| | Div. MIL | Div. HHI | @ 50 Cov. Item | Div. Gini | Div. Shannon |
|----------------------------|---------------|---------------|----------------------|---------------|-----------------|
| Random | 0.9871 | 0.9997 | 1.0000 | 0.9348 | 11.9129 |
| TopPopular | 0.1855 | 0.9837 | 0.0322 | 0.0150 | 6.0329 |
| UserKNN CF cosine | 0.6879 | 0.9938 | 0.4160 | 0.0512 | 7.9970 |
| UserKNN CF dice | 0.6462 | 0.9929 | 0.3470 | 0.0431 | 7.7664 |
| UserKNN CF jaccard | 0.6718 | 0.9934 | 0.3830 | 0.0484 | 7.9176 |
| UserKNN CF asymmetric | 0.5823 | 0.9916 | 0.2450 | 0.0333 | 7.4237 |
| UserKNN CF tversky | 0.6079 | 0.9922 | 0.3096 | 0.0373 | 7.5686 |
| ItemKNN CF cosine | 0.6920 | 0.9938 | 0.6695 | 0.0610 | 8.1128 |
| ItemKNN CF dice | 0.6342 | 0.9927 | 0.5247 | 0.0458 | 7.7590 |
| ItemKNN CF jaccard | 0.6332 | 0.9927 | 0.5598 | 0.0475 | 7.7844 |
| ItemKNN CF asymmetric | 0.6214 | 0.9924 | 0.5621 | 0.0452 | 7.7170 |
| ItemKNN CF tversky | 0.6578 | 0.9932 | 0.5088 | 0.0495 | 7.8964 |
| $P^3\alpha$ | 0.6824 | 0.9936 | 0.3449 | 0.0476 | 7.9227 |
| $RP^3\beta$ | 0.6343 | 0.9927 | 0.6430 | 0.0492 | 7.7865 |
| EASE ^R | 0.7303 | 0.9946 | 0.2687 | 0.0509 | 8.0533 |
| SLIM BPR | 0.6465 | 0.9929 | 0.3220 | 0.0424 | 7.7517 |
| SLIM ElasticNet | 0.7481 | 0.9950 | 0.3290 | 0.0599 | 8.2649 |
| MF BPR | 0.6152 | 0.9923 | 0.4088 | 0.0425 | 7.6973 |
| MF FunkSVD | 0.8022 | 0.9960 | 0.3006 | 0.0878 | 8.7693 |
| PureSVD | 0.7689 | 0.9954 | 0.1383 | 0.0520 | 8.0705 |
| NMF | 0.7306 | 0.9946 | 0.1069 | 0.0447 | 7.8312 |
| iALS | 0.8324 | 0.9966 | 0.2326 | 0.0772 | 8.6570 |
| SpectralCF | 0.2217 | 0.9844 | 0.0425 | 0.0155 | 6.1331 |
| SpectralCF article default | 0.1925 | 0.9838 | 0.0289 | 0.0150 | 6.0420 |

Table 38. Computation time for the algorithms in the selected results for the SpectralCF method on the Amazon Instant Video dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|----------------------------|--|------------------------|---------------------------|
| Random | 0.00 [sec] | 15.23 [sec] | 204 |
| TopPopular | 0.00 [sec] | 15.41 [sec] | 202 |
| UserKNN CF cosine | 0.16 ± 0.01 [sec] | 15.56 ± 0.04 [sec] | 200 |
| UserKNN CF dice | 0.17 ± 0.00 [sec] | 15.52 ± 0.03 [sec] | 200 |
| UserKNN CF jaccard | 0.17 ± 0.00 [sec] | 15.48 ± 0.02 [sec] | 201 |
| UserKNN CF asymmetric | 0.17 ± 0.01 [sec] | 15.41 ± 0.13 [sec] | 201 |
| UserKNN CF tversky | 0.17 ± 0.00 [sec] | 15.53 ± 0.03 [sec] | 201 |
| ItemKNN CF cosine | 0.38 ± 0.04 [sec] | 15.42 ± 0.09 [sec] | 202 |
| ItemKNN CF dice | 0.40 ± 0.01 [sec] | 15.53 ± 0.01 [sec] | 201 |
| ItemKNN CF jaccard | 0.40 ± 0.01 [sec] | 15.47 ± 0.05 [sec] | 202 |
| ItemKNN CF asymmetric | 0.41 ± 0.01 [sec] | 15.52 ± 0.02 [sec] | 201 |
| ItemKNN CF tversky | 0.41 ± 0.01 [sec] | 15.53 ± 0.04 [sec] | 200 |
| $P^3\alpha$ | 0.87 ± 0.03 [sec] | 15.54 ± 0.07 [sec] | 199 |
| $RP^3\beta$ | 0.95 ± 0.04 [sec] | 15.28 ± 0.17 [sec] | 201 |
| EASE ^R | 15.96 ± 0.03 [sec] | 15.57 ± 0.17 [sec] | 199 |
| SLIM BPR | 83.48 [sec] / 1.39 ± 0.59 [min] | 15.66 ± 0.05 [sec] | 199 |
| SLIM ElasticNet | 51.52 ± 3.17 [sec] | 15.62 ± 0.16 [sec] | 198 |
| MF BPR | 225.64 [sec] / 3.76 ± 2.96 [min] | 15.77 ± 0.20 [sec] | 194 |
| MF FunkSVD | 85.76 [sec] / 1.43 ± 1.50 [min] | 15.58 ± 0.16 [sec] | 198 |
| PureSVD | 0.15 ± 0.19 [sec] | 15.38 ± 0.02 [sec] | 202 |
| NMF | 28.00 ± 44.31 [sec] | 15.64 ± 0.14 [sec] | 199 |
| iALS | 193.65 [sec] / 3.23 ± 2.87 [min] | 15.73 ± 0.09 [sec] | 198 |
| SpectralCF | 1118.55 [sec] / 18.64 ± 2.65 [min] | 10.42 ± 0.21 [sec] | 295 |
| SpectralCF article default | 1335.22 [sec] / 22.25 [min] | 10.27 [sec] | 303 |

Table 39. Computation time for the algorithms in the selected results for the SpectralCF method on the Hetrec dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|----------------------------|--|------------------------|---------------------------|
| Random | 0.00 [sec] | 10.21 [sec] | 194 |
| TopPopular | 0.00 [sec] | 11.17 [sec] | 177 |
| UserKNN CF cosine | 0.22 ± 0.03 [sec] | 11.47 ± 0.06 [sec] | 172 |
| UserKNN CF dice | 0.23 ± 0.03 [sec] | 11.38 ± 0.10 [sec] | 174 |
| UserKNN CF jaccard | 0.23 ± 0.04 [sec] | 11.41 ± 0.03 [sec] | 174 |
| UserKNN CF asymmetric | 0.23 ± 0.04 [sec] | 11.44 ± 0.04 [sec] | 173 |
| UserKNN CF tversky | 0.23 ± 0.04 [sec] | 11.45 ± 0.03 [sec] | 174 |
| ItemKNN CF cosine | 1.13 ± 0.09 [sec] | 11.52 ± 0.05 [sec] | 172 |
| ItemKNN CF dice | 1.13 ± 0.06 [sec] | 11.34 ± 0.19 [sec] | 174 |
| ItemKNN CF jaccard | 1.14 ± 0.06 [sec] | 11.42 ± 0.04 [sec] | 174 |
| ItemKNN CF asymmetric | 1.13 ± 0.08 [sec] | 11.29 ± 0.44 [sec] | 172 |
| ItemKNN CF tversky | 1.15 ± 0.07 [sec] | 11.40 ± 0.06 [sec] | 174 |
| $P^3\alpha$ | 4.78 ± 0.66 [sec] | 11.40 ± 0.07 [sec] | 174 |
| $RP^3\beta$ | 4.55 ± 0.99 [sec] | 11.07 ± 0.63 [sec] | 172 |
| EASE ^R | 77.06 [sec] / 1.28 ± 0.00 [min] | 11.53 ± 0.03 [sec] | 171 |
| SLIM BPR | 130.28 [sec] / 2.17 ± 0.91 [min] | 11.39 ± 0.07 [sec] | 173 |
| SLIM ElasticNet | 306.32 [sec] / 5.11 ± 0.21 [min] | 11.30 ± 0.57 [sec] | 173 |
| MF BPR | 124.76 [sec] / 2.08 ± 2.29 [min] | 11.36 ± 0.34 [sec] | 171 |
| MF FunkSVD | 91.33 [sec] / 1.52 ± 1.67 [min] | 11.59 ± 0.20 [sec] | 170 |
| PureSVD | 0.18 ± 0.23 [sec] | 11.08 ± 0.20 [sec] | 177 |
| NMF | 36.78 ± 66.11 [sec] | 11.50 ± 0.12 [sec] | 172 |
| iALS | 187.90 [sec] / 3.13 ± 5.58 [min] | 11.45 ± 0.03 [sec] | 172 |
| SpectralCF | 1704.87 [sec] / 28.41 ± 4.33 [min] | 7.80 ± 0.19 [sec] | 249 |
| SpectralCF article default | 2197.20 [sec] / 36.62 [min] | 7.42 [sec] | 267 |

Table 40. Computation time for the algorithms in the selected results for the SpectralCF method on the Movielens 1M original dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|----------------------------|---|------------------------|---------------------------|
| Random | 0.00 [sec] | 27.37 [sec] | 200 |
| TopPopular | 0.01 [sec] | 28.35 [sec] | 193 |
| UserKNN CF cosine | 1.54 ± 0.20 [sec] | 29.21 ± 0.45 [sec] | 187 |
| UserKNN CF dice | 1.53 ± 0.22 [sec] | 29.04 ± 0.19 [sec] | 189 |
| UserKNN CF jaccard | 1.51 ± 0.21 [sec] | 29.09 ± 0.18 [sec] | 189 |
| UserKNN CF asymmetric | 1.58 ± 0.24 [sec] | 29.19 ± 0.08 [sec] | 188 |
| UserKNN CF tversky | 1.54 ± 0.25 [sec] | 29.07 ± 0.39 [sec] | 188 |
| ItemKNN CF cosine | 0.47 ± 0.07 [sec] | 29.08 ± 0.33 [sec] | 188 |
| ItemKNN CF dice | 0.41 ± 0.06 [sec] | 28.73 ± 0.15 [sec] | 191 |
| ItemKNN CF jaccard | 0.42 ± 0.06 [sec] | 28.93 ± 0.21 [sec] | 191 |
| ItemKNN CF asymmetric | 0.49 ± 0.05 [sec] | 29.26 ± 0.50 [sec] | 184 |
| ItemKNN CF tversky | 0.43 ± 0.07 [sec] | 28.95 ± 0.11 [sec] | 190 |
| $P^3\alpha$ | 2.31 ± 0.82 [sec] | 29.54 ± 0.23 [sec] | 186 |
| $RP^3\beta$ | 2.66 ± 0.90 [sec] | 29.35 ± 0.97 [sec] | 186 |
| EASE ^R | 5.07 ± 0.01 [sec] | 28.77 ± 0.24 [sec] | 189 |
| SLIM BPR | 218.79 [sec] / 3.65 ± 1.84 [min] | 29.69 ± 0.13 [sec] | 185 |
| SLIM ElasticNet | 60.10 [sec] / 1.00 ± 0.16 [min] | 28.88 ± 0.16 [sec] | 190 |
| MF BPR | 431.40 [sec] / 7.19 ± 5.28 [min] | 29.09 ± 0.09 [sec] | 188 |
| MF FunkSVD | 370.25 [sec] / 6.17 ± 7.29 [min] | 29.05 ± 0.52 [sec] | 182 |
| PureSVD | 0.27 ± 0.33 [sec] | 28.38 ± 0.24 [sec] | 192 |
| NMF | 47.42 ± 66.95 [sec] | 28.86 ± 0.43 [sec] | 190 |
| iALS | 272.61 [sec] / 4.54 ± 5.91 [min] | 28.65 ± 0.12 [sec] | 191 |
| SpectralCF | 2429.47 [sec] / 40.49 ± 24.14 [min] | 19.72 ± 0.28 [sec] | 279 |
| SpectralCF article default | 2111.55 [sec] / 35.19 [min] | 20.30 [sec] | 270 |

Table 41. Computation time for the algorithms in the selected results for the SpectralCF method on the Movielens 1M ours dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|----------------------------|---|------------------------|---------------------------|
| Random | 0.00 [sec] | 29.72 [sec] | 201 |
| TopPopular | 0.01 [sec] | 32.39 [sec] | 184 |
| UserKNN CF cosine | 1.67 ± 0.23 [sec] | 34.04 ± 0.03 [sec] | 175 |
| UserKNN CF dice | 1.49 ± 0.21 [sec] | 34.04 ± 0.10 [sec] | 175 |
| UserKNN CF jaccard | 1.50 ± 0.23 [sec] | 33.99 ± 0.41 [sec] | 175 |
| UserKNN CF asymmetric | 1.59 ± 0.19 [sec] | 34.16 ± 0.22 [sec] | 174 |
| UserKNN CF tversky | 1.56 ± 0.21 [sec] | 33.95 ± 0.74 [sec] | 174 |
| ItemKNN CF cosine | 0.45 ± 0.07 [sec] | 34.20 ± 0.87 [sec] | 173 |
| ItemKNN CF dice | 0.46 ± 0.08 [sec] | 33.75 ± 0.17 [sec] | 176 |
| ItemKNN CF jaccard | 0.45 ± 0.07 [sec] | 33.90 ± 0.19 [sec] | 176 |
| ItemKNN CF asymmetric | 0.50 ± 0.07 [sec] | 34.32 ± 0.18 [sec] | 174 |
| ItemKNN CF tversky | 0.43 ± 0.09 [sec] | 33.99 ± 0.31 [sec] | 176 |
| $P^3\alpha$ | 2.44 ± 0.93 [sec] | 33.77 ± 0.27 [sec] | 176 |
| $RP^3\beta$ | 3.11 ± 1.03 [sec] | 32.47 ± 1.99 [sec] | 175 |
| EASE ^R | 5.65 ± 0.01 [sec] | 33.43 ± 0.31 [sec] | 178 |
| SLIM BPR | 230.44 [sec] / 3.84 ± 2.74 [min] | 34.23 ± 0.10 [sec] | 174 |
| SLIM ElasticNet | 68.77 [sec] / 1.15 ± 0.24 [min] | 34.91 ± 0.52 [sec] | 169 |
| MF BPR | 358.82 [sec] / 5.98 ± 5.73 [min] | 33.91 ± 0.41 [sec] | 174 |
| MF FunkSVD | 328.94 [sec] / 5.48 ± 6.66 [min] | 32.76 ± 1.40 [sec] | 176 |
| PureSVD | 0.32 ± 0.40 [sec] | 33.13 ± 0.47 [sec] | 178 |
| NMF | 28.92 ± 61.24 [sec] | 33.66 ± 0.76 [sec] | 174 |
| iALS | 248.84 [sec] / 4.15 ± 4.96 [min] | 33.20 ± 0.64 [sec] | 176 |
| SpectralCF | 2243.74 [sec] / 37.40 ± 19.07 [min] | 21.55 ± 0.33 [sec] | 280 |
| SpectralCF article default | 2556.16 [sec] / 42.60 [min] | 21.74 [sec] | 274 |

Table 42. Hyperparameter values for our collaborative KNN baselines on all datasets.

| Algorithm | Hyperparameter | Movielens 1M ours | Movielens 1M original | Hetrec | Amazon Instant Video |
|-----------------------|-------------------|----------------------|--------------------------|------------|-------------------------|
| UserKNN CF cosine | topK | 418 | 365 | 464 | 800 |
| | shrink | 402 | 0 | 0 | 346 |
| | similarity | cosine | cosine | cosine | cosine |
| | normalize | True | True | True | False |
| | feature weighting | TF-IDF | TF-IDF | none | TF-IDF |
| UserKNN CF dice | topK | 383 | 276 | 428 | 484 |
| | shrink | 0 | 1 | 1 | 940 |
| | similarity | dice | dice | dice | dice |
| | normalize | False | True | True | False |
| UserKNN CF jaccard | topK | 300 | 337 | 456 | 444 |
| | shrink | 0 | 0 | 0 | 303 |
| | similarity | jaccard | jaccard | jaccard | jaccard |
| | normalize | False | True | False | True |
| UserKNN CF asymmetric | topK | 734 | 369 | 441 | 855 |
| | shrink | 0 | 134 | 0 | 19 |
| | similarity | asymmetric | asymmetric | asymmetric | asymmetric |
| | normalize | True | True | True | True |
| | asymmetric alpha | 0.4193 | 0.6047 | 0.5026 | 0.7882 |
| UserKNN CF tfidf | feature weighting | TF-IDF | TF-IDF | TF-IDF | none |
| UserKNN CF tversky | topK | 516 | 377 | 449 | 476 |
| | shrink | 0 | 0 | 0 | 806 |
| | similarity | tversky | tversky | tversky | tversky |
| | normalize | True | True | True | True |
| | tversky alpha | 1.2079 | 2.0000 | 2.0000 | 1.3499 |
| ItemKNN CF cosine | tversky beta | 2.0000 | 2.0000 | 2.0000 | 1.7078 |
| | topK | 197 | 615 | 322 | 998 |
| | shrink | 0 | 0 | 1000 | 21 |
| | similarity | cosine | cosine | cosine | cosine |
| | normalize | True | True | True | False |
| ItemKNN CF dice | feature weighting | TF-IDF | TF-IDF | TF-IDF | TF-IDF |
| | topK | 218 | 137 | 195 | 443 |
| | shrink | 2 | 0 | 33 | 172 |
| | similarity | dice | dice | dice | dice |
| | normalize | True | False | False | False |
| ItemKNN CF jaccard | topK | 158 | 135 | 222 | 290 |
| | shrink | 1 | 0 | 5 | 140 |
| | similarity | jaccard | jaccard | jaccard | jaccard |
| | normalize | True | False | True | True |
| ItemKNN CF asymmetric | topK | 269 | 1000 | 462 | 1000 |
| | shrink | 0 | 0 | 222 | 1000 |
| | similarity | asymmetric | asymmetric | asymmetric | asymmetric |
| | normalize | True | True | True | True |
| | asymmetric alpha | 0.3993 | 0.0466 | 0.0000 | 0.0000 |
| ItemKNN CF tfidf | feature weighting | TF-IDF | TF-IDF | TF-IDF | TF-IDF |
| ItemKNN CF tversky | topK | 48 | 143 | 142 | 1000 |
| | shrink | 77 | 0 | 23 | 1000 |
| | similarity | tversky | tversky | tversky | tversky |
| | normalize | True | True | True | True |
| | tversky alpha | 0.8429 | 0.4521 | 0.2786 | 0.0000 |
| ItemKNN CF tversky | tversky beta | 1.7696 | 2.0000 | 1.3237 | 2.0000 |

Table 43. Hyperparameter values for our non-neural machine learning and graph based baselines on all datasets.

| Algorithm | Hyperparameter | Movielens 1M ours | Movielens 1M original | Hetrec | Amazon Instant Video |
|-----------------|----------------------|----------------------|--------------------------|--------------|-------------------------|
| $P^3\alpha$ | topK | 350 | 332 | 901 | 1000 |
| | alpha | 0.6537 | 1.0075 | 0.7565 | 0.3705 |
| | normalize similarity | True | True | False | False |
| $RP^3\beta$ | topK | 853 | 537 | 1000 | 442 |
| | alpha | 0.0000 | 0.7551 | 0.8163 | 0.6540 |
| | beta | 0.4098 | 0.5412 | 0.2099 | 0.0332 |
| | normalize similarity | True | False | False | False |
| $EASE^R$ | l2 norm | 1.25E+03 | 1.72E+03 | 1.03E+03 | 3.06E+06 |
| SLIM BPR | topK | 329 | 1000 | 725 | 1000 |
| | epochs | 130 | 200 | 80 | 150 |
| | symmetric | True | True | True | False |
| | sgd mode | sgd | adagrad | adagrad | adagrad |
| | lambda i | 1.00E-02 | 1.00E-05 | 1.00E-05 | 1.00E-02 |
| | lambda j | 1.00E-02 | 1.00E-05 | 1.00E-05 | 1.00E-02 |
| | learning rate | 1.33E-02 | 1.00E-01 | 3.19E-04 | 1.00E-04 |
| SLIM ElasticNet | topK | 642 | 747 | 602 | 862 |
| | l1 ratio | 1.89E-05 | 7.37E-05 | 1.58E-05 | 6.11E-05 |
| | alpha | 0.0490 | 0.0371 | 0.1354 | 0.5507 |
| MF BPR | sgd mode | adagrad | adagrad | adagrad | adagrad |
| | epochs | 790 | 445 | 190 | 500 |
| | num factors | 200 | 200 | 200 | 200 |
| | batch size | 512 | 32 | 64 | 1 |
| | positive reg | 1.00E-02 | 1.00E-02 | 1.00E-02 | 1.00E-02 |
| | negative reg | 1.00E-05 | 1.00E-02 | 1.00E-02 | 1.00E-02 |
| | learning rate | 2.86E-02 | 1.00E-01 | 2.22E-02 | 1.00E-01 |
| | | | | | |
| MF FunkSVD | sgd mode | adam | adam | adam | adam |
| | epochs | 325 | 280 | 70 | 370 |
| | use bias | True | False | True | False |
| | batch size | 32 | 2 | 8 | 2 |
| | num factors | 19 | 8 | 98 | 13 |
| | item reg | 1.47E-04 | 1.28E-05 | 2.19E-05 | 6.87E-05 |
| | user reg | 1.88E-04 | 6.74E-04 | 6.92E-03 | 3.09E-04 |
| | learning rate | 2.45E-03 | 1.54E-03 | 4.73E-03 | 1.15E-02 |
| | negative quota | 0.2131 | 0.1045 | 0.4633 | 0.1323 |
| | | | | | |
| PureSVD | num factors | 16 | 15 | 9 | 33 |
| NMF | num factors | 9 | 20 | 22 | 37 |
| | solver | coord. descent | mult. update | mult. update | mult. update |
| | init type | random | nndsvda | random | nndsvda |
| | beta loss | frobenius | kullback-leibler | frobenius | frobenius |
| iALS | num factors | 24 | 22 | 11 | 26 |
| | confidence scaling | log | log | linear | linear |
| | alpha | 50.0000 | 1.7077 | 6.0056 | 50.0000 |
| | epsilon | 10.0000 | 0.0010 | 0.0010 | 0.0010 |
| | reg | 1.20E-04 | 1.00E-05 | 1.31E-03 | 1.00E-05 |
| | epochs | 20 | 75 | 45 | 50 |

Table 44. Hyperparameter values for the neural algorithm on all datasets.

| Algorithm | Hyperparameter | Movielens 1M ours | Movielens 1M original | Hetrec | Amazon Instant Video |
|----------------------------|----------------|----------------------|--------------------------|----------|-------------------------|
| SpectralCF | batch size | 2048 | 2048 | 512 | 1024 |
| | embedding size | 4 | 16 | 16 | 8 |
| | decay | 3.06E-02 | 3.20E-03 | 1.80E-03 | 2.78E-04 |
| | learning rate | 8.83E-04 | 7.00E-03 | 5.35E-03 | 9.68E-03 |
| | k | 2 | 3 | 2 | 3 |
| | epochs | 805 | 350 | 265 | 445 |
| SpectralCF article default | epochs | 600 | 410 | 185 | 425 |
| | batch size | 1024 | 1024 | 1024 | 1024 |
| | embedding size | 16 | 16 | 16 | 16 |
| | decay | 1.00E-03 | 1.00E-03 | 1.00E-03 | 1.00E-03 |
| | k | 3 | 3 | 3 | 3 |
| | learning rate | 1.00E-03 | 1.00E-03 | 1.00E-03 | 1.00E-03 |

G KDD: LEVERAGING META-PATH BASED CONTEXT FOR TOP-N RECOMMENDATION WITH A NEURAL CO-ATTENTION MODEL

Relevant statistics on the dataset, which we mentioned in the paper, are reported in Table 45. The results of our evaluation can be seen in Table 46 (Movielens 100k). The corresponding optimal hyperparameters are reported in Table 48 (collaborative KNNs), Table 49 (non-neural machine learning and graph based), Table 50 (content-based KNNs), Table 51 (hybrid KNNs) and Table 52 (MCRec).

Lastly, the time required to train and evaluate the models is reported in Table 47 (Movielens 100k).

Table 45. Dataset characteristics.

| Dataset | Interactions | Items | Users | Density |
|----------------|--------------|-------|--------|---------|
| Movielens 100k | 100 K | 1682 | 943 | 6.30% |
| LastFM | 92 K | 17 k | 1.8 k | 0.27% |
| YelpBusiness | 198 K | 14 k | 16.2 k | 0.08% |

Table 46. Experimental results for the MCRec method for the Movielens 100K dataset.

| | PREC | @ 10 REC | NDCG |
|--------------------------|---------------|---------------|---------------|
| Random | 0.0136 | 0.0070 | 0.0085 |
| TopPopular | 0.1907 | 0.1180 | 0.1361 |
| UserKNN CF cosine | 0.2807 | 0.1825 | 0.2260 |
| UserKNN CF dice | 0.3442 | 0.2237 | 0.2692 |
| UserKNN CF jaccard | 0.3430 | 0.2225 | 0.2687 |
| UserKNN CF asymmetric | 0.2814 | 0.1828 | 0.2264 |
| UserKNN CF tversky | 0.3426 | 0.2227 | 0.2694 |
| ItemKNN CF cosine | 0.3293 | 0.2152 | 0.2571 |
| ItemKNN CF dice | 0.3211 | 0.2040 | 0.2425 |
| ItemKNN CF jaccard | 0.3177 | 0.2043 | 0.2431 |
| ItemKNN CF asymmetric | 0.3320 | 0.2171 | 0.2601 |
| ItemKNN CF tversky | 0.3283 | 0.2145 | 0.2562 |
| $P^3\alpha$ | 0.3305 | 0.2081 | 0.2554 |
| $RP^3\beta$ | 0.3435 | 0.2191 | 0.2588 |
| EASE ^R | 0.3640 | 0.2318 | 0.2815 |
| SLIM BPR | 0.3127 | 0.2040 | 0.2460 |
| SLIM ElasticNet | 0.3770 | 0.2441 | 0.2957 |
| MF BPR | 0.2816 | 0.1860 | 0.2195 |
| MF FunkSVD | 0.3442 | 0.2203 | 0.2642 |
| PureSVD | 0.3545 | 0.2247 | 0.2719 |
| NMF | 0.3350 | 0.2139 | 0.2585 |
| iALS | 0.3596 | 0.2283 | 0.2759 |
| ItemKNN CBF cosine | 0.0455 | 0.0185 | 0.0254 |
| ItemKNN CBF dice | 0.0135 | 0.0038 | 0.0054 |
| ItemKNN CBF jaccard | 0.0135 | 0.0038 | 0.0054 |
| ItemKNN CBF asymmetric | 0.0547 | 0.0243 | 0.0319 |
| ItemKNN CBF tversky | 0.0097 | 0.0031 | 0.0042 |
| ItemKNN CFCBF cosine | 0.3398 | 0.2239 | 0.2646 |
| ItemKNN CFCBF dice | 0.3215 | 0.2043 | 0.2403 |
| ItemKNN CFCBF jaccard | 0.3200 | 0.2057 | 0.2422 |
| ItemKNN CFCBF asymmetric | 0.3390 | 0.2224 | 0.2662 |
| ItemKNN CFCBF tversky | 0.3127 | 0.2023 | 0.2439 |
| MCRec | 0.3110 | 0.2113 | 0.2466 |

Table 47. Computation time for the algorithms in the selected results for the MCRec method on the Movielens 100K dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|--------------------------|--|-----------------------------|---------------------------|
| Random | 0.00 [sec] | 0.37 [sec] | 2513 |
| TopPopular | 0.00 [sec] | 0.41 [sec] | 2319 |
| UserKNN CF cosine | 0.10 ± 0.02 [sec] | 0.86 ± 0.03 [sec] | 1077 |
| UserKNN CF dice | 0.09 ± 0.02 [sec] | 0.75 ± 0.02 [sec] | 1283 |
| UserKNN CF jaccard | 0.08 ± 0.02 [sec] | 0.78 ± 0.05 [sec] | 1160 |
| UserKNN CF asymmetric | 0.10 ± 0.01 [sec] | 0.85 ± 0.03 [sec] | 1077 |
| UserKNN CF tversky | 0.09 ± 0.02 [sec] | 0.76 ± 0.03 [sec] | 1256 |
| ItemKNN CF cosine | 0.16 ± 0.03 [sec] | 0.87 ± 0.02 [sec] | 1077 |
| ItemKNN CF dice | 0.13 ± 0.03 [sec] | 0.80 ± 0.05 [sec] | 1256 |
| ItemKNN CF jaccard | 0.13 ± 0.03 [sec] | 0.75 ± 0.02 [sec] | 1256 |
| ItemKNN CF asymmetric | 0.15 ± 0.02 [sec] | 0.84 ± 0.01 [sec] | 1117 |
| ItemKNN CF tversky | 0.14 ± 0.04 [sec] | 0.76 ± 0.04 [sec] | 1256 |
| $P^3\alpha$ | 0.88 ± 0.50 [sec] | 0.73 ± 0.02 [sec] | 1256 |
| $RP^3\beta$ | 0.97 ± 0.50 [sec] | 0.77 ± 0.02 [sec] | 1231 |
| EASE ^R | 0.57 ± 0.17 [sec] | 0.78 ± 0.16 [sec] | 1483 |
| SLIM BPR | 38.71 ± 16.65 [sec] | 1.11 ± 0.04 [sec] | 839 |
| SLIM ElasticNet | 11.45 ± 5.35 [sec] | 1.09 ± 0.01 [sec] | 862 |
| MF BPR | 82.37 [sec] / 1.37 ± 1.20 [min] | 1.56 ± 0.22 [sec] | 548 |
| MF FunkSVD | 148.82 [sec] / 2.48 ± 3.72 [min] | 1.69 ± 0.68 [sec] | 877 |
| PureSVD | 0.05 ± 0.04 [sec] | 0.60 ± 0.34 [sec] | 1945 |
| NMF | 20.65 ± 27.78 [sec] | 0.53 [sec] | 1774 |
| iALS | 26.02 ± 33.39 [sec] | 0.75 ± 0.44 [sec] | 1774 |
| ItemKNN CBF cosine | 0.93 ± 0.61 [sec] | 1.06 ± 0.05 [sec] | 928 |
| ItemKNN CBF dice | 0.92 ± 0.61 [sec] | 1.01 ± 0.04 [sec] | 900 |
| ItemKNN CBF jaccard | 1.02 ± 0.59 [sec] | 1.01 ± 0.04 [sec] | 914 |
| ItemKNN CBF asymmetric | 0.93 ± 0.56 [sec] | 1.06 ± 0.01 [sec] | 887 |
| ItemKNN CBF tversky | 1.18 ± 1.25 [sec] | 1.02 ± 0.02 [sec] | 914 |
| ItemKNN CFCBF cosine | 0.29 ± 0.05 [sec] | 1.13 ± 0.02 [sec] | 838 |
| ItemKNN CFCBF dice | 0.26 ± 0.04 [sec] | 1.09 ± 0.04 [sec] | 901 |
| ItemKNN CFCBF jaccard | 0.26 ± 0.04 [sec] | 1.10 ± 0.05 [sec] | 900 |
| ItemKNN CFCBF asymmetric | 0.28 ± 0.06 [sec] | 1.14 ± 0.04 [sec] | 838 |
| ItemKNN CFCBF tversky | 0.26 ± 0.06 [sec] | 1.12 ± 0.08 [sec] | 888 |
| MCRec | 8496.61 [sec] / 2.36 [hour] | 165.63 [sec] / 2.76 [min] | 6 |

Table 48. Hyperparameter values for our collaborative KNN baselines on all datasets.

| Algorithm | Hyperparameter | Movielens 100K |
|-----------------------|-------------------|----------------|
| UserKNN CF cosine | topK | 903 |
| | shrink | 2 |
| | similarity | cosine |
| | normalize | True |
| | feature weighting | BM25 |
| UserKNN CF dice | topK | 129 |
| | shrink | 0 |
| | similarity | dice |
| | normalize | True |
| UserKNN CF jaccard | topK | 128 |
| | shrink | 0 |
| | similarity | jaccard |
| | normalize | True |
| UserKNN CF asymmetric | topK | 1000 |
| | shrink | 1000 |
| | similarity | asymmetric |
| | normalize | True |
| | asymmetric alpha | 2.0000 |
| | feature weighting | BM25 |
| UserKNN CF tversky | topK | 125 |
| | shrink | 28 |
| | similarity | tversky |
| | normalize | True |
| | tversky alpha | 1.8829 |
| | tversky beta | 1.9666 |
| ItemKNN CF cosine | topK | 886 |
| | shrink | 403 |
| | similarity | cosine |
| | normalize | True |
| | feature weighting | BM25 |
| ItemKNN CF dice | topK | 179 |
| | shrink | 0 |
| | similarity | dice |
| | normalize | False |
| ItemKNN CF jaccard | topK | 161 |
| | shrink | 0 |
| | similarity | jaccard |
| | normalize | True |
| ItemKNN CF asymmetric | topK | 468 |
| | shrink | 706 |
| | similarity | asymmetric |
| | normalize | True |
| | asymmetric alpha | 1.5629 |
| | feature weighting | BM25 |
| ItemKNN CF tversky | topK | 122 |
| | shrink | 0 |
| | similarity | tversky |
| | normalize | True |
| | tversky alpha | 0.8648 |
| | tversky beta | 1.5755 |

Table 49. Hyperparameter values for our non-neural machine learning and graph based baselines on all datasets.

| Algorithm | Hyperparameter | Movielens 100K |
|-----------------|----------------------|----------------|
| $P^3\alpha$ | topK | 197 |
| | alpha | 0.0000 |
| | normalize similarity | True |
| $RP^3\beta$ | topK | 324 |
| | alpha | 0.8593 |
| | beta | 0.6574 |
| | normalize similarity | True |
| $EASE^R$ | l2 norm | 1.10E+03 |
| SLIM BPR | topK | 584 |
| | epochs | 120 |
| | symmetric | True |
| | sgd mode | adam |
| | lambda i | 1.00E-05 |
| | lambda j | 1.00E-05 |
| | learning rate | 1.00E-01 |
| SLIM ElasticNet | topK | 605 |
| | l1 ratio | 1.13E-04 |
| | alpha | 0.2225 |
| MF BPR | sgd mode | adagrad |
| | epochs | 355 |
| | num factors | 170 |
| | batch size | 256 |
| | positive reg | 2.16E-04 |
| | negative reg | 4.80E-05 |
| | learning rate | 3.97E-02 |
| MF FunkSVD | sgd mode | adagrad |
| | epochs | 135 |
| | use bias | False |
| | batch size | 2 |
| | num factors | 16 |
| | item reg | 1.00E-02 |
| | user reg | 1.00E-02 |
| | learning rate | 1.00E-01 |
| | negative quota | 0.0782 |
| PureSVD | num factors | 13 |
| NMF | num factors | 30 |
| | solver | coord. descent |
| | init type | random |
| | beta loss | frobenius |
| iALS | num factors | 15 |
| | confidence scaling | log |
| | alpha | 0.0010 |
| | epsilon | 0.0010 |
| | reg | 2.71E-04 |
| | epochs | 10 |

Table 50. Hyperparameter values for our content based KNN baselines on all datasets.

| Algorithm | Hyperparameter | Movielens 100K |
|------------------------|-------------------|----------------|
| ItemKNN CBF cosine | topK | 113 |
| | shrink | 349 |
| | similarity | cosine |
| | normalize | False |
| | feature weighting | BM25 |
| ItemKNN CBF dice | topK | 991 |
| | shrink | 1 |
| | similarity | dice |
| | normalize | True |
| ItemKNN CBF jaccard | topK | 991 |
| | shrink | 999 |
| | similarity | jaccard |
| | normalize | False |
| ItemKNN CBF asymmetric | topK | 403 |
| | shrink | 428 |
| | similarity | asymmetric |
| | normalize | True |
| | asymmetric alpha | 1.8470 |
| | feature weighting | BM25 |
| ItemKNN CBF tversky | topK | 983 |
| | shrink | 3 |
| | similarity | tversky |
| | normalize | True |
| | tversky alpha | 0.0501 |
| | tversky beta | 0.2996 |

Table 51. Hyperparameter values for our hybrid KNN baselines on all datasets.

| Algorithm | Hyperparameter | Movielens 100K |
|--------------------------|-------------------|----------------|
| ItemKNN CFCBF cosine | topK | 355 |
| | shrink | 228 |
| | similarity | cosine |
| | normalize | True |
| | feature weighting | BM25 |
| | ICM weight | 0.0757 |
| ItemKNN CFCBF dice | topK | 170 |
| | shrink | 0 |
| | similarity | dice |
| | normalize | True |
| | ICM weight | 0.0100 |
| ItemKNN CFCBF jaccard | topK | 164 |
| | shrink | 0 |
| | similarity | jaccard |
| | normalize | True |
| | ICM weight | 100.0000 |
| ItemKNN CFCBF asymmetric | topK | 311 |
| | shrink | 929 |
| | similarity | asymmetric |
| | normalize | True |
| | asymmetric alpha | 2.0000 |
| | feature weighting | BM25 |
| | ICM weight | 0.0100 |
| ItemKNN CFCBF tversky | topK | 70 |
| | shrink | 537 |
| | similarity | tversky |
| | normalize | True |
| | tversky alpha | 0.3134 |
| | tversky beta | 1.4108 |
| | ICM weight | 0.8798 |

Table 52. Hyperparameter values for the neural algorithm on all datasets.

| Algorithm | Hyperparameter | Movielens 100K |
|-----------|----------------|---------------------|
| MCR | epochs | 130 |
| | latent dim | 128 |
| | reg latent | 0 |
| | layers | [512, 256, 128, 64] |
| | reg layes | [0, 0, 0, 0] |
| | learning rate | 1.00E-03 |
| | batch size | 256 |
| | num negatives | 4 |

H WWW: NEURAL COLLABORATIVE FILTERING

Relevant statistics on the dataset, which we mentioned in the paper, are reported in Table 53. The results of our evaluation can be seen in Table 55 (Movielens 1M) and Table 54 (Pinterest). The corresponding optimal hyperparameters are reported in Table 58 (collaborative KNNs), Table 59 (non-neural machine learning and graph based) and Table 60 (NCF).

