Chap 4

Thursday, April 2, 2020 4:22 PM

<u>4.1.3</u>

$$|\Gamma\rangle_{RS} = \frac{1}{\sqrt{d}} \sum_{i=0}^{d-1} |i\rangle_R |i\rangle_S \qquad d = \dim(\mathcal{H}_S) \because \mathcal{H}_R \text{ is isomorphic to } \mathcal{H}_S.$$

Evaluate RHS.

$$\langle\Gamma|_{RS}\, I_R \otimes A_S\, |\Gamma\rangle_{RS} = \frac{1}{d} \sum_{i,j=0}^{d-1} \langle i|_R \langle i|_S\; I_R \otimes A_S\; |j\rangle_R |j\rangle_S$$

$$= \frac{1}{d} \sum_{i,j=0}^{d-1} \delta_{ij,R}\; \langle i|_S A_S |j\rangle_S$$

$$= \frac{1}{d} \sum_{i=0}^{d-1} \langle i|_S A_S |i\rangle_S = \mathrm{Tr}(A)$$

Hence $\mathrm{Tr}(A) = \langle\Gamma|_{RS}\, I_R \otimes A_S\, |\Gamma\rangle_{RS}$

<u>4.4.1</u>

Claim: Choi $(\mathcal{N})$ is P.S.D. $\rightarrow$ $id_R \otimes \mathcal{N}(X_{RA})$ is P.S.D. $\forall\, X_{RA}$ P.S.D.

Let $X_{RA} = \sum_{k=0}^{r-1} |\phi_k\rangle\langle\phi_k|_{RA}$. Can write like this because it's PSD, and has rank r for some $r \geq 0$

Then need to show that $id_R \otimes \mathcal{N}(X_{RA})$ is PSD.

$$id_R \otimes \mathcal{N}(X_{RA}) = id_R \otimes \mathcal{N}\left(\sum_{k=0}^{r-1} |\phi_k\rangle\langle\phi_k|_{RA}\right) = \sum_{k=0}^{d-1} id_R \otimes \mathcal{N}(|\phi_k\rangle\langle\phi_k|_{RA})$$

Suffices to prove that $id_R \otimes \mathcal{N}_{A\to B}(|\phi_k\rangle\langle\phi_k|_{RA})$ is PSD for a given k.

WLOG can write $|\phi_k\rangle_{RA} = \sum_{i=0}^{d_A-1}\sum_{j=0}^{d_A-1} \alpha_{ij}^k |i\rangle_R |j\rangle_A$.

Then $id_R \otimes \mathcal{N}_{A\to B}(|\phi_k\rangle\langle\phi_k|_{RA}) = id_R \otimes \mathcal{N}_{A\to B}\left(\sum_{i_1,j_1}\sum_{i_2,j_2} \alpha_{i_1j_1}^k \alpha_{i_2j_2}^{k*}\, |i_1\rangle_R |j_1\rangle_A \langle i_2|_R \langle j_2|_A\right)$

$$= \sum_{i_1,j_1}\sum_{i_2,j_2} \alpha_{i_1j_1}^k \alpha_{i_2j_2}^{k*}\, |i_1\rangle\langle i_2|_R \otimes \mathcal{N}_{A\to B}(|j_1\rangle\langle j_2|_A)$$

Choi of PSD can be diagonalized allows us to use 4.210 − 4.213 directly

$$= \sum_{i_1,j_1}\sum_{i_2,j_2} \alpha_{i_1j_1}^k \alpha_{i_2j_2}^{k*}\, |i_1\rangle\langle i_2|_R \otimes \sum_{l=0}^{d} V_l |j_1\rangle\langle j_2|_A V_l^\dagger$$

$$= \sum_{l=0}^{d} id_R \otimes V_{l,A} \left(\sum_{i_1,j_1}\sum_{i_2,j_2} \alpha_{i_1j_1}^k \alpha_{i_2j_2}^{k*}\, |i_1\rangle\langle i_2|_R \otimes |j_1\rangle\langle j_2|_A\right) id_R \otimes V_l^\dagger$$

$$= \sum_{l=0}^{d} id_R \otimes V_{l,A} (|\phi_k\rangle\langle\phi_k|_{RA})\, id_R \otimes V_l^\dagger \quad (\text{manifestly PSD !})$$