Chap 5

Thursday, April 2, 2020 4:07 PM

Ex 5·1·2 Choose $\dim(\mathcal{H}_R) = \dim(\mathcal{H}_A) = d$

$$|\Gamma\rangle_{RA} = \frac{1}{\sqrt{d}} \sum_{i=0}^{d-1} |i\rangle_R |i\rangle_A$$

Nts $$\mathrm{Tr}_R \left[(I_R \otimes \sqrt{\rho_A}) |\Gamma\rangle_{RA} \langle\Gamma|_{RA} (I_R \otimes \sqrt{\rho_A}) \right] = \rho_A$$

$$\text{LHS} = \frac{1}{d} \mathrm{Tr}_R \left[\sum_{i,j=0}^{d-1} |i\rangle_R \otimes \sqrt{\rho_A} |i\rangle_A \langle j|_R \otimes \langle j|_A \sqrt{\rho_A} \right]$$

$$= \frac{1}{d} \sum_{i,j=0}^{d-1} \underbrace{\langle j|i\rangle_R}_{\delta_{ij}} \sqrt{\rho_A} |i\rangle\langle j|_A \sqrt{\rho_A}$$

by linearity of trace
& defn of partial trace eq 4.154

$$= \frac{1}{d} \sum_{i=0}^{d-1} \sqrt{\rho_A} |i\rangle\langle i|_A \sqrt{\rho_A} = \sqrt{\rho_A}\, \mathbb{1}_A \sqrt{\rho_A} = \rho_A \quad \checkmark$$