Lastly, the time required to train and evaluate the models is reported in Table 55 (Movielens 1M) and Table 54 (Pinterest).

Table 53. Dataset characteristics.

| Dataset | Interactions | Items | Users | Sparsity |
|--------------|--------------|-------|--------|----------|
| Movielens 1M | 1.0 M | 3.7 k | 6.0 k | 95.53% |
| Pinterest | 1.5 M | 9.9 k | 55.1 k | 99.73% |

Table 54. Experimental results for the NeuMF method for the Pinterest dataset.

| | @ 1 | | @ 5 | | @ 10 | |
|-----------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| | HR | NDCG | HR | NDCG | HR | NDCG |
| Random | 0.0107 | 0.0107 | 0.0500 | 0.0298 | 0.0996 | 0.0456 |
| TopPopular | 0.0467 | 0.0467 | 0.1665 | 0.1064 | 0.2740 | 0.1409 |
| UserKNN CF cosine | 0.2892 | 0.2892 | 0.7006 | 0.5036 | 0.8632 | 0.5566 |
| UserKNN CF dice | 0.2880 | 0.2880 | 0.7039 | 0.5047 | 0.8649 | 0.5572 |
| UserKNN CF jaccard | 0.2898 | 0.2898 | 0.7038 | 0.5056 | 0.8655 | 0.5583 |
| UserKNN CF asymmetric | 0.2877 | 0.2877 | 0.7040 | 0.5046 | 0.8655 | 0.5573 |
| UserKNN CF tversky | 0.2889 | 0.2889 | 0.7039 | 0.5052 | 0.8660 | 0.5580 |
| ItemKNN CF cosine | 0.2900 | 0.2900 | 0.7109 | 0.5090 | 0.8762 | 0.5628 |
| ItemKNN CF dice | 0.2917 | 0.2917 | 0.7098 | 0.5092 | 0.8765 | 0.5635 |
| ItemKNN CF jaccard | 0.2910 | 0.2910 | 0.7093 | 0.5086 | 0.8763 | 0.5631 |
| ItemKNN CF asymmetric | 0.2903 | 0.2903 | 0.7117 | 0.5096 | 0.8766 | 0.5633 |
| ItemKNN CF tversky | 0.2909 | 0.2909 | 0.7093 | 0.5086 | 0.8760 | 0.5629 |
| $P^3\alpha$ | 0.2853 | 0.2853 | 0.7022 | 0.5024 | 0.8700 | 0.5571 |
| $RP^3\beta$ | 0.2966 | 0.2966 | 0.7151 | 0.5149 | 0.8796 | 0.5685 |
| EASE ^R | 0.2889 | 0.2889 | 0.7053 | 0.5057 | 0.8682 | 0.5589 |
| SLIM BPR | 0.2983 | 0.2983 | 0.7117 | 0.5138 | 0.8736 | 0.5666 |
| SLIM ElasticNet | 0.2913 | 0.2913 | 0.7059 | 0.5072 | 0.8679 | 0.5601 |
| MF BPR | 0.2655 | 0.2655 | 0.6858 | 0.4833 | 0.8651 | 0.5418 |
| MF FunkSVD | 0.2601 | 0.2601 | 0.6890 | 0.4820 | 0.8658 | 0.5398 |
| PureSVD | 0.2630 | 0.2630 | 0.6628 | 0.4706 | 0.8268 | 0.5241 |
| NMF | 0.2307 | 0.2307 | 0.6445 | 0.4434 | 0.8343 | 0.5052 |
| iALS | 0.2811 | 0.2811 | 0.7144 | 0.5061 | 0.8761 | 0.5590 |
| NeuMF | 0.2801 | 0.2801 | 0.7101 | 0.5029 | 0.8777 | 0.5576 |

Table 55. Experimental results for the NeuMF method for the Movielens 1M dataset.

| | @ 1 | | @ 5 | | @ 10 | |
|-----------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| | HR | NDCG | HR | NDCG | HR | NDCG |
| Random | 0.0098 | 0.0098 | 0.0513 | 0.0303 | 0.0985 | 0.0454 |
| TopPopular | 0.1051 | 0.1051 | 0.3048 | 0.2064 | 0.4533 | 0.2542 |
| UserKNN CF cosine | 0.1825 | 0.1825 | 0.4925 | 0.3407 | 0.6606 | 0.3951 |
| UserKNN CF dice | 0.1911 | 0.1911 | 0.5053 | 0.3522 | 0.6700 | 0.4057 |
| UserKNN CF jaccard | 0.1906 | 0.1906 | 0.5045 | 0.3521 | 0.6725 | 0.4066 |
| UserKNN CF asymmetric | 0.1921 | 0.1921 | 0.5070 | 0.3546 | 0.6768 | 0.4100 |
| UserKNN CF tversky | 0.1921 | 0.1921 | 0.5073 | 0.3536 | 0.6684 | 0.4058 |
| ItemKNN CF cosine | 0.1825 | 0.1825 | 0.4942 | 0.3414 | 0.6694 | 0.3979 |
| ItemKNN CF dice | 0.1707 | 0.1707 | 0.4856 | 0.3323 | 0.6604 | 0.3887 |
| ItemKNN CF jaccard | 0.1692 | 0.1692 | 0.4772 | 0.3268 | 0.6533 | 0.3837 |
| ItemKNN CF asymmetric | 0.1843 | 0.1843 | 0.4906 | 0.3400 | 0.6627 | 0.3956 |
| ItemKNN CF tversky | 0.1735 | 0.1735 | 0.4856 | 0.3338 | 0.6546 | 0.3884 |
| $P^3\alpha$ | 0.1791 | 0.1791 | 0.4846 | 0.3352 | 0.6460 | 0.3876 |
| $RP^3\beta$ | 0.1836 | 0.1836 | 0.4935 | 0.3419 | 0.6758 | 0.4011 |
| EASE ^R | 0.2119 | 0.2119 | 0.5502 | 0.3857 | 0.7098 | 0.4374 |
| SLIM BPR | 0.2013 | 0.2013 | 0.5320 | 0.3713 | 0.7002 | 0.4258 |
| SLIM ElasticNet | 0.2207 | 0.2207 | 0.5576 | 0.3953 | 0.7162 | 0.4468 |
| MF BPR | 0.1679 | 0.1679 | 0.4619 | 0.3186 | 0.6305 | 0.3730 |
| MF FunkSVD | 0.2008 | 0.2008 | 0.5202 | 0.3661 | 0.6844 | 0.4192 |
| PureSVD | 0.2132 | 0.2132 | 0.5339 | 0.3783 | 0.6937 | 0.4303 |
| NMF | 0.2056 | 0.2056 | 0.5171 | 0.3651 | 0.6844 | 0.4192 |
| iALS | 0.2106 | 0.2106 | 0.5505 | 0.3862 | 0.7109 | 0.4382 |
| NeuMF | 0.2088 | 0.2088 | 0.5411 | 0.3803 | 0.7093 | 0.4349 |

Table 56. Computation time for the algorithms in the selected results for the NeuMF method on the Pinterest dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|-------------------------|-------------------------------------|----------------------------------|---------------------------|
| Random | 0.04 [sec] | 337.91 [sec] / 5.63 [min] | 163 |
| TopPopular | 0.08 [sec] | 350.44 [sec] / 5.84 [min] | 157 |
| UserKNN CF cosine | 59.52 ± 3.87 [sec] | 442.63 [sec] / 7.38 ± 0.11 [min] | 122 |
| UserKNN CF dice | 60.07 [sec] / 1.00 ± 0.05 [min] | 440.65 [sec] / 7.34 ± 0.10 [min] | 124 |
| UserKNN CF jaccard | 59.33 ± 3.16 [sec] | 443.67 [sec] / 7.39 ± 0.05 [min] | 124 |
| UserKNN CF asymmetric | 60.80 [sec] / 1.01 ± 0.04 [min] | 445.33 [sec] / 7.42 ± 0.01 [min] | 124 |
| UserKNN CF tversky | 60.89 [sec] / 1.01 ± 0.06 [min] | 435.42 [sec] / 7.26 ± 0.23 [min] | 124 |
| ItemKNN CF cosine | 3.73 ± 0.40 [sec] | 435.26 [sec] / 7.25 ± 0.04 [min] | 126 |
| ItemKNN CF dice | 3.56 ± 0.37 [sec] | 433.80 [sec] / 7.23 ± 0.05 [min] | 126 |
| ItemKNN CF jaccard | 3.70 ± 0.36 [sec] | 435.32 [sec] / 7.26 ± 0.05 [min] | 126 |
| ItemKNN CF asymmetric | 3.69 ± 0.35 [sec] | 437.39 [sec] / 7.29 ± 0.05 [min] | 126 |
| ItemKNN CF tversky | 3.64 ± 0.40 [sec] | 436.99 [sec] / 7.28 ± 0.04 [min] | 126 |
| P ³ α | 17.69 ± 4.33 [sec] | 434.43 [sec] / 7.24 ± 0.02 [min] | 127 |
| RP ³ β | 17.95 ± 4.99 [sec] | 433.36 [sec] / 7.22 ± 0.06 [min] | 126 |
| EASE ^R | 123.69 [sec] / 2.06 ± 0.00 [min] | 409.54 [sec] / 6.83 ± 0.02 [min] | 135 |
| SLIM BPR | 4566.45 [sec] / 1.27 ± 0.55 [hour] | 434.78 [sec] / 7.25 ± 0.05 [min] | 127 |
| SLIM ElasticNet | 728.11 [sec] / 12.14 ± 5.73 [min] | 428.35 [sec] / 7.14 ± 0.23 [min] | 125 |
| MF BPR | 12620.32 [sec] / 3.51 ± 2.83 [hour] | 461.88 [sec] / 7.70 ± 1.94 [min] | 95 |
| MF FunkSVD | 8736.15 [sec] / 2.43 ± 1.89 [hour] | 443.13 [sec] / 7.39 ± 1.77 [min] | 150 |
| PureSVD | 6.58 ± 5.41 [sec] | 430.77 [sec] / 7.18 ± 1.83 [min] | 149 |
| NMF | 963.74 [sec] / 16.06 ± 22.37 [min] | 543.13 [sec] / 9.05 ± 1.92 [min] | 149 |
| iALS | 10812.68 [sec] / 3.00 ± 3.91 [hour] | 372.05 [sec] / 6.20 ± 0.03 [min] | 148 |
| NeuMF | 167670.36 [sec] / 1.94 [day] | 6995.33 [sec] / 1.94 [hour] | 8 |

Table 57. Computation time for the algorithms in the selected results for the NeuMF method on the Movielens 1M dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|-----------------------|---|---------------------------|---------------------------|
| Random | 0.04 [sec] | 36.31 [sec] | 166 |
| TopPopular | 0.06 [sec] | 37.88 [sec] | 159 |
| UserKNN CF cosine | 9.01 ± 0.26 [sec] | 48.07 ± 1.08 [sec] | 126 |
| UserKNN CF dice | 9.00 ± 0.31 [sec] | 47.37 ± 1.63 [sec] | 130 |
| UserKNN CF jaccard | 8.98 ± 0.29 [sec] | 46.70 ± 1.38 [sec] | 130 |
| UserKNN CF asymmetric | 9.06 ± 0.30 [sec] | 47.01 ± 1.83 [sec] | 129 |
| UserKNN CF tversky | 9.09 ± 0.39 [sec] | 46.66 ± 2.00 [sec] | 130 |
| ItemKNN CF cosine | 4.04 ± 0.17 [sec] | 49.48 ± 4.20 [sec] | 130 |
| ItemKNN CF dice | 4.07 ± 0.17 [sec] | 47.25 ± 2.28 [sec] | 133 |
| ItemKNN CF jaccard | 4.09 ± 0.19 [sec] | 47.56 ± 3.43 [sec] | 134 |
| ItemKNN CF asymmetric | 4.10 ± 0.15 [sec] | 50.21 ± 1.92 [sec] | 127 |
| ItemKNN CF tversky | 4.09 ± 0.18 [sec] | 45.82 ± 1.18 [sec] | 133 |
| $P^3\alpha$ | 7.61 ± 2.17 [sec] | 45.84 ± 0.56 [sec] | 131 |
| $RP^3\beta$ | 8.05 ± 2.55 [sec] | 46.76 ± 0.42 [sec] | 127 |
| EASE ^R | 12.34 ± 0.03 [sec] | 46.25 ± 0.47 [sec] | 130 |
| SLIM BPR | 785.57 [sec] / 13.09 ± 7.36 [min] | 49.96 ± 2.04 [sec] | 118 |
| SLIM ElasticNet | 252.72 [sec] / 4.21 ± 2.51 [min] | 46.78 ± 0.22 [sec] | 129 |
| MF BPR | 937.36 [sec] / 15.62 ± 12.59 [min] | 56.94 ± 9.66 [sec] | 98 |
| MF FunkSVD | 3594.07 [sec] / 59.90 ± 47.65 [min] | 41.99 ± 7.61 [sec] | 154 |
| PureSVD | 2.84 ± 1.96 [sec] | 44.36 ± 10.92 [sec] | 153 |
| NMF | 721.37 [sec] / 12.02 ± 22.88 [min] | 55.73 ± 12.22 [sec] | 151 |
| iALS | 1022.66 [sec] / 17.04 ± 13.53 [min] | 42.97 ± 8.91 [sec] | 154 |
| NeuMF | 15050.89 [sec] / 4.18 [hour] | 293.85 [sec] / 4.90 [min] | 21 |

Table 58. Hyperparameter values for our collaborative KNN baselines on all datasets.

| Algorithm | Hyperparameter | Movielens 1M | Pinterest |
|-----------------------|-------------------|--------------|------------|
| UserKNN CF cosine | normalize | True | True |
| | topK | 516 | 1000 |
| | feature weighting | BM25 | BM25 |
| | similarity | cosine | cosine |
| | shrink | 0 | 0 |
| UserKNN CF dice | normalize | True | True |
| | topK | 246 | 991 |
| | similarity | dice | dice |
| | shrink | 0 | 138 |
| UserKNN CF jaccard | normalize | True | True |
| | topK | 259 | 972 |
| | similarity | jaccard | jaccard |
| | shrink | 0 | 0 |
| UserKNN CF asymmetric | topK | 306 | 1000 |
| | feature weighting | TF-IDF | TF-IDF |
| | asymmetric alpha | 0.2173 | 0.0000 |
| | normalize | True | True |
| | similarity | asymmetric | asymmetric |
| UserKNN CF tversky | shrink | 0 | 1000 |
| | normalize | True | True |
| | topK | 267 | 1000 |
| | tversky alpha | 0.6394 | 2.0000 |
| | tversky beta | 0.8051 | 1.9574 |
| ItemKNN CF cosine | similarity | tversky | tversky |
| | shrink | 0 | 33 |
| | normalize | True | False |
| | topK | 111 | 1000 |
| | feature weighting | BM25 | BM25 |
| ItemKNN CF dice | similarity | cosine | cosine |
| | shrink | 298 | 4 |
| | normalize | True | True |
| | topK | 61 | 1000 |
| | similarity | dice | dice |
| ItemKNN CF jaccard | shrink | 0 | 0 |
| | normalize | False | False |
| | topK | 62 | 997 |
| | similarity | jaccard | jaccard |
| | shrink | 19 | 1 |
| ItemKNN CF asymmetric | topK | 206 | 1000 |
| | feature weighting | BM25 | BM25 |
| | asymmetric alpha | 0.6914 | 0.0000 |
| | normalize | True | True |
| | similarity | asymmetric | asymmetric |
| ItemKNN CF tversky | shrink | 1000 | 1000 |
| | normalize | True | True |
| | topK | 83 | 1000 |
| | tversky alpha | 0.0000 | 2.0000 |
| | tversky beta | 2.0000 | 2.0000 |
| | similarity | tversky | tversky |
| | shrink | 561 | 0 |

Table 59. Hyperparameter values for our non-neural machine learning and graph based baselines on all datasets.

| Algorithm | Hyperparameter | Movielens 1M | Pinterest |
|-----------------|----------------------|--------------|------------------|
| $P^3\alpha$ | alpha | 1.2177 | 2.0000 |
| | topK | 1000 | 972 |
| | normalize similarity | False | True |
| $RP^3\beta$ | alpha | 1.0807 | 0.8616 |
| | topK | 546 | 1000 |
| | normalize similarity | True | True |
| | beta | 0.7029 | 0.4255 |
| $EASE^R$ | l2 norm | 2.96E+03 | 4.59E+03 |
| SLIM BPR | learning rate | 3.08E-02 | 1.00E-01 |
| | sgd mode | adagrad | adagrad |
| | symmetric | True | True |
| | epochs | 285 | 180 |
| | lambda i | 1.00E-02 | 1.00E-02 |
| | topK | 1000 | 916 |
| | lambda j | 4.51E-03 | 3.83E-05 |
| SLIM ElasticNet | l1 ratio | 1.19E-05 | 1.15E-04 |
| | alpha | 0.0788 | 0.0526 |
| | topK | 544 | 1000 |
| MF BPR | positive reg | 2.08E-05 | 1.00E-02 |
| | num factors | 200 | 200 |
| | negative reg | 1.00E-02 | 6.59E-03 |
| | epochs | 625 | 615 |
| | batch size | 8 | 2 |
| | sgd mode | adagrad | adagrad |
| | learning rate | 5.88E-02 | 6.74E-02 |
| MF FunkSVD | sgd mode | adam | adagrad |
| | num factors | 50 | 28 |
| | negative quota | 0.1651 | 0.4052 |
| | user reg | 7.35E-04 | 6.49E-04 |
| | learning rate | 4.38E-04 | 3.17E-02 |
| | epochs | 160 | 305 |
| | batch size | 512 | 1 |
| | item reg | 4.83E-03 | 1.56E-03 |
| | use bias | True | True |
| PureSVD | num factors | 52 | 50 |
| NMF | init type | random | nndsvda |
| | beta loss | frobenius | kullback-leibler |
| | num factors | 89 | 27 |
| | solver | mult. update | mult. update |
| iALS | num factors | 46 | 30 |
| | epochs | 10 | 45 |
| | epsilon | 10.0000 | 0.0010 |
| | confidence scaling | log | linear |
| | reg | 1.00E-05 | 1.00E-02 |
| | alpha | 50.0000 | 50.0000 |

Table 60. Hyperparameter values for the neural algorithm on all datasets.

| Algorithm | Hyperparameter | Movielens 1M | Pinterest |
|-----------|------------------------|----------------|--------------|
| NeuMF | epochs | 10 | 5 |
| | epochs gmf | 10 | 45 |
| | epochs mlp | 10 | 10 |
| | batch size | 256 | 256 |
| | num factors | 64 | 16 |
| | layers | [256, 128, 64] | [64, 32, 16] |
| | reg mf | 0.00E+00 | 0.00E+00 |
| | reg layers | [0, 0, 0] | [0, 0, 0] |
| | num negatives | 4 | 4 |
| | learning rate | 1.00E-03 | 1.00E-03 |
| | learning rate pretrain | 1.00E-03 | 1.00E-03 |
| | learner | sgd | sgd |
| | learner pretrain | adam | adam |
| | pretrain | True | True |

I WWW: VARIATIONAL AUTOENCODERS FOR COLLABORATIVE FILTERING

Relevant statistics on the dataset, which we mentioned in the paper, are reported in Table 61. The results of our evaluation can be seen in Table 62 (Movielens 20M) and Table 63 (Netflix Prize). The corresponding optimal hyperparameters are reported in Table 66 (collaborative KNNs), Table 67 (non-neural machine learning and graph based) and Table 68 (Mult VAE).

Lastly, the time required to train and evaluate the models is reported in Table 64 (Movielens 20M) and Table 65 (Netflix Prize).

Table 61. Dataset characteristics.

| Dataset | Interactions | Users | Items | Density | Held out users |
|---------------|--------------|-------|-------|---------|----------------|
| Movielens 20M | 10.0M | 136k | 20k | 0.36 | 10k |
| Netflix Prize | 56.9M | 463k | 17k | 0.69 | 40k |

Table 62. Experimental results for the MultVAE method for the Movielens 20M dataset.

| | @ 20 | | @ 50 | | @ 100 | |
|-----------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| | REC | NDCG | REC | NDCG | REC | NDCG |
| Random | 0.0010 | 0.0006 | 0.0023 | 0.0012 | 0.0047 | 0.0021 |
| TopPopular | 0.1441 | 0.1201 | 0.2320 | 0.1569 | 0.3296 | 0.1901 |
| UserKNN CF cosine | - | - | - | - | - | - |
| UserKNN CF dice | - | - | - | - | - | - |
| UserKNN CF jaccard | - | - | - | - | - | - |
| UserKNN CF asymmetric | - | - | - | - | - | - |
| UserKNN CF tversky | - | - | - | - | - | - |
| ItemKNN CF cosine | 0.2897 | 0.2434 | 0.4412 | 0.3054 | 0.5652 | 0.3492 |
| ItemKNN CF dice | 0.2689 | 0.2274 | 0.4095 | 0.2851 | 0.5316 | 0.3277 |
| ItemKNN CF jaccard | 0.2667 | 0.2284 | 0.4035 | 0.2844 | 0.5254 | 0.3268 |
| ItemKNN CF asymmetric | 0.2937 | 0.2444 | 0.4486 | 0.3087 | 0.5709 | 0.3527 |
| ItemKNN CF tversky | 0.2867 | 0.2395 | 0.4393 | 0.3030 | 0.5556 | 0.3458 |
| $P^3\alpha$ | 0.2620 | 0.2168 | 0.4047 | 0.2742 | 0.5287 | 0.3182 |
| $RP^3\beta$ | 0.3006 | 0.2501 | 0.4540 | 0.3133 | 0.5797 | 0.3583 |
| EASE ^R | 0.3100 | 0.2639 | 0.4608 | 0.3267 | 0.5860 | 0.3711 |
| SLIM BPR | 0.3206 | 0.2646 | 0.4783 | 0.3291 | 0.6030 | 0.3731 |
| SLIM ElasticNet | 0.3356 | 0.2920 | 0.4893 | 0.3576 | 0.6110 | 0.4017 |
| PureSVD | 0.1620 | 0.1137 | 0.2778 | 0.1593 | 0.3974 | 0.1995 |
| iALS | 0.2030 | 0.1340 | 0.3628 | 0.1954 | 0.4976 | 0.2418 |
| Mult VAE | 0.3541 | 0.2988 | 0.5222 | 0.3690 | 0.6517 | 0.4158 |

Table 63. Experimental results for the MultVAE method for the Netflix Prize dataset.

| | @ 20 | | @ 50 | | @ 100 | |
|-------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| | REC | NDCG | REC | NDCG | REC | NDCG |
| Random | 0.0013 | 0.0011 | 0.0032 | 0.0020 | 0.0059 | 0.0032 |
| TopPopular | 0.0786 | 0.0762 | 0.1643 | 0.1159 | 0.2717 | 0.1570 |
| UserKNN CF cosine | - | - | - | - | - | - |
| UserKNN CF dice | - | - | - | - | - | - |
| UserKNN CF jaccard | - | - | - | - | - | - |
| UserKNN CF asymmetric | - | - | - | - | - | - |
| UserKNN CF tversky | - | - | - | - | - | - |
| ItemKNN CF cosine | 0.2091 | 0.1970 | 0.3387 | 0.2592 | 0.4598 | 0.3092 |
| ItemKNN CF dice | 0.1963 | 0.1862 | 0.3224 | 0.2479 | 0.4379 | 0.2983 |
| ItemKNN CF jaccard | 0.1997 | 0.1883 | 0.3248 | 0.2481 | 0.4450 | 0.2978 |
| ItemKNN CF asymmetric | 0.2119 | 0.1968 | 0.3466 | 0.2623 | 0.4764 | 0.3165 |
| ItemKNN CF tversky | 0.2075 | 0.1933 | 0.3420 | 0.2582 | 0.4708 | 0.3118 |
| P ³ α | 0.1960 | 0.1759 | 0.3325 | 0.2412 | 0.4633 | 0.2962 |
| RP ³ β | 0.2210 | 0.2053 | 0.3633 | 0.2739 | 0.4932 | 0.3281 |
| EASE ^R | 0.2393 | 0.2288 | 0.3801 | 0.2978 | 0.5072 | 0.3510 |
| SLIM BPR | 0.2394 | 0.2219 | 0.3767 | 0.2886 | 0.5004 | 0.3403 |
| SLIM ElasticNet | 0.2555 | 0.2479 | 0.4002 | 0.3203 | 0.5299 | 0.3752 |
| PureSVD | 0.1177 | 0.0908 | 0.2193 | 0.1357 | 0.3247 | 0.1765 |
| iALS | 0.1397 | 0.1014 | 0.2675 | 0.1570 | 0.3930 | 0.2066 |
| Mult VAE | 0.2615 | 0.2423 | 0.4127 | 0.3167 | 0.5456 | 0.3730 |

Table 64. Computation time for the algorithms in the selected results for the MultVAE method on the MovieLens 20M dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|-------------------------|---|-------------------------|---------------------------|
| Random | 0.23 [sec] | 13.64 [sec] | 733 |
| TopPopular | 0.38 [sec] | 12.62 [sec] | 792 |
| UserKNN CF cosine | - | - | - |
| UserKNN CF dice | - | - | - |
| UserKNN CF jaccard | - | - | - |
| UserKNN CF asymmetric | - | - | - |
| UserKNN CF tversky | - | - | - |
| ItemKNN CF cosine | 12.12 \pm 0.53 [sec] | 17.66 \pm 2.93 [sec] | 651 |
| ItemKNN CF dice | 11.85 \pm 0.45 [sec] | 13.84 \pm 0.69 [sec] | 741 |
| ItemKNN CF jaccard | 11.74 \pm 0.46 [sec] | 14.08 \pm 0.67 [sec] | 723 |
| ItemKNN CF asymmetric | 12.29 \pm 0.71 [sec] | 15.81 \pm 1.82 [sec] | 736 |
| ItemKNN CF tversky | 11.79 \pm 0.58 [sec] | 15.24 \pm 2.53 [sec] | 764 |
| P ³ α | 24.76 \pm 4.74 [sec] | 12.61 \pm 0.31 [sec] | 822 |
| RP ³ β | 25.39 \pm 5.11 [sec] | 13.09 \pm 0.68 [sec] | 767 |
| EASE ^R | 195.45 [sec] / 3.26 \pm 0.09 [min] | 17.86 \pm 0.96 [sec] | 561 |
| SLIM BPR | 2315.60 [sec] / 38.59 \pm 48.48 [min] | 13.97 \pm 1.00 [sec] | 690 |
| SLIM ElasticNet | 6508.83 [sec] / 1.81 \pm 1.26 [hour] | 13.88 \pm 0.70 [sec] | 708 |
| PureSVD | 10.78 \pm 9.06 [sec] | 33.89 \pm 10.20 [sec] | 260 |
| iALS | 1352.64 [sec] / 22.54 \pm 19.35 [min] | 34.69 \pm 8.11 [sec] | 279 |
| Mult VAE | 1296.97 [sec] / 21.62 [min] | 18.72 [sec] | 534 |

Table 65. Computation time for the algorithms in the selected results for the MultVAE method on the Netflix Prize dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|-----------------------|---|--------------------------------------|---------------------------|
| Random | 1.67 [sec] | 60.99 [sec] / 1.02 [min] | 656 |
| TopPopular | 2.95 [sec] | 59.14 [sec] | 676 |
| UserKNN CF cosine | - | - | - |
| UserKNN CF dice | - | - | - |
| UserKNN CF jaccard | - | - | - |
| UserKNN CF asymmetric | - | - | - |
| UserKNN CF tversky | - | - | - |
| ItemKNN CF cosine | 65.70 [sec] / 1.09 ± 0.03 [min] | 84.91 [sec] / 1.42 ± 0.46 [min] | 593 |
| ItemKNN CF dice | 64.29 [sec] / 1.07 ± 0.01 [min] | 63.77 [sec] / 1.06 ± 0.08 [min] | 682 |
| ItemKNN CF jaccard | 64.40 [sec] / 1.07 ± 0.01 [min] | 61.38 [sec] / 1.02 [min] | 652 |
| ItemKNN CF asymmetric | 65.41 [sec] / 1.09 ± 0.04 [min] | 97.70 [sec] / 1.63 ± 0.59 [min] | 654 |
| ItemKNN CF tversky | 64.82 [sec] / 1.08 ± 0.01 [min] | 66.06 [sec] / 1.10 ± 0.12 [min] | 657 |
| $P^3\alpha$ | 74.12 [sec] / 1.24 ± 0.08 [min] | 59.98 ± 0.57 [sec] | 666 |
| $RP^3\beta$ | 75.10 [sec] / 1.25 ± 0.09 [min] | 58.00 ± 5.61 [sec] | 648 |
| EASE ^R | 205.33 [sec] / 3.42 ± 0.04 [min] | 85.09 [sec] / 1.42 ± 0.01 [min] | 473 |
| SLIM BPR | 5741.37 [sec] / 1.59 ± 1.80 [hour] | 65.49 [sec] / 1.09 ± 0.09 [min] | 600 |
| SLIM ElasticNet | 29589.53 [sec] / 8.22 ± 7.70 [hour] | 69.53 [sec] / 1.16 ± 0.13 [min] | 580 |
| PureSVD | 85.66 [sec] / 1.43 ± 0.92 [min] | 156.60 [sec] / 2.61 ± 0.33 [min] | 245 |
| iALS | 6101.72 [sec] / 1.69 ± 1.16 [hour] | 153.64 [sec] / 2.56 ± 0.12 [min] | 265 |
| Mult VAE | 4521.33 [sec] / 1.26 [hour] | 81.53 [sec] / 1.36 [min] | 491 |

Table 66. Hyperparameter values for our collaborative KNN baselines on all datasets.

| Algorithm | Hyperparameter | Movielens 20M | Netflix Prize |
|-----------------------|-------------------|---------------|---------------|
| UserKNN CF cosine | - | - | - |
| UserKNN CF dice | - | - | - |
| UserKNN CF jaccard | - | - | - |
| UserKNN CF asymmetric | - | - | - |
| UserKNN CF tversky | - | - | - |
| ItemKNN CF cosine | topK | 278 | 140 |
| | shrink | 409 | 1000 |
| | similarity | cosine | cosine |
| | normalize | True | True |
| | feature weighting | BM25 | BM25 |
| ItemKNN CF dice | topK | 107 | 9 |
| | shrink | 3 | 983 |
| | similarity | dice | dice |
| | normalize | True | True |
| | | | |
| ItemKNN CF jaccard | topK | 118 | 54 |
| | shrink | 214 | 544 |
| | similarity | jaccard | jaccard |
| | normalize | True | True |
| | | | |
| ItemKNN CF asymmetric | topK | 52 | 64 |
| | shrink | 0 | 360 |
| | similarity | asymmetric | asymmetric |
| | normalize | True | True |
| | asymmetric alpha | 0.9034 | 0.2002 |
| | feature weighting | BM25 | none |
| ItemKNN CF tversky | topK | 25 | 54 |
| | shrink | 273 | 1000 |
| | similarity | tversky | tversky |
| | normalize | True | True |
| | tversky alpha | 0.0782 | 0.1997 |
| | tversky beta | 0.5191 | 0.9652 |
| | | | |

Table 67. Hyperparameter values for our non-neural machine learning and graph based baselines on all datasets.

| Algorithm | Hyperparameter | Movielens 20M | Netflix Prize |
|-----------------|-------------------------------|---------------|---------------|
| $P^3\alpha$ | topK | 523 | 534 |
| | alpha | 0.5575 | 1.6695 |
| | normalize similarity | True | True |
| $RP^3\beta$ | topK | 399 | 185 |
| | alpha | 0.7242 | 1.3871 |
| | beta | 0.4945 | 0.4271 |
| | normalize similarity | True | True |
| $EASE^R$ | l2 norm | 3.99E+04 | 1.12E+05 |
| SLIM BPR | topK | 847 | 491 |
| | epochs | 630 | 240 |
| | symmetric | True | True |
| | sgd mode | sgd | sgd |
| | lambda i | 5.69E-05 | 5.87E-04 |
| | lambda j | 1.00E-05 | 1.00E-02 |
| | learning rate | 4.56E-03 | 7.53E-03 |
| SLIM ElasticNet | topK | 718 | 1000 |
| | l1 ratio | 6.74E-03 | 9.65E-04 |
| | alpha | 0.0010 | 0.0010 |
| PureSVD | num factors | 25 | 51 |
| | estimate model for cold users | itemKNN | itemKNN |
| iALS | num factors | 54 | 78 |
| | confidence scaling | linear | log |
| | alpha | 11.0139 | 50.0000 |
| | epsilon | 10.0000 | 10.0000 |
| | reg | 1.00E-02 | 1.00E-05 |
| | estimate model for cold users | itemKNN | itemKNN |

Table 68. Hyperparameter values for the neural algorithm on all datasets.

| Algorithm | Hyperparameter | Movielens 20M | Netflix Prize |
|-----------|--------------------|---------------|---------------|
| Mult VAE | epochs | 95 | 80 |
| | batch size | 500 | 500 |
| | total anneal steps | 200000 | 200000 |
| | p dims | - | - |

J IJCAI: OUTER PRODUCT-BASED NEURAL COLLABORATIVE FILTERING

Relevant statistics on the dataset, which we mentioned in the paper, are reported in Table 69. The results of our evaluation can be seen in Table 70 (Gowalla) and Table 71 (Yelp). The corresponding optimal hyperparameters are reported in Table 74 (collaborative KNNs), Table 75 (non-neural machine learning and graph based) and Table 76 (ConvNCF).

Lastly, the time required to train and evaluate the models is reported in Table 72 (Gowalla) and Table 73 (Yelp).

Table 69. Dataset characteristics.

| Dataset | Interactions | Items | Users | Density |
|---------|--------------|-------|-------|---------|
| Yelp | 69K | 25815 | 25677 | 0.105 |
| Gowalla | 1249K | 52400 | 54156 | 0.044 |

Table 70. Experimental results for the ConvNCF method for the Gowalla dataset.

| | @ 5 | | @ 10 | | @ 20 | |
|-----------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| | HR | NDCG | HR | NDCG | HR | NDCG |
| Random | 0.0049 | 0.0029 | 0.0099 | 0.0045 | 0.0205 | 0.0071 |
| TopPopular | 0.2188 | 0.1652 | 0.2910 | 0.1884 | 0.3803 | 0.2110 |
| UserKNN CF cosine | 0.7131 | 0.5879 | 0.7939 | 0.6142 | 0.8532 | 0.6293 |
| UserKNN CF dice | 0.6848 | 0.5632 | 0.7649 | 0.5893 | 0.8226 | 0.6039 |
| UserKNN CF jaccard | 0.6786 | 0.5572 | 0.7597 | 0.5836 | 0.8174 | 0.5983 |
| UserKNN CF asymmetric | 0.6720 | 0.5486 | 0.7555 | 0.5758 | 0.8156 | 0.5911 |
| UserKNN CF tversky | 0.6769 | 0.5556 | 0.7579 | 0.5820 | 0.8149 | 0.5965 |
| ItemKNN CF cosine | 0.6806 | 0.5511 | 0.7668 | 0.5792 | 0.8257 | 0.5942 |
| ItemKNN CF dice | 0.6605 | 0.5231 | 0.7592 | 0.5552 | 0.8280 | 0.5728 |
| ItemKNN CF jaccard | 0.6890 | 0.5577 | 0.7752 | 0.5857 | 0.8306 | 0.5999 |
| ItemKNN CF asymmetric | 0.6953 | 0.5711 | 0.7762 | 0.5974 | 0.8332 | 0.6119 |
| ItemKNN CF tversky | 0.7047 | 0.5864 | 0.7790 | 0.6105 | 0.8331 | 0.6244 |
| $P^3\alpha$ | 0.6926 | 0.5703 | 0.7674 | 0.5948 | 0.8158 | 0.6071 |
| $RP^3\beta$ | 0.6836 | 0.5525 | 0.7723 | 0.5814 | 0.8361 | 0.5976 |
| EASE ^R | - | - | - | - | - | - |
| SLIM BPR | - | - | - | - | - | - |
| SLIM ElasticNet | 0.6365 | 0.5284 | 0.7083 | 0.5517 | 0.7608 | 0.5651 |
| MF BPR | 0.6376 | 0.4996 | 0.7416 | 0.5334 | 0.8234 | 0.5542 |
| MF FunkSVD | 0.6029 | 0.4592 | 0.7216 | 0.4979 | 0.8082 | 0.5199 |
| PureSVD | 0.5653 | 0.4482 | 0.6627 | 0.4798 | 0.7393 | 0.4993 |
| NMF | 0.5856 | 0.4607 | 0.6842 | 0.4927 | 0.7674 | 0.5138 |
| iALS | 0.6460 | 0.5081 | 0.7554 | 0.5436 | 0.8356 | 0.5641 |
| ConvNCF | 0.6702 | 0.5233 | 0.7799 | 0.5590 | 0.8623 | 0.5799 |

Table 71. Experimental results for the ConvNCF method for the Yelp dataset.

| | @ 5 | | @ 10 | | @ 20 | |
|-----------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| | HR | NDCG | HR | NDCG | HR | NDCG |
| Random | 0.0048 | 0.0030 | 0.0097 | 0.0045 | 0.0204 | 0.0072 |
| TopPopular | 0.0817 | 0.0538 | 0.1199 | 0.0661 | 0.1754 | 0.0800 |
| UserKNN CF cosine | 0.2068 | 0.1355 | 0.3126 | 0.1695 | 0.4401 | 0.2017 |
| UserKNN CF dice | 0.1994 | 0.1306 | 0.3014 | 0.1634 | 0.4271 | 0.1951 |
| UserKNN CF jaccard | 0.2006 | 0.1311 | 0.3023 | 0.1638 | 0.4286 | 0.1956 |
| UserKNN CF asymmetric | 0.2185 | 0.1441 | 0.3275 | 0.1792 | 0.4553 | 0.2115 |
| UserKNN CF tversky | 0.2046 | 0.1346 | 0.3049 | 0.1669 | 0.4320 | 0.1990 |
| ItemKNN CF cosine | 0.2521 | 0.1686 | 0.3669 | 0.2056 | 0.4974 | 0.2385 |
| ItemKNN CF dice | 0.2329 | 0.1564 | 0.3396 | 0.1908 | 0.4665 | 0.2228 |
| ItemKNN CF jaccard | 0.2414 | 0.1634 | 0.3512 | 0.1988 | 0.4786 | 0.2309 |
| ItemKNN CF asymmetric | 0.2421 | 0.1598 | 0.3514 | 0.1950 | 0.4815 | 0.2278 |
| ItemKNN CF tversky | 0.2303 | 0.1546 | 0.3346 | 0.1884 | 0.4563 | 0.2192 |
| $P^3\alpha$ | 0.2145 | 0.1394 | 0.3211 | 0.1738 | 0.4442 | 0.2049 |
| $RP^3\beta$ | 0.2202 | 0.1431 | 0.3323 | 0.1793 | 0.4667 | 0.2132 |
| EASE ^R | - | - | - | - | - | - |
| SLIM BPR | - | - | - | - | - | - |
| SLIM ElasticNet | 0.2330 | 0.1535 | 0.3475 | 0.1904 | 0.4799 | 0.2238 |
| MF BPR | 0.1557 | 0.1024 | 0.2421 | 0.1302 | 0.3599 | 0.1598 |
| MF FunkSVD | 0.1728 | 0.1121 | 0.2621 | 0.1409 | 0.3727 | 0.1688 |
| PureSVD | 0.2011 | 0.1307 | 0.3002 | 0.1626 | 0.4238 | 0.1938 |
| NMF | 0.1816 | 0.1172 | 0.2825 | 0.1496 | 0.4090 | 0.1815 |
| iALS | 0.2048 | 0.1348 | 0.3080 | 0.1680 | 0.4319 | 0.1993 |
| ConvNCF | 0.1947 | 0.1250 | 0.3059 | 0.1608 | 0.4446 | 0.1957 |

Table 72. Computation time for the algorithms in the selected results for the ConvNCF method on the Gowalla dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|-------------------------|-------------------------------------|----------------------------------|---------------------------|
| Random | 0.04 [sec] | 94.26 [sec] / 1.57 [min] | 575 |
| TopPopular | 0.08 [sec] | 112.67 [sec] / 1.88 [min] | 481 |
| UserKNN CF cosine | 24.25 ± 1.03 [sec] | 161.09 [sec] / 2.68 ± 0.07 [min] | 322 |
| UserKNN CF dice | 25.07 ± 0.93 [sec] | 161.77 [sec] / 2.70 ± 0.03 [min] | 332 |
| UserKNN CF jaccard | 25.07 ± 0.88 [sec] | 161.03 [sec] / 2.68 ± 0.08 [min] | 334 |
| UserKNN CF asymmetric | 25.75 ± 1.03 [sec] | 161.07 [sec] / 2.68 ± 0.09 [min] | 334 |
| UserKNN CF tversky | 25.62 ± 0.92 [sec] | 162.28 [sec] / 2.70 ± 0.03 [min] | 337 |
| ItemKNN CF cosine | 25.63 ± 1.65 [sec] | 162.49 [sec] / 2.71 ± 0.03 [min] | 338 |
| ItemKNN CF dice | 24.78 ± 1.52 [sec] | 166.14 [sec] / 2.77 ± 0.09 [min] | 339 |
| ItemKNN CF jaccard | 24.87 ± 1.55 [sec] | 163.76 [sec] / 2.73 ± 0.07 [min] | 341 |
| ItemKNN CF asymmetric | 25.52 ± 1.85 [sec] | 166.42 [sec] / 2.77 ± 0.08 [min] | 329 |
| ItemKNN CF tversky | 25.92 ± 1.72 [sec] | 164.91 [sec] / 2.75 ± 0.07 [min] | 324 |
| P ³ α | 90.96 [sec] / 1.52 ± 0.27 [min] | 156.74 [sec] / 2.61 ± 0.04 [min] | 348 |
| RP ³ β | 103.55 [sec] / 1.73 ± 0.31 [min] | 159.29 [sec] / 2.65 ± 0.01 [min] | 342 |
| EASE ^R | - | - | - |
| SLIM BPR | 24663.44 [sec] / 6.85 ± 4.36 [hour] | 165.90 [sec] / 2.76 ± 0.11 [min] | 316 |
| SLIM ElasticNet | 4531.84 [sec] / 1.26 ± 0.70 [hour] | 158.98 [sec] / 2.65 ± 0.17 [min] | 329 |
| MF BPR | 19828.95 [sec] / 5.51 ± 2.32 [hour] | 128.32 [sec] / 2.14 ± 0.09 [min] | 413 |
| MF FunkSVD | 14731.56 [sec] / 4.09 ± 2.15 [hour] | 136.27 [sec] / 2.27 ± 0.13 [min] | 383 |
| PureSVD | 11.12 ± 3.45 [sec] | 321.24 [sec] / 5.35 ± 0.60 [min] | 159 |
| NMF | 2368.87 [sec] / 39.48 ± 23.03 [min] | 251.74 [sec] / 4.20 ± 0.68 [min] | 195 |
| iALS | 2695.70 [sec] / 44.93 ± 26.66 [min] | 129.40 [sec] / 2.16 ± 0.05 [min] | 413 |
| ConvNCF | 44743.03 [sec] / 12.43 [hour] | 233.89 [sec] / 3.90 [min] | 232 |

Table 73. Computation time for the algorithms in the selected results for the ConvNCF method on the Yelp dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|-----------------------|---|-------------------------------------|---------------------------|
| Random | 0.02 [sec] | 44.30 [sec] | 580 |
| TopPopular | 0.03 [sec] | 49.17 [sec] | 522 |
| UserKNN CF cosine | 8.93 ± 0.65 [sec] | 74.11 [sec] / 1.24 ± 0.01 [min] | 348 |
| UserKNN CF dice | 9.26 ± 0.72 [sec] | 74.30 [sec] / 1.24 ± 0.01 [min] | 347 |
| UserKNN CF jaccard | 9.18 ± 0.69 [sec] | 73.27 [sec] / 1.22 ± 0.01 [min] | 348 |
| UserKNN CF asymmetric | 9.21 ± 0.96 [sec] | 74.33 [sec] / 1.24 ± 0.02 [min] | 346 |
| UserKNN CF tversky | 9.43 ± 0.70 [sec] | 74.67 [sec] / 1.24 ± 0.02 [min] | 350 |
| ItemKNN CF cosine | 8.04 ± 0.88 [sec] | 75.74 [sec] / 1.26 ± 0.04 [min] | 328 |
| ItemKNN CF dice | 7.95 ± 0.70 [sec] | 73.03 [sec] / 1.22 ± 0.04 [min] | 361 |
| ItemKNN CF jaccard | 7.95 ± 0.74 [sec] | 74.48 [sec] / 1.24 ± 0.04 [min] | 347 |
| ItemKNN CF asymmetric | 8.46 ± 0.79 [sec] | 77.62 [sec] / 1.29 ± 0.02 [min] | 336 |
| ItemKNN CF tversky | 8.13 ± 0.70 [sec] | 73.72 [sec] / 1.23 ± 0.03 [min] | 355 |
| $P^3\alpha$ | 33.85 ± 5.97 [sec] | 71.02 [sec] / 1.18 ± 0.01 [min] | 363 |
| $RP^3\beta$ | 37.25 ± 9.81 [sec] | 71.48 [sec] / 1.19 ± 0.04 [min] | 349 |
| EASE ^R | - | - | - |
| SLIM BPR | - | - | - |
| SLIM ElasticNet | 846.95 [sec] / 14.12 ± 6.92 [min] | 68.16 [sec] / 1.14 ± 0.07 [min] | 361 |
| MF BPR | 3981.91 [sec] / 1.11 ± 0.88 [hour] | 50.36 \pm 1.71 [sec] | 495 |
| MF FunkSVD | 3944.58 [sec] / 1.10 ± 0.85 [hour] | 51.88 \pm 1.46 [sec] | 501 |
| PureSVD | 2.35 ± 1.79 [sec] | 56.89 \pm 4.49 [sec] | 466 |
| NMF | 952.10 [sec] / 15.87 ± 6.43 [min] | 75.16 [sec] / 1.25 ± 0.47 [min] | 464 |
| iALS | 1778.07 [sec] / 29.63 ± 19.78 [min] | 47.35 \pm 1.33 [sec] | 530 |
| ConvNCF | 11465.29 [sec] / 3.18 [hour] | 102.80 [sec] / 1.71 [min] | 250 |

Table 74. Hyperparameter values for our collaborative KNN baselines on all datasets.

| Algorithm | Hyperparameter | Yelp | Gowalla |
|-----------------------|-------------------|------------|------------|
| UserKNN CF cosine | topK | 470 | 1000 |
| | shrink | 0 | 1000 |
| | similarity | cosine | cosine |
| | normalize | True | True |
| | feature weighting | none | TF-IDF |
| UserKNN CF dice | topK | 494 | 513 |
| | shrink | 0 | 10 |
| | similarity | dice | dice |
| | normalize | False | True |
| UserKNN CF jaccard | topK | 553 | 455 |
| | shrink | 2 | 5 |
| | similarity | jaccard | jaccard |
| | normalize | False | True |
| UserKNN CF asymmetric | topK | 529 | 451 |
| | shrink | 721 | 173 |
| | similarity | asymmetric | asymmetric |
| | normalize | True | True |
| | asymmetric alpha | 0.1781 | 0.6950 |
| | feature weighting | TF-IDF | TF-IDF |
| UserKNN CF tversky | topK | 474 | 368 |
| | shrink | 67 | 0 |
| | similarity | tversky | tversky |
| | normalize | True | True |
| | tversky alpha | 1.9756 | 1.3012 |
| | tversky beta | 1.9345 | 2.0000 |
| ItemKNN CF cosine | topK | 1000 | 317 |
| | shrink | 387 | 1000 |
| | similarity | cosine | cosine |
| | normalize | True | True |
| | feature weighting | TF-IDF | TF-IDF |
| ItemKNN CF dice | topK | 195 | 409 |
| | shrink | 10 | 20 |
| | similarity | dice | dice |
| | normalize | False | True |
| ItemKNN CF jaccard | topK | 479 | 302 |
| | shrink | 4 | 68 |
| | similarity | jaccard | jaccard |
| | normalize | True | True |
| ItemKNN CF asymmetric | topK | 918 | 712 |
| | shrink | 154 | 507 |
| | similarity | asymmetric | asymmetric |
| | normalize | True | True |
| | asymmetric alpha | 0.3530 | 0.2575 |
| | feature weighting | TF-IDF | TF-IDF |
| ItemKNN CF tversky | topK | 374 | 944 |
| | shrink | 0 | 16 |
| | similarity | tversky | tversky |
| | normalize | True | True |
| | tversky alpha | 0.7020 | 0.0558 |
| | tversky beta | 1.5460 | 1.9805 |

Table 75. Hyperparameter values for our non-neural machine learning and graph based baselines on all datasets.

| Algorithm | Hyperparameter | Yelp | Gowalla |
|-------------------|----------------------|------------------|------------------|
| $P^3\alpha$ | topK | 584 | 413 |
| | alpha | 0.3672 | 0.4424 |
| | normalize similarity | True | True |
| $RP^3\beta$ | topK | 1000 | 640 |
| | alpha | 0.5548 | 0.5168 |
| | beta | 0.3389 | 0.2009 |
| | normalize similarity | False | True |
| EASE ^R | - | - | - |
| SLIM ElasticNet | topK | 1000 | 916 |
| | l1 ratio | 2.66E-05 | 4.30E-04 |
| | alpha | 0.0520 | 0.0010 |
| MF BPR | sgd mode | adam | adam |
| | epochs | 1420 | 1485 |
| | num factors | 200 | 200 |
| | batch size | 32 | 16 |
| | positive reg | 1.00E-02 | 1.00E-02 |
| | negative reg | 1.00E-02 | 1.00E-02 |
| | learning rate | 5.70E-04 | 1.69E-03 |
| MF FunkSVD | sgd mode | adam | adam |
| | epochs | 365 | 410 |
| | use bias | True | True |
| | batch size | 32 | 4 |
| | num factors | 103 | 192 |
| | item reg | 1.43E-05 | 1.02E-04 |
| | user reg | 9.96E-03 | 2.88E-04 |
| | learning rate | 2.16E-03 | 1.41E-03 |
| | negative quota | 0.0642 | 0.1492 |
| PureSVD | num factors | 70 | 350 |
| NMF | num factors | 53 | 286 |
| | solver | mult. update | mult. update |
| | init type | random | nndsvda |
| | beta loss | kullback-leibler | kullback-leibler |
| iALS | num factors | 145 | 200 |
| | confidence scaling | log | log |
| | alpha | 7.3331 | 50.0000 |
| | epsilon | 0.0270 | 0.1846 |
| | reg | 4.50E-03 | 1.00E-02 |
| | epochs | 60 | 5 |
| SLIM BPR | topK | - | 795 |
| | epochs | - | 570 |
| | symmetric | - | False |
| | sgd mode | - | adagrad |
| | lambda i | - | 4.42E-04 |
| | lambda j | - | 2.16E-04 |
| | learning rate | - | 2.98E-03 |

Table 76. Hyperparameter values for the neural algorithm on all datasets.

| Algorithm | Hyperparameter | Yelp | Gowalla |
|-----------|---------------------------------|--------------------------|--------------------------|
| ConvNCF | batch size | 512 | 512 |
| | epochs | 145 | 445 |
| | epochs MFBPR | 480 | 490 |
| | embedding size | 64 | 64 |
| | hidden size | 128 | 128 |
| | negative sample per positive | 1 | 1 |
| | negative instances per positive | 4 | 4 |
| | regularization users items | 1.00E-02 | 1.00E-02 |
| | regularization weights | 10 | 10 |
| | regularization filter weights | 1 | 1 |
| | learning rate embeddings | 5.00E-02 | 5.00E-02 |
| | learning rate CNN | 5.00E-02 | 5.00E-02 |
| | channel size | [32, 32, 32, 32, 32, 32] | [32, 32, 32, 32, 32, 32] |
| | dropout | 0.0000 | 0.0000 |
| | epoch verbose | 1 | 1 |

K IJCAI: NEUREC: ON NONLINEAR TRANSFORMATION FOR PERSONALIZED RANKING

Relevant statistics on the dataset, which we mentioned in the paper, are reported in Table 77. The results of our evaluation can be seen in Table 78 (Movielens 1M), Table 79 (Movielens HetRec), Table 80 (Frappe) and Table 81 (FilmTrust). The corresponding optimal hyperparameters are reported in Table 90 (collaborative KNNs), Table 91 (non-neural machine learning and graph based) and Table 92 (NeuRec).

Lastly, the time required to train and evaluate the models is reported in Table 86 (Movielens 1M), Table 87 (Movielens HetRec), Table 88 (Frappe) and Table 89 (FilmTrust).

Table 77. Dataset characteristics.

| Dataset | Interactions | Items | Users | Density |
|------------------|--------------|-------|-------|---------|
| Movielens 1M | 1M | 3882 | 6039 | 4.25 |
| Movielens HetRec | 855K | 10109 | 2113 | 4.01 |
| Frappe | 19K | 4082 | 957 | 0.48 |
| FilmTrust | 35K | 2071 | 1508 | 1.14 |

Table 78. Experimental results for the NeuRec method for the Movielens 1M dataset.

| | @ 5 | | | | | @ 10 | | | | | @ 50 | | | | |
|-------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| | PREC | REC | MAP | NDGC | MRR | PREC | REC | MAP | NDGC | MRR | PREC | REC | MAP | NDGC | MRR |
| Random | 0.0090 | 0.0010 | 0.0044 | 0.0022 | 0.0213 | 0.0091 | 0.0024 | 0.0029 | 0.0035 | 0.0265 | 0.0091 | 0.0128 | 0.0016 | 0.0099 | 0.0353 |
| TopPopular | 0.2105 | 0.0402 | 0.1531 | 0.0689 | 0.3621 | 0.1832 | 0.0685 | 0.1168 | 0.0939 | 0.3793 | 0.1127 | 0.1985 | 0.0734 | 0.1732 | 0.3912 |
| UserKNN CF cosine | 0.4075 | 0.1034 | 0.3298 | 0.1626 | 0.6335 | 0.3468 | 0.1667 | 0.2616 | 0.2158 | 0.6441 | 0.1972 | 0.4022 | 0.1910 | 0.3583 | 0.6482 |
| UserKNN CF dice | 0.4189 | 0.1055 | 0.3415 | 0.1658 | 0.6368 | 0.3583 | 0.1714 | 0.2738 | 0.2210 | 0.6475 | 0.2051 | 0.4116 | 0.2015 | 0.3672 | 0.6517 |
| UserKNN CF jaccard | 0.4179 | 0.1050 | 0.3406 | 0.1653 | 0.6365 | 0.3578 | 0.1705 | 0.2730 | 0.2204 | 0.6474 | 0.2038 | 0.4102 | 0.1996 | 0.3662 | 0.6515 |
| UserKNN CF asymmetric | 0.4212 | 0.1065 | 0.3441 | 0.1674 | 0.6399 | 0.3617 | 0.1726 | 0.2774 | 0.2230 | 0.6509 | 0.2069 | 0.4146 | 0.2047 | 0.3704 | 0.6550 |
| UserKNN CF tversky | 0.4042 | 0.1024 | 0.3265 | 0.1611 | 0.6277 | 0.3410 | 0.1647 | 0.2571 | 0.2132 | 0.6386 | 0.1940 | 0.3968 | 0.1873 | 0.3536 | 0.6428 |
| ItemKNN CF cosine | 0.4002 | 0.0987 | 0.3237 | 0.1561 | 0.6137 | 0.3432 | 0.1585 | 0.2600 | 0.2074 | 0.6247 | 0.1968 | 0.3831 | 0.1872 | 0.3452 | 0.6297 |
| ItemKNN CF dice | 0.3709 | 0.0854 | 0.2951 | 0.1383 | 0.5714 | 0.3215 | 0.1406 | 0.2367 | 0.1862 | 0.5845 | 0.1894 | 0.3612 | 0.1690 | 0.3211 | 0.5902 |
| ItemKNN CF jaccard | 0.3747 | 0.0875 | 0.2982 | 0.1401 | 0.5751 | 0.3219 | 0.1407 | 0.2376 | 0.1869 | 0.5870 | 0.1869 | 0.3559 | 0.1676 | 0.3187 | 0.5928 |
| ItemKNN CF asymmetric | 0.3995 | 0.0984 | 0.3244 | 0.1563 | 0.6179 | 0.3452 | 0.1590 | 0.2618 | 0.2084 | 0.6293 | 0.1978 | 0.3865 | 0.1886 | 0.3474 | 0.6341 |
| ItemKNN CF tversky | 0.3718 | 0.0867 | 0.2998 | 0.1414 | 0.5878 | 0.3116 | 0.1359 | 0.2343 | 0.1846 | 0.5985 | 0.1750 | 0.3300 | 0.1582 | 0.3037 | 0.6041 |
| $P^{\frac{1}{2}}\alpha$ | 0.4041 | 0.1007 | 0.3286 | 0.1596 | 0.6250 | 0.3456 | 0.1627 | 0.2627 | 0.2121 | 0.6362 | 0.1988 | 0.3945 | 0.1919 | 0.3538 | 0.6410 |
| $RP^{\frac{1}{2}}\beta$ | 0.4080 | 0.1007 | 0.3325 | 0.1602 | 0.6260 | 0.3508 | 0.1639 | 0.2676 | 0.2137 | 0.6374 | 0.2012 | 0.3938 | 0.1949 | 0.3551 | 0.6420 |
| EASE ^R | 0.4360 | 0.1073 | 0.3608 | 0.1697 | 0.6475 | 0.3745 | 0.1731 | 0.2923 | 0.2259 | 0.6585 | 0.2208 | 0.4263 | 0.2190 | 0.3820 | 0.6624 |
| SLIM BPR | 0.3964 | 0.1034 | 0.3161 | 0.1606 | 0.6222 | 0.3358 | 0.1663 | 0.2494 | 0.2128 | 0.6335 | 0.1968 | 0.4048 | 0.1892 | 0.3568 | 0.6379 |
| SLIM ElasticNet | 0.4437 | 0.1106 | 0.3692 | 0.1749 | 0.6578 | 0.3813 | 0.1770 | 0.3003 | 0.2321 | 0.6679 | 0.2234 | 0.4333 | 0.2259 | 0.3902 | 0.6720 |
| MF BPR | 0.3576 | 0.0830 | 0.2812 | 0.1340 | 0.5628 | 0.3073 | 0.1384 | 0.2217 | 0.1807 | 0.5768 | 0.1828 | 0.3575 | 0.1593 | 0.3128 | 0.5825 |
| MF FunkSVD | 0.3936 | 0.0927 | 0.3154 | 0.1479 | 0.6000 | 0.3458 | 0.1555 | 0.2572 | 0.2014 | 0.6125 | 0.2090 | 0.4074 | 0.1926 | 0.3541 | 0.6176 |
| PureSVD | 0.4123 | 0.0987 | 0.3371 | 0.1586 | 0.6266 | 0.3575 | 0.1624 | 0.2722 | 0.2132 | 0.6380 | 0.2133 | 0.4089 | 0.2033 | 0.3651 | 0.6427 |
| NMF | 0.3811 | 0.0891 | 0.3017 | 0.1430 | 0.5817 | 0.3338 | 0.1499 | 0.2442 | 0.1948 | 0.5947 | 0.2070 | 0.4047 | 0.1872 | 0.3489 | 0.6005 |
| iALS | 0.4164 | 0.1036 | 0.3373 | 0.1635 | 0.6327 | 0.3628 | 0.1702 | 0.2743 | 0.2200 | 0.6443 | 0.2180 | 0.4265 | 0.2104 | 0.3774 | 0.6483 |
| INeuRec | 0.3280 | 0.0663 | 0.2554 | 0.1110 | 0.5003 | 0.2839 | 0.1094 | 0.2027 | 0.1500 | 0.5129 | 0.1755 | 0.3048 | 0.1397 | 0.2719 | 0.5206 |
| UNeuRec | 0.2098 | 0.0395 | 0.1560 | 0.0684 | 0.3663 | 0.1856 | 0.0688 | 0.1199 | 0.0944 | 0.3852 | 0.1143 | 0.2002 | 0.0750 | 0.1743 | 0.3968 |

Table 79. Experimental results for the NeuRec method for the HetRec dataset.

| | @ 5 | | | | | @ 10 | | | | | @ 50 | | | | |
|-----------------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| | PREC | REC | MAP | NDCG | MRR | PREC | REC | MAP | NDCG | MRR | PREC | REC | MAP | NDCG | MRR |
| Random | 0.0093 | 0.0005 | 0.0040 | 0.0011 | 0.0181 | 0.0093 | 0.0009 | 0.0027 | 0.0018 | 0.0233 | 0.0091 | 0.0048 | 0.0011 | 0.0051 | 0.0324 |
| TopPopular | 0.4556 | 0.0408 | 0.3850 | 0.0889 | 0.6264 | 0.4057 | 0.0712 | 0.3137 | 0.1237 | 0.6368 | 0.2632 | 0.2048 | 0.1768 | 0.2326 | 0.6406 |
| UserKNN CF cosine | 0.5632 | 0.0605 | 0.4977 | 0.1237 | 0.7420 | 0.4988 | 0.1001 | 0.4145 | 0.1677 | 0.7486 | 0.3131 | 0.2685 | 0.2413 | 0.3005 | 0.7517 |
| UserKNN CF dice | 0.5714 | 0.0614 | 0.5079 | 0.1250 | 0.7465 | 0.5087 | 0.1015 | 0.4279 | 0.1698 | 0.7525 | 0.3237 | 0.2736 | 0.2538 | 0.3066 | 0.7560 |
| UserKNN CF jaccard | 0.5692 | 0.0617 | 0.5063 | 0.1250 | 0.7461 | 0.5088 | 0.1016 | 0.4276 | 0.1699 | 0.7520 | 0.3233 | 0.2729 | 0.2533 | 0.3062 | 0.7554 |
| UserKNN CF asymmetric | 0.5729 | 0.0619 | 0.5097 | 0.1251 | 0.7449 | 0.5151 | 0.1012 | 0.4346 | 0.1702 | 0.7504 | 0.3283 | 0.2750 | 0.2597 | 0.3086 | 0.7537 |
| UserKNN CF tversky | 0.5670 | 0.0612 | 0.5039 | 0.1245 | 0.7474 | 0.5044 | 0.1014 | 0.4216 | 0.1693 | 0.7538 | 0.3195 | 0.2713 | 0.2487 | 0.3044 | 0.7571 |
| ItemKNN CF cosine | 0.5405 | 0.0528 | 0.4747 | 0.1119 | 0.7096 | 0.4750 | 0.0873 | 0.3922 | 0.1519 | 0.7157 | 0.2971 | 0.2427 | 0.2219 | 0.2759 | 0.7196 |
| ItemKNN CF dice | 0.5371 | 0.0513 | 0.4672 | 0.1098 | 0.6993 | 0.4741 | 0.0867 | 0.3880 | 0.1504 | 0.7062 | 0.2981 | 0.2425 | 0.2198 | 0.2754 | 0.7101 |
| ItemKNN CF jaccard | 0.5200 | 0.0504 | 0.4537 | 0.1078 | 0.6993 | 0.4560 | 0.0838 | 0.3711 | 0.1462 | 0.7063 | 0.2857 | 0.2316 | 0.2068 | 0.2647 | 0.7101 |
| ItemKNN CF asymmetric | 0.5676 | 0.0572 | 0.5041 | 0.1210 | 0.7426 | 0.4996 | 0.0941 | 0.4165 | 0.1635 | 0.7494 | 0.3185 | 0.2437 | 0.2439 | 0.2900 | 0.7520 |
| ItemKNN CF tversky | 0.5408 | 0.0539 | 0.4747 | 0.1150 | 0.7234 | 0.4791 | 0.0898 | 0.3923 | 0.1563 | 0.7295 | 0.3082 | 0.2522 | 0.2292 | 0.2863 | 0.7332 |
| $P^3\alpha$ | 0.5032 | 0.0519 | 0.4351 | 0.1070 | 0.6831 | 0.4501 | 0.0905 | 0.3592 | 0.1484 | 0.6923 | 0.2861 | 0.2481 | 0.2057 | 0.2714 | 0.6963 |
| $RP^3\beta$ | 0.5464 | 0.0558 | 0.4692 | 0.1159 | 0.7110 | 0.4970 | 0.0950 | 0.4013 | 0.1607 | 0.7172 | 0.2936 | 0.2246 | 0.2215 | 0.2727 | 0.7201 |
| EASE ^R | 0.6253 | 0.0662 | 0.5673 | 0.1358 | 0.7790 | 0.5610 | 0.1094 | 0.4865 | 0.1853 | 0.7837 | 0.3621 | 0.2949 | 0.3010 | 0.3355 | 0.7865 |
| SLIM BPR | 0.5196 | 0.0574 | 0.4383 | 0.1136 | 0.6929 | 0.4709 | 0.0989 | 0.3701 | 0.1581 | 0.6999 | 0.3050 | 0.2656 | 0.2258 | 0.2895 | 0.7030 |
| SLIM ElasticNet | 0.6283 | 0.0670 | 0.5732 | 0.1379 | 0.7879 | 0.5612 | 0.1103 | 0.4882 | 0.1874 | 0.7933 | 0.3549 | 0.2906 | 0.2958 | 0.3333 | 0.7959 |
| MF BPR | 0.4204 | 0.0360 | 0.3493 | 0.0811 | 0.6028 | 0.3750 | 0.0659 | 0.2805 | 0.1143 | 0.6138 | 0.2533 | 0.1983 | 0.1608 | 0.2215 | 0.6189 |
| MF FunkSVD | 0.4882 | 0.0447 | 0.4088 | 0.0947 | 0.6446 | 0.4520 | 0.0820 | 0.3525 | 0.1364 | 0.6538 | 0.3145 | 0.2541 | 0.2240 | 0.2730 | 0.6585 |
| PureSVD | 0.5977 | 0.0601 | 0.5364 | 0.1271 | 0.7524 | 0.5369 | 0.1010 | 0.4548 | 0.1746 | 0.7578 | 0.3521 | 0.2849 | 0.2784 | 0.3237 | 0.7609 |
| NMF | 0.5432 | 0.0570 | 0.4686 | 0.1180 | 0.7108 | 0.4892 | 0.0964 | 0.3941 | 0.1632 | 0.7165 | 0.3193 | 0.2675 | 0.2384 | 0.3023 | 0.7197 |
| iALS | 0.5900 | 0.0609 | 0.5253 | 0.1281 | 0.7542 | 0.5322 | 0.1039 | 0.4456 | 0.1770 | 0.7593 | 0.3464 | 0.2857 | 0.2715 | 0.3243 | 0.7620 |
| INeuRec | 0.5435 | 0.0489 | 0.4797 | 0.1076 | 0.7021 | 0.4884 | 0.0844 | 0.4047 | 0.1493 | 0.7094 | 0.3151 | 0.2402 | 0.2371 | 0.2769 | 0.7129 |
| UNeuRec | 0.4467 | 0.0397 | 0.3785 | 0.0877 | 0.6278 | 0.3973 | 0.0693 | 0.3077 | 0.1216 | 0.6365 | 0.2599 | 0.2029 | 0.1731 | 0.2299 | 0.6413 |

Table 80. Experimental results for the NeuRec method for the Frappe dataset.

| | @ 5 | | | | | @ 10 | | | | | @ 50 | | | | |
|-----------------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| | PREC | REC | MAP | NDCG | MRR | PREC | REC | MAP | NDCG | MRR | PREC | REC | MAP | NDCG | MRR |
| Random | 0.0017 | 0.0007 | 0.0009 | 0.0011 | 0.0047 | 0.0011 | 0.0018 | 0.0007 | 0.0016 | 0.0051 | 0.0013 | 0.0103 | 0.0010 | 0.0047 | 0.0073 |
| TopPopular | 0.1332 | 0.2010 | 0.1441 | 0.1842 | 0.3183 | 0.0920 | 0.2602 | 0.1411 | 0.2118 | 0.3302 | 0.0383 | 0.4325 | 0.1530 | 0.2721 | 0.3387 |
| UserKNN CF cosine | 0.1888 | 0.2243 | 0.1943 | 0.2213 | 0.3880 | 0.1339 | 0.2929 | 0.1845 | 0.2578 | 0.3980 | 0.0480 | 0.4635 | 0.1940 | 0.3207 | 0.4042 |
| UserKNN CF dice | 0.1810 | 0.2175 | 0.1899 | 0.2168 | 0.3821 | 0.1279 | 0.2873 | 0.1813 | 0.2532 | 0.3928 | 0.0467 | 0.4541 | 0.1906 | 0.3149 | 0.3991 |
| UserKNN CF jaccard | 0.1844 | 0.2248 | 0.1923 | 0.2203 | 0.3843 | 0.1291 | 0.2871 | 0.1823 | 0.2540 | 0.3937 | 0.0472 | 0.4568 | 0.1918 | 0.3167 | 0.4001 |
| UserKNN CF asymmetric | 0.1899 | 0.2267 | 0.1965 | 0.2232 | 0.3893 | 0.1349 | 0.2939 | 0.1873 | 0.2592 | 0.3979 | 0.0477 | 0.4564 | 0.1944 | 0.3193 | 0.4045 |
| UserKNN CF tversky | 0.1961 | 0.2225 | 0.2037 | 0.2231 | 0.3893 | 0.1401 | 0.2965 | 0.1912 | 0.2619 | 0.3996 | 0.0490 | 0.4613 | 0.1987 | 0.3233 | 0.4057 |
| ItemKNN CF cosine | 0.1947 | 0.2192 | 0.2032 | 0.2243 | 0.3980 | 0.1342 | 0.2780 | 0.1877 | 0.2564 | 0.4059 | 0.0484 | 0.4524 | 0.1965 | 0.3198 | 0.4120 |
| ItemKNN CF dice | 0.1779 | 0.2075 | 0.1856 | 0.2094 | 0.3743 | 0.1198 | 0.2643 | 0.1724 | 0.2390 | 0.3833 | 0.0453 | 0.4406 | 0.1800 | 0.3018 | 0.3911 |
| ItemKNN CF jaccard | 0.1785 | 0.2094 | 0.1855 | 0.2095 | 0.3726 | 0.1196 | 0.2669 | 0.1719 | 0.2389 | 0.3817 | 0.0452 | 0.4385 | 0.1792 | 0.3006 | 0.3891 |
| ItemKNN CF asymmetric | 0.1852 | 0.2261 | 0.1964 | 0.2236 | 0.3882 | 0.1263 | 0.2817 | 0.1839 | 0.2538 | 0.3970 | 0.0473 | 0.4554 | 0.1940 | 0.3178 | 0.4037 |
| ItemKNN CF tversky | 0.1777 | 0.2139 | 0.1840 | 0.2117 | 0.3762 | 0.1212 | 0.2698 | 0.1729 | 0.2414 | 0.3840 | 0.0462 | 0.4434 | 0.1814 | 0.3042 | 0.3912 |
| $P^3\alpha$ | 0.1933 | 0.2322 | 0.1939 | 0.2254 | 0.3838 | 0.1447 | 0.3050 | 0.1899 | 0.2664 | 0.3932 | 0.0511 | 0.4699 | 0.2029 | 0.3296 | 0.3984 |
| $RP^3\beta$ | 0.2059 | 0.2349 | 0.2084 | 0.2341 | 0.4074 | 0.1486 | 0.3067 | 0.1981 | 0.2742 | 0.4163 | 0.0507 | 0.4631 | 0.2084 | 0.3341 | 0.4208 |
| EASE ^R | 0.1978 | 0.2252 | 0.1997 | 0.2245 | 0.3877 | 0.1444 | 0.2927 | 0.1915 | 0.2630 | 0.3960 | 0.0509 | 0.4482 | 0.2022 | 0.3242 | 0.4010 |
| SLIM BPR | 0.1791 | 0.2231 | 0.1860 | 0.2167 | 0.3773 | 0.1272 | 0.2812 | 0.1771 | 0.2493 | 0.3865 | 0.0456 | 0.4464 | 0.1860 | 0.3091 | 0.3931 |
| SLIM ElasticNet | 0.2014 | 0.2299 | 0.2023 | 0.2285 | 0.3943 | 0.1464 | 0.3028 | 0.1938 | 0.2686 | 0.4030 | 0.0516 | 0.4665 | 0.2054 | 0.3312 | 0.4076 |
| MF BPR | 0.1209 | 0.1429 | 0.1045 | 0.1333 | 0.2456 | 0.0933 | 0.2101 | 0.1036 | 0.1664 | 0.2604 | 0.0396 | 0.3914 | 0.1174 | 0.2316 | 0.2701 |
| MF FunkSVD | 0.1601 | 0.2214 | 0.1600 | 0.2016 | 0.3431 | 0.1205 | 0.2973 | 0.1597 | 0.2410 | 0.3569 | 0.0467 | 0.4726 | 0.1769 | 0.3069 | 0.3624 |
| PureSVD | 0.1282 | 0.1753 | 0.1362 | 0.1687 | 0.3064 | 0.0983 | 0.2386 | 0.1356 | 0.2017 | 0.3200 | 0.0375 | 0.3993 | 0.1443 | 0.2558 | 0.3263 |
| NMF | 0.1439 | 0.1008 | 0.1310 | 0.1197 | 0.2378 | 0.1047 | 0.1389 | 0.1118 | 0.1436 | 0.2434 | 0.0336 | 0.2282 | 0.1088 | 0.1784 | 0.2492 |
| iALS | 0.1774 | 0.1804 | 0.1768 | 0.1918 | 0.3503 | 0.1335 | 0.2534 | 0.1690 | 0.2313 | 0.3599 | 0.0454 | 0.4016 | 0.1750 | 0.2867 | 0.3657 |
| INeuRec | 0.2117 | 0.2196 | 0.2073 | 0.2235 | 0.4045 | 0.1479 | 0.2862 | 0.1891 | 0.2603 | 0.4116 | 0.0478 | 0.4305 | 0.1946 | 0.3133 | 0.4161 |
| UNeuRec | 0.1679 | 0.2206 | 0.1788 | 0.2100 | 0.3650 | 0.1194 | 0.2964 | 0.1731 | 0.2470 | 0.3780 | 0.0402 | 0.4448 | 0.1785 | 0.2992 | 0.3852 |

Table 81. Experimental results for the NeuRec method for the FilmTrust dataset.

| | @ 5 | | | | | @ 10 | | | | | @ 50 | | | | |
|-----------------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| | PREC | REC | MAP | NDCG | MRR | PREC | REC | MAP | NDCG | MRR | PREC | REC | MAP | NDCG | MRR |
| Random | 0.0025 | 0.0018 | 0.0015 | 0.0019 | 0.0063 | 0.0026 | 0.0041 | 0.0012 | 0.0032 | 0.0081 | 0.0027 | 0.0217 | 0.0018 | 0.0095 | 0.0118 |
| TopPopular | 0.4200 | 0.4126 | 0.4393 | 0.4203 | 0.6145 | 0.3471 | 0.6351 | 0.4597 | 0.5450 | 0.6273 | 0.0916 | 0.8614 | 0.4954 | 0.6314 | 0.6316 |
| UserKNN CF cosine | 0.4349 | 0.4357 | 0.4735 | 0.4498 | 0.6496 | 0.3545 | 0.6357 | 0.4909 | 0.5677 | 0.6581 | 0.0921 | 0.8247 | 0.5202 | 0.6429 | 0.6615 |
| UserKNN CF dice | 0.4341 | 0.4339 | 0.4667 | 0.4457 | 0.6452 | 0.3512 | 0.6286 | 0.4822 | 0.5604 | 0.6526 | 0.0917 | 0.8241 | 0.5135 | 0.6382 | 0.6561 |
| UserKNN CF jaccard | 0.4354 | 0.4400 | 0.4713 | 0.4510 | 0.6492 | 0.3560 | 0.6455 | 0.4907 | 0.5712 | 0.6579 | 0.0932 | 0.8524 | 0.5229 | 0.6517 | 0.6620 |
| UserKNN CF asymmetric | 0.4322 | 0.4292 | 0.4656 | 0.4431 | 0.6431 | 0.3517 | 0.6309 | 0.4825 | 0.5608 | 0.6515 | 0.0919 | 0.8257 | 0.5139 | 0.6386 | 0.6554 |
| UserKNN CF tversky | 0.4338 | 0.4335 | 0.4646 | 0.4436 | 0.6434 | 0.3519 | 0.6328 | 0.4805 | 0.5601 | 0.6515 | 0.0918 | 0.8267 | 0.5116 | 0.6372 | 0.6545 |
| ItemKNN CF cosine | 0.4268 | 0.4247 | 0.4551 | 0.4354 | 0.6332 | 0.3484 | 0.6303 | 0.4724 | 0.5540 | 0.6432 | 0.0916 | 0.8478 | 0.5072 | 0.6380 | 0.6470 |
| ItemKNN CF dice | 0.4264 | 0.4205 | 0.4559 | 0.4355 | 0.6388 | 0.3507 | 0.6338 | 0.4753 | 0.5577 | 0.6492 | 0.0917 | 0.8473 | 0.5083 | 0.6394 | 0.6527 |
| ItemKNN CF jaccard | 0.4259 | 0.4202 | 0.4561 | 0.4358 | 0.6407 | 0.3511 | 0.6344 | 0.4758 | 0.5585 | 0.6512 | 0.0917 | 0.8473 | 0.5086 | 0.6398 | 0.6548 |
| ItemKNN CF asymmetric | 0.4286 | 0.4238 | 0.4564 | 0.4360 | 0.6348 | 0.3490 | 0.6303 | 0.4734 | 0.5548 | 0.6447 | 0.0923 | 0.8516 | 0.5081 | 0.6400 | 0.6487 |
| ItemKNN CF tversky | 0.4262 | 0.4208 | 0.4566 | 0.4365 | 0.6413 | 0.3509 | 0.6342 | 0.4763 | 0.5587 | 0.6513 | 0.0917 | 0.8473 | 0.5092 | 0.6402 | 0.6549 |
| $P^3\alpha$ | 0.4240 | 0.4199 | 0.4526 | 0.4321 | 0.6351 | 0.3500 | 0.6343 | 0.4719 | 0.5550 | 0.6467 | 0.0938 | 0.8605 | 0.5080 | 0.6430 | 0.6511 |
| $RP^3\beta$ | 0.4373 | 0.4365 | 0.4709 | 0.4492 | 0.6537 | 0.3575 | 0.6436 | 0.4880 | 0.5701 | 0.6631 | 0.0950 | 0.8647 | 0.5228 | 0.6562 | 0.6674 |
| EASE ^R | 0.4371 | 0.4390 | 0.4745 | 0.4528 | 0.6569 | 0.3551 | 0.6443 | 0.4915 | 0.5721 | 0.6662 | 0.0923 | 0.8511 | 0.5236 | 0.6520 | 0.6699 |
| SLIM BPR | 0.4327 | 0.4351 | 0.4643 | 0.4455 | 0.6465 | 0.3522 | 0.6399 | 0.4825 | 0.5647 | 0.6549 | 0.0922 | 0.8529 | 0.5166 | 0.6472 | 0.6589 |
| SLIM ElasticNet | 0.4418 | 0.4417 | 0.4803 | 0.4572 | 0.6600 | 0.3583 | 0.6566 | 0.4983 | 0.5796 | 0.6708 | 0.0944 | 0.8631 | 0.5309 | 0.6604 | 0.6742 |
| MF BPR | 0.4115 | 0.4047 | 0.4309 | 0.4114 | 0.5979 | 0.3433 | 0.6156 | 0.4519 | 0.5330 | 0.6088 | 0.0902 | 0.8433 | 0.4877 | 0.6193 | 0.6138 |
| MF FunkSVD | 0.4112 | 0.4004 | 0.4148 | 0.3972 | 0.5781 | 0.3452 | 0.6265 | 0.4378 | 0.5245 | 0.5917 | 0.0906 | 0.8486 | 0.4731 | 0.6095 | 0.5957 |
| PureSVD | 0.4292 | 0.4255 | 0.4563 | 0.4366 | 0.6366 | 0.3478 | 0.6255 | 0.4724 | 0.5526 | 0.6453 | 0.0912 | 0.8301 | 0.5041 | 0.6322 | 0.6490 |
| NMF | 0.2721 | 0.2407 | 0.2769 | 0.2584 | 0.4131 | 0.1983 | 0.3332 | 0.2443 | 0.3123 | 0.4234 | 0.0684 | 0.5484 | 0.2744 | 0.3968 | 0.4315 |
| iALS | 0.4038 | 0.3855 | 0.4240 | 0.4028 | 0.6021 | 0.3342 | 0.5920 | 0.4400 | 0.5201 | 0.6137 | 0.0889 | 0.8124 | 0.4720 | 0.6043 | 0.6193 |
| INeuRec | 0.4221 | 0.4089 | 0.4398 | 0.4196 | 0.6151 | 0.3466 | 0.6187 | 0.4577 | 0.5398 | 0.6261 | 0.0918 | 0.8556 | 0.4935 | 0.6285 | 0.6310 |
| UNeuRec | 0.4174 | 0.4062 | 0.4384 | 0.4181 | 0.6157 | 0.3472 | 0.6291 | 0.4596 | 0.5436 | 0.6286 | 0.0912 | 0.8570 | 0.4952 | 0.6304 | 0.6337 |

Table 82. Experimental results for the NeuRec method for the Movielens 1M dataset on beyond accuracy metrics.

| | @ 50 | | | | |
|-----------------------|---------------|---------------|---------------|---------------|-----------------|
| | Div. MIL | Div. HHI | Cov. Item | Div. Gini | Div. Shannon |
| Random | 0.9871 | 0.9997 | 1.0000 | 0.9293 | 11.9110 |
| TopPopular | 0.3793 | 0.9876 | 0.0605 | 0.0193 | 6.5336 |
| UserKNN CF cosine | 0.8454 | 0.9969 | 0.5093 | 0.1010 | 9.0133 |
| UserKNN CF dice | 0.8565 | 0.9971 | 0.5234 | 0.1128 | 9.1618 |
| UserKNN CF jaccard | 0.8469 | 0.9969 | 0.4882 | 0.1044 | 9.0536 |
| UserKNN CF asymmetric | 0.8607 | 0.9972 | 0.5260 | 0.1151 | 9.1941 |
| UserKNN CF tversky | 0.8373 | 0.9967 | 0.5201 | 0.0969 | 8.9490 |
| ItemKNN CF cosine | 0.8360 | 0.9967 | 0.4626 | 0.0938 | 8.9078 |
| ItemKNN CF dice | 0.8164 | 0.9963 | 0.4408 | 0.0863 | 8.7731 |
| ItemKNN CF jaccard | 0.7971 | 0.9959 | 0.4238 | 0.0780 | 8.6252 |
| ItemKNN CF asymmetric | 0.8364 | 0.9967 | 0.4621 | 0.0941 | 8.9124 |
| ItemKNN CF tversky | 0.6940 | 0.9939 | 0.3408 | 0.0495 | 7.9711 |
| $P^3\alpha$ | 0.8404 | 0.9968 | 0.3310 | 0.0894 | 8.8568 |
| $RP^3\beta$ | 0.8508 | 0.9970 | 0.3794 | 0.0983 | 8.9848 |
| EASE ^R | 0.8576 | 0.9971 | 0.3936 | 0.1013 | 9.0385 |
| SLIM BPR | 0.7860 | 0.9957 | 0.3776 | 0.0755 | 8.5778 |
| SLIM ElasticNet | 0.8549 | 0.9971 | 0.4250 | 0.1046 | 9.0716 |
| MF BPR | 0.8322 | 0.9966 | 0.4333 | 0.0943 | 8.9084 |
| MF FunkSVD | 0.9003 | 0.9980 | 0.3815 | 0.1326 | 9.4299 |
| PureSVD | 0.8831 | 0.9977 | 0.3591 | 0.1147 | 9.2240 |
| NMF | 0.9074 | 0.9981 | 0.4869 | 0.1563 | 9.6559 |
| iALS | 0.8958 | 0.9979 | 0.4173 | 0.1323 | 9.4256 |
| INeuRec | 0.8304 | 0.9966 | 0.4060 | 0.0866 | 8.8077 |
| UNeuRec | 0.4011 | 0.9880 | 0.0618 | 0.0200 | 6.6111 |

Table 83. Experimental results for the NeuRec method for the HetRec dataset on beyond accuracy metrics.

| | Div. MIL | Div. HHI | @ 50 Cov. Item | Div. Gini | Div. Shannon |
|-----------------------|---------------|---------------|----------------------|---------------|-----------------|
| Random | 0.9950 | 0.9999 | 1.0000 | 0.8238 | 13.2303 |
| TopPopular | 0.5873 | 0.9917 | 0.0253 | 0.0108 | 7.1171 |
| UserKNN CF cosine | 0.7525 | 0.9950 | 0.1607 | 0.0224 | 8.1707 |
| UserKNN CF dice | 0.7852 | 0.9957 | 0.1751 | 0.0265 | 8.4347 |
| UserKNN CF jaccard | 0.7819 | 0.9956 | 0.1696 | 0.0259 | 8.4063 |
| UserKNN CF asymmetric | 0.8071 | 0.9961 | 0.2263 | 0.0325 | 8.6742 |
| UserKNN CF tversky | 0.7736 | 0.9955 | 0.1677 | 0.0247 | 8.3338 |
| ItemKNN CF cosine | 0.6692 | 0.9934 | 0.1161 | 0.0151 | 7.5890 |
| ItemKNN CF dice | 0.7040 | 0.9941 | 0.1484 | 0.0183 | 7.8382 |
| ItemKNN CF jaccard | 0.6585 | 0.9932 | 0.1265 | 0.0148 | 7.5297 |
| ItemKNN CF asymmetric | 0.8973 | 0.9979 | 0.3058 | 0.0573 | 9.5589 |
| ItemKNN CF tversky | 0.7197 | 0.9944 | 0.1248 | 0.0181 | 7.8899 |
| $P^3\alpha$ | 0.6586 | 0.9932 | 0.0718 | 0.0140 | 7.5316 |
| $RP^3\beta$ | 0.8703 | 0.9974 | 0.3079 | 0.0530 | 9.3710 |
| EASE ^R | 0.8180 | 0.9964 | 0.1384 | 0.0297 | 8.6441 |
| SLIM BPR | 0.7416 | 0.9948 | 0.0905 | 0.0187 | 7.9807 |
| SLIM ElasticNet | 0.7745 | 0.9955 | 0.1269 | 0.0234 | 8.2955 |
| MF BPR | 0.5737 | 0.9915 | 0.0299 | 0.0105 | 7.0806 |
| MF FunkSVD | 0.8430 | 0.9969 | 0.1419 | 0.0338 | 8.8440 |
| PureSVD | 0.8548 | 0.9971 | 0.1862 | 0.0395 | 9.0429 |
| NMF | 0.8670 | 0.9973 | 0.3345 | 0.0515 | 9.3067 |
| iALS | 0.8663 | 0.9973 | 0.2039 | 0.0433 | 9.1722 |
| INeuRec | 0.7490 | 0.9950 | 0.1390 | 0.0237 | 8.2440 |
| UNeuRec | 0.5835 | 0.9917 | 0.0255 | 0.0108 | 7.1117 |

Table 84. Experimental results for the NeuRec method for the Frappe dataset on beyond accuracy metrics.

| | Div. MIL | Div. HHI | @ 50 Cov. Item | Div. Gini | Div. Shannon |
|-----------------------|---------------|---------------|----------------------|---------------|-----------------|
| Random | 0.9877 | 0.9997 | 0.9998 | 0.8101 | 11.9098 |
| TopPopular | 0.1825 | 0.9836 | 0.0233 | 0.0141 | 6.0351 |
| UserKNN CF cosine | 0.5542 | 0.9911 | 0.2295 | 0.0334 | 7.4110 |
| UserKNN CF dice | 0.5709 | 0.9914 | 0.2589 | 0.0371 | 7.5221 |
| UserKNN CF jaccard | 0.5660 | 0.9913 | 0.2545 | 0.0367 | 7.5062 |
| UserKNN CF asymmetric | 0.4628 | 0.9892 | 0.1509 | 0.0250 | 7.0258 |
| UserKNN CF tversky | 0.5498 | 0.9910 | 0.2266 | 0.0338 | 7.4207 |
| ItemKNN CF cosine | 0.4824 | 0.9896 | 0.1987 | 0.0278 | 7.1419 |
| ItemKNN CF dice | 0.3896 | 0.9878 | 0.1646 | 0.0223 | 6.8019 |
| ItemKNN CF jaccard | 0.3909 | 0.9878 | 0.1651 | 0.0223 | 6.8052 |
| ItemKNN CF asymmetric | 0.4302 | 0.9886 | 0.1617 | 0.0238 | 6.9264 |
| ItemKNN CF tversky | 0.4034 | 0.9881 | 0.1867 | 0.0236 | 6.8639 |
| $P^3\alpha$ | 0.6171 | 0.9923 | 0.2947 | 0.0434 | 7.7499 |
| $RP^3\beta$ | 0.5919 | 0.9918 | 0.4130 | 0.0519 | 7.7600 |
| EASE ^R | 0.6617 | 0.9932 | 0.2237 | 0.0424 | 7.8208 |
| SLIM BPR | 0.5038 | 0.9901 | 0.2464 | 0.0318 | 7.2674 |
| SLIM ElasticNet | 0.5825 | 0.9916 | 0.2097 | 0.0348 | 7.5115 |
| MF BPR | 0.6772 | 0.9935 | 0.4682 | 0.0646 | 8.1158 |
| MF FunkSVD | 0.6645 | 0.9933 | 0.5470 | 0.0935 | 8.4578 |
| PureSVD | 0.2378 | 0.9847 | 0.0517 | 0.0156 | 6.2599 |
| NMF | 0.8099 | 0.9962 | 0.3555 | 0.0848 | 8.7953 |
| iALS | 0.7219 | 0.9944 | 0.1889 | 0.0458 | 7.9647 |
| INeuRec | 0.7558 | 0.9951 | 0.2197 | 0.0542 | 8.2077 |
| UNeuRec | 0.3286 | 0.9866 | 0.0345 | 0.0167 | 6.4323 |

Table 85. Experimental results for the NeuRec method for the FilmTrust dataset on beyond accuracy metrics.

| | Div. MIL | Div. HHI | @ 50 Cov. Item | Div. Gini | Div. Shannon |
|-----------------------|---------------|---------------|----------------------|---------------|-----------------|
| Random | 0.9758 | 0.9995 | 1.0000 | 0.8975 | 10.9915 |
| TopPopular | 0.3783 | 0.9876 | 0.0526 | 0.0353 | 6.4165 |
| UserKNN CF cosine | 0.6027 | 0.9920 | 0.6321 | 0.0823 | 7.6004 |
| UserKNN CF dice | 0.5818 | 0.9916 | 0.6089 | 0.0777 | 7.4987 |
| UserKNN CF jaccard | 0.5538 | 0.9911 | 0.5171 | 0.0643 | 7.3065 |
| UserKNN CF asymmetric | 0.5782 | 0.9916 | 0.4674 | 0.0664 | 7.3958 |
| UserKNN CF tversky | 0.5794 | 0.9916 | 0.5963 | 0.0754 | 7.4756 |
| ItemKNN CF cosine | 0.4224 | 0.9884 | 0.2245 | 0.0393 | 6.6506 |
| ItemKNN CF dice | 0.4262 | 0.9885 | 0.2501 | 0.0398 | 6.6709 |
| ItemKNN CF jaccard | 0.4269 | 0.9885 | 0.2685 | 0.0401 | 6.6779 |
| ItemKNN CF asymmetric | 0.4266 | 0.9885 | 0.2139 | 0.0396 | 6.6682 |
| ItemKNN CF tversky | 0.4248 | 0.9885 | 0.2438 | 0.0396 | 6.6627 |
| $P^3\alpha$ | 0.5451 | 0.9909 | 0.5886 | 0.0696 | 7.3812 |
| $RP^3\beta$ | 0.5782 | 0.9916 | 0.7682 | 0.0974 | 7.6630 |
| $EASE^R$ | 0.4722 | 0.9894 | 0.2926 | 0.0454 | 6.9094 |
| SLIM BPR | 0.4616 | 0.9892 | 0.3317 | 0.0445 | 6.8299 |
| SLIM ElasticNet | 0.5671 | 0.9913 | 0.4684 | 0.0626 | 7.3467 |
| MF BPR | 0.5428 | 0.9908 | 0.7846 | 0.0860 | 7.5281 |
| MF FunkSVD | 0.4010 | 0.9880 | 0.0758 | 0.0368 | 6.5373 |
| PureSVD | 0.6616 | 0.9932 | 0.2603 | 0.0756 | 7.6955 |
| NMF | 0.8670 | 0.9973 | 0.7088 | 0.2127 | 9.1850 |
| iALS | 0.6957 | 0.9939 | 0.2984 | 0.0808 | 7.8120 |
| INeuRec | 0.4314 | 0.9886 | 0.1845 | 0.0403 | 6.7224 |
| UNeuRec | 0.4502 | 0.9890 | 0.0671 | 0.0398 | 6.6356 |

Table 86. Computation time for the algorithms in the selected results for the NeuRec method on the Movielens 1M dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|-----------------------|---|------------------------|---------------------------|
| Random | 0.02 [sec] | 52.96 [sec] | 114 |
| TopPopular | 0.03 [sec] | 51.76 [sec] | 117 |
| UserKNN CF cosine | 3.50 ± 0.15 [sec] | 54.05 ± 0.34 [sec] | 112 |
| UserKNN CF dice | 3.32 ± 0.24 [sec] | 53.36 ± 0.64 [sec] | 115 |
| UserKNN CF jaccard | 3.30 ± 0.22 [sec] | 53.55 ± 0.68 [sec] | 114 |
| UserKNN CF asymmetric | 3.35 ± 0.25 [sec] | 53.54 ± 0.31 [sec] | 113 |
| UserKNN CF tversky | 3.38 ± 0.27 [sec] | 53.32 ± 0.90 [sec] | 112 |
| ItemKNN CF cosine | 1.58 ± 0.14 [sec] | 54.81 ± 1.12 [sec] | 111 |
| ItemKNN CF dice | 1.53 ± 0.12 [sec] | 54.57 ± 1.19 [sec] | 114 |
| ItemKNN CF jaccard | 1.51 ± 0.13 [sec] | 53.87 ± 1.13 [sec] | 114 |
| ItemKNN CF asymmetric | 1.52 ± 0.17 [sec] | 53.16 ± 1.05 [sec] | 112 |
| ItemKNN CF tversky | 1.52 ± 0.15 [sec] | 53.93 ± 0.54 [sec] | 113 |
| $P^3\alpha$ | 3.88 ± 1.58 [sec] | 52.34 ± 0.77 [sec] | 116 |
| $RP^3\beta$ | 4.49 ± 1.98 [sec] | 51.69 ± 0.88 [sec] | 117 |
| EASE ^R | 7.50 ± 0.04 [sec] | 53.57 ± 0.02 [sec] | 113 |
| SLIM BPR | 540.48 [sec] / 9.01 ± 9.45 [min] | 53.73 ± 1.12 [sec] | 110 |
| SLIM ElasticNet | 157.83 [sec] / 2.63 ± 1.60 [min] | 51.36 ± 2.17 [sec] | 112 |
| MF BPR | 403.10 [sec] / 6.72 ± 5.21 [min] | 51.98 ± 0.87 [sec] | 114 |
| MF FunkSVD | 1471.34 [sec] / 24.52 ± 36.83 [min] | 51.81 ± 0.17 [sec] | 116 |
| PureSVD | 0.70 ± 0.66 [sec] | 52.60 ± 0.15 [sec] | 114 |
| NMF | 295.91 [sec] / 4.93 ± 3.69 [min] | 52.88 ± 0.17 [sec] | 114 |
| iALS | 350.79 [sec] / 5.85 ± 5.04 [min] | 52.84 ± 0.05 [sec] | 114 |
| INeuRec | 71409.67 [sec] / 19.84 [hour] | 42.31 [sec] | 143 |
| UNeuRec | 57989.25 [sec] / 16.11 [hour] | 42.22 [sec] | 143 |

Table 87. Computation time for the algorithms in the selected results for the NeuRec method on the HetRec dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|-----------------------|--------------------------------------|------------------------|---------------------------|
| Random | 0.02 [sec] | 41.87 [sec] | 50 |
| TopPopular | 0.03 [sec] | 40.99 [sec] | 52 |
| UserKNN CF cosine | 1.12 ± 0.05 [sec] | 42.23 ± 1.16 [sec] | 49 |
| UserKNN CF dice | 1.15 ± 0.07 [sec] | 42.47 ± 0.44 [sec] | 50 |
| UserKNN CF jaccard | 1.15 ± 0.07 [sec] | 42.41 ± 0.49 [sec] | 50 |
| UserKNN CF asymmetric | 1.16 ± 0.08 [sec] | 42.10 ± 1.37 [sec] | 50 |
| UserKNN CF tversky | 1.16 ± 0.08 [sec] | 42.46 ± 0.46 [sec] | 50 |
| ItemKNN CF cosine | 4.34 ± 0.52 [sec] | 45.85 ± 1.56 [sec] | 47 |
| ItemKNN CF dice | 4.43 ± 0.59 [sec] | 43.91 ± 1.37 [sec] | 49 |
| ItemKNN CF jaccard | 4.38 ± 0.61 [sec] | 43.30 ± 0.70 [sec] | 49 |
| ItemKNN CF asymmetric | 4.14 ± 0.55 [sec] | 43.05 ± 3.72 [sec] | 52 |
| ItemKNN CF tversky | 4.37 ± 0.54 [sec] | 43.20 ± 2.21 [sec] | 51 |
| $P^3\alpha$ | 16.00 ± 5.00 [sec] | 41.18 ± 0.26 [sec] | 51 |
| $RP^3\beta$ | 13.93 ± 6.45 [sec] | 41.57 ± 0.99 [sec] | 52 |
| EASE ^R | 86.06 [sec] / 1.43 ± 0.00 [min] | 44.61 ± 0.23 [sec] | 47 |
| SLIM BPR | 265.23 [sec] / 4.42 ± 3.41 [min] | 43.10 ± 0.64 [sec] | 48 |
| SLIM ElasticNet | 310.27 [sec] / 5.17 ± 2.36 [min] | 42.49 ± 1.28 [sec] | 49 |
| MF BPR | 80.12 [sec] / 1.34 ± 1.07 [min] | 42.17 ± 0.28 [sec] | 50 |
| MF FunkSVD | 208.95 [sec] / 3.48 ± 5.74 [min] | 42.47 ± 0.20 [sec] | 50 |
| PureSVD | 0.59 ± 0.40 [sec] | 42.15 ± 0.04 [sec] | 50 |
| NMF | 326.21 [sec] / 5.44 ± 4.32 [min] | 42.71 ± 0.84 [sec] | 50 |
| iALS | 328.16 [sec] / 5.47 ± 5.00 [min] | 42.58 ± 0.05 [sec] | 50 |
| INeuRec | 50152.19 [sec] / 13.93 [hour] | 35.26 [sec] | 60 |
| UNeuRec | 74545.39 [sec] / 20.71 [hour] | 39.53 [sec] | 53 |

Table 88. Computation time for the algorithms in the selected results for the NeuRec method on the Frappe dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|-----------------------|--------------------------------------|-----------------------|---------------------------|
| Random | 0.00 [sec] | 5.78 [sec] | 124 |
| TopPopular | 0.00 [sec] | 4.94 [sec] | 145 |
| UserKNN CF cosine | 0.06 ± 0.01 [sec] | 5.07 ± 0.03 [sec] | 141 |
| UserKNN CF dice | 0.06 ± 0.01 [sec] | 5.13 ± 0.03 [sec] | 141 |
| UserKNN CF jaccard | 0.06 ± 0.01 [sec] | 5.09 ± 0.03 [sec] | 141 |
| UserKNN CF asymmetric | 0.06 ± 0.01 [sec] | 5.09 ± 0.01 [sec] | 140 |
| UserKNN CF tversky | 0.06 ± 0.01 [sec] | 5.07 ± 0.03 [sec] | 141 |
| ItemKNN CF cosine | 0.24 ± 0.01 [sec] | 5.00 ± 0.02 [sec] | 143 |
| ItemKNN CF dice | 0.24 ± 0.01 [sec] | 4.98 ± 0.01 [sec] | 143 |
| ItemKNN CF jaccard | 0.24 ± 0.01 [sec] | 4.98 ± 0.01 [sec] | 144 |
| ItemKNN CF asymmetric | 0.24 ± 0.01 [sec] | 4.99 ± 0.02 [sec] | 143 |
| ItemKNN CF tversky | 0.25 ± 0.01 [sec] | 4.99 ± 0.00 [sec] | 144 |
| $P^3\alpha$ | 0.84 ± 0.13 [sec] | 4.93 ± 0.03 [sec] | 145 |
| $RP^3\beta$ | 0.88 ± 0.19 [sec] | 4.96 ± 0.03 [sec] | 144 |
| EASE ^R | 6.22 ± 0.04 [sec] | 5.00 ± 0.01 [sec] | 143 |
| SLIM BPR | 22.86 ± 9.88 [sec] | 4.96 ± 0.05 [sec] | 144 |
| SLIM ElasticNet | 26.27 ± 3.82 [sec] | 5.01 ± 0.01 [sec] | 143 |
| MF BPR | 21.62 ± 41.61 [sec] | 5.01 ± 0.03 [sec] | 143 |
| MF FunkSVD | 39.35 ± 32.47 [sec] | 5.05 ± 0.03 [sec] | 141 |
| PureSVD | 0.06 ± 0.09 [sec] | 4.93 ± 0.02 [sec] | 145 |
| NMF | 24.63 ± 19.89 [sec] | 5.01 ± 0.04 [sec] | 144 |
| iALS | 138.27 [sec] / 2.30 ± 2.19 [min] | 5.17 ± 0.18 [sec] | 142 |
| INeuRec | 825.96 [sec] / 13.77 [min] | 4.13 [sec] | 174 |
| UNeuRec | 356.02 [sec] / 5.93 [min] | 4.12 [sec] | 174 |

Table 89. Computation time for the algorithms in the selected results for the NeuRec method on the FilmTrust dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|-----------------------|---------------------------|-----------------------|---------------------------|
| Random | 0.00 [sec] | 6.43 [sec] | 197 |
| TopPopular | 0.00 [sec] | 6.21 [sec] | 204 |
| UserKNN CF cosine | 0.19 ± 0.03 [sec] | 6.41 ± 0.07 [sec] | 196 |
| UserKNN CF dice | 0.20 ± 0.04 [sec] | 6.47 ± 0.05 [sec] | 195 |
| UserKNN CF jaccard | 0.20 ± 0.03 [sec] | 6.46 ± 0.02 [sec] | 197 |
| UserKNN CF asymmetric | 0.19 ± 0.04 [sec] | 6.47 ± 0.06 [sec] | 196 |
| UserKNN CF tversky | 0.20 ± 0.04 [sec] | 6.49 ± 0.01 [sec] | 195 |
| ItemKNN CF cosine | 0.10 ± 0.01 [sec] | 6.33 ± 0.05 [sec] | 200 |
| ItemKNN CF dice | 0.11 ± 0.01 [sec] | 6.35 ± 0.00 [sec] | 199 |
| ItemKNN CF jaccard | 0.10 ± 0.01 [sec] | 6.33 ± 0.02 [sec] | 200 |
| ItemKNN CF asymmetric | 0.11 ± 0.01 [sec] | 6.33 ± 0.05 [sec] | 199 |
| ItemKNN CF tversky | 0.11 ± 0.01 [sec] | 6.36 ± 0.00 [sec] | 199 |
| $P^3\alpha$ | 0.55 ± 0.11 [sec] | 6.31 ± 0.04 [sec] | 202 |
| $RP^3\beta$ | 0.58 ± 0.13 [sec] | 6.31 ± 0.03 [sec] | 201 |
| EASE ^R | 0.97 ± 0.03 [sec] | 6.26 ± 0.07 [sec] | 201 |
| SLIM BPR | 14.96 ± 9.29 [sec] | 6.34 ± 0.06 [sec] | 199 |
| SLIM ElasticNet | 9.53 ± 1.99 [sec] | 6.31 ± 0.04 [sec] | 201 |
| MF BPR | 38.98 ± 45.37 [sec] | 6.23 ± 0.14 [sec] | 200 |
| MF FunkSVD | 31.97 ± 39.84 [sec] | 6.26 ± 0.02 [sec] | 202 |
| PureSVD | 0.05 ± 0.06 [sec] | 6.15 ± 0.14 [sec] | 202 |
| NMF | 32.20 ± 35.82 [sec] | 6.32 ± 0.17 [sec] | 197 |
| iALS | 58.77 ± 37.14 [sec] | 6.25 ± 0.05 [sec] | 201 |
| INeuRec | 424.72 [sec] / 7.08 [min] | 5.07 [sec] | 250 |
| UNeuRec | 230.86 [sec] / 3.85 [min] | 5.11 [sec] | 248 |

Table 90. Hyperparameter values for our collaborative KNN baselines on all datasets.

| Algorithm | Hyperparameter | Frappe | Filmtrust | Movielens 1M | Hetrec |
|-----------------------|-------------------|------------|------------|--------------|------------|
| UserKNN CF cosine | topK | 235 | 490 | 565 | 609 |
| | shrink | 3 | 921 | 749 | 0 |
| | similarity | cosine | cosine | cosine | cosine |
| | normalize | True | True | True | True |
| | feature weighting | TF-IDF | TF-IDF | TF-IDF | none |
| UserKNN CF dice | topK | 224 | 593 | 182 | 277 |
| | shrink | 0 | 336 | 4 | 1 |
| | similarity | dice | dice | dice | dice |
| | normalize | False | True | False | False |
| UserKNN CF jaccard | topK | 237 | 510 | 244 | 290 |
| | shrink | 0 | 5 | 7 | 0 |
| | similarity | jaccard | jaccard | jaccard | jaccard |
| | normalize | False | False | True | False |
| UserKNN CF asymmetric | topK | 398 | 594 | 193 | 279 |
| | shrink | 0 | 1000 | 348 | 306 |
| | similarity | asymmetric | asymmetric | asymmetric | asymmetric |
| | normalize | True | True | True | True |
| | asymmetric alpha | 0.0000 | 2.0000 | 0.0744 | 0.3358 |
| UserKNN CF tfidf | feature weighting | TF-IDF | TF-IDF | TF-IDF | TF-IDF |
| UserKNN CF tversky | topK | 309 | 598 | 580 | 358 |
| | shrink | 0 | 32 | 169 | 18 |
| | similarity | tversky | tversky | tversky | tversky |
| | normalize | True | True | True | True |
| | tversky alpha | 0.0000 | 0.0113 | 2.0000 | 1.9153 |
| | tversky beta | 2.0000 | 0.2846 | 2.0000 | 1.5318 |
| ItemKNN CF cosine | topK | 1000 | 320 | 307 | 264 |
| | shrink | 524 | 497 | 1 | 1000 |
| | similarity | cosine | cosine | cosine | cosine |
| | normalize | True | False | True | True |
| | feature weighting | TF-IDF | TF-IDF | TF-IDF | TF-IDF |
| ItemKNN CF dice | topK | 973 | 676 | 99 | 103 |
| | shrink | 273 | 746 | 82 | 226 |
| | similarity | dice | dice | dice | dice |
| | normalize | True | True | True | True |
| ItemKNN CF jaccard | topK | 763 | 301 | 112 | 87 |
| | shrink | 304 | 734 | 59 | 974 |
| | similarity | jaccard | jaccard | jaccard | jaccard |
| | normalize | True | True | True | True |
| ItemKNN CF asymmetric | topK | 779 | 1000 | 258 | 5 |
| | shrink | 1000 | 1000 | 0 | 1000 |
| | similarity | asymmetric | asymmetric | asymmetric | asymmetric |
| | normalize | True | True | True | True |
| | asymmetric alpha | 0.2985 | 0.0000 | 0.4750 | 0.0000 |
| ItemKNN CF tfidf | feature weighting | TF-IDF | TF-IDF | TF-IDF | TF-IDF |
| ItemKNN CF tversky | topK | 258 | 1000 | 331 | 30 |
| | shrink | 196 | 859 | 0 | 394 |
| | similarity | tversky | tversky | tversky | tversky |
| | normalize | True | True | True | True |
| | tversky alpha | 0.3885 | 0.9712 | 0.9659 | 0.7213 |
| | tversky beta | 1.6358 | 0.6517 | 2.0000 | 1.9521 |

Table 91. Hyperparameter values for our non-neural machine learning and graph based baselines on all datasets.

| Algorithm | Hyperparameter | Frappe | Filmtrust | Movielens 1M | Hetrec |
|-----------------|----------------------|----------------|--------------|------------------|----------------|
| $P^3\alpha$ | topK | 347 | 270 | 162 | 857 |
| | alpha | 0.6960 | 1.5154 | 1.4534 | 1.8765 |
| | normalize similarity | False | True | True | True |
| $RP^3\beta$ | topK | 640 | 263 | 109 | 5 |
| | alpha | 0.6964 | 1.0001 | 1.1222 | 0.6525 |
| | beta | 0.2177 | 0.4673 | 0.1090 | 0.3863 |
| | normalize similarity | False | True | True | False |
| $EASE^R$ | l2 norm | 2.72E+02 | 2.37E+03 | 2.31E+03 | 1.77E+03 |
| SLIM BPR | topK | 1000 | 1000 | 1000 | 1000 |
| | epochs | 25 | 35 | 195 | 160 |
| | symmetric | True | True | True | True |
| | sgd mode | adagrad | adagrad | sgd | sgd |
| | lambda i | 1.00E-02 | 1.00E-02 | 1.00E-02 | 1.00E-02 |
| | lambda j | 1.00E-02 | 1.00E-05 | 1.00E-02 | 1.00E-02 |
| | learning rate | 1.00E-04 | 3.60E-02 | 8.77E-03 | 1.55E-02 |
| SLIM ElasticNet | topK | 671 | 458 | 659 | 344 |
| | l1 ratio | 1.13E-05 | 1.00E-05 | 1.19E-03 | 3.19E-03 |
| | alpha | 0.3182 | 0.1082 | 0.1394 | 0.5223 |
| MF BPR | sgd mode | adagrad | adam | adagrad | adagrad |
| | epochs | 95 | 135 | 1005 | 400 |
| | num factors | 200 | 104 | 200 | 1 |
| | batch size | 256 | 2 | 16 | 8 |
| | positive reg | 3.97E-03 | 4.85E-05 | 2.45E-03 | 1.00E-05 |
| | negative reg | 1.00E-02 | 1.00E-05 | 1.00E-02 | 1.00E-02 |
| | learning rate | 8.84E-02 | 4.67E-04 | 3.37E-02 | 4.08E-02 |
| MF FunkSVD | sgd mode | adagrad | adam | adam | adam |
| | epochs | 170 | 70 | 420 | 65 |
| | use bias | True | False | False | True |
| | batch size | 1 | 8 | 128 | 1024 |
| | num factors | 200 | 1 | 32 | 63 |
| | item reg | 1.00E-05 | 1.00E-02 | 1.05E-05 | 1.22E-04 |
| | user reg | 1.00E-02 | 1.00E-05 | 7.21E-04 | 9.95E-04 |
| | learning rate | 1.00E-01 | 1.31E-03 | 1.95E-04 | 3.29E-04 |
| | negative quota | 0.5000 | 0.1059 | 0.1103 | 0.1179 |
| PureSVD | num factors | 1 | 5 | 30 | 43 |
| NMF | num factors | 42 | 208 | 62 | 139 |
| | solver | coord. descent | mult. update | mult. update | coord. descent |
| | init type | nndsvda | nndsvda | random | nndsvda |
| | beta loss | frobenius | frobenius | kullback-leibler | frobenius |
| iALS | num factors | 22 | 10 | 35 | 61 |
| | confidence scaling | log | linear | log | linear |
| | alpha | 0.0045 | 0.3831 | 0.8507 | 0.0094 |
| | epsilon | 0.1795 | 0.0984 | 0.3278 | 9.7751 |
| | reg | 1.45E-03 | 1.38E-03 | 1.41E-03 | 1.50E-05 |
| | epochs | 5 | 15 | 60 | 60 |

Table 92. Hyperparameter values for the neural algorithm on all datasets.

| Algorithm | Hyperparameter | Frappe | Filmtrust | Movielens 1M | Hetrec |
|-----------|---------------------|----------|-----------|--------------|----------|
| INeuRec | num neurons | 300 | 150 | 300 | 300 |
| | num factors | 50 | 40 | 50 | 50 |
| | dropout percentage | 0.0300 | 0.0000 | 0.0300 | 0.0300 |
| | learning rate | 1.00E-04 | 5.00E-05 | 1.00E-04 | 1.00E-04 |
| | regularization rate | 1.00E-02 | 1.00E-01 | 1.00E-01 | 1.00E-01 |
| | epochs | 50 | 25 | 5 | 10 |
| | batch size | 1024 | 1024 | 1024 | 1024 |
| | display epoch | - | - | - | - |
| | display step | - | - | - | - |
| | verbose | True | True | True | True |
| UNeuRec | num neurons | 300 | 150 | 300 | 300 |
| | num factors | 50 | 40 | 50 | 50 |
| | dropout percentage | 0.0300 | 0.0000 | 0.0300 | 0.0300 |
| | learning rate | 1.00E-04 | 5.00E-05 | 1.00E-04 | 1.00E-04 |
| | regularization rate | 1.00E-02 | 1.00E-01 | 1.00E-01 | 1.00E-01 |
| | epochs | 5 | 5 | 50 | 145 |
| | batch size | 1024 | 1024 | 1024 | 1024 |
| | display epoch | - | - | - | - |
| | display step | - | - | - | - |
| | verbose | True | True | True | True |

L IJCAI: DEEP MATRIX FACTORIZATION MODELS FOR RECOMMENDER SYSTEMS

Relevant statistics on the dataset, which we mentioned in the paper, are reported in Table 93. The results of our evaluation can be seen in Table 94 (Amazon Music original), Table 95 (Amazon Music ours), Table 96 (Amazon Movie), Table 97 (Movielens 100k) and Table 98 (Movielens 1M). The corresponding optimal hyperparameters are reported in Table 104 (collaborative KNNs), Table 105 (non-neural machine learning and graph based) and Table 106 (NeuRec).

Lastly, the time required to train and evaluate the models is reported in Table 99 (Amazon Music original), Table 100 (Amazon Music ours), Table 101 (Amazon Movie), Table 102 (Movielens 100k) and Table 103 (Movielens 1M).

Table 93. Dataset characteristics.

| Dataset | Interactions | Items | Users | Density |
|-----------------------|--------------|-------|-------|---------|
| Amazon Music original | 46.5K | 18813 | 844 | 0.293 |
| Amazon Music ours | 37K | 23184 | 844 | 0.189 |
| Amazon Movie | 878K | 83512 | 15067 | 0.070 |
| Movielens 100k | 100K | 1682 | 943 | 6.305 |
| Movielens 1M | 1M | 3706 | 6040 | 4.468 |

Table 94. Experimental results for the DMF method for the Amazon Music original dataset.

| | @ 10 | |
|-----------------------|---------------|---------------|
| | HR | NDCG |
| Random | 0.0972 | 0.0466 |
| TopPopular | 0.4372 | 0.2489 |
| UserKNN CF cosine | 0.5509 | 0.3955 |
| UserKNN CF dice | 0.5462 | 0.3873 |
| UserKNN CF jaccard | 0.5498 | 0.3874 |
| UserKNN CF asymmetric | 0.5509 | 0.3950 |
| UserKNN CF tversky | 0.5427 | 0.3870 |
| ItemKNN CF cosine | 0.5450 | 0.3944 |
| ItemKNN CF dice | 0.5308 | 0.3804 |
| ItemKNN CF jaccard | 0.5284 | 0.3790 |
| ItemKNN CF asymmetric | 0.5415 | 0.3907 |
| ItemKNN CF tversky | 0.5284 | 0.3781 |
| $P^3\alpha$ | 0.5509 | 0.3975 |
| $RP^3\beta$ | 0.5498 | 0.3972 |
| EASE ^R | 0.5284 | 0.3892 |
| SLIM BPR | 0.5403 | 0.3771 |
| SLIM ElasticNet | 0.5355 | 0.3922 |
| MF BPR | 0.4443 | 0.3139 |
| MF FunkSVD | 0.4479 | 0.3220 |
| PureSVD | 0.4502 | 0.3386 |
| NMF | 0.5130 | 0.3557 |
| iALS | 0.5533 | 0.4033 |
| DMF NCE | 0.3863 | 0.2773 |
| DMF BCE | 0.5415 | 0.3846 |

Table 95. Experimental results for the DMF method for the Amazon Music ours dataset.

| | @ 10 | |
|-----------------------|---------------|---------------|
| | HR | NDCG |
| Random | 0.1126 | 0.0526 |
| TopPopular | 0.5308 | 0.3037 |
| UserKNN CF cosine | 0.6694 | 0.4798 |
| UserKNN CF dice | 0.6576 | 0.4740 |
| UserKNN CF jaccard | 0.6564 | 0.4731 |
| UserKNN CF asymmetric | 0.6730 | 0.4813 |
| UserKNN CF tversky | 0.6517 | 0.4754 |
| ItemKNN CF cosine | 0.6647 | 0.4880 |
| ItemKNN CF dice | 0.6540 | 0.4698 |
| ItemKNN CF jaccard | 0.6576 | 0.4721 |
| ItemKNN CF asymmetric | 0.6647 | 0.4853 |
| ItemKNN CF tversky | 0.6481 | 0.4665 |
| $P^3\alpha$ | 0.6588 | 0.4823 |
| $RP^3\beta$ | 0.6754 | 0.4912 |
| EASE ^R | - | - |
| SLIM BPR | 0.6694 | 0.4720 |
| SLIM ElasticNet | 0.6469 | 0.4744 |
| MF BPR | 0.5367 | 0.3689 |
| MF FunkSVD | 0.5474 | 0.3870 |
| PureSVD | 0.5912 | 0.4190 |
| NMF | 0.6540 | 0.4486 |
| iALS | 0.6600 | 0.4880 |
| DMF NCE | 0.4799 | 0.3371 |
| DMF BCE | 0.6671 | 0.4819 |

Table 96. Experimental results for the DMF method for the Amazon Movie dataset.

| | @ 10 | |
|-----------------------|---------------|---------------|
| | HR | NDCG |
| Random | 0.1007 | 0.0457 |
| TopPopular | 0.5794 | 0.3489 |
| UserKNN CF cosine | 0.7327 | 0.5132 |
| UserKNN CF dice | 0.7064 | 0.4963 |
| UserKNN CF jaccard | 0.7066 | 0.4968 |
| UserKNN CF asymmetric | 0.7325 | 0.5132 |
| UserKNN CF tversky | 0.7033 | 0.4952 |
| ItemKNN CF cosine | 0.6983 | 0.4967 |
| ItemKNN CF dice | 0.6947 | 0.4868 |
| ItemKNN CF jaccard | 0.6972 | 0.4902 |
| ItemKNN CF asymmetric | 0.6986 | 0.4914 |
| ItemKNN CF tversky | 0.6660 | 0.4789 |
| $P^3\alpha$ | 0.6972 | 0.5028 |
| $RP^3\beta$ | 0.7107 | 0.5078 |
| EASE ^R | - | - |
| SLIM BPR | - | - |
| SLIM ElasticNet | 0.6981 | 0.5005 |
| MF BPR | 0.6422 | 0.4161 |
| MF FunkSVD | 0.5972 | 0.4091 |
| PureSVD | 0.6021 | 0.4156 |
| NMF | 0.6252 | 0.4217 |
| iALS | 0.7352 | 0.5230 |
| DMF NCE | 0.6832 | 0.4677 |
| DMF BCE | 0.7816 | 0.5417 |

Table 97. Experimental results for the DMF method for the Movielens 100k dataset.

| | @ 10 | |
|-----------------------|---------------|---------------|
| | HR | NDCG |
| Random | 0.0914 | 0.0416 |
| TopPopular | 0.4145 | 0.2342 |
| UserKNN CF cosine | 0.5834 | 0.3400 |
| UserKNN CF dice | 0.5834 | 0.3325 |
| UserKNN CF jaccard | 0.5760 | 0.3335 |
| UserKNN CF asymmetric | 0.5994 | 0.3492 |
| UserKNN CF tversky | 0.5802 | 0.3382 |
| ItemKNN CF cosine | 0.5781 | 0.3363 |
| ItemKNN CF dice | 0.5962 | 0.3484 |
| ItemKNN CF jaccard | 0.5834 | 0.3422 |
| ItemKNN CF asymmetric | 0.5855 | 0.3416 |
| ItemKNN CF tversky | 0.6026 | 0.3506 |
| $P^3\alpha$ | 0.5717 | 0.3421 |
| $RP^3\beta$ | 0.5685 | 0.3270 |
| EASE ^R | 0.6111 | 0.3591 |
| SLIM BPR | 0.6206 | 0.3578 |
| SLIM ElasticNet | 0.6238 | 0.3765 |
| MF BPR | 0.5951 | 0.3365 |
| MF FunkSVD | 0.5707 | 0.3354 |
| PureSVD | 0.5877 | 0.3555 |
| NMF | 0.5855 | 0.3515 |
| iALS | 0.6142 | 0.3691 |
| DMF NCE | 0.5930 | 0.3410 |
| DMF BCE | 0.6026 | 0.3623 |

Table 98. Experimental results for the DMF method for the Movielens 1M dataset.

| | @ 10 | |
|-----------------------|---------------|---------------|
| | HR | NDCG |
| Random | 0.1052 | 0.0472 |
| TopPop | 0.4418 | 0.2475 |
| UserKNN CF cosine | 0.6293 | 0.3766 |
| UserKNN CF dice | 0.6324 | 0.3822 |
| UserKNN CF jaccard | 0.6323 | 0.3828 |
| UserKNN CF asymmetric | 0.6324 | 0.3779 |
| UserKNN CF tversky | 0.6362 | 0.3840 |
| ItemKNN CF cosine | 0.6347 | 0.3808 |
| ItemKNN CF dice | 0.6293 | 0.3692 |
| ItemKNN CF jaccard | 0.6255 | 0.3682 |
| ItemKNN CF asymmetric | 0.6190 | 0.3704 |
| ItemKNN CF tversky | 0.6326 | 0.3757 |
| P3alpha | 0.6097 | 0.3639 |
| RP3beta | 0.6304 | 0.3726 |
| EASE R | 0.6691 | 0.4093 |
| SLIM BPR | 0.6719 | 0.4068 |
| SLIM ElasticNet | 0.6825 | 0.4209 |
| MF BPR | 0.6323 | 0.3729 |
| MF FunkSVD | 0.6499 | 0.3912 |
| PureSVD | 0.6570 | 0.4015 |
| NMF | 0.6422 | 0.3862 |
| IALS | 0.6947 | 0.4257 |
| DMF NCE | 0.6266 | 0.3768 |
| DMF BCE | 0.6731 | 0.4033 |

Table 99. Computation time for the algorithms in the selected results for the DMF method on the Amazon Music original dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|-----------------------|--------------------------------------|-----------------------|---------------------------|
| Random | 0.00 [sec] | 1.36 [sec] | 621 |
| TopPopular | 0.00 [sec] | 1.53 [sec] | 551 |
| UserKNN CF cosine | 0.05 ± 0.02 [sec] | 2.28 ± 0.01 [sec] | 368 |
| UserKNN CF dice | 0.05 ± 0.00 [sec] | 2.26 ± 0.03 [sec] | 378 |
| UserKNN CF jaccard | 0.05 ± 0.00 [sec] | 2.27 ± 0.02 [sec] | 376 |
| UserKNN CF asymmetric | 0.05 ± 0.00 [sec] | 2.26 ± 0.04 [sec] | 370 |
| UserKNN CF tversky | 0.05 ± 0.00 [sec] | 2.27 ± 0.03 [sec] | 380 |
| ItemKNN CF cosine | 3.33 ± 0.41 [sec] | 2.36 ± 0.01 [sec] | 357 |
| ItemKNN CF dice | 3.45 ± 0.15 [sec] | 2.28 ± 0.01 [sec] | 369 |
| ItemKNN CF jaccard | 3.46 ± 0.13 [sec] | 2.31 ± 0.04 [sec] | 360 |
| ItemKNN CF asymmetric | 3.55 ± 0.15 [sec] | 2.32 ± 0.05 [sec] | 358 |
| ItemKNN CF tversky | 3.53 ± 0.17 [sec] | 2.29 ± 0.04 [sec] | 370 |
| $P^3\alpha$ | 14.26 ± 1.70 [sec] | 2.31 ± 0.02 [sec] | 363 |
| $RP^3\beta$ | 14.36 ± 2.21 [sec] | 2.30 ± 0.06 [sec] | 362 |
| EASE ^R | 125.05 [sec] / 2.08 ± 0.01 [min] | 2.15 ± 0.01 [sec] | 392 |
| SLIM BPR | 304.44 [sec] / 5.07 ± 2.50 [min] | 2.18 ± 0.01 [sec] | 387 |
| SLIM ElasticNet | 410.27 [sec] / 6.84 ± 5.29 [min] | 2.14 ± 0.03 [sec] | 388 |
| MF BPR | 77.68 [sec] / 1.29 ± 0.90 [min] | 1.58 ± 0.03 [sec] | 527 |
| MF FunkSVD | 212.46 [sec] / 3.54 ± 3.48 [min] | 1.65 ± 0.05 [sec] | 497 |
| PureSVD | 0.39 ± 0.34 [sec] | 1.60 ± 0.03 [sec] | 532 |
| NMF | 50.60 ± 46.44 [sec] | 1.77 ± 0.10 [sec] | 508 |
| iALS | 441.76 [sec] / 7.36 ± 5.91 [min] | 1.57 ± 0.01 [sec] | 535 |
| DMF NCE | 1549.96 [sec] / 25.83 [min] | 10.86 [sec] | 78 |
| DMF BCE | 2902.64 [sec] / 48.38 [min] | 8.04 [sec] | 105 |

Table 100. Computation time for the algorithms in the selected results for the DMF method on the Amazon Music ours dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|-----------------------|--|-----------------------|---------------------------|
| Random | 0.00 [sec] | 1.29 [sec] | 653 |
| TopPopular | 0.00 [sec] | 1.69 [sec] | 498 |
| UserKNN CF cosine | 0.05 ± 0.00 [sec] | 2.37 ± 0.01 [sec] | 355 |
| UserKNN CF dice | 0.04 ± 0.00 [sec] | 2.34 ± 0.02 [sec] | 361 |
| UserKNN CF jaccard | 0.04 ± 0.00 [sec] | 2.35 ± 0.01 [sec] | 361 |
| UserKNN CF asymmetric | 0.04 ± 0.00 [sec] | 2.36 ± 0.03 [sec] | 354 |
| UserKNN CF tversky | 0.04 ± 0.00 [sec] | 2.35 ± 0.03 [sec] | 363 |
| ItemKNN CF cosine | 3.65 ± 0.65 [sec] | 2.41 ± 0.03 [sec] | 347 |
| ItemKNN CF dice | 4.01 ± 0.08 [sec] | 2.40 ± 0.02 [sec] | 351 |
| ItemKNN CF jaccard | 4.01 ± 0.06 [sec] | 2.40 ± 0.01 [sec] | 353 |
| ItemKNN CF asymmetric | 4.07 ± 0.09 [sec] | 2.41 ± 0.01 [sec] | 349 |
| ItemKNN CF tversky | 4.13 ± 0.07 [sec] | 2.39 ± 0.05 [sec] | 351 |
| $P^3\alpha$ | 13.21 ± 0.96 [sec] | 2.38 ± 0.02 [sec] | 357 |
| $RP^3\beta$ | 14.29 ± 1.10 [sec] | 2.41 ± 0.02 [sec] | 349 |
| EASE ^R | - | - | - |
| SLIM BPR | 342.16 [sec] / 5.70 ± 3.42 [min] | 2.41 ± 0.02 [sec] | 347 |
| SLIM ElasticNet | 1400.37 [sec] / 23.34 ± 2.14 [min] | 2.43 ± 0.01 [sec] | 345 |
| MF BPR | 90.35 [sec] / 1.51 ± 1.43 [min] | 1.73 ± 0.02 [sec] | 484 |
| MF FunkSVD | 146.22 [sec] / 2.44 ± 1.95 [min] | 1.75 ± 0.01 [sec] | 481 |
| PureSVD | 0.41 ± 0.48 [sec] | 1.76 ± 0.07 [sec] | 492 |
| NMF | 67.64 [sec] / 1.13 ± 0.96 [min] | 1.91 ± 0.10 [sec] | 453 |
| iALS | 184.95 [sec] / 3.08 ± 2.43 [min] | 1.72 ± 0.00 [sec] | 490 |
| DMF NCE | 952.84 [sec] / 15.88 [min] | 11.40 [sec] | 74 |
| DMF BCE | 2183.37 [sec] / 36.39 [min] | 8.43 [sec] | 100 |

Table 101. Computation time for the algorithms in the selected results for the DMF method on the Amazon Movie dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|-----------------------|---|-------------------------------------|---------------------------|
| Random | 0.03 [sec] | 28.58 [sec] | 527 |
| TopPopular | 0.06 [sec] | 43.04 [sec] | 350 |
| UserKNN CF cosine | 4.67 ± 0.53 [sec] | 63.92 [sec] / 1.07 ± 0.05 [min] | 225 |
| UserKNN CF dice | 4.57 ± 0.52 [sec] | 63.27 [sec] / 1.05 ± 0.06 [min] | 228 |
| UserKNN CF jaccard | 4.59 ± 0.48 [sec] | 64.06 [sec] / 1.07 ± 0.06 [min] | 228 |
| UserKNN CF asymmetric | 4.72 ± 0.49 [sec] | 66.17 [sec] / 1.10 ± 0.02 [min] | 225 |
| UserKNN CF tversky | 4.63 ± 0.40 [sec] | 63.04 [sec] / 1.05 ± 0.05 [min] | 232 |
| ItemKNN CF cosine | 55.00 ± 5.26 [sec] | 70.23 [sec] / 1.17 ± 0.08 [min] | 203 |
| ItemKNN CF dice | 56.26 ± 1.87 [sec] | 68.45 [sec] / 1.14 ± 0.08 [min] | 216 |
| ItemKNN CF jaccard | 56.34 ± 1.87 [sec] | 69.12 [sec] / 1.15 ± 0.05 [min] | 226 |
| ItemKNN CF asymmetric | 56.82 ± 1.89 [sec] | 71.95 [sec] / 1.20 ± 0.04 [min] | 206 |
| ItemKNN CF tversky | 57.14 ± 2.12 [sec] | 62.86 [sec] / 1.05 ± 0.05 [min] | 239 |
| $P^3\alpha$ | 196.62 [sec] / 3.28 ± 0.32 [min] | 59.93 \pm 1.25 [sec] | 249 |
| $RP^3\beta$ | 214.41 [sec] / 3.57 ± 0.31 [min] | 60.62 [sec] / 1.01 ± 0.04 [min] | 239 |
| EASE ^R | - | - | - |
| SLIM BPR | - | - | - |
| SLIM ElasticNet | 17335.50 [sec] / 4.82 ± 1.00 [hour] | 64.46 [sec] / 1.07 ± 0.11 [min] | 211 |
| MF BPR | 5985.66 [sec] / 1.66 ± 0.93 [hour] | 43.42 \pm 0.22 [sec] | 345 |
| MF FunkSVD | 6606.42 [sec] / 1.84 ± 1.22 [hour] | 47.53 \pm 2.25 [sec] | 309 |
| PureSVD | 4.39 \pm 3.54 [sec] | 45.12 \pm 1.44 [sec] | 343 |
| NMF | 2030.98 [sec] / 33.85 ± 20.16 [min] | 45.26 [sec] | 333 |
| iALS | 3384.98 [sec] / 56.42 ± 45.83 [min] | 43.14 \pm 0.15 [sec] | 350 |
| DMF NCE | 70909.11 [sec] / 19.70 [hour] | 833.82 [sec] / 13.90 [min] | 18 |
| DMF BCE | 72580.90 [sec] / 20.16 [hour] | 380.28 [sec] / 6.34 [min] | 40 |

Table 102. Computation time for the algorithms in the selected results for the DMF method on the Movielens 100k dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|-----------------------|--|-----------------------|---------------------------|
| Random | 0.00 [sec] | 1.33 [sec] | 706 |
| TopPopular | 0.00 [sec] | 1.39 [sec] | 676 |
| UserKNN CF cosine | 0.14 ± 0.02 [sec] | 2.09 ± 0.04 [sec] | 448 |
| UserKNN CF dice | 0.14 ± 0.02 [sec] | 2.13 ± 0.05 [sec] | 449 |
| UserKNN CF jaccard | 0.15 ± 0.02 [sec] | 2.13 ± 0.07 [sec] | 454 |
| UserKNN CF asymmetric | 0.15 ± 0.02 [sec] | 2.15 ± 0.08 [sec] | 445 |
| UserKNN CF tversky | 0.15 ± 0.02 [sec] | 2.09 ± 0.03 [sec] | 452 |
| ItemKNN CF cosine | 0.20 ± 0.04 [sec] | 2.14 ± 0.05 [sec] | 429 |
| ItemKNN CF dice | 0.18 ± 0.04 [sec] | 2.18 ± 0.11 [sec] | 460 |
| ItemKNN CF jaccard | 0.19 ± 0.04 [sec] | 2.16 ± 0.14 [sec] | 458 |
| ItemKNN CF asymmetric | 0.20 ± 0.04 [sec] | 2.20 ± 0.09 [sec] | 454 |
| ItemKNN CF tversky | 0.18 ± 0.04 [sec] | 2.14 ± 0.10 [sec] | 454 |
| $P^3\alpha$ | 0.99 ± 0.63 [sec] | 2.08 ± 0.05 [sec] | 460 |
| $RP^3\beta$ | 1.21 ± 0.70 [sec] | 2.12 ± 0.04 [sec] | 446 |
| EASE ^R | 0.41 ± 0.01 [sec] | 1.88 ± 0.00 [sec] | 500 |
| SLIM BPR | 64.02 [sec] / 1.07 ± 0.69 [min] | 2.15 ± 0.07 [sec] | 440 |
| SLIM ElasticNet | 9.40 ± 4.87 [sec] | 2.08 ± 0.01 [sec] | 454 |
| MF BPR | 44.66 ± 46.61 [sec] | 1.45 ± 0.04 [sec] | 631 |
| MF FunkSVD | 156.60 [sec] / 2.61 ± 2.88 [min] | 1.46 ± 0.01 [sec] | 644 |
| PureSVD | 0.06 ± 0.05 [sec] | 1.43 ± 0.01 [sec] | 661 |
| NMF | 64.98 [sec] / 1.08 ± 1.03 [min] | 1.45 ± 0.02 [sec] | 646 |
| iALS | 39.52 ± 27.96 [sec] | 1.44 ± 0.00 [sec] | 655 |
| DMF NCE | 13775.00 [sec] / 3.83 [hour] | 24.09 [sec] | 39 |
| DMF BCE | 23861.05 [sec] / 6.63 [hour] | 22.85 [sec] | 41 |

Table 103. Computation time for the algorithms in the selected results for the DMF method on the Movielens 1M dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|-----------------------|---|-------------------------------|---------------------------|
| Random | 0.03 [sec] | 8.68 [sec] | 695 |
| TopPop | 0.05 [sec] | 9.07 [sec] | 665 |
| UserKNN CF cosine | 4.72 ± 0.25 [sec] | 14.59 ± 1.40 [sec] | 380 |
| UserKNN CF dice | 4.43 ± 0.21 [sec] | 14.60 ± 0.55 [sec] | 426 |
| UserKNN CF jaccard | 4.43 ± 0.22 [sec] | 14.97 ± 0.70 [sec] | 427 |
| UserKNN CF asymmetric | 4.51 ± 0.20 [sec] | 15.31 ± 0.62 [sec] | 388 |
| UserKNN CF tversky | 4.55 ± 0.22 [sec] | 14.97 ± 0.36 [sec] | 410 |
| ItemKNN CF cosine | 2.06 ± 0.11 [sec] | 15.87 ± 1.14 [sec] | 410 |
| ItemKNN CF dice | 2.02 ± 0.11 [sec] | 14.59 ± 1.18 [sec] | 437 |
| ItemKNN CF jaccard | 2.03 ± 0.11 [sec] | 14.40 ± 0.66 [sec] | 441 |
| ItemKNN CF asymmetric | 2.10 ± 0.11 [sec] | 15.84 ± 1.07 [sec] | 402 |
| ItemKNN CF tversky | 2.03 ± 0.13 [sec] | 14.24 ± 0.36 [sec] | 435 |
| P3alpha | 4.79 ± 1.34 [sec] | 14.13 ± 0.30 [sec] | 417 |
| RP3beta | 4.80 ± 1.32 [sec] | 14.26 ± 0.51 [sec] | 426 |
| EASE R | 4.69 ± 0.02 [sec] | 13.07 ± 0.18 [sec] | 458 |
| SLIM BPR | 1067.49 [sec] / 17.79 ± 18.34 [min] | 15.51 ± 0.33 [sec] | 383 |
| SLIM ElasticNet | 204.56 [sec] / 3.41 ± 3.33 [min] | 14.18 ± 0.01 [sec] | 425 |
| MF BPR | 450.68 [sec] / 7.51 ± 6.28 [min] | 9.52 ± 0.12 [sec] | 624 |
| MF FunkSVD | 2080.92 [sec] / 34.68 ± 26.08 [min] | 9.49 ± 0.11 [sec] | 638 |
| PureSVD | 0.83 ± 0.64 [sec] | 9.56 ± 0.11 [sec] | 635 |
| NMF | 355.21 [sec] / 5.92 ± 13.25 [min] | 9.56 ± 0.02 [sec] | 630 |
| IALS | 273.69 [sec] / 4.56 ± 2.68 [min] | 9.52 ± 0.02 [sec] | 631 |
| DMF NCE | 671545.15 [sec] / 7.77 [day] | 548.79 [sec] / 9.15 [min] | 11 |
| DMF BCE | 351928.51 [sec] / 4.07 [day] | 482.96 [sec] / 8.05 [min] | 12 |

Table 104. Hyperparameter values for our collaborative KNN baselines on all datasets.

| Algorithm | Hyperparameter | Amazon Music original | Amazon Music ours | Movielens 100K | Amazon Movie | Movielens 1M |
|-----------------------|-------------------|--------------------------|----------------------|-------------------|-----------------|-----------------|
| UserKNN CF cosine | topK | 883 | 614 | 159 | 1000 | 770 |
| | shrink | 990 | 15 | 0 | 0 | 0 |
| | similarity | cosine | cosine | cosine | cosine | cosine |
| | normalize | True | True | True | True | True |
| | feature weighting | BM25 | BM25 | BM25 | BM25 | BM25 |
| UserKNN CF dice | topK | 163 | 202 | 148 | 955 | 177 |
| | shrink | 0 | 6 | 5 | 11 | 0 |
| | similarity | dice | dice | dice | dice | dice |
| | normalize | True | False | False | True | True |
| UserKNN CF jaccard | topK | 195 | 204 | 100 | 999 | 178 |
| | shrink | 1 | 0 | 3 | 23 | 0 |
| | similarity | jaccard | jaccard | jaccard | jaccard | jaccard |
| | normalize | False | True | True | True | False |
| UserKNN CF asymmetric | topK | 651 | 786 | 169 | 1000 | 613 |
| | shrink | 846 | 1000 | 0 | 0 | 0 |
| | similarity | asymmetric | asymmetric | asymmetric | asymmetric | asymmetric |
| | normalize | True | True | True | True | True |
| | asymmetric alpha | 0.4426 | 0.7110 | 0.1994 | 0.4960 | 0.3317 |
| UserKNN CF tfidf | feature weighting | BM25 | BM25 | BM25 | BM25 | BM25 |
| UserKNN CF tfidf | topK | 152 | 147 | 107 | 858 | 337 |
| | shrink | 0 | 92 | 0 | 31 | 0 |
| | similarity | tfidf | tfidf | tfidf | tfidf | tfidf |
| | normalize | True | True | True | True | True |
| | tfidf alpha | 2.0000 | 1.9995 | 1.5901 | 1.0846 | 2.0000 |
| UserKNN CF tfidf | tfidf beta | 2.0000 | 1.3750 | 2.0000 | 1.8286 | 2.0000 |
| ItemKNN CF cosine | topK | 929 | 1000 | 303 | 1000 | 191 |
| | shrink | 285 | 893 | 0 | 1000 | 1 |
| | similarity | cosine | cosine | cosine | cosine | cosine |
| | normalize | False | True | True | False | True |
| | feature weighting | BM25 | TF-IDF | BM25 | BM25 | BM25 |
| ItemKNN CF dice | topK | 232 | 500 | 12 | 845 | 66 |
| | shrink | 99 | 0 | 37 | 18 | 4 |
| | similarity | dice | dice | dice | dice | dice |
| | normalize | False | False | False | True | True |
| ItemKNN CF jaccard | topK | 726 | 422 | 28 | 582 | 63 |
| | shrink | 9 | 11 | 2 | 42 | 5 |
| | similarity | jaccard | jaccard | jaccard | jaccard | jaccard |
| | normalize | True | False | False | False | True |
| ItemKNN CF asymmetric | topK | 778 | 669 | 24 | 978 | 303 |
| | shrink | 641 | 1000 | 1000 | 859 | 0 |
| | similarity | asymmetric | asymmetric | asymmetric | asymmetric | asymmetric |
| | normalize | True | True | True | True | True |
| | asymmetric alpha | 0.2886 | 0.3042 | 0.2911 | 0.0000 | 0.3402 |
| ItemKNN CF asymmetric | feature weighting | TF-IDF | TF-IDF | TF-IDF | none | BM25 |
| ItemKNN CF tfidf | topK | 207 | 626 | 118 | 360 | 58 |
| | shrink | 949 | 95 | 0 | 71 | 143 |
| | similarity | tfidf | tfidf | tfidf | tfidf | tfidf |
| | normalize | True | True | True | True | True |
| | tfidf alpha | 2.0000 | 1.2255 | 0.0876 | 0.1388 | 0.0000 |
| ItemKNN CF tfidf | tfidf beta | 2.0000 | 2.0000 | 0.6605 | 0.9260 | 0.3728 |

Table 105. Hyperparameter values for our non-neural machine learning and graph based baselines on all datasets.

| Algorithm | Hyperparameter | Amazon Music original | Amazon Music ours | Movielens 100K | Amazon Movie | Movielens 1M |
|-----------------|----------------------|--------------------------|----------------------|-------------------|------------------|-----------------|
| $P^3\alpha$ | topK | 871 | 453 | 93 | 1000 | 1000 |
| | alpha | 0.8151 | 0.4565 | 0.0000 | 0.5195 | 1.2781 |
| | normalize similarity | True | True | True | False | False |
| $RP^3\beta$ | topK | 864 | 900 | 311 | 1000 | 405 |
| | alpha | 0.8665 | 0.9432 | 0.3966 | 0.4285 | 0.9764 |
| | beta | 0.1478 | 0.2462 | 0.5232 | 0.2689 | 0.5807 |
| | normalize similarity | True | True | True | False | False |
| $EASE^R$ | l2 norm | 5.79E+03 | - | 1.66E+04 | - | 6.07E+04 |
| SLIM BPR | topK | 789 | 676 | 241 | - | 765 |
| | epochs | 175 | 255 | 260 | - | 345 |
| | symmetric | True | True | True | - | True |
| | sgd mode | adam | adagrad | adagrad | - | adagrad |
| | lambda i | 6.07E-04 | 2.02E-05 | 2.85E-03 | - | 1.00E-02 |
| | lambda j | 2.71E-03 | 5.73E-04 | 1.23E-04 | - | 1.00E-05 |
| | learning rate | 1.00E-01 | 7.89E-03 | 2.91E-02 | - | 4.12E-02 |
| SLIM ElasticNet | topK | 1000 | 728 | 96 | 1000 | 694 |
| | l1 ratio | 8.28E-05 | 4.68E-05 | 2.76E-03 | 4.41E-04 | 1.86E-04 |
| | alpha | 1.0000 | 1.0000 | 0.9354 | 1.0000 | 0.6571 |
| MF BPR | sgd mode | adagrad | adagrad | adam | adagrad | sgd |
| | epochs | 420 | 495 | 365 | 1490 | 810 |
| | num factors | 183 | 200 | 200 | 200 | 200 |
| | batch size | 128 | 1 | 256 | 256 | 2 |
| | positive reg | 1.00E-02 | 1.00E-05 | 1.00E-05 | 1.00E-02 | 3.82E-05 |
| | negative reg | 1.00E-05 | 1.14E-03 | 1.00E-05 | 1.00E-02 | 1.00E-02 |
| | learning rate | 5.72E-02 | 7.38E-02 | 2.36E-03 | 6.68E-02 | 8.09E-02 |
| | | | | | | |
| MF FunkSVD | sgd mode | adam | sgd | sgd | adam | sgd |
| | epochs | 450 | 230 | 130 | 245 | 400 |
| | use bias | True | False | True | True | False |
| | batch size | 16 | 2 | 8 | 32 | 1 |
| | num factors | 200 | 190 | 70 | 177 | 41 |
| | item reg | 3.33E-05 | 1.00E-02 | 5.26E-04 | 2.69E-05 | 2.21E-04 |
| | user reg | 1.00E-02 | 1.00E-02 | 7.28E-03 | 1.00E-02 | 1.54E-03 |
| | learning rate | 3.64E-04 | 2.43E-02 | 1.48E-02 | 1.32E-03 | 1.15E-03 |
| | negative quota | 0.0432 | 0.3709 | 0.1257 | 0.0409 | 0.1628 |
| | | | | | | |
| PureSVD | num factors | 81 | 17 | 24 | 77 | 57 |
| NMF | num factors | 121 | 164 | 40 | 99 | 64 |
| | solver | mult. update | mult. update | coord. descent | mult. update | coord. descent |
| | init type | random | random | nndsvda | nndsvda | nndsvda |
| | beta loss | frobenius | frobenius | frobenius | kullback-leibler | frobenius |
| iALS | num factors | 31 | 28 | 25 | 52 | 63 |
| | confidence scaling | log | log | log | log | log |
| | alpha | 3.2850 | 50.0000 | 0.0150 | 2.8548 | 0.3345 |
| | epsilon | 0.0157 | 9.4295 | 9.3913 | 0.0010 | 0.0010 |
| | reg | 1.00E-02 | 1.00E-02 | 1.00E-05 | 1.00E-05 | 1.00E-05 |
| | epochs | 65 | 110 | 20 | 60 | 20 |

Table 106. Hyperparameter values for the neural algorithm on all datasets.

| Algorithm | Hyperparameter | amazon music original | amazon music ours | movielens100k | amazon movie | movielens1m |
|-----------|-----------------|-----------------------|-------------------|---------------|--------------|-------------|
| DMF NCE | epochs | 10 | 5 | 75 | 10 | 120 |
| | learning rate | 1.00E-04 | 1.00E-04 | 1.00E-04 | 1.00E-04 | 1.00E-04 |
| | batch size | 256 | 256 | 256 | 256 | 256 |
| | num negatives | 7 | 7 | 7 | 7 | 7 |
| | last layer size | 128 | 128 | 64 | 64 | 64 |
| DMF BCE | epochs | 75 | 80 | 165 | 35 | 65 |
| | learning rate | 1.00E-04 | 1.00E-04 | 1.00E-04 | 1.00E-04 | 1.00E-04 |
| | batch size | 256 | 256 | 256 | 256 | 256 |
| | num negatives | 7 | 7 | 7 | 7 | 7 |
| | last layer size | 128 | 128 | 64 | 64 | 64 |
| | max rating | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

M IJCAI: COUPLED CF: LEARNING EXPLICIT AND IMPLICIT USER-ITEM COUPLINGS IN RECOMMENDATION FOR DEEP COLLABORATIVE FILTERING

Relevant statistics on the dataset, which we mentioned in the paper, are reported in Table 107. The results of our evaluation can be seen in Table 108 (Movielens 1M) and Table 109 (Tafeng). The corresponding optimal hyperparameters are reported in Table 114 (collaborative KNNs), Table 115 (non-neural machine learning and graph based), Table 116 (content-based KNNs), Table 117 (item-based hybrid KNNs), Table 118 (user-based hybrid KNNs), Table 119 (content-based KNNs for Tafeng), Table 120 (hybrid KNNs for Tafeng) and Table 121 (CoupledCF).

Lastly, the time required to train and evaluate the models is reported in Table 112 (Movielens 1M) and Table 113 (Tafeng).

Table 107. Dataset characteristics.

| Dataset | Interactions | Items | Users | Density |
|--------------|--------------|-------|-------|---------|
| Movielens 1M | 1M | 3953 | 6041 | 4.18 |
| Tafeng | 743K | 23813 | 32267 | 0.097 |

Table 108. Experimental results for the CoupledCF method for the Movielens 1M dataset.

| | @ 1 | | @ 5 | | @ 10 | |
|--------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| | HR | NDCG | HR | NDCG | HR | NDCG |
| Random | 0.0101 | 0.0101 | 0.0513 | 0.0301 | 0.1008 | 0.0459 |
| TopPopular | 0.1084 | 0.1084 | 0.3224 | 0.2174 | 0.4740 | 0.2661 |
| UserKNN CF cosine | 0.1985 | 0.1985 | 0.5189 | 0.3653 | 0.6808 | 0.4178 |
| UserKNN CF dice | 0.1962 | 0.1962 | 0.5167 | 0.3631 | 0.6714 | 0.4133 |
| UserKNN CF jaccard | 0.1947 | 0.1947 | 0.5187 | 0.3633 | 0.6748 | 0.4139 |
| UserKNN CF asymmetric | 0.2000 | 0.2000 | 0.5220 | 0.3677 | 0.6896 | 0.4220 |
| UserKNN CF tversky | 0.1972 | 0.1972 | 0.5194 | 0.3649 | 0.6781 | 0.4165 |
| ItemKNN CF cosine | 0.1810 | 0.1810 | 0.5111 | 0.3514 | 0.6844 | 0.4077 |
| ItemKNN CF dice | 0.1818 | 0.1818 | 0.5038 | 0.3468 | 0.6775 | 0.4031 |
| ItemKNN CF jaccard | 0.1765 | 0.1765 | 0.5003 | 0.3434 | 0.6738 | 0.3996 |
| ItemKNN CF asymmetric | 0.1783 | 0.1783 | 0.4781 | 0.3321 | 0.6478 | 0.3872 |
| ItemKNN CF tversky | 0.1800 | 0.1800 | 0.5000 | 0.3436 | 0.6659 | 0.3975 |
| $P^3\alpha$ | 0.1829 | 0.1829 | 0.4965 | 0.3451 | 0.6589 | 0.3978 |
| $RP^3\beta$ | 0.1801 | 0.1801 | 0.5083 | 0.3498 | 0.6623 | 0.3996 |
| EASE ^R | 0.2187 | 0.2187 | 0.5675 | 0.3988 | 0.7260 | 0.4502 |
| SLIM BPR | 0.2070 | 0.2070 | 0.5363 | 0.3769 | 0.6990 | 0.4297 |
| SLIM ElasticNet | 0.2258 | 0.2258 | 0.5778 | 0.4073 | 0.7281 | 0.4561 |
| MF BPR | 0.1806 | 0.1806 | 0.4985 | 0.3437 | 0.6662 | 0.3981 |
| MF FunkSVD | 0.2051 | 0.2051 | 0.5368 | 0.3771 | 0.6944 | 0.4284 |
| PureSVD | 0.2167 | 0.2167 | 0.5540 | 0.3916 | 0.7055 | 0.4408 |
| NMF | 0.2000 | 0.2000 | 0.5334 | 0.3723 | 0.6861 | 0.4217 |
| iALS | 0.2220 | 0.2220 | 0.5680 | 0.4015 | 0.7225 | 0.4516 |
| ItemKNN CBF cosine | 0.0652 | 0.0652 | 0.2167 | 0.1414 | 0.3364 | 0.1800 |
| ItemKNN CBF dice | 0.0598 | 0.0598 | 0.2030 | 0.1321 | 0.3185 | 0.1692 |
| ItemKNN CBF jaccard | 0.0606 | 0.0606 | 0.2017 | 0.1321 | 0.3164 | 0.1690 |
| ItemKNN CBF asymmetric | 0.0636 | 0.0636 | 0.2164 | 0.1412 | 0.3406 | 0.1812 |
| ItemKNN CBF tversky | 0.0596 | 0.0596 | 0.2026 | 0.1320 | 0.3192 | 0.1693 |
| UserKNN CBF cosine | 0.1182 | 0.1182 | 0.3373 | 0.2291 | 0.4990 | 0.2813 |
| UserKNN CBF dice | 0.1175 | 0.1175 | 0.3371 | 0.2288 | 0.4982 | 0.2807 |
| UserKNN CBF jaccard | 0.1192 | 0.1192 | 0.3377 | 0.2295 | 0.5010 | 0.2820 |
| UserKNN CBF asymmetric | 0.1177 | 0.1177 | 0.3374 | 0.2289 | 0.4980 | 0.2806 |
| UserKNN CBF tversky | 0.1200 | 0.1200 | 0.3381 | 0.2300 | 0.5013 | 0.2826 |
| ItemKNN CFCBF cosine | 0.1818 | 0.1818 | 0.5159 | 0.3526 | 0.6833 | 0.4068 |
| ItemKNN CFCBF dice | 0.1813 | 0.1813 | 0.5022 | 0.3465 | 0.6770 | 0.4032 |
| ItemKNN CFCBF jaccard | 0.1791 | 0.1791 | 0.4997 | 0.3433 | 0.6707 | 0.3988 |
| ItemKNN CFCBF asymmetric | 0.1820 | 0.1820 | 0.5048 | 0.3477 | 0.6732 | 0.4023 |
| ItemKNN CFCBF tversky | 0.1765 | 0.1765 | 0.4768 | 0.3309 | 0.6536 | 0.3882 |
| UserKNN CFCBF cosine | 0.1889 | 0.1889 | 0.5154 | 0.3572 | 0.6790 | 0.4102 |
| UserKNN CFCBF dice | 0.1993 | 0.1993 | 0.5175 | 0.3649 | 0.6805 | 0.4177 |
| UserKNN CFCBF jaccard | 0.1983 | 0.1983 | 0.5171 | 0.3645 | 0.6803 | 0.4175 |
| UserKNN CFCBF asymmetric | 0.1892 | 0.1892 | 0.5144 | 0.3555 | 0.6820 | 0.4099 |
| UserKNN CFCBF tversky | 0.1967 | 0.1967 | 0.5182 | 0.3641 | 0.6877 | 0.4193 |
| DeepCF | 0.1959 | 0.1959 | 0.5522 | 0.3795 | 0.7171 | 0.4330 |
| CoupledCF | 0.2071 | 0.2071 | 0.5465 | 0.3817 | 0.7079 | 0.4342 |

Table 109. Experimental results for the CoupledCF method for the Tafeng dataset.

| | @ 1 | | @ 5 | | @ 10 | |
|--------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| | HR | NDCG | HR | NDCG | HR | NDCG |
| Random | 0.0555 | 0.0555 | 0.0936 | 0.0741 | 0.1406 | 0.0891 |
| TopPopular | 0.2809 | 0.2809 | 0.5287 | 0.4094 | 0.6614 | 0.4523 |
| UserKNN CF cosine | 0.3498 | 0.3498 | 0.5737 | 0.4674 | 0.6703 | 0.4987 |
| UserKNN CF dice | 0.3477 | 0.3477 | 0.5680 | 0.4632 | 0.6661 | 0.4950 |
| UserKNN CF jaccard | 0.3473 | 0.3473 | 0.5678 | 0.4629 | 0.6659 | 0.4946 |
| UserKNN CF asymmetric | 0.3487 | 0.3487 | 0.5717 | 0.4657 | 0.6686 | 0.4971 |
| UserKNN CF tversky | 0.3472 | 0.3472 | 0.5679 | 0.4628 | 0.6658 | 0.4946 |
| ItemKNN CF cosine | 0.3573 | 0.3573 | 0.5727 | 0.4709 | 0.6648 | 0.5007 |
| ItemKNN CF dice | 0.3458 | 0.3458 | 0.5773 | 0.4673 | 0.6770 | 0.4996 |
| ItemKNN CF jaccard | 0.3484 | 0.3484 | 0.5766 | 0.4680 | 0.6717 | 0.4989 |
| ItemKNN CF asymmetric | 0.3615 | 0.3615 | 0.5726 | 0.4733 | 0.6541 | 0.4997 |
| ItemKNN CF tversky | 0.3516 | 0.3516 | 0.5788 | 0.4713 | 0.6647 | 0.4992 |
| $P^3\alpha$ | 0.3523 | 0.3523 | 0.5844 | 0.4749 | 0.6775 | 0.5051 |
| $RP^3\beta$ | 0.3400 | 0.3400 | 0.5140 | 0.4322 | 0.5898 | 0.4567 |
| EASE ^R | 0.3503 | 0.3503 | 0.5719 | 0.4666 | 0.6696 | 0.4982 |
| SLIM BPR | 0.3500 | 0.3500 | 0.5792 | 0.4705 | 0.6723 | 0.5006 |
| SLIM ElasticNet | 0.3496 | 0.3496 | 0.5721 | 0.4666 | 0.6724 | 0.4991 |
| MF BPR | 0.2413 | 0.2413 | 0.4641 | 0.3562 | 0.5808 | 0.3939 |
| MF FunkSVD | 0.2853 | 0.2853 | 0.5300 | 0.4123 | 0.6614 | 0.4548 |
| PureSVD | 0.2751 | 0.2751 | 0.5222 | 0.4032 | 0.6535 | 0.4457 |
| NMF | 0.2476 | 0.2476 | 0.4603 | 0.3584 | 0.5769 | 0.3961 |
| iALS | 0.3296 | 0.3296 | 0.5634 | 0.4530 | 0.6621 | 0.4849 |
| ItemKNN CBF cosine | 0.0545 | 0.0545 | 0.0905 | 0.0722 | 0.1342 | 0.0861 |
| ItemKNN CBF dice | 0.0537 | 0.0537 | 0.0916 | 0.0723 | 0.1350 | 0.0861 |
| ItemKNN CBF jaccard | 0.0537 | 0.0537 | 0.0917 | 0.0723 | 0.1354 | 0.0862 |
| ItemKNN CBF asymmetric | 0.0547 | 0.0547 | 0.0946 | 0.0741 | 0.1425 | 0.0894 |
| ItemKNN CBF tversky | 0.0537 | 0.0537 | 0.0918 | 0.0723 | 0.1353 | 0.0862 |
| UserKNN CBF cosine | 0.2604 | 0.2604 | 0.4728 | 0.3705 | 0.5831 | 0.4061 |
| UserKNN CBF dice | 0.2562 | 0.2562 | 0.4635 | 0.3640 | 0.5701 | 0.3984 |
| UserKNN CBF jaccard | 0.2559 | 0.2559 | 0.4631 | 0.3636 | 0.5692 | 0.3978 |
| UserKNN CBF asymmetric | 0.2632 | 0.2632 | 0.4768 | 0.3743 | 0.5885 | 0.4103 |
| UserKNN CBF tversky | 0.2564 | 0.2564 | 0.4643 | 0.3644 | 0.5708 | 0.3988 |
| ItemKNN CFCBF cosine | 0.3573 | 0.3573 | 0.5727 | 0.4709 | 0.6650 | 0.5008 |
| ItemKNN CFCBF dice | 0.3329 | 0.3329 | 0.5642 | 0.4541 | 0.6677 | 0.4878 |
| ItemKNN CFCBF jaccard | 0.3348 | 0.3348 | 0.5669 | 0.4562 | 0.6646 | 0.4879 |
| ItemKNN CFCBF asymmetric | 0.3611 | 0.3611 | 0.5735 | 0.4736 | 0.6561 | 0.5003 |
| ItemKNN CFCBF tversky | 0.3357 | 0.3357 | 0.5675 | 0.4573 | 0.6651 | 0.4890 |
| UserKNN CFCBF cosine | 0.3536 | 0.3536 | 0.5858 | 0.4766 | 0.6874 | 0.5095 |
| UserKNN CFCBF dice | 0.3289 | 0.3289 | 0.5576 | 0.4490 | 0.6594 | 0.4819 |
| UserKNN CFCBF jaccard | 0.3272 | 0.3272 | 0.5551 | 0.4469 | 0.6570 | 0.4799 |
| UserKNN CFCBF asymmetric | 0.3584 | 0.3584 | 0.5915 | 0.4814 | 0.6907 | 0.5136 |
| UserKNN CFCBF tversky | 0.3259 | 0.3259 | 0.5529 | 0.4451 | 0.6554 | 0.4782 |
| DeepCF | 0.2869 | 0.2869 | 0.5366 | 0.4169 | 0.6637 | 0.4581 |
| CoupledCF | 0.2767 | 0.2767 | 0.5272 | 0.4065 | 0.6597 | 0.4494 |

Table 110. Experimental results for the CoupledCF method for the Movielens 1M dataset on beyond accuracy metrics.

| | Div. MIL | Div. HHI | @ 5 Cov. Item | Div. Gini | Div. Shannon |
|--------------------------|---------------|---------------|---------------------|---------------|-----------------|
| Random | 0.9987 | 0.9997 | 0.9992 | 0.7934 | 11.8479 |
| TopPopular | 0.9848 | 0.9969 | 0.1703 | 0.0751 | 8.5819 |
| UserKNN CF cosine | 0.9937 | 0.9987 | 0.5231 | 0.2079 | 10.0972 |
| UserKNN CF dice | 0.9939 | 0.9988 | 0.5512 | 0.2208 | 10.1796 |
| UserKNN CF jaccard | 0.9938 | 0.9987 | 0.5416 | 0.2156 | 10.1464 |
| UserKNN CF asymmetric | 0.9935 | 0.9987 | 0.5052 | 0.2021 | 10.0569 |
| UserKNN CF tversky | 0.9936 | 0.9987 | 0.5196 | 0.2054 | 10.0798 |
| ItemKNN CF cosine | 0.9936 | 0.9987 | 0.5143 | 0.2018 | 10.0616 |
| ItemKNN CF dice | 0.9925 | 0.9985 | 0.4677 | 0.1748 | 9.8552 |
| ItemKNN CF jaccard | 0.9924 | 0.9984 | 0.4642 | 0.1718 | 9.8301 |
| ItemKNN CF asymmetric | 0.9920 | 0.9984 | 0.4270 | 0.1572 | 9.7089 |
| ItemKNN CF tversky | 0.9946 | 0.9989 | 0.4956 | 0.2215 | 10.1913 |
| $P^3\alpha$ | 0.9921 | 0.9984 | 0.3514 | 0.1509 | 9.6396 |
| $RP^3\beta$ | 0.9931 | 0.9986 | 0.4116 | 0.1752 | 9.8555 |
| EASE ^R | 0.9941 | 0.9988 | 0.4849 | 0.2077 | 10.1009 |
| SLIM BPR | 0.9919 | 0.9983 | 0.4129 | 0.1565 | 9.7003 |
| SLIM ElasticNet | 0.9943 | 0.9988 | 0.5229 | 0.2225 | 10.1981 |
| MF BPR | 0.9939 | 0.9987 | 0.4867 | 0.2044 | 10.0794 |
| MF FunkSVD | 0.9953 | 0.9990 | 0.4971 | 0.2459 | 10.3268 |
| PureSVD | 0.9954 | 0.9990 | 0.5325 | 0.2564 | 10.3933 |
| NMF | 0.9958 | 0.9991 | 0.6345 | 0.2957 | 10.5957 |
| iALS | 0.9957 | 0.9991 | 0.5783 | 0.2772 | 10.5042 |
| ItemKNN CBF cosine | 0.9920 | 0.9984 | 0.4505 | 0.1666 | 9.7814 |
| ItemKNN CBF dice | 0.9945 | 0.9989 | 0.6036 | 0.2427 | 10.3121 |
| ItemKNN CBF jaccard | 0.9946 | 0.9989 | 0.5892 | 0.2444 | 10.3235 |
| ItemKNN CBF asymmetric | 0.9922 | 0.9984 | 0.4543 | 0.1711 | 9.8178 |
| ItemKNN CBF tversky | 0.9944 | 0.9989 | 0.6064 | 0.2421 | 10.3090 |
| UserKNN CBF cosine | 0.9876 | 0.9975 | 0.2459 | 0.0946 | 8.9657 |
| UserKNN CBF dice | 0.9876 | 0.9975 | 0.2472 | 0.0949 | 8.9707 |
| UserKNN CBF jaccard | 0.9877 | 0.9975 | 0.2474 | 0.0956 | 8.9811 |
| UserKNN CBF asymmetric | 0.9877 | 0.9975 | 0.2466 | 0.0950 | 8.9713 |
| UserKNN CBF tversky | 0.9877 | 0.9975 | 0.2484 | 0.0951 | 8.9725 |
| ItemKNN CFCBF cosine | 0.9943 | 0.9988 | 0.4693 | 0.2093 | 10.1088 |
| ItemKNN CFCBF dice | 0.9926 | 0.9985 | 0.4789 | 0.1766 | 9.8691 |
| ItemKNN CFCBF jaccard | 0.9921 | 0.9984 | 0.4526 | 0.1648 | 9.7716 |
| ItemKNN CFCBF asymmetric | 0.9932 | 0.9986 | 0.4629 | 0.1840 | 9.9320 |
| ItemKNN CFCBF tversky | 0.9911 | 0.9982 | 0.3810 | 0.1399 | 9.5424 |
| UserKNN CFCBF cosine | 0.9931 | 0.9986 | 0.5029 | 0.1950 | 10.0030 |
| UserKNN CFCBF dice | 0.9936 | 0.9987 | 0.5242 | 0.2086 | 10.0991 |
| UserKNN CFCBF jaccard | 0.9937 | 0.9987 | 0.5317 | 0.2116 | 10.1190 |
| UserKNN CFCBF asymmetric | 0.9932 | 0.9986 | 0.5282 | 0.2008 | 10.0445 |
| UserKNN CFCBF tversky | 0.9933 | 0.9986 | 0.4958 | 0.1954 | 10.0097 |
| DeepCF | 0.9952 | 0.9990 | 0.6033 | 0.2655 | 10.4448 |
| CoupledCF | 0.9954 | 0.9990 | 0.6246 | 0.2750 | 10.4951 |

Table 111. Experimental results for the CoupledCF method for the Tafeng dataset on beyond accuracy metrics.

| | Div. MIL | Div. HHI | @ 5 Cov. Item | Div. Gini | Div. Shannon |
|--------------------------|---------------|---------------|---------------------|---------------|-----------------|
| Random | 0.9998 | 1.0000 | 0.9979 | 0.7737 | 14.4165 |
| TopPopular | 0.9965 | 0.9992 | 0.1360 | 0.0535 | 10.6959 |
| UserKNN CF cosine | 0.9978 | 0.9995 | 0.5041 | 0.1268 | 11.8704 |
| UserKNN CF dice | 0.9978 | 0.9995 | 0.5499 | 0.1376 | 11.9577 |
| UserKNN CF jaccard | 0.9978 | 0.9995 | 0.5518 | 0.1382 | 11.9637 |
| UserKNN CF asymmetric | 0.9978 | 0.9995 | 0.4966 | 0.1266 | 11.8757 |
| UserKNN CF tversky | 0.9978 | 0.9995 | 0.5530 | 0.1386 | 11.9670 |
| ItemKNN CF cosine | 0.9981 | 0.9996 | 0.6799 | 0.1898 | 12.3269 |
| ItemKNN CF dice | 0.9976 | 0.9995 | 0.5450 | 0.1250 | 11.7631 |
| ItemKNN CF jaccard | 0.9977 | 0.9995 | 0.5951 | 0.1441 | 11.9154 |
| ItemKNN CF asymmetric | 0.9982 | 0.9996 | 0.6819 | 0.2056 | 12.4523 |
| ItemKNN CF tversky | 0.9978 | 0.9995 | 0.6190 | 0.1529 | 11.9853 |
| $P^3\alpha$ | 0.9978 | 0.9995 | 0.5740 | 0.1369 | 11.9159 |
| $RP^3\beta$ | 0.9990 | 0.9998 | 0.9148 | 0.3974 | 13.4528 |
| EASE ^R | 0.9978 | 0.9995 | 0.4534 | 0.1180 | 11.8149 |
| SLIM BPR | 0.9977 | 0.9995 | 0.5831 | 0.1374 | 11.8920 |
| SLIM ElasticNet | 0.9977 | 0.9995 | 0.4654 | 0.1116 | 11.7116 |
| MF BPR | 0.9970 | 0.9993 | 0.2171 | 0.0661 | 11.0286 |
| MF FunkSVD | 0.9965 | 0.9992 | 0.1392 | 0.0537 | 10.7046 |
| PureSVD | 0.9966 | 0.9993 | 0.1433 | 0.0539 | 10.7071 |
| NMF | 0.9982 | 0.9996 | 0.3697 | 0.1145 | 11.8346 |
| iALS | 0.9985 | 0.9997 | 0.3529 | 0.1401 | 12.1119 |
| ItemKNN CBF cosine | 0.9996 | 0.9999 | 0.8006 | 0.4320 | 13.7032 |
| ItemKNN CBF dice | 0.9997 | 0.9999 | 0.8070 | 0.4726 | 13.8282 |
| ItemKNN CBF jaccard | 0.9997 | 0.9999 | 0.8079 | 0.4732 | 13.8300 |
| ItemKNN CBF asymmetric | 0.9996 | 0.9999 | 0.9018 | 0.4758 | 13.8621 |
| ItemKNN CBF tversky | 0.9997 | 0.9999 | 0.8062 | 0.4725 | 13.8279 |
| UserKNN CBF cosine | 0.9977 | 0.9995 | 0.5528 | 0.1316 | 11.8754 |
| UserKNN CBF dice | 0.9979 | 0.9995 | 0.5952 | 0.1459 | 11.9962 |
| UserKNN CBF jaccard | 0.9979 | 0.9995 | 0.5986 | 0.1476 | 12.0116 |
| UserKNN CBF asymmetric | 0.9977 | 0.9995 | 0.5346 | 0.1265 | 11.8308 |
| UserKNN CBF tversky | 0.9978 | 0.9995 | 0.5899 | 0.1441 | 11.9805 |
| ItemKNN CFCBF cosine | 0.9981 | 0.9996 | 0.6822 | 0.1903 | 12.3289 |
| ItemKNN CFCBF dice | 0.9976 | 0.9995 | 0.5518 | 0.1255 | 11.7596 |
| ItemKNN CFCBF jaccard | 0.9977 | 0.9995 | 0.5838 | 0.1360 | 11.8450 |
| ItemKNN CFCBF asymmetric | 0.9982 | 0.9996 | 0.6811 | 0.2015 | 12.4229 |
| ItemKNN CFCBF tversky | 0.9976 | 0.9995 | 0.5782 | 0.1346 | 11.8304 |
| UserKNN CFCBF cosine | 0.9977 | 0.9995 | 0.4440 | 0.1159 | 11.7711 |
| UserKNN CFCBF dice | 0.9976 | 0.9995 | 0.4100 | 0.1057 | 11.6555 |
| UserKNN CFCBF jaccard | 0.9976 | 0.9995 | 0.4258 | 0.1094 | 11.6953 |
| UserKNN CFCBF asymmetric | 0.9977 | 0.9995 | 0.4444 | 0.1161 | 11.7731 |
| UserKNN CFCBF tversky | 0.9976 | 0.9995 | 0.4474 | 0.1141 | 11.7434 |
| DeepCF | 0.9971 | 0.9994 | 0.2314 | 0.0724 | 11.1734 |
| CoupledCF | 0.9969 | 0.9993 | 0.1859 | 0.0627 | 10.9586 |

Table 112. Computation time for the algorithms in the selected results for the CoupledCF method on the MovieLens 1M dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|--------------------------|---|------------------------|---------------------------|
| Random | 0.02 [sec] | 10.19 [sec] | 593 |
| TopPopular | 0.03 [sec] | 11.38 [sec] | 531 |
| UserKNN CF cosine | 3.14 ± 0.17 [sec] | 15.78 ± 0.38 [sec] | 386 |
| UserKNN CF dice | 3.03 ± 0.15 [sec] | 15.60 ± 0.64 [sec] | 403 |
| UserKNN CF jaccard | 3.04 ± 0.16 [sec] | 15.73 ± 0.59 [sec] | 396 |
| UserKNN CF asymmetric | 3.12 ± 0.18 [sec] | 15.73 ± 0.71 [sec] | 393 |
| UserKNN CF tversky | 3.15 ± 0.17 [sec] | 16.03 ± 0.70 [sec] | 395 |
| ItemKNN CF cosine | 1.48 ± 0.10 [sec] | 16.46 ± 1.46 [sec] | 400 |
| ItemKNN CF dice | 1.45 ± 0.08 [sec] | 15.43 ± 1.18 [sec] | 410 |
| ItemKNN CF jaccard | 1.45 ± 0.08 [sec] | 15.15 ± 0.60 [sec] | 416 |
| ItemKNN CF asymmetric | 1.56 ± 0.10 [sec] | 16.67 ± 0.60 [sec] | 362 |
| ItemKNN CF tversky | 1.41 ± 0.08 [sec] | 15.77 ± 1.39 [sec] | 412 |
| $P^3\alpha$ | 3.23 ± 0.85 [sec] | 14.91 ± 0.21 [sec] | 409 |
| $RP^3\beta$ | 3.35 ± 1.07 [sec] | 14.94 ± 0.33 [sec] | 413 |
| EASE ^R | 4.04 ± 0.02 [sec] | 14.85 ± 0.20 [sec] | 404 |
| SLIM BPR | 721.91 [sec] / 12.03 ± 9.52 [min] | 16.28 ± 0.45 [sec] | 362 |
| SLIM ElasticNet | 171.44 [sec] / 2.86 ± 1.86 [min] | 14.69 ± 0.73 [sec] | 402 |
| MF BPR | 471.61 [sec] / 7.86 ± 5.82 [min] | 16.58 ± 2.70 [sec] | 332 |
| MF FunkSVD | 1860.97 [sec] / 31.02 ± 34.81 [min] | 14.78 ± 2.96 [sec] | 479 |
| PureSVD | 0.82 ± 0.46 [sec] | 13.42 ± 2.36 [sec] | 484 |
| NMF | 547.17 [sec] / 9.12 ± 6.08 [min] | 17.01 ± 2.98 [sec] | 330 |
| iALS | 464.07 [sec] / 7.73 ± 4.84 [min] | 12.50 ± 0.11 [sec] | 479 |
| ItemKNN CBF cosine | 0.27 ± 0.06 [sec] | 14.85 ± 0.60 [sec] | 396 |
| ItemKNN CBF dice | 0.26 ± 0.05 [sec] | 14.41 ± 0.12 [sec] | 423 |
| ItemKNN CBF jaccard | 0.27 ± 0.05 [sec] | 14.19 ± 0.30 [sec] | 421 |
| ItemKNN CBF asymmetric | 0.29 ± 0.06 [sec] | 15.42 ± 0.35 [sec] | 384 |
| ItemKNN CBF tversky | 0.26 ± 0.05 [sec] | 14.26 ± 0.40 [sec] | 418 |
| UserKNN CBF cosine | 0.71 ± 0.06 [sec] | 17.62 ± 0.55 [sec] | 340 |
| UserKNN CBF dice | 0.75 ± 0.09 [sec] | 17.88 ± 0.24 [sec] | 341 |
| UserKNN CBF jaccard | 0.77 ± 0.09 [sec] | 17.59 ± 0.35 [sec] | 343 |
| UserKNN CBF asymmetric | 0.78 ± 0.12 [sec] | 17.95 ± 0.25 [sec] | 339 |
| UserKNN CBF tversky | 0.76 ± 0.09 [sec] | 17.83 ± 0.36 [sec] | 332 |
| ItemKNN CFCBF cosine | 1.48 ± 0.09 [sec] | 16.26 ± 0.84 [sec] | 395 |
| ItemKNN CFCBF dice | 1.48 ± 0.09 [sec] | 14.60 ± 0.58 [sec] | 423 |
| ItemKNN CFCBF jaccard | 1.48 ± 0.08 [sec] | 14.50 ± 0.13 [sec] | 422 |
| ItemKNN CFCBF asymmetric | 1.54 ± 0.08 [sec] | 15.55 ± 0.31 [sec] | 390 |
| ItemKNN CFCBF tversky | 1.47 ± 0.09 [sec] | 15.28 ± 1.47 [sec] | 414 |
| UserKNN CFCBF cosine | 3.10 ± 0.13 [sec] | 17.10 ± 0.61 [sec] | 365 |
| UserKNN CFCBF dice | 3.11 ± 0.17 [sec] | 17.11 ± 0.42 [sec] | 349 |
| UserKNN CFCBF jaccard | 3.07 ± 0.18 [sec] | 16.83 ± 0.64 [sec] | 371 |
| UserKNN CFCBF asymmetric | 3.26 ± 0.29 [sec] | 17.50 ± 0.74 [sec] | 361 |
| UserKNN CFCBF tversky | 3.23 ± 0.25 [sec] | 16.82 ± 0.66 [sec] | 368 |
| DeepCF | 3901.52 [sec] / 1.08 [hour] | 25.05 [sec] | 241 |
| CoupledCF | 14029.82 [sec] / 3.90 [hour] | 34.18 [sec] | 177 |

Table 113. Computation time for the algorithms in the selected results for the CoupledCF method on the Tafeng dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|--------------------------|-------------------------------------|---------------------------------|---------------------------|
| Random | 0.01 [sec] | 57.31 [sec] | 536 |
| TopPopular | 0.02 [sec] | 63.39 [sec] / 1.06 [min] | 485 |
| UserKNN CF cosine | 10.08 ± 0.90 [sec] | 82.01 [sec] / 1.37 ± 0.07 [min] | 358 |
| UserKNN CF dice | 10.34 ± 0.66 [sec] | 82.74 [sec] / 1.38 ± 0.07 [min] | 360 |
| UserKNN CF jaccard | 10.45 ± 0.65 [sec] | 84.58 [sec] / 1.41 ± 0.01 [min] | 361 |
| UserKNN CF asymmetric | 10.24 ± 0.80 [sec] | 82.03 [sec] / 1.37 ± 0.04 [min] | 360 |
| UserKNN CF tversky | 10.45 ± 0.78 [sec] | 81.36 [sec] / 1.36 ± 0.07 [min] | 357 |
| ItemKNN CF cosine | 4.28 ± 0.35 [sec] | 84.92 [sec] / 1.42 ± 0.05 [min] | 354 |
| ItemKNN CF dice | 4.34 ± 0.27 [sec] | 85.91 [sec] / 1.43 ± 0.01 [min] | 359 |
| ItemKNN CF jaccard | 4.37 ± 0.30 [sec] | 83.91 [sec] / 1.40 ± 0.09 [min] | 358 |
| ItemKNN CF asymmetric | 4.46 ± 0.31 [sec] | 82.72 [sec] / 1.38 ± 0.11 [min] | 358 |
| ItemKNN CF tversky | 4.21 ± 0.31 [sec] | 85.76 [sec] / 1.43 ± 0.01 [min] | 362 |
| P ³ α | 16.15 ± 3.44 [sec] | 77.19 [sec] / 1.29 ± 0.05 [min] | 384 |
| RP ³ β | 19.20 ± 3.05 [sec] | 78.05 [sec] / 1.30 ± 0.03 [min] | 387 |
| EASE ^R | 310.60 [sec] / 5.18 ± 0.07 [min] | 75.08 [sec] / 1.25 ± 0.05 [min] | 418 |
| SLIM BPR | 2499.23 [sec] / 41.65 ± 18.16 [min] | 78.52 [sec] / 1.31 ± 0.01 [min] | 392 |
| SLIM ElasticNet | 527.87 [sec] / 8.80 ± 4.22 [min] | 80.33 [sec] / 1.34 ± 0.08 [min] | 360 |
| MF BPR | 1856.56 [sec] / 30.94 ± 28.65 [min] | 69.71 [sec] / 1.16 ± 0.19 [min] | 476 |
| MF FunkSVD | 1721.76 [sec] / 28.70 ± 36.20 [min] | 73.04 [sec] / 1.22 ± 0.20 [min] | 460 |
| PureSVD | 1.19 ± 1.72 [sec] | 63.30 [sec] / 1.06 ± 0.02 [min] | 481 |
| NMF | 396.04 [sec] / 6.60 ± 3.69 [min] | 75.72 [sec] / 1.26 ± 0.29 [min] | 473 |
| iALS | 1870.58 [sec] / 31.18 ± 27.80 [min] | 65.58 [sec] / 1.09 ± 0.00 [min] | 469 |
| ItemKNN CBF cosine | 13.56 ± 7.13 [sec] | 68.00 [sec] / 1.13 ± 0.01 [min] | 451 |
| ItemKNN CBF dice | 12.10 ± 6.57 [sec] | 66.94 [sec] / 1.12 ± 0.01 [min] | 461 |
| ItemKNN CBF jaccard | 13.21 ± 6.67 [sec] | 67.27 [sec] / 1.12 ± 0.01 [min] | 453 |
| ItemKNN CBF asymmetric | 13.24 ± 6.89 [sec] | 68.53 [sec] / 1.14 ± 0.04 [min] | 437 |
| ItemKNN CBF tversky | 15.57 ± 6.33 [sec] | 68.15 [sec] / 1.14 ± 0.01 [min] | 447 |
| UserKNN CBF cosine | 12.43 ± 1.13 [sec] | 87.55 [sec] / 1.46 ± 0.07 [min] | 340 |
| UserKNN CBF dice | 12.35 ± 0.65 [sec] | 89.77 [sec] / 1.50 ± 0.02 [min] | 339 |
| UserKNN CBF jaccard | 12.95 ± 0.88 [sec] | 90.25 [sec] / 1.50 ± 0.04 [min] | 336 |
| UserKNN CBF asymmetric | 13.36 ± 0.86 [sec] | 91.22 [sec] / 1.52 ± 0.02 [min] | 339 |
| UserKNN CBF tversky | 13.02 ± 0.82 [sec] | 90.31 [sec] / 1.51 ± 0.10 [min] | 341 |
| ItemKNN CFCBF cosine | 13.59 ± 1.28 [sec] | 81.15 [sec] / 1.35 ± 0.03 [min] | 372 |
| ItemKNN CFCBF dice | 13.71 ± 0.60 [sec] | 77.25 [sec] / 1.29 ± 0.03 [min] | 393 |
| ItemKNN CFCBF jaccard | 13.67 ± 0.64 [sec] | 77.42 [sec] / 1.29 ± 0.02 [min] | 398 |
| ItemKNN CFCBF asymmetric | 14.24 ± 0.90 [sec] | 79.89 [sec] / 1.33 ± 0.08 [min] | 375 |
| ItemKNN CFCBF tversky | 13.20 ± 1.08 [sec] | 76.06 [sec] / 1.27 ± 0.04 [min] | 398 |
| UserKNN CFCBF cosine | 15.01 ± 0.56 [sec] | 89.86 [sec] / 1.50 ± 0.02 [min] | 345 |
| UserKNN CFCBF dice | 16.99 ± 0.92 [sec] | 94.92 [sec] / 1.58 ± 0.09 [min] | 336 |
| UserKNN CFCBF jaccard | 16.73 ± 1.06 [sec] | 92.12 [sec] / 1.54 ± 0.06 [min] | 309 |
| UserKNN CFCBF asymmetric | 16.42 ± 1.22 [sec] | 90.62 [sec] / 1.51 ± 0.01 [min] | 340 |
| UserKNN CFCBF tversky | 16.78 ± 1.19 [sec] | 92.53 [sec] / 1.54 ± 0.09 [min] | 340 |
| DeepCF | 3182.28 [sec] / 53.04 [min] | 131.53 [sec] / 2.19 [min] | 234 |
| CoupledCF | 4632.16 [sec] / 1.29 [hour] | 198.59 [sec] / 3.31 [min] | 155 |

Table 114. Hyperparameter values for our collaborative KNN baselines on all datasets.

| Algorithm | Hyperparameter | Movielens 1M | Tafeng |
|-----------------------|-------------------|--------------|------------|
| UserKNN CF cosine | topK | 363 | 1000 |
| | shrink | 0 | 327 |
| | similarity | cosine | cosine |
| | normalize | True | True |
| | feature weighting | TF-IDF | TF-IDF |
| UserKNN CF dice | topK | 165 | 1000 |
| | shrink | 0 | 0 |
| | similarity | dice | dice |
| | normalize | True | True |
| UserKNN CF jaccard | topK | 187 | 1000 |
| | shrink | 0 | 0 |
| | similarity | jaccard | jaccard |
| | normalize | False | False |
| UserKNN CF asymmetric | topK | 268 | 1000 |
| | shrink | 212 | 82 |
| | similarity | asymmetric | asymmetric |
| | normalize | True | True |
| | asymmetric alpha | 0.0000 | 0.4272 |
| | feature weighting | TF-IDF | TF-IDF |
| UserKNN CF tversky | topK | 221 | 1000 |
| | shrink | 0 | 0 |
| | similarity | tversky | tversky |
| | normalize | True | True |
| | tversky alpha | 1.5095 | 2.0000 |
| | tversky beta | 2.0000 | 2.0000 |
| ItemKNN CF cosine | topK | 66 | 1000 |
| | shrink | 302 | 1000 |
| | similarity | cosine | cosine |
| | normalize | True | True |
| | feature weighting | TF-IDF | TF-IDF |
| ItemKNN CF dice | topK | 50 | 995 |
| | shrink | 5 | 65 |
| | similarity | dice | dice |
| | normalize | False | True |
| ItemKNN CF jaccard | topK | 43 | 999 |
| | shrink | 2 | 35 |
| | similarity | jaccard | jaccard |
| | normalize | True | False |
| ItemKNN CF asymmetric | topK | 446 | 1000 |
| | shrink | 906 | 975 |
| | similarity | asymmetric | asymmetric |
| | normalize | True | True |
| | asymmetric alpha | 0.9254 | 0.3635 |
| | feature weighting | BM25 | TF-IDF |
| ItemKNN CF tversky | topK | 42 | 784 |
| | shrink | 298 | 100 |
| | similarity | tversky | tversky |
| | normalize | True | True |
| | tversky alpha | 0.0000 | 0.9383 |
| | tversky beta | 0.9369 | 1.9574 |

Table 115. Hyperparameter values for our non-neural machine learning and graph based baselines on all datasets.

| Algorithm | Hyperparameter | Movielens 1M | Tafeng |
|-----------------|----------------------|------------------|----------------|
| $P^3\alpha$ | topK | 342 | 1000 |
| | alpha | 1.5568 | 0.5325 |
| | normalize similarity | True | False |
| $RP^3\beta$ | topK | 200 | 1000 |
| | alpha | 1.2485 | 0.4449 |
| | beta | 0.2230 | 0.3373 |
| | normalize similarity | True | True |
| $EASE^R$ | l2 norm | 2.94E+03 | 4.21E+03 |
| SLIM BPR | topK | 1000 | 808 |
| | epochs | 270 | 425 |
| | symmetric | True | False |
| | sgd mode | sgd | adagrad |
| | lambda i | 1.00E-02 | 1.00E-05 |
| | lambda j | 1.00E-05 | 1.00E-02 |
| | learning rate | 4.31E-03 | 1.00E-04 |
| SLIM ElasticNet | topK | 453 | 841 |
| | l1 ratio | 9.77E-04 | 7.24E-05 |
| | alpha | 0.0564 | 0.1668 |
| MF BPR | sgd mode | sgd | sgd |
| | epochs | 1005 | 420 |
| | num factors | 131 | 1 |
| | batch size | 2 | 4 |
| | positive reg | 1.00E-05 | 1.00E-05 |
| | negative reg | 1.00E-02 | 1.00E-02 |
| | learning rate | 7.63E-02 | 6.60E-02 |
| MF FunkSVD | sgd mode | adam | adam |
| | epochs | 260 | 40 |
| | use bias | True | True |
| | batch size | 256 | 32 |
| | num factors | 44 | 1 |
| | item reg | 7.79E-04 | 4.54E-04 |
| | user reg | 2.90E-05 | 1.49E-03 |
| | learning rate | 5.26E-04 | 1.00E-04 |
| | negative quota | 0.0672 | 0.5000 |
| PureSVD | num factors | 65 | 1 |
| NMF | num factors | 112 | 44 |
| | solver | mult. update | coord. descent |
| | init type | random | random |
| | beta loss | kullback-leibler | frobenius |
| iALS | num factors | 78 | 26 |
| | confidence scaling | linear | linear |
| | alpha | 1.7666 | 16.1876 |
| | epsilon | 10.0000 | 0.1320 |
| | reg | 1.00E-05 | 1.22E-05 |
| | epochs | 40 | 40 |

Table 116. Hyperparameter values for our content based KNN baselines on all datasets.

| Algorithm | Hyperparameter | Movielens 1M | Tafeng |
|--------------------------------|-------------------|--------------|------------|
| ItemKNN CBF ICM all cosine | topK | 1000 | 281 |
| | shrink | 1000 | 507 |
| | similarity | cosine | cosine |
| | normalize | True | False |
| | feature weighting | TF-IDF | BM25 |
| ItemKNN CBF ICM all dice | topK | 318 | 118 |
| | shrink | 323 | 0 |
| | similarity | dice | dice |
| | normalize | False | False |
| ItemKNN CBF ICM all jaccard | topK | 283 | 350 |
| | shrink | 553 | 0 |
| | similarity | jaccard | jaccard |
| | normalize | False | True |
| ItemKNN CBF ICM all asymmetric | topK | 986 | 179 |
| | shrink | 785 | 1000 |
| | similarity | asymmetric | asymmetric |
| | normalize | True | True |
| | asymmetric alpha | 2.0000 | 2.0000 |
| | feature weighting | TF-IDF | BM25 |
| ItemKNN CBF ICM all tversky | topK | 322 | 119 |
| | shrink | 354 | 809 |
| | similarity | tversky | tversky |
| | normalize | True | True |
| | tversky alpha | 0.8065 | 1.5801 |
| | tversky beta | 0.1989 | 1.9008 |
| UserKNN CBF cosine | topK | 801 | 1000 |
| | shrink | 1000 | 0 |
| | similarity | cosine | cosine |
| | normalize | True | True |
| | feature weighting | none | BM25 |
| UserKNN CBF dice | topK | 787 | 999 |
| | shrink | 10 | 959 |
| | similarity | dice | dice |
| | normalize | False | True |
| UserKNN CBF jaccard | topK | 793 | 999 |
| | shrink | 0 | 41 |
| | similarity | jaccard | jaccard |
| | normalize | False | False |
| UserKNN CBF asymmetric | topK | 788 | 1000 |
| | shrink | 992 | 510 |
| | similarity | asymmetric | asymmetric |
| | normalize | True | True |
| | asymmetric alpha | 0.0306 | 0.0000 |
| UserKNN CBF tversky | feature weighting | none | TF-IDF |
| | topK | 1000 | 1000 |
| | shrink | 0 | 1000 |
| | similarity | tversky | tversky |
| | normalize | True | True |
| | tversky alpha | 2.0000 | 0.0000 |
| | tversky beta | 2.0000 | 0.0000 |

Table 117. Hyperparameter values for our hybrid KNN baselines on all datasets.

| Algorithm | Hyperparameter | Movielens 1M | Tafeng |
|----------------------------------|-------------------|--------------|------------|
| ItemKNN CFCBF ICM all cosine | topK | 168 | 1000 |
| | shrink | 875 | 1000 |
| | similarity | cosine | cosine |
| | normalize | False | False |
| | feature weighting | BM25 | BM25 |
| | ICM weight | 1.6619 | 100.0000 |
| ItemKNN CFCBF ICM all dice | topK | 49 | 901 |
| | shrink | 0 | 179 |
| | similarity | dice | dice |
| | normalize | True | True |
| | ICM weight | 94.1764 | 0.0416 |
| ItemKNN CFCBF ICM all jaccard | topK | 55 | 640 |
| | shrink | 0 | 192 |
| | similarity | jaccard | jaccard |
| | normalize | False | False |
| | ICM weight | 100.0000 | 0.0478 |
| ItemKNN CFCBF ICM all asymmetric | topK | 231 | 1000 |
| | shrink | 1000 | 1000 |
| | similarity | asymmetric | asymmetric |
| | normalize | True | True |
| | asymmetric alpha | 0.5629 | 0.0000 |
| | feature weighting | BM25 | BM25 |
| | ICM weight | 0.2198 | 100.0000 |
| ItemKNN CFCBF ICM all tversky | topK | 46 | 1000 |
| | shrink | 874 | 308 |
| | similarity | tversky | tversky |
| | normalize | True | True |
| | tversky alpha | 0.2974 | 0.7078 |
| | tversky beta | 0.6227 | 2.0000 |
| | ICM weight | 0.6018 | 0.0100 |

Table 118. Hyperparameter values for our hybrid KNN baselines on all datasets.

| Algorithm | Hyperparameter | MovieLens 1M | Tafeng |
|--------------------------|-------------------|--------------|------------|
| UserKNN CFCBF cosine | topK | 263 | 683 |
| | shrink | 0 | 510 |
| | similarity | cosine | cosine |
| | normalize | True | True |
| | feature weighting | BM25 | BM25 |
| | UCM weight | 0.0262 | 4.9570 |
| UserKNN CFCBF dice | topK | 202 | 997 |
| | shrink | 1 | 18 |
| | similarity | dice | dice |
| | normalize | False | True |
| | UCM weight | 2.5156 | 0.0115 |
| UserKNN CFCBF jaccard | topK | 190 | 993 |
| | shrink | 2 | 5 |
| | similarity | jaccard | jaccard |
| | normalize | True | True |
| | UCM weight | 0.8025 | 43.6604 |
| UserKNN CFCBF asymmetric | topK | 350 | 670 |
| | shrink | 0 | 451 |
| | similarity | asymmetric | asymmetric |
| | normalize | True | True |
| | asymmetric alpha | 0.0000 | 0.7234 |
| | feature weighting | BM25 | BM25 |
| | UCM weight | 0.0100 | 2.5995 |
| UserKNN CFCBF tversky | topK | 224 | 1000 |
| | shrink | 0 | 0 |
| | similarity | tversky | tversky |
| | normalize | True | True |
| | tversky alpha | 0.8562 | 1.5529 |
| | tversky beta | 1.7705 | 1.8388 |
| | UCM weight | 0.0100 | 100.0000 |

Table 119. Hyperparameter values for our content based KNN baselines on all datasets.

| Algorithm | Hyperparameter | Movielens 1M | Tafeng |
|-------------------------------------|-------------------|--------------|------------|
| ItemKNN CBF ICM original cosine | topK | - | 561 |
| | shrink | - | 528 |
| | similarity | - | cosine |
| | normalize | - | False |
| | feature weighting | - | none |
| ItemKNN CBF ICM original dice | topK | - | 636 |
| | shrink | - | 369 |
| | similarity | - | dice |
| | normalize | - | True |
| ItemKNN CBF ICM original jaccard | topK | - | 991 |
| | shrink | - | 998 |
| | similarity | - | jaccard |
| | normalize | - | False |
| ItemKNN CBF ICM original asymmetric | topK | - | 500 |
| | shrink | - | 545 |
| | similarity | - | asymmetric |
| | normalize | - | True |
| | asymmetric alpha | - | 0.4976 |
| | feature weighting | - | none |
| ItemKNN CBF ICM original tversky | topK | - | 980 |
| | shrink | - | 12 |
| | similarity | - | tversky |
| | normalize | - | True |
| | tversky alpha | - | 0.0031 |
| | tversky beta | - | 1.8880 |

Table 120. Hyperparameter values for our hybrid KNN baselines on all datasets.

| Algorithm | Hyperparameter | MovieLens 1M | Tafeng |
|---------------------------------------|-------------------|--------------|------------|
| ItemKNN CFCBF ICM original cosine | topK | - | 1000 |
| | shrink | - | 1000 |
| | similarity | - | cosine |
| | normalize | - | True |
| | feature weighting | - | TF-IDF |
| | ICM weight | - | 9.1735 |
| ItemKNN CFCBF ICM original dice | topK | - | 890 |
| | shrink | - | 434 |
| | similarity | - | dice |
| | normalize | - | True |
| | ICM weight | - | 100.0000 |
| ItemKNN CFCBF ICM original jaccard | topK | - | 830 |
| | shrink | - | 363 |
| | similarity | - | jaccard |
| | normalize | - | False |
| | ICM weight | - | 0.0100 |
| ItemKNN CFCBF ICM original asymmetric | topK | - | 1000 |
| | shrink | - | 1000 |
| | similarity | - | asymmetric |
| | normalize | - | True |
| | asymmetric alpha | - | 0.3744 |
| | feature weighting | - | TF-IDF |
| | ICM weight | - | 0.0100 |
| ItemKNN CFCBF ICM original tversky | topK | - | 882 |
| | shrink | - | 684 |
| | similarity | - | tversky |
| | normalize | - | True |
| | tversky alpha | - | 1.2102 |
| | tversky beta | - | 1.9959 |
| | ICM weight | - | 4.6249 |

Table 121. Hyperparameter values for the neural algorithm on all datasets.

| Algorithm | Hyperparameter | Movielens 1M | Tafeng |
|-----------|-------------------|--------------|----------|
| DeepCF | learning rate | 1.00E-03 | 5.00E-03 |
| | epochs | 15 | 15 |
| | n negative sample | 4 | 4 |
| | temp file folder | - | - |
| | dataset name | Movielens1M | Tafeng |
| | number model | 3 | 3 |
| | verbose | 0 | 0 |
| | plot model | False | False |
| CoupledCF | learning rate | 1.00E-03 | 5.00E-03 |
| | epochs | 40 | 5 |
| | n negative sample | 4 | 4 |
| | temp file folder | - | - |
| | dataset name | Movielens1M | Tafeng |
| | number model | 2 | 2 |
| | verbose | 0 | 0 |
| | plot model | False | False |

N IJCAI: DELF: A DUAL-EMBEDDING BASED DEEP LATENT FACTOR MODEL FOR RECOMMENDATION

Relevant statistics on the dataset, which we mentioned in the paper, are reported in Table 122. The results of our evaluation can be seen in Table 123 (Amazon Music) and Table 124 (Movielens 1M). The corresponding optimal hyperparameters are reported in Table 127 (collaborative KNNs), Table 128 (non-neural machine learning and graph based) and Table 129 (DELFI).

Lastly, the time required to train and evaluate the models is reported in Table 126 (Amazon Music) and Table 125 (Movielens 1M).

Table 122. Dataset characteristics.

| Dataset | | Interactions | Items | Users | Density |
|--------------|--------------|--------------|--------|--------|----------------------|
| Amazon Music | original | 836K | 266414 | 478235 | $6.56 \cdot 10^{-4}$ |
| Amazon Music | preprocessed | 76K | 41488 | 1835 | 0.100 |
| Movielens 1M | - | 1M | 3706 | 6040 | 4.468 |

Table 123. Experimental results for the DELF method for the Amazon Music dataset.

| | @ 5 | | @ 10 | | @ 20 | |
|-------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| | HR | NDCG | HR | NDCG | HR | NDCG |
| Random | 0.0490 | 0.0293 | 0.1014 | 0.0459 | 0.1973 | 0.0699 |
| TopPopular | 0.2452 | 0.1726 | 0.3057 | 0.1921 | 0.3744 | 0.2094 |
| UserKNN CF cosine | 0.3248 | 0.2544 | 0.3760 | 0.2708 | 0.4376 | 0.2864 |
| UserKNN CF dice | 0.3210 | 0.2522 | 0.3760 | 0.2700 | 0.4371 | 0.2854 |
| UserKNN CF jaccard | 0.3210 | 0.2526 | 0.3760 | 0.2704 | 0.4371 | 0.2858 |
| UserKNN CF asymmetric | 0.3188 | 0.2516 | 0.3749 | 0.2698 | 0.4365 | 0.2853 |
| UserKNN CF tversky | 0.3221 | 0.2527 | 0.3760 | 0.2701 | 0.4371 | 0.2855 |
| ItemKNN CF cosine | 0.3204 | 0.2528 | 0.3733 | 0.2698 | 0.4371 | 0.2858 |
| ItemKNN CF dice | 0.3117 | 0.2441 | 0.3717 | 0.2632 | 0.4338 | 0.2789 |
| ItemKNN CF jaccard | 0.3090 | 0.2439 | 0.3602 | 0.2604 | 0.4256 | 0.2767 |
| ItemKNN CF asymmetric | 0.3204 | 0.2566 | 0.3711 | 0.2731 | 0.4327 | 0.2886 |
| ItemKNN CF tversky | 0.3046 | 0.2431 | 0.3619 | 0.2615 | 0.4278 | 0.2780 |
| P ³ α | 0.3188 | 0.2524 | 0.3684 | 0.2684 | 0.4300 | 0.2839 |
| RP ³ β | 0.3155 | 0.2494 | 0.3684 | 0.2663 | 0.4272 | 0.2811 |
| EASE ^R | - | - | - | - | - | - |
| SLIM BPR | 0.3139 | 0.2446 | 0.3717 | 0.2632 | 0.4392 | 0.2801 |
| SLIM ElasticNet | 0.3199 | 0.2577 | 0.3678 | 0.2730 | 0.4354 | 0.2900 |
| MF BPR | 0.2376 | 0.1896 | 0.2768 | 0.2023 | 0.3520 | 0.2213 |
| MF FunkSVD | 0.2545 | 0.2035 | 0.2916 | 0.2155 | 0.3417 | 0.2280 |
| PureSVD | 0.2627 | 0.2141 | 0.3084 | 0.2290 | 0.3537 | 0.2405 |
| NMF | 0.2921 | 0.2306 | 0.3510 | 0.2498 | 0.4087 | 0.2644 |
| iALS | 0.3319 | 0.2604 | 0.3717 | 0.2732 | 0.4229 | 0.2860 |
| DELF MLP | 0.2943 | 0.2264 | 0.3548 | 0.2460 | 0.4327 | 0.2656 |
| DELF EF | 0.3837 | 0.2668 | 0.5706 | 0.3269 | 0.7820 | 0.3804 |

Table 124. Experimental results for the DELF method for the Movielens 1M dataset.

| | @ 5 | | @ 10 | | @ 20 | |
|-------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| | HR | NDCG | HR | NDCG | HR | NDCG |
| Random | 0.0525 | 0.0307 | 0.1002 | 0.0460 | 0.1972 | 0.0703 |
| TopPopular | 0.3302 | 0.2229 | 0.4696 | 0.2674 | 0.6577 | 0.3148 |
| UserKNN CF cosine | 0.5186 | 0.3633 | 0.6796 | 0.4156 | 0.8246 | 0.4524 |
| UserKNN CF dice | 0.5150 | 0.3611 | 0.6796 | 0.4145 | 0.8218 | 0.4506 |
| UserKNN CF jaccard | 0.5166 | 0.3622 | 0.6788 | 0.4148 | 0.8227 | 0.4513 |
| UserKNN CF asymmetric | 0.5205 | 0.3635 | 0.6852 | 0.4168 | 0.8329 | 0.4542 |
| UserKNN CF tversky | 0.5161 | 0.3620 | 0.6748 | 0.4136 | 0.8256 | 0.4518 |
| ItemKNN CF cosine | 0.4936 | 0.3426 | 0.6677 | 0.3989 | 0.8243 | 0.4387 |
| ItemKNN CF dice | 0.4895 | 0.3370 | 0.6667 | 0.3943 | 0.8276 | 0.4352 |
| ItemKNN CF jaccard | 0.4958 | 0.3408 | 0.6725 | 0.3979 | 0.8197 | 0.4354 |
| ItemKNN CF asymmetric | 0.4946 | 0.3437 | 0.6718 | 0.4009 | 0.8266 | 0.4401 |
| ItemKNN CF tversky | 0.4936 | 0.3418 | 0.6620 | 0.3964 | 0.8038 | 0.4324 |
| P ³ α | 0.4945 | 0.3438 | 0.6574 | 0.3965 | 0.7952 | 0.4313 |
| RP ³ β | 0.5138 | 0.3559 | 0.6809 | 0.4102 | 0.8276 | 0.4475 |
| EASE ^R | 0.5609 | 0.3954 | 0.7248 | 0.4486 | 0.8559 | 0.4818 |
| SLIM BPR | 0.5380 | 0.3742 | 0.7077 | 0.4292 | 0.8452 | 0.4640 |
| SLIM ElasticNet | 0.5706 | 0.4038 | 0.7306 | 0.4557 | 0.8586 | 0.4882 |
| MF BPR | 0.4844 | 0.3310 | 0.6595 | 0.3877 | 0.8152 | 0.4275 |
| MF FunkSVD | 0.5312 | 0.3708 | 0.6948 | 0.4239 | 0.8245 | 0.4569 |
| PureSVD | 0.5513 | 0.3891 | 0.7021 | 0.4382 | 0.8303 | 0.4708 |
| NMF | 0.5339 | 0.3746 | 0.6965 | 0.4272 | 0.8385 | 0.4635 |
| iALS | 0.5643 | 0.3975 | 0.7228 | 0.4489 | 0.8354 | 0.4776 |
| DELF MLP | 0.5234 | 0.3592 | 0.6892 | 0.4132 | 0.8356 | 0.4503 |
| DELF EF | 0.4718 | 0.3210 | 0.6423 | 0.3762 | 0.7942 | 0.4146 |

Table 125. Computation time for the algorithms in the selected results for the DELF method on the Movielens 1M dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|-----------------------|---|-------------------------------|---------------------------|
| Random | 0.03 [sec] | 8.92 [sec] | 677 |
| TopPopular | 0.05 [sec] | 9.73 [sec] | 621 |
| UserKNN CF cosine | 4.63 ± 0.23 [sec] | 16.11 ± 0.45 [sec] | 383 |
| UserKNN CF dice | 4.44 ± 0.19 [sec] | 15.72 ± 0.68 [sec] | 389 |
| UserKNN CF jaccard | 4.45 ± 0.20 [sec] | 15.18 ± 0.35 [sec] | 393 |
| UserKNN CF asymmetric | 4.50 ± 0.19 [sec] | 15.84 ± 0.66 [sec] | 387 |
| UserKNN CF tversky | 4.56 ± 0.22 [sec] | 15.86 ± 0.87 [sec] | 386 |
| ItemKNN CF cosine | 2.13 ± 0.12 [sec] | 16.08 ± 0.26 [sec] | 375 |
| ItemKNN CF dice | 2.09 ± 0.12 [sec] | 15.20 ± 0.88 [sec] | 414 |
| ItemKNN CF jaccard | 2.10 ± 0.13 [sec] | 15.73 ± 1.94 [sec] | 417 |
| ItemKNN CF asymmetric | 2.19 ± 0.15 [sec] | 16.66 ± 0.98 [sec] | 375 |
| ItemKNN CF tversky | 2.13 ± 0.10 [sec] | 15.14 ± 0.65 [sec] | 420 |
| $P^3\alpha$ | 4.64 ± 1.45 [sec] | 14.66 ± 0.36 [sec] | 415 |
| $RP^3\beta$ | 5.03 ± 1.47 [sec] | 14.87 ± 0.49 [sec] | 407 |
| EASE ^R | 4.99 ± 0.01 [sec] | 13.87 ± 0.11 [sec] | 431 |
| SLIM BPR | 781.38 [sec] / 13.02 ± 10.84 [min] | 15.74 ± 1.06 [sec] | 354 |
| SLIM ElasticNet | 207.72 [sec] / 3.46 ± 2.28 [min] | 15.33 ± 0.52 [sec] | 401 |
| MF BPR | 537.89 [sec] / 8.96 ± 6.05 [min] | 10.06 ± 0.24 [sec] | 598 |
| MF FunkSVD | 2005.64 [sec] / 33.43 ± 36.79 [min] | 10.10 ± 0.08 [sec] | 600 |
| PureSVD | 0.91 ± 0.66 [sec] | 10.15 ± 0.10 [sec] | 597 |
| NMF | 277.51 [sec] / 4.63 ± 14.49 [min] | 10.33 ± 0.09 [sec] | 587 |
| iALS | 315.26 [sec] / 5.25 ± 4.21 [min] | 10.13 ± 0.04 [sec] | 594 |
| DELF MLP | 5479.96 [sec] / 1.52 [hour] | 337.65 [sec] / 5.63 [min] | 18 |
| DELF EF | 7183.43 [sec] / 2.00 [hour] | 326.14 [sec] / 5.44 [min] | 19 |

Table 126. Computation time for the algorithms in the selected results for the DELF method on the Amazon Music dataset.

| | Train Time | Recommendation Time | Recommendation Throughput |
|-----------------------|--|---------------------------|---------------------------|
| Random | 0.00 [sec] | 2.84 [sec] | 646 |
| TopPopular | 0.00 [sec] | 3.44 [sec] | 533 |
| UserKNN CF cosine | 0.10 ± 0.04 [sec] | 4.99 ± 0.04 [sec] | 366 |
| UserKNN CF dice | 0.10 ± 0.00 [sec] | 4.98 ± 0.06 [sec] | 366 |
| UserKNN CF jaccard | 0.10 ± 0.01 [sec] | 5.00 ± 0.01 [sec] | 367 |
| UserKNN CF asymmetric | 0.10 ± 0.01 [sec] | 4.97 ± 0.08 [sec] | 365 |
| UserKNN CF tversky | 0.10 ± 0.00 [sec] | 5.01 ± 0.03 [sec] | 365 |
| ItemKNN CF cosine | 12.41 ± 1.25 [sec] | 5.14 ± 0.05 [sec] | 354 |
| ItemKNN CF dice | 12.74 ± 0.14 [sec] | 5.11 ± 0.03 [sec] | 357 |
| ItemKNN CF jaccard | 12.72 ± 0.12 [sec] | 5.01 [sec] | 366 |
| ItemKNN CF asymmetric | 12.74 ± 0.18 [sec] | 5.14 ± 0.02 [sec] | 356 |
| ItemKNN CF tversky | 13.10 ± 0.17 [sec] | 5.13 ± 0.02 [sec] | 360 |
| $P^3\alpha$ | 39.70 ± 1.80 [sec] | 5.08 ± 0.01 [sec] | 361 |
| $RP^3\beta$ | 41.92 ± 1.71 [sec] | 5.07 ± 0.00 [sec] | 362 |
| EASE ^R | - | - | - |
| SLIM BPR | 1145.90 [sec] / 19.10 ± 9.02 [min] | 5.13 ± 0.09 [sec] | 357 |
| SLIM ElasticNet | 1153.77 [sec] / 19.23 ± 6.67 [min] | 5.19 ± 0.05 [sec] | 354 |
| MF BPR | 112.91 [sec] / 1.88 ± 2.68 [min] | 3.60 ± 0.02 [sec] | 509 |
| MF FunkSVD | 285.07 [sec] / 4.75 ± 4.56 [min] | 3.88 ± 0.14 [sec] | 462 |
| PureSVD | 0.74 ± 0.65 [sec] | 3.74 ± 0.12 [sec] | 498 |
| NMF | 568.27 [sec] / 9.47 ± 7.02 [min] | 4.14 ± 0.39 [sec] | 489 |
| iALS | 888.30 [sec] / 14.80 ± 12.32 [min] | 3.63 ± 0.02 [sec] | 507 |
| DELF MLP | 6778.45 [sec] / 1.88 [hour] | 405.44 [sec] / 6.76 [min] | 5 |
| DELF EF | 9427.74 [sec] / 2.62 [hour] | 310.86 [sec] / 5.18 [min] | 6 |

Table 127. Hyperparameter values for our collaborative KNN baselines on all datasets.

| Algorithm | Hyperparameter | amazon music | Movielens 1M |
|-----------------------|-------------------|--------------|--------------|
| UserKNN CF cosine | topK | 1000 | 461 |
| | shrink | 0 | 0 |
| | similarity | cosine | cosine |
| | normalize | True | True |
| | feature weighting | none | TF-IDF |
| UserKNN CF dice | topK | 916 | 339 |
| | shrink | 9 | 0 |
| | similarity | dice | dice |
| | normalize | False | True |
| UserKNN CF jaccard | topK | 1000 | 329 |
| | shrink | 0 | 0 |
| | similarity | jaccard | jaccard |
| | normalize | False | True |
| UserKNN CF asymmetric | topK | 1000 | 374 |
| | shrink | 1000 | 0 |
| | similarity | asymmetric | asymmetric |
| | normalize | True | True |
| | asymmetric alpha | 2.0000 | 0.1258 |
| UserKNN CF tfidf | feature weighting | none | TF-IDF |
| | topK | 997 | 414 |
| | shrink | 9 | 71 |
| | similarity | tversky | tversky |
| | normalize | True | True |
| UserKNN CF tfidf | tversky alpha | 0.2340 | 1.1580 |
| | tversky beta | 0.1063 | 1.9364 |
| ItemKNN CF cosine | topK | 998 | 283 |
| | shrink | 978 | 765 |
| | similarity | cosine | cosine |
| | normalize | True | True |
| | feature weighting | TF-IDF | BM25 |
| ItemKNN CF dice | topK | 1000 | 78 |
| | shrink | 29 | 2 |
| | similarity | dice | dice |
| | normalize | True | True |
| ItemKNN CF jaccard | topK | 335 | 42 |
| | shrink | 39 | 0 |
| | similarity | jaccard | jaccard |
| | normalize | True | False |
| ItemKNN CF asymmetric | topK | 1000 | 277 |
| | shrink | 544 | 644 |
| | similarity | asymmetric | asymmetric |
| | normalize | True | True |
| | asymmetric alpha | 0.0000 | 0.6317 |
| ItemKNN CF tfidf | feature weighting | TF-IDF | BM25 |
| | topK | 409 | 66 |
| | shrink | 51 | 0 |
| | similarity | tversky | tversky |
| | normalize | True | True |
| ItemKNN CF tfidf | tversky alpha | 0.0216 | 0.5465 |
| | tversky beta | 1.9479 | 2.0000 |

Table 128. Hyperparameter values for our non-neural machine learning and graph based baselines on all datasets.

| Algorithm | Hyperparameter | amazon music | Movielens 1M |
|-------------------|----------------------|----------------|----------------|
| $P^3\alpha$ | topK | 914 | 406 |
| | alpha | 0.5072 | 1.3317 |
| | normalize similarity | True | True |
| $RP^3\beta$ | topK | 833 | 265 |
| | alpha | 0.6294 | 1.2847 |
| | beta | 0.0343 | 0.5993 |
| | normalize similarity | True | True |
| SLIM BPR | topK | 1000 | 1000 |
| | epochs | 140 | 595 |
| | symmetric | True | True |
| | sgd mode | adam | adagrad |
| | lambda i | 9.31E-04 | 1.00E-02 |
| | lambda j | 1.00E-02 | 9.42E-03 |
| | learning rate | 1.00E-01 | 9.93E-03 |
| SLIM ElasticNet | topK | 1000 | 502 |
| | l1 ratio | 1.30E-04 | 1.86E-05 |
| | alpha | 0.2789 | 0.0689 |
| MF BPR | sgd mode | adam | adam |
| | epochs | 845 | 800 |
| | num factors | 88 | 171 |
| | batch size | 128 | 1024 |
| | positive reg | 9.10E-04 | 1.00E-05 |
| | negative reg | 4.69E-03 | 1.00E-05 |
| | learning rate | 2.16E-03 | 2.16E-03 |
| MF FunkSVD | sgd mode | adam | adagrad |
| | epochs | 225 | 320 |
| | use bias | True | False |
| | batch size | 2 | 128 |
| | num factors | 132 | 47 |
| | item reg | 1.00E-02 | 2.14E-05 |
| | user reg | 1.00E-02 | 1.28E-03 |
| | learning rate | 1.11E-03 | 3.39E-02 |
| | negative quota | 0.3648 | 0.0941 |
| PureSVD | num factors | 58 | 49 |
| NMF | num factors | 64 | 77 |
| | solver | coord. descent | coord. descent |
| | init type | nndsvda | random |
| | beta loss | frobenius | frobenius |
| iALS | num factors | 13 | 60 |
| | confidence scaling | log | log |
| | alpha | 50.0000 | 0.5425 |
| | epsilon | 0.5407 | 0.0010 |
| | reg | 1.00E-05 | 1.00E-05 |
| | epochs | 90 | 10 |
| EASE ^R | l2 norm | - | 3.25E+03 |

Table 129. Hyperparameter values for the neural algorithm on all datasets.

| Algorithm | Hyperparameter | amazon music | Movielens 1M |
|-----------|-----------------------|----------------|----------------|
| DELF MLP | epochs | 30 | 25 |
| | learning rate | 1.00E-03 | 1.00E-03 |
| | batch size | 256 | 256 |
| | num negatives | 4 | 4 |
| | layers | [256, 128, 64] | [256, 128, 64] |
| | regularization layers | [0, 0, 0] | [0, 0, 0] |
| | learner | adam | adam |
| | verbose | False | False |
| DELF EF | epochs | 55 | 45 |
| | learning rate | 1.00E-03 | 1.00E-03 |
| | batch size | 256 | 256 |
| | num negatives | 4 | 4 |
| | layers | [256, 128, 64] | [256, 128, 64] |
| | regularization layers | [0, 0, 0] | [0, 0, 0] |
| | learner | adam | adam |
| | verbose | False | False |

O HYPERPARAMETER RANGE

Table 130. Hyperparameter values for our KNN and graph based baselines.

| Algorithm | Hyperparameter | Range | Type | Distribution |
|--------------------------------|-----------------------------------|--------------------|-------------|--------------|
| UserKNN, ItemKNN cosine | topK | 5 - 1000 | Integer | uniform |
| | shrink | 0 - 1000 | Integer | uniform |
| | similarity | cosine | Categorical | |
| | normalize ^a | True, False | Categorical | |
| | feature weighting | none, TF-IDF, BM25 | Categorical | |
| UserKNN, ItemKNN dice | topK | 5 - 1000 | Integer | uniform |
| | shrink | 0 - 1000 | Integer | uniform |
| | similarity | dice | Categorical | |
| | normalize ^a | True, False | Categorical | |
| UserKNN, ItemKNN jaccard | topK | 5 - 1000 | Integer | uniform |
| | shrink | 0 - 1000 | Integer | uniform |
| | similarity | jaccard | Categorical | |
| | normalize ^a | True, False | Categorical | |
| UserKNN, ItemKNN asymmetric | topK | 5 - 1000 | Integer | uniform |
| | shrink | 0 - 1000 | Integer | uniform |
| | similarity | asymmetric | Categorical | |
| | normalize ^a | True | Categorical | |
| | asymmetric alpha | 0 - 2 | Real | uniform |
| UserKNN, ItemKNN tversky | feature weighting | none, TF-IDF, BM25 | Categorical | |
| | topK | 5 - 1000 | Integer | uniform |
| | shrink | 0 - 1000 | Integer | uniform |
| | similarity | tversky | Categorical | |
| | normalize ^a | True | Categorical | |
| | tversky alpha | 0 - 2 | Real | uniform |
| P3alpha | tversky beta | 0 - 2 | Real | uniform |
| | topK | 5 - 1000 | Integer | uniform |
| | alpha | 0 - 2 | Real | uniform |
| RP3beta | normalize similarity ^b | True, False | Categorical | |
| | topK | 5 - 1000 | Integer | uniform |
| | alpha | 0 - 2 | Real | uniform |
| | beta | 0 - 2 | Real | uniform |
| | normalize similarity ^b | True, False | Categorical | |

^aThe *normalize* hyperparameter in KNNs refers to the use of the denominator when computing the similarity.

^bThe *normalize similarity* hyperparameter in P3alpha and RP3beta refers to applying L1 regularisation on the rows of the similarity matrix

Table 131. Hyperparameter values for our machine learning baselines.

| Algorithm | Hyperparameter | Range | Type | Distribution |
|----------------|-----------------------------|--|-------------|----------------|
| SLIM BPR | topK | 5 - 1000 | Integer | uniform |
| | epochs | 1 - 1500 | Integer | early-stopping |
| | symmetric | True, False | Categorical | |
| | sgd mode | sgd, adam, adagrad | Categorical | |
| | lambda i | $10^{-5} - 10^{-2}$ | Real | log-uniform |
| | lambda j | $10^{-5} - 10^{-2}$ | Real | log-uniform |
| | learning rate | $10^{-4} - 10^{-1}$ | Real | log-uniform |
| SLIMElasticNet | topK | 5 - 1000 | Integer | uniform |
| | l1 ratio | $10^{-5} - 10^0$ | Real | log-uniform |
| | alpha | $10^{-3} - 10^0$ | Real | uniform |
| MF BPR | num factors | 1 - 200 ^a | Integer | uniform |
| | epochs | 1 - 1500 | Integer | early-stopping |
| | sgd mode | sgd, adam, adagrad | Categorical | |
| | batch size | $2^0 - 2^{10}$ | Integer | log-uniform |
| | positive reg | $10^{-5} - 10^{-2}$ | Real | log-uniform |
| | negative reg | $10^{-5} - 10^{-2}$ | Real | log-uniform |
| | learning rate | $10^{-4} - 10^{-1}$ | Real | log-uniform |
| MF FunkSVD | num factors | 1 - 200 ^a | Integer | uniform |
| | epochs | 1 - 500 ^b | Integer | early-stopping |
| | use bias | True, False | Categorical | |
| | sgd mode | sgd, adam, adagrad | Categorical | |
| | batch size | $2^0 - 2^{10}$ | Integer | log-uniform |
| | item reg | $10^{-5} - 10^{-2}$ | Real | log-uniform |
| | user reg | $10^{-5} - 10^{-2}$ | Real | log-uniform |
| | learning rate | $10^{-4} - 10^{-1}$ | Real | log-uniform |
| | negative quota ^c | 0.00 - 0.50 | Real | uniform |
| PureSVD | num factors | 1 - 350 | Integer | uniform |
| NMF | num factors | 1 - 350 | Integer | uniform |
| | solver | mult. update, coord. descent | Categorical | |
| | init type | nndsvda, random | Categorical | |
| | beta loss | kullback-leibler, frobenius | Categorical | |
| IALS | num factors | 1 - 200 ^a | Integer | uniform |
| | epochs | 1 - 500 ^b | Integer | early-stopping |
| | confidence scaling | linear, log | Categorical | |
| | alpha | $10^{-3} - 5 \cdot 10^{+1} \text{ }^d$ | Real | log-uniform |
| | epsilon | $10^{-3} - 10^{+1} \text{ }^d$ | Real | log-uniform |
| | reg | $10^{-5} - 10^{-2}$ | Real | log-uniform |
| EASE R | l2 norm | $10^0 - 10^{+7}$ | Real | log-uniform |

^aThe number of factors is lower than PureSVD or NFM due to the algorithm being slower.^bThe number of epochs is lower than SLIM BPR or MF BPR due to the algorithm being slower.^cThe *negative quota* is the percentage of samples chosen among items unobserved by the user, having a target rating of 0.^dThe maximum value of this hyperparameter had been suggested in the article proposing the algorithm.

REFERENCES

- [1] Fabio Aiolli. 2013. Efficient top-n recommendation for very large scale binary rated datasets. In *Proceedings of the 7th ACM Conference on Recommender Systems (RecSys '13)*. ACM, 273–280.
- [2] Vito W. Anelli, Vito Bellini, Tommaso Di Noia, Wanda La Bruna, Paolo Tomeo, and Eugenio Di Sciascio. 2017. An Analysis on Time- and Session-aware Diversification in Recommender Systems. In *Proceedings of the 25th Conference on User Modeling, Adaptation and Personalization (UMAP '17)*. ACM, New York, NY, USA, 270–274. <https://doi.org/10.1145/3079628.3079703>
- [3] Krisztian Balog, Filip Radlinski, and Shushan Arakelyan. 2019. Transparent, Scrutable and Explainable User Models for Personalized Recommendation. In *Proceedings of the 42nd International ACM SIGIR Conference on Research and Development in Information Retrieval (SIGIR '19)*. ACM, New York, NY, USA, 265–274. <https://doi.org/10.1145/3331184.3331211>
- [4] Robert M Bell and Yehuda Koren. 2007. Improved neighborhood-based collaborative filtering. In *KDD Cup and Workshop at the 13th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining (KDD '07)*. 7–14.
- [5] Daniel Billsus and Michael J. Pazzani. 1998. Learning Collaborative Information Filters. In *Proceedings of the 15th International Conference on Machine Learning (ICML '98)*. 46–54.
- [6] John S. Breese, David Heckerman, and Carl Kadie. 1998. Empirical Analysis of Predictive Algorithms for Collaborative Filtering. In *Proceedings of the 14th Conference on Uncertainty in Artificial Intelligence (UAI '98)*. 43–52.
- [7] Andrzej Cichocki and Anh-Huy Phan. 2009. Fast local algorithms for large scale nonnegative matrix and tensor factorizations. *IEICE transactions on fundamentals of electronics, communications and computer sciences* 92, 3 (2009), 708–721.
- [8] Colin Cooper, Sang Hyuk Lee, Tomasz Radzik, and Yiannis Siantos. 2014. Random walks in recommender systems: exact computation and simulations. In *Proceedings of the 23rd International Conference on World Wide Web (WWW '14)*. 811–816.
- [9] Paolo Cremonesi, Yehuda Koren, and Roberto Turrin. 2010. Performance of Recommender Algorithms on Top-n Recommendation Tasks. In *Proceedings of the 4th ACM Conference on Recommender Systems (RecSys '10)*. 39–46.
- [10] Lee R. Dice. 1945. Measures of the Amount of Ecologic Association Between Species. *Ecology* 26, 3 (1945), 297–302. <https://doi.org/10.2307/1932409> arXiv:<https://esajournals.onlinelibrary.wiley.com/doi/pdf/10.2307/1932409>
- [11] Maurizio Ferrari Dacrema, Simone Boglio, Paolo Cremonesi, and Dietmar Jannach. [n.d.]. A Troubling Analysis of Reproducibility and Progress in Recommender Systems Algorithms Research. ([n. d.]).
- [12] Maurizio Ferrari Dacrema, Paolo Cremonesi, and Dietmar Jannach. 2019. Are We Really Making Much Progress? A Worrying Analysis of Recent Neural Recommendation Approaches. *Proceedings of the 13th ACM Conference on Recommender Systems (RecSys '19)* (2019). <https://doi.org/10.1145/3298689.3347058> Source: https://github.com/MaurizioFD/RecSys2019_DeepLearning_Evaluation.
- [13] Yifan Hu, Yehuda Koren, and Chris Volinsky. 2008. Collaborative Filtering for Implicit Feedback Datasets.. In *Proceedings of the 8th IEEE International Conference on Data Mining (ICDM '08)*, Vol. 8. Citeseer, 263–272.
- [14] Lukas Lerche and Dietmar Jannach. 2014. Using Graded Implicit Feedback for Bayesian Personalized Ranking. In *Proceedings of the 8th ACM Conference on Recommender Systems (RecSys '14)*. ACM, New York, NY, USA, 353–356. <https://doi.org/10.1145/2645710.2645759>
- [15] Mark Levy and Kris Jack. 2013. Efficient top-n recommendation by linear regression. In *RecSys Large Scale Recommender Systems Workshop*.
- [16] G. Linden, B. Smith, and J. York. 2003. Amazon.com recommendations: item-to-item collaborative filtering. *IEEE Internet Computing* 7, 1 (2003), 76–80.
- [17] Pasquale Lops, Marco De Gemmis, and Giovanni Semeraro. 2011. Content-based recommender systems: State of the art and trends. In *Recommender systems handbook*. Springer, 73–105.
- [18] Bamshad Mobasher, Xin Jin, and Yanzan Zhou. 2004. Semantically Enhanced Collaborative Filtering on the Web. In *Web Mining: From Web to Semantic Web*, Bettina Berendt, Andreas Hotho, Dunja Mladenić, Maarten van Someren, Myra Spiliopoulou, and Gerd Stumme (Eds.). Springer Berlin Heidelberg, Berlin, Heidelberg, 57–76.
- [19] Xia Ning and George Karypis. 2011. SLIM: Sparse linear methods for top-n recommender systems. In *Proceedings of the 11th IEEE International Conference on Data Mining (ICDM '11)*. 497–506.
- [20] Bibek Paudel, Fabian Christoffel, Chris Newell, and Abraham Bernstein. 2017. Updatable, Accurate, Diverse, and Scalable Recommendations for Interactive Applications. *ACM Transactions on Interactive Intelligent Systems (TiiS)* 7, 1 (2017), 1.
- [21] Ali Mustafa Qamar, Éric Gaussier, Jean-Pierre Chevallet, and Joo-Hwee Lim. 2008. Similarity Learning for Nearest Neighbor Classification. In *Proceedings of the 8th IEEE International Conference on Data Mining (ICDM '08)*. 983–988. <https://doi.org/10.1109/ICDM.2008.81>
- [22] Steffen Rendle, Christoph Freudenthaler, Zeno Gantner, and Lars Schmidt-Thieme. 2009. BPR: Bayesian personalized ranking from implicit feedback. In *Proceedings of the 25th Conference on Uncertainty in Artificial Intelligence (UAI '09)*.

452–461.

- [23] Paul Resnick, Neophytos Iacovou, Mitesh Suchak, Peter Bergstrom, and John Riedl. 1994. GroupLens: An Open Architecture for Collaborative Filtering of Netnews. In *Proceedings of the 1994 ACM Conference on Computer-Supported Cooperative Work (CSCW '94)*. 175–186.
- [24] Badrul Sarwar, George Karypis, Joseph Konstan, and John Riedl. 2001. Item-based Collaborative Filtering Recommendation Algorithms. In *Proceedings of the 10th International Conference on World Wide Web (WWW '01)*. 285–295.
- [25] Harald Steck. 2019. Embarrassingly Shallow Autoencoders for Sparse Data. In *Proceedings of the 28th International Conference on World Wide Web (WWW '19) (TheWebConf 2019)*. 3251–3257.
- [26] Alessandro Suglia, Claudio Greco, Cataldo Musto, Marco de Gemmis, Pasquale Lops, and Giovanni Semeraro. 2017. A Deep Architecture for Content-based Recommendations Exploiting Recurrent Neural Networks. In *Proceedings of the 25th Conference on User Modeling, Adaptation and Personalization (UMAP '17)*. ACM, New York, NY, USA, 202–211. <https://doi.org/10.1145/3079628.3079684>
- [27] Amos Tversky. 1977. Features of Similarity. *Psychological Review* 84, 4 (1977), 327–352.
- [28] Jun Wang, Stephen Robertson, Arjen P de Vries, and Marcel JT Reinders. 2008. Probabilistic relevance ranking for collaborative filtering. *Information Retrieval* 11, 6 (2008), 477–497.